Chapter 7.1 Day 1

Introduction to Hypothesis Testing

One of a statistician's most important jobs: Draw inferences about population based samples taken from the population. Usually we are interested in population parameters like mean or proportion of success.

μ p

We approach these inferences in one of two ways:

1. CONFIDENCE INTERVALS (Ch 6) – use the sample to get "close enough" value to the actual parameter
2. HYPOTHESIS TESTING (Ch 7) – testing a claim about the value of a population parameter

NULL AND ALTERNATIVE HYPOTHESIS

DEFINITION

1. A **null hypothesis** H_0 is a statistical hypothesis that contains a statement of equality, such as $\leq$, $=$, or $\geq$.
2. The **alternative hypothesis** H_a is the complement of the null hypothesis. It is a statement that must be true if H_0 is false and it contains a statement of strict inequality, such as $>$, $\neq$, or $<$.

The symbol H_0 is read as "H sub-zero" or "H naught" and H_a is read as "H sub-a."

Possible pairings for null hypothesis and alternative hypothesis

$$\begin{cases} H_0: \mu \leq k \\ H_a: \mu > k \end{cases} \qquad \begin{cases} H_0: \mu \geq k \\ H_a: \mu < k \end{cases} \qquad \text{and} \qquad \begin{cases} H_0: \mu = k \\ H_a: \mu \neq k \end{cases}$$

Example 1: The statement represents a claim. Write its complement and state which is H_0 and which is H_a.

a) $\mu < 24$

$H_0: \mu \geq 24$

$H_a: \mu < 24$ (claim)

b) $p \geq 0.35$

$H_0: p \geq 0.35$ (claim)

$H_a: p < 0.35$

c) $\sigma \neq 2.5$

$H_0: \sigma = 2.5$

$H_a: \sigma \neq 2.5$ (claim)

d) $\mu = 82.6$

$H_0: \mu = 82.6$ (claim)

$H_a: \mu \neq 82.6$

e) $p \leq 0.21$

$H_0: p \leq 0.21$ (claim)

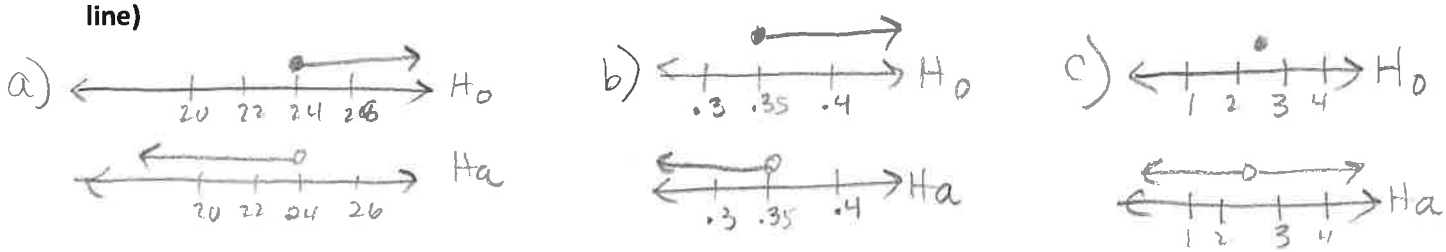
$H_a: p > 0.21$

f) $\mu > 225$

$H_0: \mu \leq 225$

$H_a: \mu > 225$ (claim)

Example 2: Graph each null and alternative hypothesis from Example 1 for parts a through c. (graph on a number line)



Example 3: Write each claim as a mathematical statement. State the null and alternative hypotheses, and identify which represents the claim.

a) The average height of a professional male basketball player was 6.5 feet 10 years ago.

$\mu = 6.5$

$H_0: \mu = 6.5$ (claim)

$H_a: \mu \neq 6.5$

b) A car dealership announces that the mean time for an oil change is less than 15 minutes.

$\mu < 15$ $\quad H_0: \mu \geq 15$

$H_a: \mu < 15$ (claim)

c) A realtor publicizes that the proportion of homeowners who feel their house is too small for their family is more than 24%.

$p > .24$ $\quad H_0: p \leq .24$

$H_a: p > .24$ (claim)

No matter which hypothesis represents the claim, you always begin a hypthesis test by assumeing that the equality condition in the null hypothesis is true. So, when you perform a hypothesis test, you make one of the two decisions:

1. reject the null hypothesis

Or

2. fail to reject the null hypothesis

Because your decision is based on a smaple rather than the entire population, there is always the possibility you will make the wrong deicsion. You might reject a null hypothesis when it is actually true. Or, you might fail to reject a null hypothesis when it is acatually false.

DEFINITION

A **type I error** occurs if the null hypothesis is rejected when it is true.

A **type II error** occurs if the null hypothesis is not rejected when it is false.

Eample:
Justice system:
type I: Guilty person found innocent
type II: Innocent person found guilty.

The table shows the four possible outcomes of a hypothesis test.

	Truth of H_0	
Decision	H_0 is true.	H_0 is false.
Do not reject H_0.	Correct decision	Type II error
Reject H_0.	Type I error	Correct decision

Example 4: A company specializing in parachute assembly states that its main parachute failure rate is not more than 1%. You perform a hypothesis test to determine whether the company's claim is false. When will a type I or type II error occur? Which error is more serious.

	Truth of Ho	
Decision	Ho is True	Ho is False
Do not reject Ho	Correct	Type II
reject Ho	Type I	Correct

claim: $p \leq .01$

$H_0: p \leq .01$

$H_a: p > .01$

Type I: When the actual proportion is not more than 1% but we reject Ho

Type II: When the actual prop. is greater than 1%, but fail to reject Ho.

Type II is more serious because you would be misleading the consumer, possibly causing serious injury or death