Chapter 6.3

Confidence Intervals for Population Proportions

SITUATION 3: You have a situation with a binomial probability distribution.

In other words, you want to estimate the proportion or percentage of success from a population instead of a mean (average).

For Example: What percentage of voters support the Libertarian Candidate for governor?

p is the population proportion

Recall:
Binomial Probability Distribution

1. **Fixed number of trials, n**
2. **Two possible outcomes, success & failure**
3. **Probability of success = p**
4. **Probability of failure = 1 – p**
5. **Looking for the Probability of x successes out of n trials P(n,x)**

You can *assume* that for a large enough number of trials, the binomial distribution *is close enough* to a normal distribution and can use this information to estimate the value of p.

POINT ESTIMATES FOR "P"

$\hat{p} = \frac{x}{n}$ $\quad \hat{q} = 1 - \hat{p}$

Where
x = number of successes from the sample
n = the number of trials = the sample size

Conditions: To approximate to a normal distribution the following MUST be true:

$$n\hat{p} \geq 5 \text{ and } n\hat{q} \geq 5$$

$\hat{p}$ read p-hat (sample proportion of success)
$\hat{q}$ read q-hat (sample proportions of failures)

Example 1: Let p be the population proportion for the situation. Find the point estimate of p and q. Then verify that $n\hat{p} > 5$ *and* $n\hat{q} > 5$.

means find $\hat{p}$ and $\hat{q}$

a) In a survey of 250 college seniors, 170 report that they wish that they had taken more foreign language courses.

$\hat{p} = \frac{170}{250} = \frac{17}{25}$ or $.68$

$\hat{q} = 1 - \frac{17}{25} = \frac{8}{25}$ or $1 - .68 = .32$

$250(.68) = 170 \geq 5$ ✓
$250(.32) = 80 \geq 5$ ✓
yes

b) In a survey of 400 new parents, 396 report they received an informational packet about the La Leche League support group ~~form~~ from their labor and delivery nurses.

$\hat{p} = \frac{396}{400} = .99$

$\hat{q} = 1 - .99 = .01$

$400(.99) = 396 \geq 5$ ✓
$400(.01) = 4 < 5$ ✗

NO
cannot use standard normal dist.

$n \cdot \hat{p} = 800(.75) = 600 > 5$ so we can
$n\hat{q} = 800(.25) = 200 > 5$ use normal dist.

Example 2: Suppose that 800 students were selected at random from a student body and given shots to prevent a certain type of flu. After a waiting period it was discovered that 600 of the students did not get the flu. Construct a 98% confidence interval for the population proportion. Interpret the results.

Confidence Interval for "P"

$$\hat{p} - E < p < \hat{p} + E$$

where $E \approx z_c \sqrt{\frac{\hat{p}\hat{q}}{n}}$

$\hat{p} = \frac{600}{800} = .75$ $\quad \frac{1-.98}{2} = \frac{.02}{2} = .0100$ $\quad z_c = 2.33$

$\hat{q} = 1 - .75 = .25$

$E = 2.33\sqrt{\frac{.75(.25)}{800}} = .0361$

$.75 - .036 < p < .75 + .036$

$.714 < p < .786$

With 98% confidence, you can say the pop. proportion for students that did not get the flu is between 71.4% and 78.6%

Example 3: A random sample of 188 books purchased at a local bookstore showed that 66 of the books were murder mysteries. Let p represent the proportion of books sold by this store that are murder mysteries. Construct a 90% confidence interval for p and interpret the results.

$\hat{p} = \frac{66}{188} = .35$ $\quad \frac{1-.90}{2} = \frac{.1}{2} = .0500$ $\quad E = 1.645\sqrt{\frac{.35(.65)}{188}} = .057$

$\hat{q} = 1 - .35 = .65$ $\quad z_c = 1.645$ $\quad .35 - .057 < p < .35 + .057$

$188(.35) = 65.8 \geq 5$ ✓ $\quad 188(.65) = 122.2 \geq 5$ ✓

$.293 < p < .407$

With 90% confidence, you can say the pop. proportion of sold murder mystery books is between 29.3% and 40.7%.

Example 4: Use the following confidence interval to find the margin of error and the sample proportion. $(5.361, 10.483)$.

$\hat{p} = \frac{5.361 + 10.483}{2} = 7.922$

$E = 7.922 - 5.361 = 2.561$

MINIMUM SAMPLE SIZE FOR ESTIMATING "P"

Example 5:

You want to do a study on the percentage of people who lie to their dentists about flossing. You wish to estimate your percentage with 95% accuracy within 5% of the population proportion.

$c = .95$ $\quad$ use $\hat{p} = .5$ $\hat{q} = .5$

a) **No preliminary estimate is available. Find the minimum** sample size needed.

$E = .05$

$n = (.5)(.5)\left(\frac{1.96}{.05}\right)^2 = 384.16$

$\frac{1-.95}{2} = \frac{.05}{2} = .0250$ $\quad z_c = 1.96$

$\boxed{n = 385}$

b) A commercial for Listerine claims that 1 out of every 5 people lie about flossing to their dentist (In other words, 20% of people lie about flossing). Find the minimum sample size needed.

$\hat{p} = \frac{1}{5} = .2$

$\hat{q} = 1 - .2 = .8$

$n = (.2)(.8)\left(\frac{1.96}{.05}\right)^2 = 245.8...$

$\boxed{n = 246}$

FINDING A MINIMUM SAMPLE SIZE TO ESTIMATE p

Given a *c*-confidence level and a margin of error E, the minimum sample size *n* needed to estimate the population proportion *p* is

$$n = \hat{p}\hat{q}\left(\frac{z_c}{E}\right)^2$$

If *n* is not a whole number, then round *n* up to the next whole number. Also, note that this formula assumes that you have preliminary estimates of *p* and *q*. If not, use $\hat{p} = 0.5$ and $\hat{q} = 0.5$.

$$n = (0.5)(0.5)\left(\frac{z_c}{E}\right)^2$$