Chapter 6.1 Day 2

Confidence Intervals for the Mean (σ known)

More Examples for applying confidence intervals.

1. A car repair shop is doing a study on the cost of replacing a car transmission. In a sample of 72 transmission repairs, the mean cost was $2650. Construct a 90% and 95% confidence intervals for the population mean. Interpret the results and compare the widths of the confidence intervals. Assume the population standard deviation is $425.

$n = 72$ $\bar{x} = 2650$ $\sigma = 425$

$\frac{1-.9}{2} = \frac{.1}{2} = .05$ $z_c = \pm 1.645$

$\frac{1-.95}{2} = .0250$ $z_c = 1.96$

90%

$E = 1.645 \cdot \frac{425}{\sqrt{72}} = 82.39$

$2650 - 82.39 < \mu < 2650 + 82.39$

$\$2567.61 < \mu < \$2732.39$

$(\$2567.61, \$2732.39)$

95%

$E = 1.96 \cdot \frac{425}{\sqrt{72}} = 98.17$

$2650 - 98.17 < \mu < 2650 + 98.17$

$\$2551.83 < \mu < \$2748.17$

$(\$2551.83, \$2748.17)$

With 90% confidence you can say the pop. mean is in ($2567.61, $2732.39)

With 95% confidence you can say the pop. mean is in ($2551.83, $2748.17)

The 95% CI is wider

2. A sample of 52 juice drinks has a mean of 88 calories. Construct a 92% confidence interval for the population mean and interpret the results. Assume the standard deviation of the population is 31.5.

$n = 52$ $\bar{x} = 88$ $\sigma = 31.5$

$\frac{1-.92}{2} = \frac{.08}{2} = .0400$ $z_c = 1.75$

$E = 1.75 \cdot \frac{31.5}{\sqrt{52}} = 7.6$

$88 - 7.6 < \mu < 88 + 7.6$

$80.4 < \mu < 95.6$

With 92% confidence, the population mean is between 80.4 calories and 95.6 calories.

3. The population standard deviation of morning commute time for teachers is 7.2 minutes. Determine the minimum sample size needed to estimate the population mean score within 5 minutes with 95% confidence.

always round up

$\frac{1-.95}{2}$ $\frac{.05}{2} = .025$

$\sigma = 7.2$ min.
$E = 5$
$z_c = 1.96$

$n = \left(\frac{1.96 \cdot 7.2}{5}\right)^2 = 7.96... \boxed{\sim 8}$

The sample mean is 15.6 minutes. Using the minimum sample size with a 95% level of confidence, does it seem possible that the population mean could be within 3% of the sample mean?

$\bar{x} = 15.6$ $n = 8$ $z_c = 1.96$

$E = 1.96 \frac{(7.2)}{\sqrt{8}} = 4.989$

$15.6 - 4.989 < \mu < 15.6 + 4.989$

$10.611 < \mu < 20.589$

$15.6 \cdot .03$
$.468$

$15.6 - .468 = 15.132$
$15.6 + .468 = 16.068$

yes, because these are within the CI

4. The following data represents the amount of time (in minutes) each day a sample of college students spent online watching videos. Construct a 99% confidence interval for the population mean time students spent watching online videos. Interpret the results.

15.0	16.25	18.0	12.5	14.75	14.5
17.2	16.6	15.8	15.5	14.2	16.75
15.4	19.8	18.2	16.4	17.8	16.5
15.9	17.0	17.6	15.8	14.6	16.2
16.1	20.0	19.4	13.6	17.6	15.75

assume $\sigma = 1.7$

$\bar{x} = 16.36$

$n = 30$

$\frac{1-.99}{2} = \frac{.01}{2} = .005$

$z_c = 2.575$

$E = 2.575\left(\frac{1.7}{\sqrt{30}}\right) = .799$

$16.36 - .799 < \mu < 16.36 + .799$

$15.561 < \mu < 17.159$

(15.6, 17.2) minutes

With 99% confidence, the population mean is between 15.6 minutes and 17.2 minutes.

5. A tennis ball manufacturer wants to estimate the ~~man~~ mean circumference of tennis balls within 0.05 inch. Assume the population of circumferences is normally distributed.

a. Determine the minimum sample size required to construct a 99% confidence interval for the population mean. Assume the population standard deviation is 0.10 inch.

99%

$\frac{1-.99}{2} = .005$

$z_c = 2.575$

$n = \left(\frac{2.575 \cdot .10}{.05}\right)^2 \approx 27$

26.5225

b. The sample mean is 8.3 inches. With a sample size of 34, a 99% level of confidence, and a population standard deviation of 0.10 inch, does it seem possible that the population mean could be exactly 8.258 inches? Explain.

$\bar{x} = 8.3 \quad n = 34 \quad z_c = 2.575 \quad \sigma = .1$

$E = \frac{2.575 \cdot .1}{\sqrt{34}} = .044$

$8.3 - .044 < \mu < 8.3 + .044$

$8.256 < \mu < 8.344$

Yes, The 99% CI is (8.256, 8.344) and 8.258 falls in that interval.