Chapter 5.3

Normal Distributions: Finding Values

Section 5.2

Have Raw Score "X" → Convert to "Z" → Find Probability using table

Section 5.3

Have Desired % or Probability → Use table to determine "Z" → Convert to Raw Score "X"

Sometimes we want to determine a RAW SCORE "X" that satisfies some condition about the percentage of data under the normal curve.

What's the lowest test score I can receive and still be in the top 10% on a college entrance exam?

I want to do a study on the health habits of patients as pertains to weight. What weights are in the middle 60% of my patients?

What "warranty guarantee period" should I advertise so I only have to refund the bottom 5% of all products purchased?

FINDING A Z-SCORE GIVEN AN AREA

AREA TO THE LEFT

Example 1:

Find the Z-score that corresponds to

a) 1.54% of the area to the left

1.54% = .0154

Z = −2.16

b) .0076 of the area to the left

Z ≈ −2.43

c) 1.64% of the area to the left

1.64% = .0164

Z = −2.135

Areas of a Standard Normal Distribution

(a) Table of Areas to the Left of z

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183

AREA TO THE RIGHT

Example 2:

Find the Z-score that corresponds to

a) 20.33% to the right

.2033

$1 - .2033 = .7967$

$Z = 0.83$

b) 10% to the right

.1

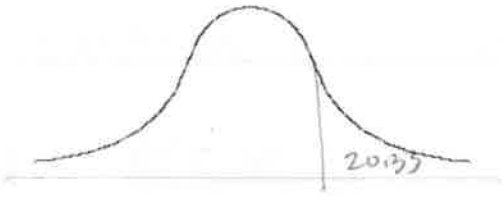
$1 - .1 = .9000$

$Z = 1.28$

c) 0.4701 to the right Not on this table — use handout I gave you

½ way between .4721 and .4681

So $Z = -.075$



Areas of a Standard Normal Distribution *continued*

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319

a

b is closest to this

CENTERED AROUND THE MEAN

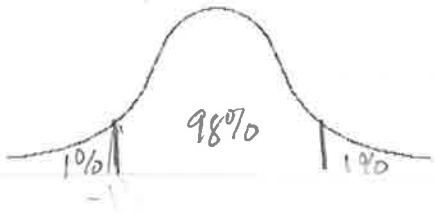
Example 3:

Find the Z-scores that correspond to the middle 98% of data.

left of 1% or .0100

So $Z = -2.33$

and $Z = 2.33$



Areas of a Standard Normal Distribution

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−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084

FINDING A z-SCORE GIVEN A PERCENTILE (use the table handed out previously)

Example 4: Find the z-score that corresponds to each percentile.

Change the Percentile to a % then find it on the table

a. P_5

5th percentile = .0500

$\frac{1}{2}$ way between .0505 + .0495

So $z = -1.645$

b. P_{50}

.5000

$z = 0$

c. P_{90}

.9000

closest to .8997

So $z = 1.28$

INVERSE NORMAL DISTRIBUTION

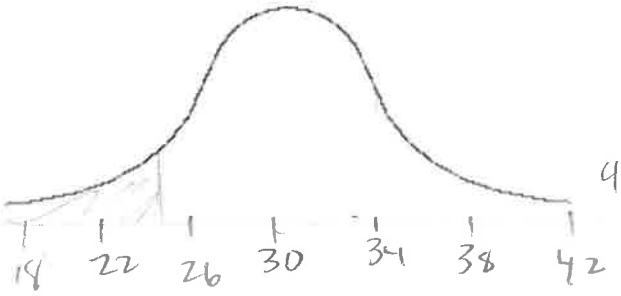
Example 5: Magic Video Games, Inc., sells an expensive video games package. Because the package is so expensive, the company wants to advertise an impressive guarantee for the life expectancy of the computer system. The guarantee policy will refund the full purchase price if the computer fails during the guarantee period. The research department has done tests that show that the mean life for the computer is 30 months, with the standard deviation of 4 months. The computer life is normally distributed. How long can the guarantee period be if management does not want to refund the purchase price on more than 7% of the Magic Video Packages?

How to find x, given a desired probability.

1. Determine what percentage of area under the normal curve you ACTUALLY want based on left/right/between and your given probability.

2. Use your table to estimate z.

3. Translate your z answer into x using the equation: $x = z\sigma + \mu$ or $z = \frac{x - \mu}{\sigma}$



7% = .0700 $z = -1.48$ from table

$z = \frac{x - \mu}{\sigma}$

$4 \cdot (-1.48) = \frac{x - 30}{4} \cdot 4$

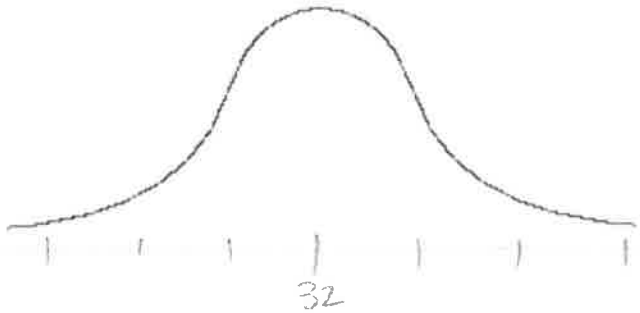
$-5.92 = x - 30$

$+30 \quad +30$

$x = 35.92$

The guarantee period should be about 24 months.

Example 6: The weights of bags of baby carrots are normally distributed with a mean of 32 ounces and a standard deviation of 0.36 ounce. Bags in the upper 4.5% are too heavy and must be repackaged. What is the most a bag of baby carrots can weigh and not need to be repackaged?



$1 - .045 = .9550$

$z \approx 1.7$ from table

$.36(1.7) = \frac{x - 32}{.36} \cdot .36$

$.612 = x - 32$

$+32 \quad +32$

$x = 32.612$

The bag can weigh at most 32.6 ounces to not be repackaged.