Chapter 7.1 Day 2

Introduction to Hypothesis Testing

Recall from yesterday:

TYPES OF ERRORS

A hypothesis test always starts by assuming the Null Hypothesis is true.

At the end of the test there are only two outcomes: Reject the Null Hypothesis
Fail to reject the Null Hypothesis

Since your decision is based on a sample there is a possibility that you could be **wrong**.

The table shows the four possible outcomes of a hypothesis test.

	Truth of H_0	
Decision	H_0 is true.	H_0 is false.
Do not reject H_0.	Correct decision	Type II error
Reject H_0.	Type I error	Correct decision

Because there is variation from sample to sample, there is always a possibility that you will reject a null hypothesis when it is actually true. You can decrease the probability of this happening by lowering the level of significance. The **Level of Significance α** is the probability of rejecting H_o (the null hypothesis) when it is true.

THE LEVEL OF SIGNIFICANCE AND THE P-VALUE OF A TEST

The Probability of Making a Type 1 error is called: **α = the level of significance**

The idea: Set α to a small number (three commonly used levels of significance are α = .01 or α = .05 or α = .10)
This makes the probability of INCORRECTLY rejecting the null hypothesis very small.

After stating the null and alternative hypothesis and specifying the level of significance, the next step is to obtain a random sample from the population and calculate the sample statistic (such as $\bar{x}, \hat{p}, or\ s^2$) corresponding to the parameter in the null hypothesis (such as $\mu, p, or\ \sigma^2$). This sample statistic is called the **test statistic**.

The **"P-value" of a test is the probability of obtaining a sample statistic** with a value as extreme or more extreme than the one determined from the sample data.

If the probability of getting the sample statistic is highly unusual you need to reject the null hypothesis...

The level of significance is the line you draw to explain what ~~"highly unusual~~ means."

given in our problems

Making a Decision

Decision Rule Based on P-value

- Compare the P-value with α.
  - If $P \le \alpha$, then reject H_0.
  - If $P > \alpha$, then fail to reject H_0.

Example 1: The P-value of your sample test statistic is 0.0135. Should you reject or fail to reject the null hypothesis if your level of significance is:

P

a) $\alpha = 0.05$

$P \leq \alpha$

$.0135 \leq .05$ reject Ho

b) $\alpha = 0.01$

$P > .01$

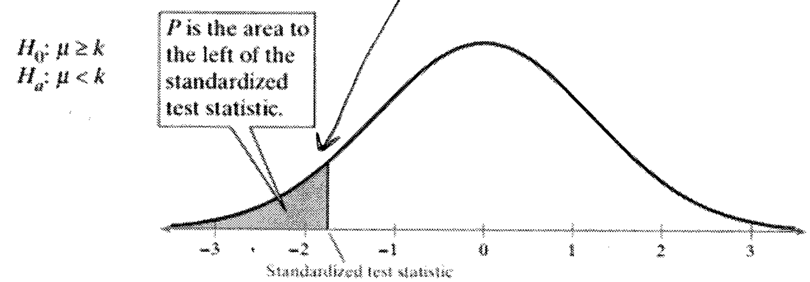
$.0135 > .01$ fail to reject Ho

The *P*-value of a hypothesis test depends on the nature of the test. There are three types of hypothesis testing (listed below). The type of test depends on the location of the region of the sampling distribution that favors a rejection of H_0. This region is indicated by the alternative hypothesis.

can look at Ha to determine type of test

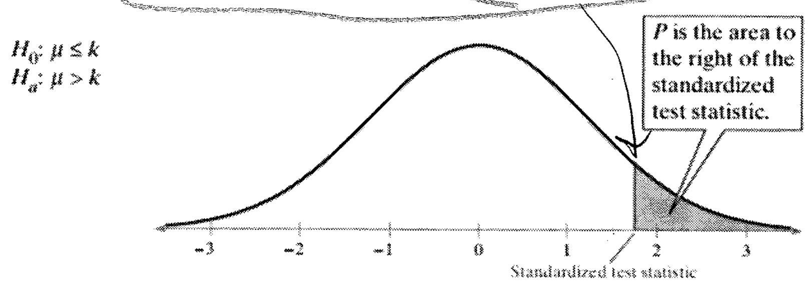
DEFINITION

1. If the alternative hypothesis H_a contains the less-than inequality symbol ($<$), then the hypothesis test is a **left-tailed test.**



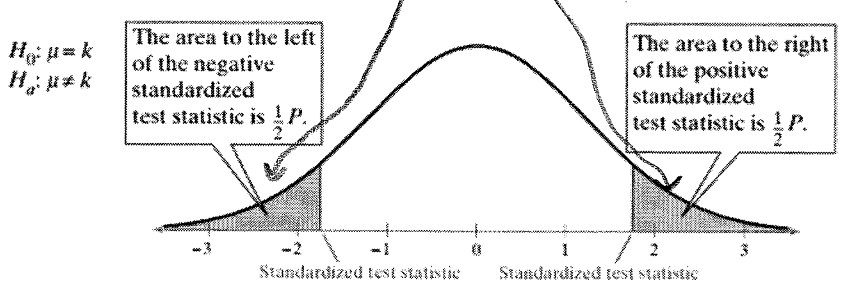
Left-Tailed Test

2. If the alternative hypothesis H_a contains the greater-than inequality symbol ($>$), then the hypothesis test is a **right-tailed test.**



Right-Tailed Test

3. If the alternative hypothesis H_a contains the not-equal-to symbol ($\neq$), then the hypothesis test is a **two-tailed test.** In a two-tailed test, each tail has an area of $\frac{1}{2}P$.



Two-Tailed Test

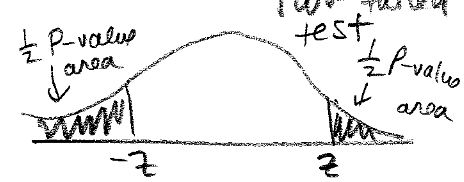
Example 2: For each claim state H_0 and H_a in words and in symbols. The determine whether the hypothesis test is a left-tailed, right-tailed, or two-tailed test. Sketch a normal sampling distribution and shade the area for the *P*-value.

a) A school publicizes the proportion of its students who are involved in at least one extracurricular activity is 61%.

$H_0: p = .61$

$H_a: p \neq .61$

two-tailed test



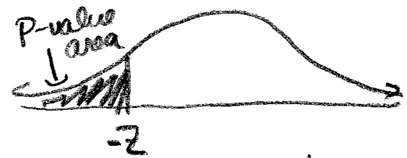
H_0: The proportion of Students who are involved in at least one extracurricular activity is 61%.

H_a: The proportion of Students who are involved in at least one extracurricular activity is not 61%.

b) A car dealership announces that the mean time for an oil change is less than 15 minutes.

$\mu < 15$ $\quad H_0: \mu \geq 15$

$H_a: \mu < 15$



Left tailed test

H_0: The mean time for an oil change is greater than or equal to 15 minutes.

H_a: The mean time for an oil change is less than 15 minutes.

c) A company advertises that the mean life of its furnaces is more than 18 years.

$\mu > 18$

$H_0: \mu \leq 18$ The mean life of the furnace is less than or equal to 18 years

$H_a: \mu > 18$ The mean life of the furnace is greater than 18 years.

Right-tailed test

Interpreting the Decision

	Claim	
Decision	**Claim is H_0.**	**Claim is H_a.**
Reject H_0.	There is enough evidence to reject the claim.	There is enough evidence to support the claim.
Fail to reject H_0.	There is not enough evidence to reject the claim.	There is not enough evidence to support the claim.

Example 3: You perform a hypothesis test for each claim. How should you interpret your decision if you reject H_0? If you fail to reject H_0?

a) H_0 (claim): A school publicizes that the proportion of its students who are involved in at least one extracurricular activity is 61%. $\quad H_0: p = .61$

If you reject H_0 then you should conclude, "there is enough evidence to reject the claim that 61% of the students are involved in at least one extracurricular activity. If you fail to reject H_0, then you should conclude, "there is not enough evidence to reject the claim that 61% of the students are involved in at least one extracurricular activity."

b) H_a (claim): A car dealership announces that the mean time for an oil change is less than 15 minutes.

$H_a: \mu < 15$

$H_0: \mu \geq 15$

If you reject H_0, then you should conclude, "there is enough evidence to support the dealer's claim that the mean time for an oil change is less than 15 minutes." If you fail to reject H_0, then you should conclude, "there is not enough evidence to support the dealer's claim that the mean time for an oil change is less than 15 minutes.

Strategies for Hypothesis Testing

The strategy you will use in hypothesis testing should depend on whether you are trying to support or reject a claim. You cannot use a hypothesis test to support your claim when your claim is the null hypothesis. So, as a researcher, to perform a hypothesis test where the possible outcome will support a claim, word the claim so it is the alternative hypothesis. To perform a hypothesis test where the possible outcome will reject a claim word it so the claim is the null hypothesis.

Writing the Hypothesis:

Example 4: A medical research team is investigating the benefits of a new surgical treatment. One of the claims is that the mean recovery time for the patients after the new treatment is less than 96 hours. H_0 $H_a: \mu < 96$

a. How would you write the null and alternative hypotheses when you are on the research team and want to support the claim? How should you interpret a decision that rejects the null hypothesis?

Since you want to support the claim, make the Claim $H_a: \mu < 96$ hours (claim)
The complement will be $H_0: \mu \geq 96$ hours.
If you reject H_0, then you will support the claim that "the mean recovery time for the patients after a new treatment is less than 96 hours.

b. How would you write the null and alternative hypotheses when you are on an opposing team and want to reject the claim? How should you interpret a decision that rejects the null hypothses?

Since you want to reject from opposing team make it the null hypothesis
$H_0: \mu \leq 96$ (claim) and then H_a is $\mu > 96$.
If you reject H_0, then you will reject the claim that "the mean recovery time for the patients after a new treatment is less than or equal to 96 hours.