# Chapter 6.2

# Confidence Intervals for the Mean (σ Unknown)

SITUATION 1: Want to estimate μ for population. Use: $E = z_c \frac{\sigma}{\sqrt{n}}$ to find

Know standard deviation of population (σ).

6.1 Know $\bar{x}$ (mean of sample size n)

$$\bar{x} - E < \mu < \bar{x} + E$$

SITUATION 2: Want to estimate μ for population.

Don't know standard deviation of population (σ).

6.2 Know $\bar{x}$ (average of sample size n) and $s$ (standard deviation of sample size n)

USE: A non-normal distribution called ***The Student's t distribution***

**THE STUDENT'S *t* DISTRIBUTION**

*History Lesson: William S. Gossett, Statistician for Guinness Brewing Company*

*Gossett and other employees "discouraged" publication of research. Gossett believed the research was important and published anyway under the name: "Student"*

*Now instead of "Gossett's t-distribution", statistical literature refers to it as "The Student's t-distribution"*

Assume that x has a normal distribution with mean μ. For samples of size $n$ with sample mean $\bar{x}$ and sample standard deviation s, the $t$ ***variable*** has a Student's t distribution with degrees of freedom $d.f. = n - 1$

d.f.

**CONFIDENCE INTERVAL:** $\bar{x} - E < \mu < \bar{x} + E$

**MARGIN OF ERROR:** $E = t_c \frac{s}{\sqrt{n}}$

**Get $t_c$ from t distribution table:**

**FIND ROW:**

Use d.f. = n -1

Use closest SMALLER d.f. is your value isn't there.

**FIND COLUMN:**

Use c = confidence level from problem (ignore one-tail area, two-tail area for now)

**Critical Values for Student's t Distribution**

| one-tail area | 0.250 | 0.125 | 0.100 | 0.075 | 0.050 | 0.025 | 0.010 | 0.005 |
|---|---|---|---|---|---|---|---|---|
| two-tail area | 0.500 | 0.250 | 0.200 | 0.150 | 0.100 | 0.050 | 0.020 | 0.010 |
| d.f. \ c | 0.500 | 0.750 | 0.800 | 0.850 | 0.900 | 0.950 | 0.980 | 0.990 |
| 1 | 1.000 | 2.414 | 3.078 | 4.165 | 6.314 | 12.706 | 31.821 | 63.657 |
| 2 | 0.816 | 1.604 | 1.886 | 2.282 | 2.920 | 4.303 | 6.965 | 9.925 |
| 3 | 0.765 | 1.423 | 1.638 | 1.924 | 2.353 | 3.182 | 4.541 | 5.841 |
| 4 | 0.741 | 1.344 | 1.533 | 1.778 | 2.132 | 2.776 | 3.747 | 4.604 |
| 5 | 0.727 | 1.301 | 1.476 | 1.699 | 2.015 | 2.571 | 3.365 | 4.032 |
| 6 | 0.718 | 1.273 | 1.440 | 1.650 | 1.943 | 2.447 | 3.143 | 3.707 |
| 7 | 0.711 | 1.254 | 1.415 | 1.617 | 1.895 | 2.365 | 2.998 | 3.499 |
| 8 | 0.706 | 1.240 | 1.397 | 1.592 | 1.860 | 2.306 | 2.896 | 3.355 |
| 9 | 0.703 | 1.230 | 1.383 | 1.574 | 1.833 | 2.262 | 2.821 | 3.250 |
| 10 | 0.700 | 1.221 | 1.372 | 1.559 | 1.812 | 2.228 | 2.764 | 3.169 |
| 11 | 0.697 | 1.214 | 1.363 | 1.548 | 1.796 | 2.201 | 2.718 | 3.106 |
| 12 | 0.695 | 1.209 | 1.356 | 1.538 | 1.782 | 2.179 | 2.681 | 3.055 |

**Example 1:** Find the critical value for n = 11 and c = 0.75

d.f = 10    $t_c = 1.221$

**Example 2**: Find the critical value $t_c$ for a 0.90 confidence level for a t distribution with sample size n = 8.

d.f. = 7

$t_c = 1.895$

**Example 3**: Find the values $t_{0.98}$ for a sample of size 5.

d.f = 4

$t_c = \cancel{2.776}$ 3.747

**Example 4**: Find the critical value $t_c$ for a 0.95 confidence level for a t distribution with sample size n = 32.

in book d.f. = 31 $t_c = 2.040$

**Critical Values for Student's *t* Distribution**

| one-tail area | 0.250 | 0.125 | 0.100 | 0.075 | 0.050 | 0.025 | 0.010 | 0.005 |
|---|---|---|---|---|---|---|---|---|
| two-tail area | 0.500 | 0.250 | 0.200 | 0.150 | 0.100 | 0.050 | 0.020 | 0.010 |
| d.f. \ c | 0.500 | 0.750 | 0.800 | 0.850 | 0.900 | 0.950 | 0.980 | 0.990 |
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**MARGIN OF ERROR FOR μ WHEN σ IS UNKNOWN**

$$E = t_c \frac{s}{\sqrt{n}}$$

Find the margin of error for the values of c, s, and n.

**Example 5**: c = 0.98, s = 6.5, n = 14

d.f = 13

$t_c = 2.650$

$E = 2.650 \cdot \frac{6.5}{\sqrt{14}} = 4.6$

**CONFIDENCE INTERVAL FOR μ WHEN σ IS UNKNOWN**

$$\bar{x} - E < \mu < \bar{x} + E$$

**Example 6**: A company has a new process for manufacturing large artificial sapphires. In a trial run, 37 sapphires are produced. The mean weight for these 37 gems is $\bar{x} = 6.75$ *carats*, and the sample standard deviation is $s = 0.33$ *carats*. Let μ be the mean weight for the distribution of all sapphires produced by the new process.

Find a 95% confidence interval for μ. c = .95 d.f = 36

n = 37 $\bar{x} = 6.75$ S = 0.33

$t_c = 2.028$

$E = 2.028\left(\frac{0.33}{\sqrt{37}}\right) = .11$

$6.75 - .11 < \mu < 6.75 + .11$

$6.64 < \mu < 6.86$

or

(6.64, 6.86)

with 95% confidence, you can say pop. mean weight of sapphires produced by the new process are between 6.64 and 6.86 carats

**Example 7:** An archeologist discovers only seven fossil skeletons from a previously unknown species of miniature horse. For this sample, the mean is $\bar{x} = 46.14$ and the sample standard deviation is $s = 1.19$. Find a 99% confidence interval for the entire population of such horses.

$n = 7$
$\bar{x} = 46.14$
$s = 1.19$
$c = .99$
$d.f. = 6$
$t_c = 3.707$

$E = 3.707\left(\frac{1.19}{\sqrt{7}}\right) = 1.67$

$46.14 - 1.67 < \mu < 46.14 + 1.67$

$44.47 < \mu < 47.81$

With 99% confidence, you can say the pop. mean for these horses is in the interval 44.47 to 47.81.

Step 1: Check your assumptions
- ☐ simple random sample
- ☐ $\sigma$ is unknown

Step 2: Identify $s$, $n$, $t_c$

Step 3: Calculate the margin of error, E using formula

Step 4: Identify $\bar{x} \pm E$ and set up your confidence interval

Step 5: Interpret your interval in the context of the problem

*"I am C% confident that the interval _____ to _____ contains the true mean."*

**Example 8:** Use the confidence interval to find the margin of error and the sample mean.
(7.85, 19.35)

$\bar{x} = \frac{\text{left} + \text{right}}{2} = \frac{7.85 + 19.35}{2} = 13.6$

$E = \bar{x} - \text{left}$ or $\text{right} - \bar{x}$ $\quad 13.6 - 7.85 = 5.75$

The sample mean is 13.6 and the margin of error is 5.75.

**WHEN TO CHOOSE THE STANDARD NORMAL DISTRIBUTION OR THE t-DISTRIBUTION**

remember:
$\sigma$ is the POPULATION STANDARD DEVIATION