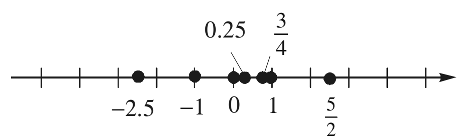
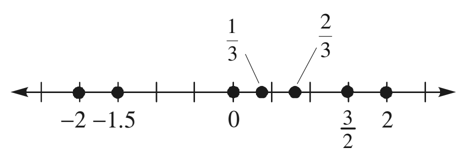
Appendix A
Review

Section A.1

1. variable
2. origin
3. strict
4. base; exponent (or power)
5. True.
6. True
7. False; the absolute value of a real number is nonnegative. $|0| = 0$ which is not a positive number.
8. False; to multiply two expressions with the same base, retain the base and *add* the exponents.
9. $A \cup B = \{1, 3, 4, 5, 9\} \cup \{2, 4, 6, 7, 8\}$
$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
10. $A \cup C = \{1, 3, 4, 5, 9\} \cup \{1, 3, 4, 6\}$
$= \{1, 3, 4, 5, 6, 9\}$
11. $A \cap B = \{1, 3, 4, 5, 9\} \cap \{2, 4, 6, 7, 8\} = \{4\}$
12. $A \cap C = \{1, 3, 4, 5, 9\} \cap \{1, 3, 4, 6\} = \{1, 3, 4\}$
13. $(A \cup B) \cap C$
$= (\{1, 3, 4, 5, 9\} \cup \{2, 4, 6, 7, 8\}) \cap \{1, 3, 4, 6\}$
$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \cap \{1, 3, 4, 6\}$
$= \{1, 3, 4, 6\}$
14. $(A \cap B) \cup C$
$= (\{1, 3, 4, 5, 9\} \cap \{2, 4, 6, 7, 8\}) \cup \{1, 3, 4, 6\}$
$= \{4\} \cup \{1, 3, 4, 6\}$
$= \{1, 3, 4, 6\}$
15. $\overline{A} = \{0, 2, 6, 7, 8\}$
16. $\overline{C} = \{0, 2, 5, 7, 8, 9\}$
17. $\overline{A \cap B} = \overline{\{1, 3, 4, 5, 9\} \cap \{2, 4, 6, 7, 8\}}$
$= \overline{\{4\}} = \{0, 1, 2, 3, 5, 6, 7, 8, 9\}$
18. $\overline{B \cup C} = \overline{\{2, 4, 6, 7, 8\} \cup \{1, 3, 4, 6\}}$
$= \overline{\{1, 2, 3, 4, 6, 7, 8\}} = \{0, 5, 9\}$
19. $\overline{A} \cup \overline{B} = \{0, 2, 6, 7, 8\} \cup \{0, 1, 3, 5, 9\}$
$= \{0, 1, 2, 3, 5, 6, 7, 8, 9\}$
20. $\overline{B} \cap \overline{C} = \{0, 1, 3, 5, 9\} \cap \{0, 2, 5, 7, 8, 9\}$
$= \{0, 5, 9\}$
21. 

22. 

23. $\frac{1}{2} > 0$
24. $5 < 6$
25. $-1 > -2$
26. $-3 < -\frac{5}{2}$
27. $\pi > 3.14$
28. $\sqrt{2} > 1.41$
29. $\frac{1}{2} = 0.5$
30. $\frac{1}{3} > 0.33$
31. $\frac{2}{3} < 0.67$

32. $\frac{1}{4} = 0.25$

33. $x > 0$

34. $z < 0$

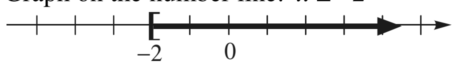
35. $x < 2$

36. $y > -5$

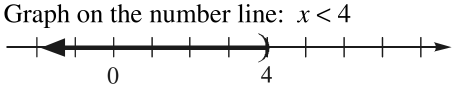
37. $x \le 1$

38. $x \ge 2$

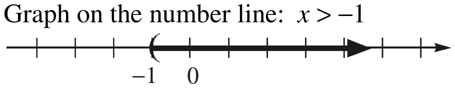
39. Graph on the number line: $x \ge -2$



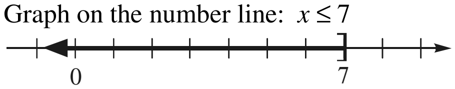
40. Graph on the number line: $x < 4$



41. Graph on the number line: $x > -1$



42. Graph on the number line: $x \le 7$



43. $d(C, D) = d(0, 1) = |1 - 0| = |1| = 1$

44. $d(C, A) = d(0, -3) = |-3 - 0| = |-3| = 3$

45. $d(D, E) = d(1, 3) = |3 - 1| = |2| = 2$

46. $d(C, E) = d(0, 3) = |3 - 0| = |3| = 3$

47. $d(A, E) = d(-3, 3) = |3 - (-3)| = |6| = 6$

48. $d(D, B) = d(1, -1) = |-1 - 1| = |-2| = 2$

49. $x + 2y = -2 + 2 \cdot 3 = -2 + 6 = 4$

50. $3x + y = 3(-2) + 3 = -6 + 3 = -3$

51. $5xy + 2 = 5(-2)(3) + 2 = -30 + 2 = -28$

52. $-2x + xy = -2(-2) + (-2)(3) = 4 - 6 = -2$

53. $\frac{2x}{x - y} = \frac{2(-2)}{-2 - 3} = \frac{-4}{-5} = \frac{4}{5}$

54. $\frac{x + y}{x - y} = \frac{-2 + 3}{-2 - 3} = \frac{1}{-5} = -\frac{1}{5}$

55. $\frac{3x + 2y}{2 + y} = \frac{3(-2) + 2(3)}{2 + 3} = \frac{-6 + 6}{5} = \frac{0}{5} = 0$

56. $\frac{2x - 3}{y} = \frac{2(-2) - 3}{3} = \frac{-4 - 3}{3} = -\frac{7}{3}$

57. $|x + y| = |3 + (-2)| = |1| = 1$

58. $|x - y| = |3 - (-2)| = |5| = 5$

59. $|x| + |y| = |3| + |-2| = 3 + 2 = 5$

60. $|x| - |y| = |3| - |-2| = 3 - 2 = 1$

61. $\frac{|x|}{x} = \frac{|3|}{3} = \frac{3}{3} = 1$

62. $\frac{|y|}{y} = \frac{|-2|}{-2} = \frac{2}{-2} = -1$

63.
$$\begin{aligned} |4x - 5y| &= |4(3) - 5(-2)| \\ &= |12 + 10| \\ &= |22| \\ &= 22 \end{aligned}$$

64. $|3x + 2y| = |3(3) + 2(-2)| = |9 - 4| = |5| = 5$

65.
$$\begin{aligned} \big||4x| - |5y|\big| &= \big||4(3)| - |5(-2)|\big| \\ &= \big||12| - |-10|\big| \\ &= |12 - 10| \\ &= |2| \\ &= 2 \end{aligned}$$

66.
$$\begin{aligned} 3|x| + 2|y| &= 3|3| + 2|-2| \\ &= 3 \cdot 3 + 2 \cdot 2 \\ &= 9 + 4 \\ &= 13 \end{aligned}$$

67. $\dfrac{x^2-1}{x}$
Part (c) must be excluded. The value $x=0$ must be excluded from the domain because it causes division by 0.

68. $\dfrac{x^2+1}{x}$
Part (c) must be excluded. The value $x=0$ must be excluded from the domain because it causes division by 0.

69. $\dfrac{x}{x^2-9}=\dfrac{x}{(x-3)(x+3)}$
Part (a) must be excluded. The values $x=-3$ and $x=3$ must be excluded from the domain because they cause division by 0.

70. $\dfrac{x}{x^2+9}$
None of the given values are excluded. The domain is all real numbers.

71. $\dfrac{x^2}{x^2+1}$
None of the given values are excluded. The domain is all real numbers.

72. $\dfrac{x^3}{x^2-1}=\dfrac{x^3}{(x-1)(x+1)}$
Parts (b) and (d) must be excluded. The values $x=1$, and $x=-1$ must be excluded from the domain because they cause division by 0.

73. $\dfrac{x^2+5x-10}{x^3-x}=\dfrac{x^2+5x-10}{x(x-1)(x+1)}$
Parts (b), (c), and (d) must be excluded. The values $x=0$, $x=1$, and $x=-1$ must be excluded from the domain because they cause division by 0.

74. $\dfrac{-9x^2-x+1}{x^3+x}=\dfrac{-9x^2-x+1}{x(x^2+1)}$
Part (c) must be excluded. The value $x=0$ must be excluded from the domain because it causes division by 0.

75. $\dfrac{4}{x-5}$
$x=5$ must be exluded because it makes the denominator equal 0.
Domain $=\{x \mid x\neq 5\}$

76. $\dfrac{-6}{x+4}$
$x=-4$ must be excluded sine it makes the denominator equal 0.
Domain $=\{x \mid x\neq -4\}$

77. $\dfrac{x}{x+4}$
$x=-4$ must be excluded sine it makes the denominator equal 0.
Domain $=\{x \mid x\neq -4\}$

78. $\dfrac{x-2}{x-6}$
$x=6$ must be excluded sine it makes the denominator equal 0.
Domain $=\{x \mid x\neq 6\}$

79. $C=\dfrac{5}{9}(F-32)=\dfrac{5}{9}(32-32)=\dfrac{5}{9}(0)=0°\text{C}$

80. $C=\dfrac{5}{9}(F-32)=\dfrac{5}{9}(212-32)=\dfrac{5}{9}(180)=100°\text{C}$

81. $C=\dfrac{5}{9}(F-32)=\dfrac{5}{9}(77-32)=\dfrac{5}{9}(45)=25°\text{C}$

82. $C=\dfrac{5}{9}(F-32)=\dfrac{5}{9}(-4-32)$
$=\dfrac{5}{9}(-36)$
$=-20°\text{C}$

83. $(-4)^2=(-4)(-4)=16$

84. $-4^2=-(4)^2=-16$

85. $4^{-2}=\dfrac{1}{4^2}=\dfrac{1}{16}$

86. $-4^{-2}=-\dfrac{1}{4^2}=-\dfrac{1}{16}$

87. $3^{-6} \cdot 3^4 = 3^{-6+4} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

88. $4^{-2} \cdot 4^3 = 4^{-2+3} = 4^1 = 4$

89. $\left(3^{-2}\right)^{-1} = 3^{(-2)(-1)} = 3^2 = 9$

90. $\left(2^{-1}\right)^{-3} = 2^{(-1)(-3)} = 2^3 = 8$

91. $\sqrt{25} = \sqrt{5^2} = 5$

92. $\sqrt{36} = \sqrt{6^2} = 6$

93. $\sqrt{(-4)^2} = |-4| = 4$

94. $\sqrt{(-3)^2} = |-3| = 3$

95. $\left(8x^3\right)^2 = 8^2\left(x^3\right)^2 = 64x^6$

96. $\left(-4x^2\right)^{-1} = \frac{1}{-4x^2} = -\frac{1}{4x^2}$

97. $\left(x^2y^{-1}\right)^2 = \left(x^2\right)^2 \cdot \left(y^{-1}\right)^2 = x^4y^{-2} = \frac{x^4}{y^2}$

98. $\left(x^{-1}y\right)^3 = \left(x^{-1}\right)^3 \cdot y^3 = x^{-3}y^3 = \frac{y^3}{x^3}$

99. $\frac{x^2y^3}{xy^4} = x^{2-1}y^{3-4} = x^1y^{-1} = \frac{x}{y}$

100. $\frac{x^{-2}y}{xy^2} = x^{-2-1}y^{1-2} = x^{-3}y^{-1} = \frac{1}{x^3y}$

101.
$$\begin{aligned}\frac{(-2)^3x^4(yz)^2}{3^2xy^3z} &= \frac{-8x^4y^2z^2}{9xy^3z}\\ &= \frac{-8}{9}x^{4-1}y^{2-3}z^{2-1}\\ &= \frac{-8}{9}x^3y^{-1}z^1\\ &= -\frac{8x^3z}{9y}\end{aligned}$$

102.
$$\begin{aligned}\frac{4x^{-2}(yz)^{-1}}{2^3x^4y} &= \frac{4x^{-2}y^{-1}z^{-1}}{8x^4y}\\ &= \frac{4}{8}x^{-2-4}y^{-1-1}z^{-1}\\ &= \frac{1}{2}x^{-6}y^{-2}z^{-1}\\ &= \frac{1}{2x^6y^2z}\end{aligned}$$

103. $\left(\frac{3x^{-1}}{4y^{-1}}\right)^{-2} = \left(\frac{3y}{4x}\right)^{-2} = \left(\frac{4x}{3y}\right)^2 = \frac{4^2x^2}{3^2y^2} = \frac{16x^2}{9y^2}$

104.
$$\begin{aligned}\left(\frac{5x^{-2}}{6y^{-2}}\right)^{-3} &= \left(\frac{5y^2}{6x^2}\right)^{-3} = \left(\frac{6x^2}{5y^2}\right)^3\\ &= \frac{6^3\left(x^2\right)^3}{5^3\left(y^2\right)^3} = \frac{216x^6}{125y^6}\end{aligned}$$

105. $2xy^{-1} = \frac{2x}{y} = \frac{2(2)}{(-1)} = -4$

106. $-3x^{-1}y = \frac{-3y}{x} = \frac{-3(-1)}{(2)} = \frac{3}{2}$

107. $x^2 + y^2 = (2)^2 + (-1)^2 = 4 + 1 = 5$

108. $x^2y^2 = (2)^2(-1)^2 = 4 \cdot 1 = 4$

109. $(xy)^2 = (2 \cdot (-1))^2 = (-2)^2 = 4$

110. $(x+y)^2 = (2 + (-1))^2 = (1)^2 = 1$

111. $\sqrt{x^2} = |x| = |2| = 2$

112. $\left(\sqrt{x}\right)^2 = x = 2$

113. $\sqrt{x^2 + y^2} = \sqrt{(2)^2 + (-1)^2} = \sqrt{4+1} = \sqrt{5}$

114. $\sqrt{x^2} + \sqrt{y^2} = |x| + |y| = |2| + |-1| = 2 + 1 = 3$

115. $x^y = 2^{-1} = \frac{1}{2}$

116. $y^x = (-1)^2 = 1$

117. If $x = 2$,

$$\begin{aligned} 2x^3 - 3x^2 + 5x - 4 &= 2\cdot 2^3 - 3\cdot 2^2 + 5\cdot 2 - 4 \\ &= 16 - 12 + 10 - 4 \\ &= 10 \end{aligned}$$

If $x = 1$,

$$\begin{aligned} 2x^3 - 3x^2 + 5x - 4 &= 2\cdot 1^3 - 3\cdot 1^2 + 5\cdot 1 - 4 \\ &= 2 - 3 + 5 - 4 \\ &= 0 \end{aligned}$$

118. If $x = 1$,

$$\begin{aligned} 4x^3 + 3x^2 - x + 2 &= 4\cdot 1^3 + 3\cdot 1^2 - 1 + 2 \\ &= 4 + 3 - 1 + 2 \\ &= 8 \end{aligned}$$

If $x = 2$,

$$\begin{aligned} 4x^3 + 3x^2 - x + 2 &= 4\cdot 2^3 + 3\cdot 2^2 - 2 + 2 \\ &= 32 + 12 - 2 + 2 \\ &= 44 \end{aligned}$$

119. $\dfrac{(666)^4}{(222)^4} = \left(\dfrac{666}{222}\right)^4 = 3^4 = 81$

120.
$$\begin{aligned} (0.1)^3(20)^3 &= \left(\frac{1}{10}\right)^3 \cdot (2\cdot 10)^3 \\ &= \frac{1}{10^3}\cdot 2^3 \cdot 10^3 \\ &= 2^3 = 8 \end{aligned}$$

121. $(8.2)^6 \approx 304{,}006.671$

122. $(3.7)^5 \approx 693.440$

123. $(6.1)^{-3} \approx 0.004$

124. $(2.2)^{-5} \approx 0.019$

125. $(-2.8)^6 \approx 481.890$

126. $-(2.8)^6 \approx -481.890$

127. $(-8.11)^{-4} \approx 0.000$

128. $-(8.11)^{-4} \approx -0.000$

129. $A = lw$

130. $P = 2(l + w)$

131. $C = \pi d$

132. $A = \dfrac{1}{2}bh$

133. $A = \dfrac{\sqrt{3}}{4}x^2$

134. $P = 3x$

135. $V = \dfrac{4}{3}\pi r^3$

136. $S = 4\pi r^2$

137. $V = x^3$

138. $S = 6x^2$

139. **a.** If $x = 1000$,

$$\begin{aligned} C &= 4000 + 2x \\ &= 4000 + 2(1000) \\ &= 4000 + 2000 \\ &= \$6000 \end{aligned}$$

The cost of producing 1000 watches is $6000.

b. If $x = 2000$,

$$\begin{aligned} C &= 4000 + 2x \\ &= 4000 + 2(2000) \\ &= 4000 + 4000 \\ &= \$8000 \end{aligned}$$

The cost of producing 2000 watches is $8000.

140. $210 + 80 - 120 + 25 - 60 - 32 - 5 = \$98$

His balance at the end of the month was $98.

141. We want the difference between x and 4 to be at least 6 units. Since we don't care whether the value for x is larger or smaller than 4, we take the absolute value of the difference. We want the inequality to be non-strict since we are dealing with an 'at least' situation. Thus, we have
$|x-4| \ge 6$

142. We want the difference between x and 2 to be more than 5 units. Since we don't care whether the value for x is larger or smaller than 2, we take the absolute value of the difference. We want the inequality to be strict since we are dealing with a 'more than' situation. Thus, we have
$|x-2| > 5$

143. **a.** $|x-110| = |108-110| = |-2| = 2 \le 5$
108 volts is acceptable.

b. $|x-110| = |104-110| = |-6| = 6 > 5$
104 volts is *not* acceptable.

144. **a.** $|x-220| = |214-220| = |-6| = 6 \le 8$
214 volts is acceptable.

b. $|x-220| = |209-220| = |-11| = 11 > 8$
209 volts is *not* acceptable.

145. **a.** $|x-3| = |2.999-3|$
$= |-0.001|$
$= 0.001 \le 0.01$
A radius of 2.999 centimeters is acceptable.

b. $|x-3| = |2.89-3|$
$= |-0.11|$
$= 0.11 \not\le 0.01$
A radius of 2.89 centimeters is *not* acceptable.

146. **a.** $|x-98.6| = |97-98.6|$
$= |-1.6|$
$= 1.6 \ge 1.5$
97°F is unhealthy.

b. $|x-98.6| = |100-98.6|$
$= |1.4|$
$= 1.4 < 1.5$
100°F is *not* unhealthy.

147. $\frac{1}{3} = 0.333333\ldots > 0.333$
$\frac{1}{3}$ is larger by approximately $0.0003333\ldots$

148. $\frac{2}{3} = 0.666666\ldots > 0.666$
$\frac{2}{3}$ is larger by approximately $0.0006666\ldots$

149. No. For any positive number a, the value $\frac{a}{2}$ is smaller and therefore closer to 0.

150. We are given that $1 < x^2 < 10$. This implies that $1 < x < \sqrt{10}$. Since $x < \sqrt{10} \approx 3.162$ and $x > \pi \approx 3.142$, the number could be 3.15 or 3.16 (which are between 1 and 10 as required). The number could also be 3.14 since numbers such as 3.146 which lie between π and $\sqrt{10}$ would equal 3.14 when truncated to two decimal places.

151. Answers will vary.

152. Answers will vary.
5 < 8 is a true statement because 5 is further to the left than 8 on a real number line.

Section A.2

1. right; hypotenuse

2. $A = \frac{1}{2}bh$

3. $C = 2\pi r$

4. similar

5. True

6. True; $6^2 + 8^2 = 36 + 64 = 100 = 10^2$

7. False; the volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.

8. True; The lengths of the corresponding sides are equal.

9. True; Two corresponding angles are equal.

10. False; The sides are not proportional.

11. $a = 5,\ b = 12,$

$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 5^2 + 12^2 \\ &= 25 + 144 \\ &= 169 \Rightarrow c = 13 \end{aligned}$$

12. $a = 6,\ b = 8,$

$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 6^2 + 8^2 \\ &= 36 + 64 \\ &= 100 \Rightarrow c = 10 \end{aligned}$$

13. $a = 10,\ b = 24,$

$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 10^2 + 24^2 \\ &= 100 + 576 \\ &= 676 \Rightarrow c = 26 \end{aligned}$$

14. $a = 4,\ b = 3,$

$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 4^2 + 3^2 \\ &= 16 + 9 \\ &= 25 \Rightarrow c = 5 \end{aligned}$$

15. $a = 7,\ b = 24,$

$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 7^2 + 24^2 \\ &= 49 + 576 \\ &= 625 \Rightarrow c = 25 \end{aligned}$$

16. $a = 14,\ b = 48,$

$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 14^2 + 48^2 \\ &= 196 + 2304 \\ &= 2500 \Rightarrow c = 50 \end{aligned}$$

17. $5^2 = 3^2 + 4^2$

$25 = 9 + 16$

$25 = 25$

The given triangle is a right triangle. The hypotenuse is 5.

18. $10^2 = 6^2 + 8^2$

$100 = 36 + 64$

$100 = 100$

The given triangle is a right triangle. The hypotenuse is 10.

19. $6^2 = 4^2 + 5^2$

$36 = 16 + 25$

$36 = 41$ false

The given triangle is not a right triangle.

20. $3^2 = 2^2 + 2^2$

$9 = 4 + 4$

$9 = 8$ false

The given triangle is not a right triangle.

21. $25^2 = 7^2 + 24^2$

$625 = 49 + 576$

$625 = 625$

The given triangle is a right triangle. The hypotenuse is 25.

22. $26^2 = 10^2 + 24^2$

$676 = 100 + 576$

$676 = 676$

The given triangle is a right triangle. The hypotenuse is 26.

23. $6^2 = 3^2 + 4^2$

$36 = 9 + 16$

$36 = 25$ false

The given triangle is not a right triangle.

24. $7^2 = 5^2 + 4^2$

$49 = 25 + 16$

$49 = 41$ false

The given triangle is not a right triangle.

25. $A = l \cdot w = 4 \cdot 2 = 8 \text{ in}^2$

26. $A = l \cdot w = 9 \cdot 4 = 36 \text{ cm}^2$

27. $A = \frac{1}{2} b \cdot h = \frac{1}{2}(2)(4) = 4 \text{ in}^2$

28. $A = \frac{1}{2} b \cdot h = \frac{1}{2}(4)(9) = 18 \text{ cm}^2$

29. $A = \pi r^2 = \pi(5)^2 = 25\pi \text{ m}^2$
$C = 2\pi r = 2\pi(5) = 10\pi \text{ m}$

30. $A = \pi r^2 = \pi(2)^2 = 4\pi \text{ ft}^2$
$C = 2\pi r = 2\pi(2) = 4\pi \text{ ft}$

31. $V = lwh = 8 \cdot 4 \cdot 7 = 224 \text{ ft}^3$

$$\begin{aligned} S &= 2lw + 2lh + 2wh \\ &= 2(8)(4) + 2(8)(7) + 2(4)(7) \\ &= 64 + 112 + 56 \\ &= 232 \text{ ft}^2 \end{aligned}$$

32. $V = lwh = 9 \cdot 4 \cdot 8 = 288 \text{ in}^3$

$$\begin{aligned} S &= 2lw + 2lh + 2wh \\ &= 2(9)(4) + 2(9)(8) + 2(4)(8) \\ &= 72 + 144 + 64 \\ &= 280 \text{ in}^2 \end{aligned}$$

33. $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \cdot 4^3 = \frac{256}{3}\pi \text{ cm}^3$
$S = 4\pi r^2 = 4\pi \cdot 4^2 = 64\pi \text{ cm}^2$

34. $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \cdot 3^3 = 36\pi \text{ f}t^3$
$S = 4\pi r^2 = 4\pi \cdot 3^2 = 36\pi \text{ ft}^2$

35. $V = \pi r^2 h = \pi(9)^2(8) = 648\pi \text{ in}^3$

$$\begin{aligned} S &= 2\pi r^2 + 2\pi rh \\ &= 2\pi(9)^2 + 2\pi(9)(8) \\ &= 162\pi + 144\pi \\ &= 306\pi \text{ in}^2 \end{aligned}$$

36. $V = \pi r^2 h = \pi(8)^2(9) = 576\pi \text{ in}^3$

$$\begin{aligned} S &= 2\pi r^2 + 2\pi rh \\ &= 2\pi(8)^2 + 2\pi(8)(9) \\ &= 128\pi + 144\pi \\ &= 272\pi \text{ in}^2 \end{aligned}$$

37. The diameter of the circle is 2, so its radius is 1.
$A = \pi r^2 = \pi(1)^2 = \pi$ square units

38. The diameter of the circle is 2, so its radius is 1.
$A = 2^2 - \pi(1)^2 = 4 - \pi$ square units

39. The diameter of the circle is the length of the diagonal of the square.

$$\begin{aligned} d^2 &= 2^2 + 2^2 \\ &= 4 + 4 \\ &= 8 \\ d &= \sqrt{8} = 2\sqrt{2} \\ r &= \frac{d}{2} = \frac{2\sqrt{2}}{2} = \sqrt{2} \end{aligned}$$

The area of the circle is:
$A = \pi r^2 = \pi\left(\sqrt{2}\right)^2 = 2\pi$ square units

40. The diameter of the circle is the length of the diagonal of the square.

$$\begin{aligned} d^2 &= 2^2 + 2^2 \\ &= 4 + 4 \\ &= 8 \\ d &= \sqrt{8} = 2\sqrt{2} \\ r &= \frac{d}{2} = \frac{2\sqrt{2}}{2} = \sqrt{2} \end{aligned}$$

The area is:
$A = \pi\left(\sqrt{2}\right)^2 - 2^2 = 2\pi - 4$ square units

41. Since the triangles are similar, the lengths of corresponding sides are proportional. Therefore, we get

$$\begin{aligned} \frac{8}{4} &= \frac{x}{2} \\ \frac{8 \cdot 2}{4} &= x \\ 4 &= x \end{aligned}$$

In addition, corresponding angles must have the same angle measure. Therefore, we have $A = 90°$, $B = 60°$, and $C = 30°$.

42. Since the triangles are similar, the lengths of corresponding sides are proportional. Therefore, we get

$$\frac{6}{12}=\frac{x}{16}$$
$$\frac{6\cdot 16}{12}=x$$
$$8=x$$

In addition, corresponding angles must have the same angle measure. Therefore, we have $A=30°$, $B=75°$, and $C=75°$.

43. Since the triangles are similar, the lengths of corresponding sides are proportional. Therefore, we get

$$\frac{30}{20}=\frac{x}{45}$$
$$\frac{30\cdot 45}{20}=x$$
$$\frac{135}{2}=x \text{ or } x=67.5$$

In addition, corresponding angles must have the same angle measure. Therefore, we have $A=60°$, $B=95°$, and $C=25°$.

44. Since the triangles are similar, the lengths of corresponding sides are proportional. Therefore, we get

$$\frac{8}{10}=\frac{x}{50}$$
$$\frac{8\cdot 50}{10}=x$$
$$40=x$$

In addition, corresponding angles must have the same angle measure. Therefore, we have $A=50°$, $B=125°$, and $C=5°$.

45. The total distance traveled is 4 times the circumference of the wheel.

$$\text{Total Distance}=4C=4(\pi d)=4\pi\cdot 16$$
$$=64\pi\approx 201.1 \text{ inches}\approx 16.8 \text{ feet}$$

46. The distance traveled in one revolution is the circumference of the disk 4π.
The number of revolutions =

$$\frac{\text{dist. traveled}}{\text{circumference}}=\frac{20}{4\pi}=\frac{5}{\pi}\approx 1.6 \text{ revolutions}$$

47. Area of the border = area of EFGH – area of ABCD $=10^2-6^2=100-36=64 \text{ ft}^2$

48. FG = 4 feet; BG = 4 feet and BC = 10 feet, so CG= 6 feet. The area of the triangle CGF is:

$$A=\frac{1}{2}\cdot(4)(6)=12 \text{ ft}^2$$

49. Area of the window = area of the rectangle + area of the semicircle.

$$A=(6)(4)+\frac{1}{2}\cdot\pi\cdot 2^2=24+2\pi\approx 30.28 \text{ ft}^2$$

Perimeter of the window = 2 heights + width + one-half the circumference.

$$P=2(6)+4+\frac{1}{2}\cdot\pi(4)=12+4+2\pi$$
$$=16+2\pi\approx 22.28 \text{ feet}$$

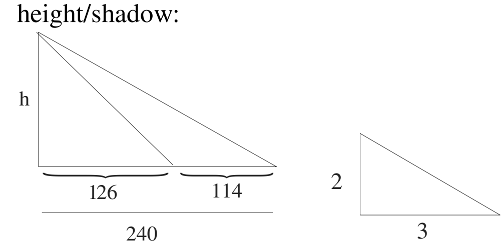
50. Area of the deck = area of the pool and deck – area of the pool.

$$A=\pi(13)^2-\pi(10)^2=169\pi-100\pi$$
$$=69\pi \text{ ft}^2\approx 216.77 \text{ ft}^2$$

The amount of fence is the circumference of the circle with radius 13 feet.

$$C=2\pi(13)=26\pi \text{ ft}\approx 81.68 \text{ ft}$$

51. We can form similar triangles using the Great Pyramid's height/shadow and Thales' height/shadow:



This allows us to write

$$\frac{h}{240}=\frac{2}{3}$$
$$h=\frac{2\cdot 240}{3}=160$$

The height of the Great Pyramid is 160 paces.

52. Let x = the approximate distance from San Juan to Hamilton and y = the approximate distance from Hamilton to Fort Lauderdale. Using similar triangles, we get

$$\frac{1046}{58}=\frac{x}{53.5} \qquad \frac{1046}{58}=\frac{y}{57}$$

$$\frac{1046\cdot 53.5}{58}=x \qquad \frac{1046\cdot 57}{58}=y$$

$$964.8\approx x \qquad 1028.0\approx y$$

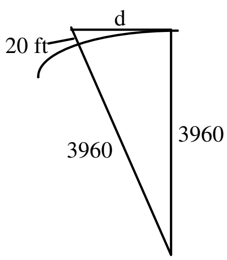
The approximate distance between San Juan and Hamilton is 965 miles and the approximate distance between Hamilton and Fort Lauderdale is 1028 miles.

53. Convert 20 feet to miles, and solve the Pythagorean Theorem to find the distance:

$$20 \text{ feet } = 20 \text{ feet } \cdot\frac{1 \text{ mile}}{5280 \text{ feet}}=0.003788 \text{ miles}$$

$$d^2=(3960+0.003788)^2-3960^2=30 \text{ sq. miles}$$

$$d\approx 5.477 \text{ miles}$$

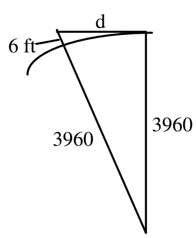


54. Convert 6 feet to miles, and solve the Pythagorean Theorem to find the distance:

$$6 \text{ feet } = 6 \text{ feet } \cdot\frac{1 \text{ mile}}{5280 \text{ feet}}=0.001136 \text{ miles}$$

$$d^2=(3960+0.001136)^2-3960^2=9 \text{ sq. miles}$$

$$d\approx 3 \text{ miles}$$



55. Convert 100 feet to miles, and solve the Pythagorean Theorem to find the distance:

$$100 \text{ feet } = 100 \text{ feet } \cdot\frac{1 \text{ mile}}{5280 \text{ feet}}=0.018939 \text{ miles}$$

$$d^2=(3960+0.018939)^2-3960^2\approx 150 \text{ sq. miles}$$

$$d\approx 12.2 \text{ miles}$$

Convert 150 feet to miles, and solve the Pythagorean Theorem to find the distance:

$$150 \text{ feet } = 150 \text{ feet } \cdot\frac{1 \text{ mile}}{5280 \text{ feet}}=0.028409 \text{ miles}$$

$$d^2=(3960+0.028409)^2-3960^2\approx 225 \text{ sq. miles}$$

$$d\approx 15.0 \text{ miles}$$

56. Given $m>0$, $n>0$ and $m>n$,

if $a=m^2-n^2, b=2mn$ and $c=m^2+n^2$, then

$$a^2+b^2=\left(m^2-n^2\right)^2+\left(2mn\right)^2$$
$$=m^4-2m^2n^2+n^4+4m^2n^2$$
$$=m^4+2m^2n^2+n^4$$

and $c^2=\left(m^2+n^2\right)^2=m^4+2m^2n^2+n^4$

$\therefore a^2+b^2=c^2\rightarrow a,b$ and c represent the sides of a right triangle.

57. Let l = length of the rectangle and w = width of the rectangle.
Notice that

$$(l+w)^2-(l-w)^2$$
$$=[(l+w)+(l-w)][(l+w)-(l-w)]$$
$$=(2l)(2w)=4lw=4A$$

So $A=\frac{1}{4}[(l+w)^2-(l-w)^2]$

Since $(l-w)^2\geq 0$, the largest area will occur when $l-w=0$ or $l=w$; that is, when the rectangle is a square. But

$$1000=2l+2w=2(l+w)$$
$$500=l+w=2l$$
$$250=l=w$$

The largest possible area is $250^2=62500$ sq ft.
A circular pool with circumference = 1000 feet yields the equation: $2\pi r=1000\Rightarrow r=\frac{500}{\pi}$

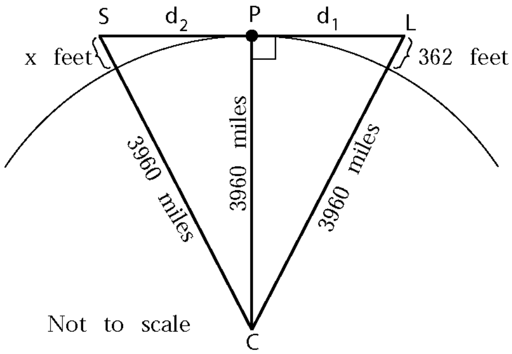
The area enclosed by the circular pool is:

$$A=\pi r^2=\pi\left(\frac{500}{\pi}\right)^2=\frac{500^2}{\pi}\approx 79577.47 \text{ ft}^2$$

Thus, a circular pool will enclose the most area.

58. Consider the diagram showing the lighthouse at point L, relative to the center of Earth, using the radius of Earth as 3960 miles. Let P refer to the furthest point on the horizon from which the light is visible. Note also that

$362 \text{ feet} = \dfrac{362}{5280} \text{ miles.}$



Apply the Pythagorean Theorem to ΔCPL :

$$(3960)^2 + (d_1)^2 = \left(3960 + \frac{362}{5280}\right)^2$$

$$(d_1)^2 = \left(3960 + \frac{362}{5280}\right)^2 - (3960)^2$$

$$d_1 = \sqrt{\left(3960 + \frac{362}{5280}\right)^2 - (3960)^2} \approx 23.30 \text{ mi.}$$

Therefore, the light from the lighthouse can be seen at point P on the horizon, where point P is approximately 23.30 miles away from the lighthouse. Brochure information is slightly overstated.

<u>Verify the ship information:</u>
Let S refer to the ship's location, and let x equal the height, in feet, of the ship.
We need $d_1 + d_2 \geq 40$.
Since $d_1 \approx 23.30$ miles we need
$d_2 \geq 40 - 23.30 = 16.70$ miles.

Apply the Pythagorean Theorem to ΔCPS :

$$(3960)^2 + (16.7)^2 = (3960 + x)^2$$

$$\sqrt{(3960)^2 + (16.7)^2} = 3960 + x$$

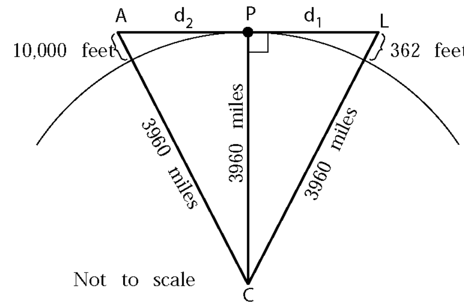
$$\sqrt{(3960)^2 + (16.7)^2} - 3960 = x$$

$$x \approx 0.035 \text{ miles}$$

$$x \approx 185.93 \text{ feet.}$$

The ship would have to be at least 186 feet tall to see the lighthouse from 40 miles away.

<u>Verify the airplane information:</u>



Let A refer to the airplane's location. The distance from the plane to point P is d_2.
We want to show that $d_1 + d_2 \geq 120$.
Assume the altitude of the airplane is
$10{,}000 \text{ feet} = \dfrac{10000}{5280} \text{ miles.}$

Apply the Pythagorean Theorem to ΔCPA :

$$(3960)^2 + (d_2)^2 = \left(3960 + \frac{10000}{5280}\right)^2$$

$$(d_2)^2 = \left(3960 + \frac{10000}{5280}\right)^2 - (3960)^2$$

$$d_2 = \sqrt{\left(3960 + \frac{10000}{5280}\right)^2 - (3960)^2}$$

$$\approx 122.49 \text{ miles.}$$

Therefore,
$d_1 + d_2 \approx 23.30 + 122.49 = 145.79 \geq 120.$
The brochure information is slightly understated. Note that a plane at an altitude of 6233 feet could see the lighthouse from 120 miles away.

Section A.3

1. 4; 3

2. $x^4 - 16$

3. $x^3 - 8$

4. False; monomials cannot have negative degrees.

5. False; $x^3 + a^3 = (x + a)(x^2 - ax + a^2)$

6. quotient, divisor, remainder

7. $3x(x-2)(x+2)$

8. add; $\left[\frac{1}{2}(5)\right]^2 = \frac{25}{4}$

9. True; $x^2 + 4$ is prime over the set of real numbers.

10. False; $3x^3 - 2x^2 - 6x + 4 = (3x-2)(x^2-2)$

11. $2x^3$ Monomial; Variable: x; Coefficient: 2; Degree: 3

12. $-4x^2$ Monomial; Variable: x; Coefficient: –4; Degree: 2

13. $\frac{8}{x} = 8x^{-1}$ Not a monomial; when written in the form ax^k, the variable has a negative exponent.

14. $-2x^{-3}$ Not a monomial; when written in the form ax^k, the variable has a negative exponent.

15. $-2x^3 + 5x^2$ Not a monomial; the expression contains more than one term. This expression is a binomial.

16. $6x^5 - 8x^2$ Not a monomial; the expression contains more than one term. This expression is a binomial.

17. $\frac{8x}{x^2-1}$ Not a monomial; the polynomial in the denominator has a degree greater than 0. The expression cannot be written in the form ax^k where $k \geq 0$ is an integer.

18. $-\frac{2x^2}{x^3+1}$ Not a monomial; the polynomial in the denominator has a degree greater than 0. The expression cannot be written in the form ax^k where $k \geq 0$ is an integer.

19. $x^2 + 2x - 5$ Not a monomial; the expression contains more than one term. This expression is a trinomial.

20. $3x^2 + 4$ Not a monomial; the expression contains more than one term. This expression is a binomial.

21. $3x^2 - 5$ Polynomial; Degree: 2

22. $1 - 4x$ Polynomial; Degree: 1

23. 5 Polynomial; Degree: 0

24. $-\pi$ Polynomial; Degree: 0

25. $3x^2 - \frac{5}{x}$ Not a polynomial; the variable in the denominator results in an exponent that is not a nonnegative integer.

26. $\frac{3}{x} + 2$ Not a polynomial; the variable in the denominator results in an exponent that is not a nonnegative integer.

27. $2y^3 - \sqrt{2}$ Polynomial; Degree: 3

28. $10z^2 + z$ Polynomial; Degree: 2

29. $\frac{x^2+5}{x^3-1}$ Not a polynomial; the polynomial in the denominator has a degree greater than 0.

30. $\frac{3x^3+2x-1}{x^2+x+1}$ Not a polynomial; the polynomial in the denominator has a degree greater than 0.

31. $(x^2+4x+5)+(3x-3)$
$= x^2 + (4x+3x) + (5-3)$
$= x^2 + 7x + 2$

32. $(x^3+3x^2+2)+(x^2-4x+4)$
$= x^3 + (3x^2+x^2) + (-4x) + (2+4)$
$= x^3 + 4x^2 - 4x + 6$

33. $(x^3-2x^2+5x+10)-(2x^2-4x+3)$
$= x^3 - 2x^2 + 5x + 10 - 2x^2 + 4x - 3$
$= x^3 + (-2x^2 - 2x^2) + (5x+4x) + (10-3)$
$= x^3 - 4x^2 + 9x + 7$

34. $(x^2-3x-4)-(x^3-3x^2+x+5)$
$= x^2 - 3x - 4 - x^3 + 3x^2 - x - 5$
$= -x^3 + (x^2+3x^2) + (-3x-x) + (-4-5)$
$= -x^3 + 4x^2 - 4x - 9$

35. $6(x^3+x^2-3)-4(2x^3-3x^2)$
$=6x^3+6x^2-18-8x^3+12x^2$
$=-2x^3+18x^2-18$

36. $8(4x^3-3x^2-1)-6(4x^3+8x-2)$
$=32x^3-24x^2-8-24x^3-48x+12$
$=8x^3-24x^2-48x+4$

37. $9\left(y^2-3y+4\right)-6\left(1-y^2\right)$
$=9y^2-27y+36-6+6y^2$
$=15y^2-27y+30$

38. $8\left(1-y^3\right)+4\left(1+y+y^2+y^3\right)$
$=8-8y^3+4+4y+4y^2+4y^3$
$=-4y^3+4y^2+4y+12$

39. $x(x^2+x-4)=x^3+x^2-4x$

40. $4x^2(x^3-x+2)=4x^5-4x^3+8x^2$

41. $(x+2)(x+4)=x^2+4x+2x+8$
$=x^2+6x+8$

42. $(x+3)(x+5)=x^2+5x+3x+15$
$=x^2+8x+15$

43. $(2x+5)(x+2)=2x^2+4x+5x+10$
$=2x^2+9x+10$

44. $(3x+1)(2x+1)=6x^2+3x+2x+1$
$=6x^2+5x+1$

45. $(x-7)(x+7)=x^2-7^2=x^2-49$

46. $(x-1)(x+1)=x^2-1^2=x^2-1$

47. $(2x+3)(2x-3)=(2x)^2-3^2=4x^2-9$

48. $(3x+2)(3x-2)=(3x)^2-2^2=9x^2-4$

49. $(x+4)^2=x^2+2\cdot x\cdot 4+4^2=x^2+8x+16$

50. $(x-5)^2=x^2-2\cdot x\cdot 5+5^2=x^2-10x+25$

51. $(2x-3)^2=(2x)^2-2(2x)(3)+3^2$
$=4x^2-12x+9$

52. $(3x-4)^2=(3x)^2-2(3x)(4)+4^2$
$=9x^2-24x+16$

53. $(x-2)^3=x^3-3\cdot x^2\cdot 2+3\cdot x\cdot 2^2-2^3$
$=x^3-6x^2+12x-8$

54. $(x+1)^3=x^3+3\cdot x^2\cdot 1+3\cdot x\cdot 1^2+1^3$
$=x^3+3x^2+3x+1$

55. $(2x+1)^3=(2x)^3+3(2x)^2(1)+3(2x)\cdot 1^2+1^3$
$=8x^3+12x^2+6x+1$

56. $(3x-2)^3=(3x)^3-3(3x)^2(2)+3(3x)\cdot 2^2-2^3$
$=27x^3-54x^2+36x-8$

57.
$$\begin{array}{r}
4x^2-11x+23 \\
x+2\overline{)4x^3-3x^2+x+1} \\
\underline{4x^3+8x^2} \\
-11x^2+x \\
\underline{-11x^2-22x} \\
23x+1 \\
\underline{23x+46} \\
-45
\end{array}$$

Check:
$(x+2)(4x^2-11x+23)+(-45)$
$=4x^3-11x^2+23x+8x^2-22x+46-45$
$=4x^3-3x^2+x+1$
The quotient is $4x^2-11x+23$; the remainder is –45.

58.

$$\begin{array}{r|l} & 3x^2 - 7x + 15 \\ \hline x+2 & 3x^3 - x^2 + x - 2 \\ & \underline{3x^3 + 6x^2} \\ & -7x^2 + x \\ & \underline{-7x^2 - 14x} \\ & 15x - 2 \\ & \underline{15x + 30} \\ & -32 \end{array}$$

Check:

$(x+2)(3x^2-7x+15)+(-32)$

$=3x^3-7x^2+15x+6x^2-14x+30-32$

$=3x^3-x^2+x-2$

The quotient is $3x^2-7x+15$; the remainder is –32.

59.

$$\begin{array}{r|l} & 4x - 3 \\ \hline x^2 & 4x^3 - 3x^2 + x + 1 \\ & \underline{4x^3} \\ & -3x^2 + x + 1 \\ & \underline{-3x^2} \\ & x + 1 \end{array}$$

Check:

$(x^2)(4x-3)+(x+1)=4x^3-3x^2+x+1$

The quotient is $4x-3$; the remainder is $x+1$.

60.

$$\begin{array}{r|l} & 3x - 1 \\ \hline x^2 & 3x^3 - x^2 + x - 2 \\ & \underline{3x^3} \\ & -x^2 + x - 2 \\ & \underline{-x^2} \\ & x - 2 \end{array}$$

Check:

$(x^2)(3x-1)+(x-2)=3x^3-x^2+x-2$

The quotient is $3x-1$; the remainder is $x-2$.

61.

$$\begin{array}{r|l} & 5x^2 - 13 \\ \hline x^2+2 & 5x^4 + 0x^3 - 3x^2 + x + 1 \\ & \underline{5x^4 \qquad + 10x^2} \\ & -13x^2 + x + 1 \\ & \underline{-13x^2 \qquad - 26} \\ & x + 27 \end{array}$$

Check:

$\left(x^2+2\right)\left(5x^2-13\right)+\left(x+27\right)$

$=5x^4+10x^2-13x^2-26+x+27$

$=5x^4-3x^2+x+1$

The quotient is $5x^2-13$; the remainder is $x+27$.

62.

$$\begin{array}{r|l} & 5x^2 - 11 \\ \hline x^2+2 & 5x^4 + 0x^3 - x^2 + x - 2 \\ & \underline{5x^4 \qquad + 10x^2} \\ & -11x^2 + x - 2 \\ & \underline{-11x^2 \qquad - 22} \\ & x + 20 \end{array}$$

Check:

$\left(x^2+2\right)\left(5x^2-11\right)+\left(x+20\right)$

$=5x^4+10x^2-11x^2-22+x+20$

$=5x^4-x^2+x-2$

The quotient is $5x^2-11$; the remainder is $x+20$.

63.

$$\begin{array}{r|l} & 2x^2 \\ \hline 2x^3-1 & 4x^5 + 0x^4 + 0x^3 - 3x^2 + x + 1 \\ & \underline{4x^5 \qquad\qquad - 2x^2} \\ & -x^2 + x + 1 \end{array}$$

Check:

$\left(2x^3-1\right)\left(2x^2\right)+\left(-x^2+x+1\right)$

$=4x^5-2x^2-x^2+x+1=4x^5-3x^2+x+1$

The quotient is $2x^2$; the remainder is $-x^2+x+1$.

64.
$$\begin{array}{r} x^2 \\ 3x^3-1\overline{)3x^5+0x^4+0x^3-x^2+x-2} \\ \underline{3x^5 \qquad\qquad\quad -x^2 } \\ x-2 \end{array}$$

Check:

$\left(3x^3-1\right)\left(x^2\right)+\left(x-2\right)=3x^5-x^2+x-2$

The quotient is x^2; the remainder is $x-2$.

65.
$$\begin{array}{r} x^2-2x+\tfrac{1}{2} \\ 2x^2+x+1\overline{)2x^4-3x^3+0x^2+x+1} \\ \underline{2x^4+x^3+x^2} \\ -4x^3-x^2+x \\ \underline{-4x^3-2x^2-2x} \\ x^2+3x+1 \\ \underline{x^2+\frac{1}{2}x+\frac{1}{2}} \\ \frac{5}{2}x+\frac{1}{2} \end{array}$$

Check:

$\left(2x^2+x+1\right)\left(x^2-2x+\frac{1}{2}\right)+\frac{5}{2}x+\frac{1}{2}$

$=2x^4-4x^3+x^2+x^3-2x^2+\frac{1}{2}x$

$\quad +x^2-2x+\frac{1}{2}+\frac{5}{2}x+\frac{1}{2}$

$=2x^4-3x^3+x+1$

The quotient is $x^2-2x+\frac{1}{2}$; the remainder is $\frac{5}{2}x+\frac{1}{2}$.

66.
$$\begin{array}{r} x^2-\frac{2}{3}x-\frac{1}{9} \\ 3x^2+x+1\overline{)3x^4-x^3+0x^2+x-2} \\ \underline{3x^4+x^3+x^2} \\ -2x^3-x^2+x \\ \underline{-2x^3-\frac{2}{3}x^2-\frac{2}{3}x} \\ -\frac{1}{3}x^2+\frac{5}{3}x-2 \\ \underline{-\frac{1}{3}x^2-\frac{1}{9}x-\frac{1}{9}} \\ \frac{16}{9}x-\frac{17}{9} \end{array}$$

Check:

$\left(3x^2+x+1\right)\left(x^2-\frac{2}{3}x-\frac{1}{9}\right)+\left(\frac{16}{9}x-\frac{17}{9}\right)$

$=3x^4+x^3+x^2-2x^3-\frac{2}{3}x^2-\frac{2}{3}x$

$\quad -\frac{1}{3}x^2-\frac{1}{9}x-\frac{1}{9}+\frac{16}{9}x-\frac{17}{9}$

$=3x^4-x^3+x-2$

The quotient is $x^2-\frac{2}{3}x-\frac{1}{9}$; the remainder is $\frac{16}{9}x-\frac{17}{9}$.

67.
$$\begin{array}{r} -4x^2-3x-3 \\ x-1\overline{)-4x^3+x^2+0x-4} \\ \underline{-4x^3+4x^2} \\ -3x^2 \\ \underline{-3x^2+3x} \\ -3x-4 \\ \underline{-3x+3} \\ -7 \end{array}$$

Check:

$(x-1)(-4x^2-3x-3)+(-7)$

$=-4x^3-3x^2-3x+4x^2+3x+3-7$

$=-4x^3+x^2-4$

The quotient is $-4x^2-3x-3$; the remainder is –7.

68.
$$\begin{array}{r} -3x^3-3x^2-3x-5 \\ x-1\overline{)-3x^4+0x^3+0x^2-2x-1} \\ \underline{-3x^4+3x^3} \\ -3x^3 \\ \underline{-3x^3+3x^2} \\ -3x^2-2x \\ \underline{-3x^2+3x} \\ -5x-1 \\ \underline{-5x+5} \\ -6 \end{array}$$

Check:

$(x-1)(-3x^3-3x^2-3x-5)+(-6)$

$=-3x^4-3x^3-3x^2-5x+3x^3+3x^2$

$\quad +3x+5-6$

$=-3x^4-2x-1$

The quotient is $-3x^3-3x^2-3x-5$; the remainder is –6.

69.

$$\begin{array}{r} x^2-x-1 \\ x^2+x+1\overline{)x^4+0x^3-x^2+0x+1} \\ \underline{x^4+x^3+x^2} \\ -x^3-2x^2 \\ \underline{-x^3-x^2-x} \\ -x^2+x+1 \\ \underline{-x^2-x-1} \\ 2x+2 \end{array}$$

Check:

$(x^2+x+1)(x^2-x-1)+2x+2$

$=x^4+x^3+x^2-x^3-x^2-x-x^2-x$

$\quad -1+2x+2$

$=x^4-x^2+1$

The quotient is x^2-x-1; the remainder is $2x+2$.

70.

$$\begin{array}{r} x^2+x-1 \\ x^2-x+1\overline{)x^4+0x^3-x^2+0x+1} \\ \underline{x^4-x^3+x^2} \\ x^3-2x^2 \\ \underline{x^3-x^2+x} \\ -x^2-x+1 \\ \underline{-x^2+x-1} \\ -2x+2 \end{array}$$

Check:

$(x^2-x+1)(x^2+x-1)+(-2x+2)$

$=x^4+x^3-x^2-x^3-x^2+x+x^2+x$

$\quad -1-2x+2$

$=x^4-x^2+1$

The quotient is x^2+x-1; the remainder is $-2x+2$.

71.

$$\begin{array}{r} x^2+ax+a^2 \\ x-a\overline{)x^3+0x^2+0x-a^3} \\ \underline{x^3-ax^2} \\ ax^2 \\ \underline{ax^2-a^2x} \\ a^2x-a^3 \\ \underline{a^2x-a^3} \\ 0 \end{array}$$

Check:

$(x-a)(x^2+ax+a^2)+0$

$=x^3+ax^2+a^2x-ax^2-a^2x-a^3$

$=x^3-a^3$

The quotient is x^2+ax+a^2; the remainder is 0.

72.

$$\begin{array}{r} x^4+ax^3+a^2x^2+a^3x+a^4 \\ x-a\overline{)x^5+0x^4+0x^3+0x^2+0x-a^5} \\ \underline{x^5-ax^4} \\ ax^4 \\ \underline{ax^4-a^2x^3} \\ a^2x^3 \\ \underline{a^2x^3-a^3x^2} \\ a^3x^2 \\ \underline{a^3x^2-a^4x} \\ a^4x-a^5 \\ \underline{a^4x-a^5} \\ 0 \end{array}$$

Check:

$(x-a)(x^4+ax^3+a^2x^2+a^3x+a^4)+0$
$= x^5+ax^4+a^2x^3+a^3x^2+a^4x-ax^4$
$\quad -a^2x^3-a^3x^2-a^4x-a^5$
$= x^5-a^5$

The quotient is $x^4+ax^3+a^2x^2+a^3x+a^4$; the remainder is 0.

73. $x^2-36=(x-6)(x+6)$

74. $x^2-9=(x-3)(x+3)$

75. $2-8x^2=2(1-4x^2)=2(1-2x)(1+2x)$

76. $3-27x^2=3(1-9x^2)=3(1-3x)(1+3x)$

77. $x^2+11x+10=(x+1)(x+10)$

78. $x^2+5x+4=(x+4)(x+1)$

79. $x^2-10x+21=(x-7)(x-3)$

80. $x^2-6x+8=(x-2)(x-4)$

81. $4x^2-8x+32=4(x^2-2x+8)$

82. $3x^2-12x+15=3(x^2-4x+5)$

83. $x^2+4x+16$ is prime over the reals because there are no factors of 16 whose sum is 4.

84. $x^2+12x+36=(x+6)^2$

85. $15+2x-x^2=-(x^2-2x-15)=-(x-5)(x+3)$

86. $14+6x-x^2=-(x^2-6x-14)$ is prime over the integers because there are no factors of –14 whose sum is –6.

87. $3x^2-12x-36=3(x^2-4x-12)$
$= 3(x-6)(x+2)$

88. $x^3+8x^2-20x=x(x^2+8x-20)$
$= x(x+10)(x-2)$

89. $y^4+11y^3+30y^2=y^2(y^2+11y+30)$
$= y^2(y+5)(y+6)$

90. $3y^3-18y^2-48y=3y(y^2-6y-16)$
$= 3y(y+2)(y-8)$

91. $4x^2+12x+9=(2x+3)^2$

92. $9x^2-12x+4=(3x-2)^2$

93. $6x^2+8x+2=2(3x^2+4x+1)$
$= 2(3x+1)(x+1)$

94. $8x^2+6x-2=2(4x^2+3x-1)$
$= 2(4x-1)(x+1)$

95. $x^4-81=(x^2)^2-9^2=(x^2-9)(x^2+9)$
$= (x-3)(x+3)(x^2+9)$

96. $x^4-1=(x^2)^2-1^2=(x^2-1)(x^2+1)$
$= (x-1)(x+1)(x^2+1)$

97. $x^6-2x^3+1=(x^3-1)^2$
$= \left[(x-1)(x^2+x+1)\right]^2$
$= (x-1)^2(x^2+x+1)^2$

98. $x^6+2x^3+1=(x^3+1)^2$
$= \left[(x+1)(x^2-x+1)\right]^2$
$= (x+1)^2(x^2-x+1)^2$

99. $x^7-x^5=x^5(x^2-1)=x^5(x-1)(x+1)$

100. $x^8-x^5=x^5(x^3-1)=x^5(x-1)(x^2+x+1)$

101. $16x^2+24x+9=(4x+3)^2$

102. $9x^2-24x+16=(3x-4)^2$

103. $5+16x-16x^2=-(16x^2-16x-5)$
$= -(4x-5)(4x+1)$

104. $5+11x-16x^2=-(16x^2-11x-5)$
$=-(16x+5)(x-1)$

105. $4y^2-16y+15=(2y-5)(2y-3)$

106. $9y^2+9y-4=(3y+4)(3y-1)$

107. $1-8x^2-9x^4=-(9x^4+8x^2-1)$
$=-(9x^2-1)(x^2+1)$
$=-(3x-1)(3x+1)(x^2+1)$

108. $4-14x^2-8x^4=-2(4x^4+7x^2-2)$
$=-2(4x^2-1)(x^2+2)$
$=-2(2x-1)(2x+1)(x^2+2)$

109. $x(x+3)-6(x+3)=(x+3)(x-6)$

110. $5(3x-7)+x(3x-7)=(3x-7)(x+5)$

111. $(x+2)^2-5(x+2)=(x+2)\left[(x+2)-5\right]$
$=(x+2)(x-3)$

112. $(x-1)^2-2(x-1)=(x-1)\left[(x-1)-2\right]$
$=(x-1)(x-3)$

113. $(3x-2)^3-27$
$=(3x-2)^3-3^3$
$=\left[(3x-2)-3\right]\left[(3x-2)^2+3(3x-2)+9\right]$
$=(3x-5)\left(9x^2-12x+4+9x-6+9\right)$
$=(3x-5)\left(9x^2-3x+7\right)$

114. $(5x+1)^3-1$
$=(5x+1)^3-1^3$
$=\left[(5x+1)-1\right]\left[(5x+1)^2+(1)(5x+1)+1\right]$
$=5x\left(25x^2+10x+1+5x+1+1\right)$
$=5x\left(25x^2+15x+3\right)$

115. $3\left(x^2+10x+25\right)-4(x+5)$
$=3(x+5)^2-4(x+5)$
$=(x+5)\left[3(x+5)-4\right]$
$=(x+5)(3x+15-4)$
$=(x+5)(3x+11)$

116. $7\left(x^2-6x+9\right)+5(x-3)$
$=7(x-3)^2+5(x-3)$
$=(x-3)\left[7(x-3)+5\right]$
$=(x-3)(7x-21+5)$
$=(x-3)(7x-16)$

117. $x^3+2x^2-x-2=x^2(x+2)-1(x+2)$
$=(x+2)(x^2-1)$
$=(x+2)(x-1)(x+1)$

118. $x^3-3x^2-x+3=x^2(x-3)-1(x-3)$
$=(x-3)(x^2-1)$
$=(x-3)(x-1)(x+1)$

119. $x^4-x^3+x-1=x^3(x-1)+1(x-1)$
$=(x-1)(x^3+1)$
$=(x-1)(x+1)(x^2-x+1)$

120. $x^4+x^3+x+1=x^3(x+1)+1(x+1)$
$=(x+1)(x^3+1)$
$=(x+1)(x+1)(x^2-x+1)$
$=(x+1)^2(x^2-x+1)$

121. Since B is 10 then we need half of 10 squared to be the last term in our trinomial. Thus
$\frac{1}{2}(10)=5;\ (5)^2=25$
$x^2+10x+25=(x+5)^2$

122. Since B is 14 then we need half of 14 squared to be the last term in our trinomial. Thus
$\frac{1}{2}(14)=7;\ (7)^2=49$
$p^2+14p+49=(p+7)^2$

123. Since B is -6 then we need half of -6 squared to be the last term in our trinomial. Thus

$\frac{1}{2}(-6) = -3;\ (-3)^2 = 9$

$y^2 - 6y + 9 = (y-3)^2$

124. Since B is -4 then we need half of -4 squared to be the last term in our trinomial. Thus

$\frac{1}{2}(-4) = -2;\ (-2)^2 = 4$

$x^2 - 4x + 4 = (x-2)^2$

125. Since B is $-\frac{1}{2}$ then we need half of $-\frac{1}{2}$ squared to be the last term in our trinomial. Thus

$\frac{1}{2}(-\frac{1}{2}) = -\frac{1}{4};\ (-\frac{1}{4})^2 = \frac{1}{16}$

$x^2 - \frac{1}{2}x + \frac{1}{16} = (x - \frac{1}{4})^2$

126. Since B is $\frac{1}{3}$ then we need half of $\frac{1}{3}$ squared to be the last term in our trinomial. Thus

$\frac{1}{2}(\frac{1}{3}) = \frac{1}{6};\ (\frac{1}{6})^2 = \frac{1}{36}$

$x^2 + \frac{1}{3}x + \frac{1}{36} = (x + \frac{1}{6})^2$

127. $2(3x+4)^2 + (2x+3)\cdot 2(3x+4)\cdot 3$

$= 2(3x+4)\big((3x+4)+(2x+3)\cdot 3\big)$

$= 2(3x+4)(3x+4+6x+9)$

$= 2(3x+4)(9x+13)$

128. $5(2x+1)^2 + (5x-6)\cdot 2(2x+1)\cdot 2$

$= (2x+1)\big(5(2x+1)+(5x-6)\cdot 4\big)$

$= (2x+1)(10x+5+20x-24)$

$= (2x+1)(30x-19)$

129. $2x(2x+5) + x^2\cdot 2 = 2x\big((2x+5)+x\big)$

$= 2x(2x+5+x)$

$= 2x(3x+5)$

130. $3x^2(8x-3) + x^3\cdot 8 = x^2\big(3(8x-3)+8x\big)$

$= x^2(24x-9+8x)$

$= x^2(32x-9)$

131. $2(x+3)(x-2)^3 + (x+3)^2\cdot 3(x-2)^2$

$= (x+3)(x-2)^2\big(2(x-2)+(x+3)\cdot 3\big)$

$= (x+3)(x-2)^2(2x-4+3x+9)$

$= (x+3)(x-2)^2(5x+5)$

$= 5(x+3)(x-2)^2(x+1)$

132. $4(x+5)^3(x-1)^2 + (x+5)^4\cdot 2(x-1)$

$= 2(x+5)^3(x-1)\big(2(x-1)+(x+5)\big)$

$= 2(x+5)^3(x-1)(2x-2+x+5)$

$= 2(x+5)^3(x-1)(3x+3)$

$= 2\cdot 3(x+5)^3(x-1)(x+1)$

$= 6(x+5)^3(x-1)(x+1)$

133. $(4x-3)^2 + x\cdot 2(4x-3)\cdot 4$

$= (4x-3)\big((4x-3)+8x\big)$

$= (4x-3)(4x-3+8x)$

$= (4x-3)(12x-3)$

$= 3(4x-3)(4x-1)$

134. $3x^2(3x+4)^2 + x^3\cdot 2(3x+4)\cdot 3$

$= 3x^2(3x+4)\big((3x+4)+2x\big)$

$= 3x^2(3x+4)(3x+4+2x)$

$= 3x^2(3x+4)(5x+4)$

135. $2(3x-5)\cdot 3(2x+1)^3 + (3x-5)^2\cdot 3(2x+1)^2\cdot 2$

$= 6(3x-5)(2x+1)^2\big((2x+1)+(3x-5)\big)$

$= 6(3x-5)(2x+1)^2(2x+1+3x-5)$

$= 6(3x-5)(2x+1)^2(5x-4)$

136. $3(4x+5)^2\cdot 4(5x+1)^2 + (4x+5)^3\cdot 2(5x+1)\cdot 5$

$= 2(4x+5)^2(5x+1)\big(6(5x+1)+5(4x+5)\big)$

$= 2(4x+5)^2(5x+1)(30x+6+20x+25)$

$= 2(4x+5)^2(5x+1)(50x+31)$

137.

Factors of 4:	1, 4	2, 2	–1, –4	–2, –2
Sum:	5	4	–5	–4

None of the sums of the factors is 0, so $x^2 + 4$ is prime.

Alternatively, the possibilities are $(x\pm1)(x\pm4)=x^2\pm5x+4$ or $(x\pm2)(x\pm2)=x^2\pm4x+4$, none of which equals x^2+4.

138. Factors of 1: 1, 1 −1, −1
Sum: 2 −2
None of the sums of the factors is 1, so x^2+x+1 is prime.
Alternatively, the possibilities are $(x\pm1)^2=x^2\pm2x+1$, neither of which equals x^2+x+1.

139. When we multiply polynomials $p_1(x)$ and $p_2(x)$, each term of $p_1(x)$ will be multiplied by each term of $p_2(x)$. So when the highest-powered term of $p_1(x)$ multiplies by the highest powered term of $p_2(x)$, the exponents on the variables in those terms will add according to the basic rules of exponents. Therefore, the highest powered term of the product polynomial will have degree equal to the sum of the degrees of $p_1(x)$ and $p_2(x)$.

140. When we add two polynomials $p_1(x)$ and $p_2(x)$, where the degree of $p_1(x)\neq$ the degree of $p_2(x)$, each term of $p_1(x)$ will be added to each term of $p_2(x)$. Since only the terms with equal degrees will combine via addition, the degree of the sum polynomial will be the degree of the highest powered term overall, that is, the degree of the polynomial that had the higher degree.

141. When we add two polynomials $p_1(x)$ and $p_2(x)$, where the degree of $p_1(x)=$ the degree of $p_2(x)$, the new polynomial will have degree $\le$ the degree of $p_1(x)$ and $p_2(x)$.

142. Answers will vary.

143. Answers will vary.

144. Answers will vary.

Section A.4

1. quotient; divisor; remainder

2. $-3\overline{)\,2 \quad 0 \quad -5 \quad 1}$

3. True

4. True

5.
$$\begin{array}{r|rrrr} 2 & 1 & -1 & 2 & 4 \\ & & 2 & 2 & 8 \\ \hline & 1 & 1 & 4 & 12 \end{array}$$
Quotient: x^2+x+4
Remainder: 12

6.
$$\begin{array}{r|rrrr} -1 & 1 & 2 & -3 & 1 \\ & & -1 & -1 & 4 \\ \hline & 1 & 1 & -4 & 5 \end{array}$$
Quotient: x^2+x-4
Remainder: 5

7.
$$\begin{array}{r|rrrr} 3 & 3 & 2 & -1 & 3 \\ & & 9 & 33 & 96 \\ \hline & 3 & 11 & 32 & 99 \end{array}$$
Quotient: $3x^2+11x+32$
Remainder: 99

8.
$$\begin{array}{r|rrrr} -2 & -4 & 2 & -1 & 1 \\ & & 8 & -20 & 42 \\ \hline & -4 & 10 & -21 & 43 \end{array}$$
Quotient: $-4x^2+10x-21$
Remainder: 43

9.
$$\begin{array}{r|rrrrrr} -3 & 1 & 0 & -4 & 0 & 1 & 0 \\ & & -3 & 9 & -15 & 45 & -138 \\ \hline & 1 & -3 & 5 & -15 & 46 & -138 \end{array}$$
Quotient: $x^4-3x^3+5x^2-15x+46$
Remainder: −138

10.
$$\begin{array}{r|rrrrr} 2 & 1 & 0 & 1 & 0 & 2 \\ & & 2 & 4 & 10 & 20 \\ \hline & 1 & 2 & 5 & 10 & 22 \end{array}$$
Quotient: $x^3+2x^2+5x+10$
Remainder: 22

11. $$\begin{array}{r|rrrrrrr} 1 & 4 & 0 & -3 & 0 & 1 & 0 & 5 \\ & & 4 & 4 & 1 & 1 & 2 & 2 \\ \hline & 4 & 4 & 1 & 1 & 2 & 2 & 7 \end{array}$$

Quotient: $4x^5+4x^4+x^3+x^2+2x+2$

Remainder: 7

12. $$\begin{array}{r|rrrrrr} -1 & 1 & 0 & 5 & 0 & 0 & -10 \\ & & -1 & 1 & -6 & 6 & -6 \\ \hline & 1 & -1 & 6 & -6 & 6 & -16 \end{array}$$

Quotient: $x^4-x^3+6x^2-6x+6$

Remainder: –16

13. $$\begin{array}{r|rrrr} -1.1 & 0.1 & 0 & 0.2 & 0 \\ & & -0.11 & 0.121 & -0.3531 \\ \hline & 0.1 & -0.11 & 0.321 & -0.3531 \end{array}$$

Quotient: $0.1x^2-0.11x+0.321$

Remainder: –0.3531

14. $$\begin{array}{r|rrr} -2.1 & 0.1 & 0 & -0.2 \\ & & -0.21 & 0.441 \\ \hline & 0.1 & -0.21 & 0.241 \end{array}$$

Quotient: $0.1x-0.21$

Remainder: 0.241

15. $$\begin{array}{r|rrrrrr} 1 & 1 & 0 & 0 & 0 & 0 & -1 \\ & & 1 & 1 & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & 1 & 1 & 0 \end{array}$$

Quotient: $x^4+x^3+x^2+x+1$

Remainder: 0

16. $$\begin{array}{r|rrrrrr} -1 & 1 & 0 & 0 & 0 & 0 & 1 \\ & & -1 & 1 & -1 & 1 & -1 \\ \hline & 1 & -1 & 1 & -1 & 1 & 0 \end{array}$$

Quotient: $x^4-x^3+x^2-x+1$

Remainder: 0

17. $$\begin{array}{r|rrrr} 2 & 4 & -3 & -8 & 4 \\ & & 8 & 10 & 4 \\ \hline & 4 & 5 & 2 & 8 \end{array}$$

Remainder = $8 \neq 0$. Therefore, $x-2$ is not a factor of $4x^3-3x^2-8x+4$.

18. $$\begin{array}{r|rrrr} -3 & -4 & 5 & 0 & 8 \\ & & 12 & -51 & 153 \\ \hline & -4 & 17 & -51 & 161 \end{array}$$

Remainder = $161 \neq 0$. Therefore, $x+3$ is not a factor of $-4x^3+5x^2+8$.

19. $$\begin{array}{r|rrrrr} 2 & 3 & -6 & 0 & -5 & 10 \\ & & 6 & 0 & 0 & -10 \\ \hline & 3 & 0 & 0 & -5 & 0 \end{array}$$

Remainder = 0. Therefore, $x-2$ is a factor of $3x^4-6x^3-5x+10$.

20. $$\begin{array}{r|rrrrr} 2 & 4 & 0 & -15 & 0 & -4 \\ & & 8 & 16 & 2 & 4 \\ \hline & 4 & 8 & 1 & 2 & 0 \end{array}$$

Remainder = 0. Therefore, $x-2$ is a factor of $4x^4-15x^2-4$.

21. $$\begin{array}{r|rrrrrrr} -3 & 3 & 0 & 0 & 82 & 0 & 0 & 27 \\ & & -9 & 27 & -81 & -3 & 9 & -27 \\ \hline & 3 & -9 & 27 & 1 & -3 & 9 & 0 \end{array}$$

Remainder = 0. Therefore, $x+3$ is a factor of $3x^6+82x^3+27$.

22. $$\begin{array}{r|rrrrrrr} -3 & 2 & 0 & -18 & 0 & 1 & 0 & -9 \\ & & -6 & 18 & 0 & 0 & -3 & 9 \\ \hline & 2 & -6 & 0 & 0 & 1 & -3 & 0 \end{array}$$

Remainder = 0. Therefore, $x+3$ is a factor of $2x^6-18x^4+x^2-9$.

23. $$\begin{array}{r|rrrrrrr} -4 & 4 & 0 & -64 & 0 & 1 & 0 & -15 \\ & & -16 & 64 & 0 & 0 & -4 & 16 \\ \hline & 4 & -16 & 0 & 0 & 1 & -4 & 1 \end{array}$$

Remainder = $1 \neq 0$. Therefore, $x+4$ is not a factor of $4x^6-64x^4+x^2-15$.

24. $-4\overline{)\begin{array}{ccccccc}1 & 0 & -16 & 0 & 1 & 0 & -16\\ & -4 & 16 & 0 & 0 & -4 & 16\\ \hline 1 & -4 & 0 & 0 & 1 & -4 & 0\end{array}}$

Remainder = 0. Therefore, $x+4$ is a factor $x^6-16x^4+x^2-16$.

25. $\frac{1}{2}\overline{)\begin{array}{ccccc}2 & -1 & 0 & 2 & -1\\ & 1 & 0 & 0 & 1\\ \hline 2 & 0 & 0 & 2 & 0\end{array}}$

Remainder = 0; therefore $x-\frac{1}{2}$ is a factor of $2x^4-x^3+2x-1$.

26. $-\frac{1}{3}\overline{)\begin{array}{ccccc}3 & 1 & 0 & -3 & 1\\ & -1 & 0 & 0 & 1\\ \hline 3 & 0 & 0 & -3 & 2\end{array}}$

Remainder = 2 $\neq 0$; therefore $x+\frac{1}{3}$ is not a factor of $3x^4+x^3-3x+1$.

27. $-2\overline{)\begin{array}{cccc}1 & -2 & 3 & 5\\ & -2 & 8 & -22\\ \hline 1 & -4 & 11 & -17\end{array}}$

$\frac{x^3-2x^2+3x+5}{x+2}=x^2-4x+11+\frac{-17}{x+2}$

$a+b+c+d=1-4+11-17=-9$

28. Answers will vary.

Section A.5

1. lowest terms

2. Least Common Multiple

3. True; $\frac{2x^3-4x}{x-2}=\frac{2x\left(x^2-2\right)}{x-2}$

4. False;

$2x^3+6x^2=2x^2\left(x+3\right)$

$6x^4+4x^3=2x^3\left(3x+2\right)$

$LCM=2x^3\left(x+3\right)\left(3x+2\right)$

5. $\frac{3x+9}{x^2-9}=\frac{3(x+3)}{(x-3)(x+3)}=\frac{3}{x-3}$

6. $\frac{4x^2+8x}{12x+24}=\frac{4x(x+2)}{12(x+2)}=\frac{x}{3}$

7. $\frac{x^2-2x}{3x-6}=\frac{x(x-2)}{3(x-2)}=\frac{x}{3}$

8. $\frac{15x^2+24x}{3x^2}=\frac{3x(5x+8)}{3x^2}=\frac{5x+8}{x}$

9. $\frac{24x^2}{12x^2-6x}=\frac{24x^2}{6x(2x-1)}=\frac{4x}{2x-1}$

10. $\frac{x^2+4x+4}{x^2-4}=\frac{(x+2)(x+2)}{(x-2)(x+2)}=\frac{x+2}{x-2}$

11. $\frac{y^2-25}{2y^2-8y-10}=\frac{(y+5)(y-5)}{2\left(y^2-4y-5\right)}$

$=\frac{(y+5)(y-5)}{2(y-5)(y+1)}$

$=\frac{y+5}{2(y+1)}$

12. $\frac{3y^2-y-2}{3y^2+5y+2}=\frac{(3y+2)(y-1)}{(3y+2)(y+1)}=\frac{y-1}{y+1}$

13. $\frac{3x+6}{5x^2}\cdot\frac{x}{x^2-4}=\frac{3(x+2)}{5x^2}\cdot\frac{x}{(x-2)(x+2)}$

$=\frac{3}{5x(x-2)}$

14. $\frac{3}{2x}\cdot\frac{x^2}{6x+10}=\frac{3}{2}\cdot\frac{x}{2(3x+5)}=\frac{3x}{4(3x+5)}$

15. $\frac{4x^2}{x^2-16}\cdot\frac{x^3-64}{2x}$

$=\frac{4x^2}{(x-4)(x+4)}\cdot\frac{(x-4)\left(x^2+4x+16\right)}{2x}$

$=\frac{2x\cdot 2x(x-4)\left(x^2+4x+16\right)}{2x(x-4)(x+4)}$

$=\frac{2x\left(x^2+4x+16\right)}{x+4}$

16. $\dfrac{12}{x^2+x}\cdot\dfrac{x^3+1}{4x-2}=\dfrac{12}{x(x+1)}\cdot\dfrac{(x+1)(x^2-x+1)}{2(2x-1)}$

$=\dfrac{2\cdot 6(x+1)(x^2-x+1)}{2x(x+1)(2x-1)}$

$=\dfrac{6(x^2-x+1)}{x(2x-1)}$

17. $\dfrac{\frac{8x}{x^2-1}}{\frac{10x}{x+1}}=\dfrac{8x}{x^2-1}\cdot\dfrac{x+1}{10x}$

$=\dfrac{8x}{(x-1)(x+1)}\cdot\dfrac{x+1}{10x}$

$=\dfrac{4}{5(x-1)}$

18. $\dfrac{\frac{x-2}{4x}}{\frac{x^2-4x+4}{12x}}=\dfrac{x-2}{4x}\cdot\dfrac{12x}{x^2-4x+4}$

$=\dfrac{x-2}{4x}\cdot\dfrac{12x}{(x-2)(x-2)}$

$=\dfrac{3}{x-2}$

19. $\dfrac{\frac{4-x}{4+x}}{\frac{4x}{x^2-16}}=\dfrac{4-x}{4+x}\cdot\dfrac{x^2-16}{4x}$

$=\dfrac{4-x}{4+x}\cdot\dfrac{(x+4)(x-4)}{4x}$

$=\dfrac{(4-x)(x-4)}{4x}$

$=-\dfrac{(x-4)^2}{4x}$

20. $\dfrac{\frac{3+x}{3-x}}{\frac{x^2-9}{9x^3}}=\dfrac{3+x}{3-x}\cdot\dfrac{9x^3}{x^2-9}$

$=\dfrac{3+x}{3-x}\cdot\dfrac{9x^3}{(x+3)(x-3)}$

$=\dfrac{9x^3}{(3-x)(x-3)}$

$=\dfrac{9x^3}{-(x-3)^2}$

$=-\dfrac{9x^3}{(x-3)^2}$

21. $\dfrac{x^2}{2x-3}-\dfrac{4}{2x-3}=\dfrac{x^2-4}{2x-3}=\dfrac{(x+2)(x-2)}{2x-3}$

22. $\dfrac{3x^2}{2x-1}-\dfrac{9}{2x-1}=\dfrac{3x^2-9}{2x-1}=\dfrac{3(x^2-3)}{2x-1}$

23. $\dfrac{x}{x^2-4}+\dfrac{1}{x}=\dfrac{x^2+x^2-4}{x(x^2-4)}$

$=\dfrac{2x^2-4}{x(x^2-4)}$

$=\dfrac{2(x^2-2)}{x(x-2)(x+2)}$

24. $\dfrac{x-1}{x^3}+\dfrac{x}{x^2+1}=\dfrac{(x-1)(x^2+1)+x^4}{x^3(x^2+1)}$

$=\dfrac{x^3-x^2+x-1+x^4}{x^3(x^2+1)}$

$=\dfrac{x^4+x^3-x^2+x-1}{x^3(x^2+1)}$

25. $\dfrac{x}{x^2-7x+6}-\dfrac{x}{x^2-2x-24}$

$=\dfrac{x}{(x-6)(x-1)}-\dfrac{x}{(x-6)(x+4)}$

$=\dfrac{x(x+4)}{(x-6)(x-1)(x+4)}-\dfrac{x(x-1)}{(x-6)(x+4)(x-1)}$

$=\dfrac{x^2+4x-x^2+x}{(x-6)(x+4)(x-1)}=\dfrac{5x}{(x-6)(x+4)(x-1)}$

26. $\frac{x}{x-3}-\frac{x+1}{x^2+5x-24}$

$=\frac{x}{(x-3)}-\frac{x+1}{(x-3)(x+8)}$

$=\frac{x(x+8)}{(x-3)(x+8)}-\frac{x+1}{(x-3)(x+8)}$

$=\frac{x^2+8x-x-1}{(x-3)(x+8)}=\frac{x^2+7x-1}{(x-3)(x+8)}$

27. $\frac{4x}{x^2-4}-\frac{2}{x^2+x-6}$

$=\frac{4x}{(x-2)(x+2)}-\frac{2}{(x+3)(x-2)}$

$=\frac{4x(x+3)}{(x-2)(x+2)(x+3)}-\frac{2(x+2)}{(x+3)(x-2)(x+2)}$

$=\frac{4x^2+12x-2x-4}{(x-2)(x+2)(x+3)}$

$=\frac{4x^2+10x-4}{(x-2)(x+2)(x+3)}$

$=\frac{2(2x^2+5x-2)}{(x-2)(x+2)(x+3)}$

28. $\frac{3x}{x-1}-\frac{x-4}{x^2-2x+1}=\frac{3x}{(x-1)}-\frac{x-4}{(x-1)^2}$

$=\frac{3x(x-1)}{(x-1)(x-1)}-\frac{x-4}{(x-1)^2}$

$=\frac{3x^2-3x-x+4}{(x-1)^2}$

$=\frac{3x^2-4x+4}{(x-1)^2}$

29. $\frac{3}{(x-1)^2(x+1)}+\frac{2}{(x-1)(x+1)^2}$

$=\frac{3(x+1)+2(x-1)}{(x-1)^2(x+1)^2}$

$=\frac{3x+3+2x-2}{(x-1)^2(x+1)^2}$

$=\frac{5x+1}{(x-1)^2(x+1)^2}$

30. $\frac{2}{(x+2)^2(x-1)}-\frac{6}{(x+2)(x-1)^2}$

$=\frac{2(x-1)-6(x+2)}{(x+2)^2(x-1)^2}$

$=\frac{2x-2-6x-12}{(x+2)^2(x-1)^2}$

$=\frac{-4x-14}{(x+2)^2(x-1)^2}$

$=\frac{-2(2x+7)}{(x+2)^2(x-1)^2}$

31. $\frac{1+\frac{1}{x}}{1-\frac{1}{x}}=\frac{\left(\frac{x}{x}+\frac{1}{x}\right)}{\left(\frac{x}{x}-\frac{1}{x}\right)}=\frac{\left(\frac{x+1}{x}\right)}{\left(\frac{x-1}{x}\right)}=\frac{x+1}{x}\cdot\frac{x}{x-1}=\frac{x+1}{x-1}$

32. $\frac{4+\frac{1}{x^2}}{3-\frac{1}{x^2}}=\frac{\left(\frac{4x^2}{x^2}+\frac{1}{x^2}\right)}{\left(\frac{3x^2}{x^2}-\frac{1}{x^2}\right)}=\frac{\left(\frac{4x^2+1}{x^2}\right)}{\left(\frac{3x^2-1}{x^2}\right)}$

$=\frac{4x^2+1}{x^2}\cdot\frac{x^2}{3x^2-1}$

$=\frac{4x^2+1}{3x^2-1}$

33. $$\frac{\frac{x-2}{x+2}+\frac{x-1}{x+1}}{\frac{x}{x+1}-\frac{2x-3}{x}}=\frac{\left(\frac{(x-2)(x+1)}{(x+2)(x+1)}+\frac{(x-1)(x+2)}{(x+1)(x+2)}\right)}{\left(\frac{x^2}{(x+1)(x)}-\frac{(2x-3)(x+1)}{x(x+1)}\right)}$$
$$=\frac{\left(\frac{x^2-x-2+x^2+x-2}{(x+2)(x+1)}\right)}{\left(\frac{x^2-(2x^2-x-3)}{x(x+1)}\right)}$$
$$=\frac{\left(\frac{2x^2-4}{(x+2)(x+1)}\right)}{\left(\frac{-x^2+x+3}{x(x+1)}\right)}$$
$$=\frac{2(x^2-2)}{(x+2)(x+1)}\cdot\frac{x(x+1)}{-(x^2-x-3)}$$
$$=\frac{2x(x^2-2)}{-(x+2)(x^2-x-3)}=\frac{-2x(x^2-2)}{(x+2)(x^2-x-3)}$$

34. $$\frac{\frac{2x+5}{x}-\frac{x}{x-3}}{\frac{x^2}{x-3}-\frac{(x+1)^2}{x+3}}=\frac{\left(\frac{(2x+5)(x-3)}{x(x-3)}-\frac{x(x)}{x(x-3)}\right)}{\left(\frac{x^2(x+3)}{(x-3)(x+3)}-\frac{(x-3)(x+1)^2}{(x-3)(x+3)}\right)}$$
$$=\frac{\left(\frac{2x^2-x-15-x^2}{x(x-3)}\right)}{\left(\frac{x^3+3x^2-(x^3-x^2-5x-3)}{(x-3)(x+3)}\right)}$$
$$=\frac{\left(\frac{x^2-x-15}{x(x-3)}\right)}{\left(\frac{4x^2+5x+3}{(x-3)(x+3)}\right)}$$
$$=\frac{x^2-x-15}{x(x-3)}\cdot\frac{(x-3)(x+3)}{4x^2+5x+3}$$
$$=\frac{(x^2-x-15)(x+3)}{x(4x^2+5x+3)}$$

35. $$\frac{(2x+3)\cdot 3-(3x-5)\cdot 2}{(3x-5)^2}=\frac{6x+9-6x+10}{(3x-5)^2}=\frac{19}{(3x-5)^2}$$

36. $$\frac{(4x+1)\cdot 5-(5x-2)\cdot 4}{(5x-2)^2}=\frac{20x+5-20x+8}{(5x-2)^2}=\frac{13}{(5x-2)^2}$$

37. $$\frac{x\cdot 2x-\left(x^2+1\right)\cdot 1}{\left(x^2+1\right)^2}=\frac{2x^2-x^2-1}{\left(x^2+1\right)^2}=\frac{x^2-1}{\left(x^2+1\right)^2}=\frac{(x-1)(x+1)}{\left(x^2+1\right)^2}$$

38. $$\frac{x\cdot 2x-\left(x^2-4\right)\cdot 1}{\left(x^2-4\right)^2}=\frac{2x^2-x^2+4}{\left(x^2-4\right)^2}=\frac{x^2+4}{\left(x^2-4\right)^2}=\frac{x^2+4}{(x+2)^2(x-2)^2}$$

39. $$\frac{(3x+1)\cdot 2x-x^2\cdot 3}{(3x+1)^2}=\frac{6x^2+2x-3x^2}{(3x+1)^2}=\frac{3x^2+2x}{(3x+1)^2}=\frac{x(3x+2)}{(3x+1)^2}$$

40. $$\frac{(2x-5)\cdot 3x^2-x^3\cdot 2}{(2x-5)^2}=\frac{6x^3-15x^2-2x^3}{(2x-5)^2}=\frac{4x^3-15x^2}{(2x-5)^2}=\frac{x^2(4x-15)}{(2x-5)^2}$$

41. $$\frac{(x^2+1)\cdot 3-(3x+4)\cdot 2x}{(x^2+1)^2}=\frac{3x^2+3-6x^2-8x}{(x^2+1)^2}$$
$$=\frac{-3x^2-8x+3}{(x^2+1)^2}$$
$$=\frac{-(3x^2+8x-3)}{(x^2+1)^2}$$
$$=-\frac{(3x-1)(x+3)}{(x^2+1)^2}$$

42. $$\frac{(x^2+9)\cdot 2-(2x-5)\cdot 2x}{(x^2+9)^2}=\frac{2x^2+18-4x^2+10x}{(x^2+9)^2}$$
$$=\frac{-2x^2+10x+18}{(x^2+9)^2}$$
$$=\frac{-2(x^2-5x-9)}{(x^2+9)^2}$$

43. $$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$
$$\frac{1}{f}=(n-1)\left(\frac{R_2+R_1}{R_1\cdot R_2}\right)$$
$$\frac{R_1\cdot R_2}{f}=(n-1)(R_2+R_1)$$
$$\frac{f}{R_1\cdot R_2}=\frac{1}{(n-1)(R_2+R_1)}$$
$$f=\frac{R_1\cdot R_2}{(n-1)(R_2+R_1)}$$
$$f=\frac{0.1(0.2)}{(1.5-1)(0.2+0.1)}$$
$$=\frac{0.02}{0.5(0.3)}=\frac{0.02}{0.15}=\frac{2}{15}\text{ meters}$$

44. $$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}=\frac{R_2R_3+R_1R_3+R_1R_2}{R_1R_2R_3}$$
$$R=\frac{R_1R_2R_3}{R_2R_3+R_1R_3+R_1R_2}$$
$$=\frac{5\cdot 4\cdot 10}{4\cdot 10+5\cdot 10+5\cdot 4}$$
$$=\frac{200}{110}=\frac{20}{11}\text{ ohms}$$

45. $$1+\frac{1}{x}=\frac{x+1}{x}\Rightarrow a=1,b=1,c=0$$
$$1+\frac{1}{1+\frac{1}{x}}=1+\frac{1}{\left(\frac{x+1}{x}\right)}=1+\frac{x}{x+1}$$
$$=\frac{x+1+x}{x+1}=\frac{2x+1}{x+1}$$
$$\Rightarrow a=2,b=1,c=1$$

$$1+\frac{1}{1+\frac{1}{1+\frac{1}{x}}}=1+\frac{1}{\left(\frac{2x+1}{x+1}\right)}=1+\frac{x+1}{2x+1}$$
$$=\frac{2x+1+x+1}{2x+1}=\frac{3x+2}{2x+1}$$
$$\Rightarrow a=3,b=2,c=1$$

$$1+\frac{1}{1+\frac{1}{1+\frac{1}{1+\frac{1}{x}}}}=1+\frac{1}{\left(\frac{3x+2}{2x+1}\right)}=1+\frac{2x+1}{3x+2}$$
$$=\frac{3x+2+2x+1}{3x+2}=\frac{5x+3}{3x+2}$$
$$\Rightarrow a=5,b=3,c=2$$

If we continue this process, the values of a, b and c produce the following sequences:

$a: 1,2,3,5,8,13,21,....$

$b: 1,1,2,3,5,8,13,21,.....$

$c: 0,1,1,2,3,5,8,13,21,.....$

In each case we have a *Fibonacci Sequence*, where the next value in the list is obtained from the sum of the previous 2 values in the list.

46. Answers will vary.

47. Answers will vary.

Section A.6

1. $x^2 - 4 = (x-2)(x+2)$

$x^2 - 3x + 2 = (x-1)(x-2)$

The least common denominator is $(x-1)(x-2)(x+2)$.

2. We are looking for two factors of $a \cdot c = (2)(-3) = -6$ whose sum is -1. Since the product is negative, the two factors have opposite signs. Since the sum is negative, the factor with the larger absolute value will be negative.

$(-3)(2) = -6$ and $-3 + 2 = -1$

$$\begin{aligned} 2x^2 - x - 3 &= 2x^2 + 2x - 3x - 3 \\ &= 2x(x+1) - 3(x+1) \\ &= (x+1)(2x-3) \end{aligned}$$

3. $(x-3)(3x+5) = 0$

$x - 3 = 0$ or $3x + 5 = 0$

$x = 3 \qquad 3x = -5$

$x = -\frac{5}{3}$

Solution set: $\left\{-\frac{5}{3}, 3\right\}$

4. True

5. equivalent equations

6. identity

7. False; the solution is $\frac{8}{3}$.

$$\begin{aligned} 3x - 8 &= 0 \\ 3x &= 8 \\ x &= \frac{8}{3} \end{aligned}$$

8. True; e.g. $2x + 1 = 2x + 3$.

9. add; $\frac{25}{4}$

10. discriminant; negative

11. False; a quadratic equation may also have 1 repeated real solution or no real solutions.

12. False; the equation will have two real solutions but not necessarily negatives of one another.

13. $3x = 21$

$$\frac{3x}{3} = \frac{21}{3}$$

$x = 7$

The solution set is $\{7\}$

14. $3x = -24$

$$\frac{3x}{3} = \frac{-24}{3}$$

$x = -8$

The solution set is $\{-8\}$

15.

$$\begin{aligned} 5x + 15 &= 0 \\ 5x + 15 - 15 &= 0 - 15 \\ 5x &= -15 \\ \frac{5x}{5} &= \frac{-15}{5} \\ x &= -3 \end{aligned}$$

The solution set is $\{-3\}$

16.

$$\begin{aligned} 3x + 18 &= 0 \\ 3x + 18 - 18 &= 0 - 18 \\ 3x &= -18 \\ \frac{3x}{3} &= \frac{-18}{3} \\ x &= -6 \end{aligned}$$

The solution set is $\{-6\}$

17.

$$\begin{aligned} 2x - 3 &= 5 \\ 2x - 3 + 3 &= 5 + 3 \\ 2x &= 8 \\ \frac{2x}{2} &= \frac{8}{2} \\ x &= 4 \end{aligned}$$

The solution set is $\{4\}$

18.
$$\begin{aligned} 3x+4&=-8\\ 3x+4-4&=-8-4\\ 3x&=-12\\ \frac{3x}{3}&=\frac{-12}{3}\\ x&=-4 \end{aligned}$$
The solution set is $\{-4\}$

19.
$$\begin{aligned} \frac{1}{3}x&=\frac{5}{12}\\ 3\cdot\frac{1}{3}x&=3\cdot\frac{5}{12}\\ x&=\frac{5}{4} \end{aligned}$$
The solution set is $\left\{\frac{5}{4}\right\}$

20.
$$\begin{aligned} \frac{2}{3}x&=\frac{9}{2}\\ \frac{3}{2}\cdot\frac{2}{3}x&=\frac{3}{2}\cdot\frac{9}{2}\\ x&=\frac{27}{4} \end{aligned}$$
The solution set is $\left\{\frac{27}{4}\right\}$

21.
$$\begin{aligned} 6-x&=2x+9\\ 6-x-6&=2x+9-6\\ -x&=2x+3\\ -x-2x&=2x+3-2x\\ -3x&=3\\ \frac{-3x}{-3}&=\frac{3}{-3}\\ x&=-1 \end{aligned}$$
The solution set is $\{-1\}$.

22.
$$\begin{aligned} 3-2x&=2-x\\ 3-2x-3&=2-x-3\\ -2x&=-x-1\\ -2x+x&=-x-1+x\\ -x&=-1\\ -1(-x)&=-1(-1)\\ x&=1 \end{aligned}$$
The solution set is $\{1\}$.

23.
$$\begin{aligned} 2(3+2x)&=3(x-4)\\ 6+4x&=3x-12\\ 6+4x-6&=3x-12-6\\ 4x&=3x-18\\ 4x-3x&=3x-18-3x\\ x&=-18 \end{aligned}$$
The solution set is $\{-18\}$.

24.
$$\begin{aligned} 3(2-x)&=2x-1\\ 6-3x&=2x-1\\ 6-3x-6&=2x-1-6\\ -3x&=2x-7\\ -3x-2x&=2x-7-2x\\ -5x&=-7\\ \frac{-5x}{-5}&=\frac{-7}{-5}\\ x&=\frac{7}{5} \end{aligned}$$
The solution set is $\left\{\frac{7}{5}\right\}$.

25.
$$\begin{aligned} 8x-(2x+1)&=3x-10\\ 8x-2x-1&=3x-10\\ 6x-1&=3x-10\\ 6x-1+1&=3x-10+1\\ 6x&=3x-9\\ 6x-3x&=3x-9-3x\\ 3x&=-9\\ \frac{3x}{3}&=\frac{-9}{3}\\ x&=-3 \end{aligned}$$
The solution set is $\{-3\}$.

26.
$$\begin{aligned} 5-(2x-1)&=10\\ 5-2x+1&=10\\ -2x+6&=10\\ -2x+6-6&=10-6\\ -2x&=4\\ \frac{-2x}{-2}&=\frac{4}{-2}\\ x&=-2 \end{aligned}$$
The solution set is $\{-2\}$.

27. $\frac{1}{2}x-4=\frac{3}{4}x$

$4\left(\frac{1}{2}x-4\right)=4\left(\frac{3}{4}x\right)$

$2x-16=3x$

$2x-16+16=3x+16$

$2x=3x+16$

$2x-3x=3x+16-3x$

$-x=16$

$-1(-x)=-1(16)$

$x=-16$

The solution set is $\{-16\}$.

28. $1-\frac{1}{2}x=5$

$2\left(1-\frac{1}{2}x\right)=2(5)$

$2-x=10$

$2-x-2=10-2$

$-x=8$

$-1(-x)=-1(8)$

$x=-8$

The solution set is $\{-8\}$.

29. $0.9t=0.4+0.1t$

$0.9t-0.1t=0.4+0.1t-0.1t$

$0.8t=0.4$

$\frac{0.8t}{0.8}=\frac{0.4}{0.8}$

$t=0.5$

The solution set is $\{0.5\}$.

30. $0.9t=1+t$

$0.9t-t=1+t-t$

$-0.1t=1$

$\frac{-0.1t}{-0.1}=\frac{1}{-0.1}$

$t=-10$

The solution set is $\{-10\}$.

31. $\frac{2}{y}+\frac{4}{y}=3$

$\frac{6}{y}=3$

$\left(\frac{6}{y}\right)^{-1}=3^{-1}$

$\frac{y}{6}=\frac{1}{3}$

$6\cdot\frac{y}{6}=6\cdot\frac{1}{3}$

$y=2$

The solution set is $\{2\}$.

32. $\frac{4}{y}-5=\frac{5}{2y}$

$2y\left(\frac{4}{y}-5\right)=2y\left(\frac{5}{2y}\right)$

$8-10y=5$

$8-10y-8=5-8$

$-10y=-3$

$\frac{-10y}{-10}=\frac{-3}{-10}$

$y=\frac{3}{10}$

The solution set is $\left\{\frac{3}{10}\right\}$.

33. $(x+7)(x-1)=(x+1)^2$

$x^2-x+7x-7=x^2+2x+1$

$x^2+6x-7=x^2+2x+1$

$x^2+6x-7-x^2=x^2+2x+1-x^2$

$6x-7=2x+1$

$6x-7-2x=2x+1-2x$

$4x-7=1$

$4x-7+7=1+7$

$4x=8$

$\frac{4x}{4}=\frac{8}{4}$

$x=2$

The solution set is $\{2\}$.

34. $(x+2)(x-3)=(x-3)^2$

$$x^2+2x-3x-6=x^2-6x+9$$
$$x^2-x-6=x^2-6x+9$$
$$x^2-x-6-x^2=x^2-6x+9-x^2$$
$$-x-6=-6x+9$$
$$-x-6+6x=-6x+9+6x$$
$$5x-6=9$$
$$5x-6+6=9+6$$
$$5x=15$$
$$\frac{5x}{5}=\frac{15}{5}$$
$$x=3$$

The solution set is $\{3\}$.

35. $z(z^2+1)=3+z^3$

$$z^3+z=3+z^3$$
$$z^3+z-z^3=3+z^3-z^3$$
$$z=3$$

The solution set is $\{3\}$.

36. $w(4-w^2)=8-w^3$

$$4w-w^3=8-w^3$$
$$4w=8$$
$$w=2$$

The solution set is $\{2\}$.

37. $x^2=9x$

$$x^2-9x=0$$
$$x(x-9)=0$$
$$x=0 \quad \text{or} \quad x-9=0$$
$$x=9$$

The solution set is $\{0,9\}$.

38. $x^3=x^2$

$$x^3-x^2=0$$
$$x^2(x-1)=0$$
$$x^2=0 \quad \text{or} \quad x-1=0$$
$$x=0 \qquad x=1$$

The solution set is $\{0,1\}$.

39. $t^3-9t^2=0$

$$t^2(t-9)=0$$
$$t^2=0 \quad \text{or} \quad t-9=0$$
$$t=0 \qquad t=9$$

The solution set is $\{0,9\}$.

40. $4z^3-8z^2=0$

$$4z^2(z-2)=0$$
$$4z^2=0 \quad \text{or} \quad z-2=0$$
$$z=0 \qquad z=2$$

The solution set is $\{0,2\}$.

41. $\frac{3}{2x-3}=\frac{2}{x+5}$

$$3(x+5)=2(2x-3)$$
$$3x+15=4x-6$$
$$3x+15-3x=4x-6-3x$$
$$15=x-6$$
$$15+6=x-6+6$$
$$21=x$$

The solution set is $\{21\}$.

42. $\frac{-2}{x+4}=\frac{-3}{x+1}$

$$-2(x+1)=-3(x+4)$$
$$-2x-2=-3x-12$$
$$-2x-2+3x=-3x-12+3x$$
$$x-2=-12$$
$$x-2+2=-12+2$$
$$x=-10$$

The solution set is $\{-10\}$.

43. $(x+2)(3x)=(x+2)(6)$

$$3x^2+6x=6x+12$$
$$3x^2+6x-6x=6x+12-6x$$
$$3x^2=12$$
$$x^2=4$$
$$x=\pm 2$$

The solution set is $\{-2,2\}$.

44. $(x-5)(2x)=(x-5)(4)$

$2x^2-10x=4x-20$

$2x^2-14x+20=0$

$x^2-7x+10=0$

$(x-5)(x-2)=0$

$x-5=0$ or $x-2=0$

$x=5$ $\quad x=2$

The solution set is $\{2,5\}$.

45. $\dfrac{2}{x-2}=\dfrac{3}{x+5}+\dfrac{10}{(x+5)(x-2)}$

LCD = $(x+5)(x-2)$

$\dfrac{2(x+5)}{(x+5)(x-2)}=\dfrac{3(x-2)}{(x+5)(x-2)}+\dfrac{10}{(x+5)(x-2)}$

$2(x+5)=3(x-2)+10$

$2x+10=3x-6+10$

$2x+10=3x+4$

$10=x+4$

$6=x$

The solution set is $\{6\}$.

46. $\dfrac{1}{2x+3}+\dfrac{1}{x-1}=\dfrac{1}{(2x+3)(x-1)}$

LCD = $(2x+3)(x-1)$

$\dfrac{x-1}{(2x+3)(x-1)}+\dfrac{2x+3}{(2x+3)(x-1)}=\dfrac{1}{(2x+3)(x-1)}$

$x-1+2x+3=1$

$3x+2=1$

$3x=-1$

$x=-\dfrac{1}{3}$

The solution set is $\left\{-\dfrac{1}{3}\right\}$.

47. $|2x|=6$

$2x=6$ or $2x=-6$

$x=3$ $\quad x=-3$

The solution set is $\{-3, 3\}$.

48. $|3x|=12$

$3x=12$ or $3x=-12$

$x=4$ $\quad x=-4$

The solution set is $\{-4,4\}$.

49. $|2x+3|=5$

$2x+3=5$ or $2x+3=-5$

$2x=2$ or $2x=-8$

$x=1$ or $x=-4$

The solution set is $\{-4, 1\}$.

50. $|3x-1|=2$

$3x-1=2$ or $3x-1=-2$

$3x=3$ or $3x=-1$

$x=1$ or $x=-\dfrac{1}{3}$

The solution set is $\left\{-\dfrac{1}{3},1\right\}$.

51. $|1-4t|=5$

$1-4t=5$ or $1-4t=-5$

$-4t=4$ or $-4t=-6$

$t=-1$ or $t=\dfrac{3}{2}$

The solution set is $\left\{-1,\dfrac{3}{2}\right\}$.

52. $|1-2z|=3$

$1-2z=3$ or $1-2z=-3$

$-2z=2$ or $-2z=-4$

$z=-1$ or $z=2$

The solution set is $\{-1, 2\}$.

53. $|-2x|=8$

$-2x=8$ or $-2x=-8$

$x=-4$ or $x=4$

The solution set is $\{-4, 4\}$.

54. $|-x|=1$

$-x=1$ or $-x=-1$

$x=-1$ $\quad x=1$

The solution set is $\{-1, 1\}$.

55. $|-2|x=4$

$2x=4$

$x=2$

The solution set is $\{2\}$.

56. $|3|x=9$

$3x=9$

$x=3$

The solution set is $\{3\}$.

57. $|x-2| = -\frac{1}{2}$

Since absolute values are never negative, this equation has no solution.

58. $|2-x| = -1$

Since absolute values are never negative, this equation has no solution.

59. $\left|x^2-4\right| = 0$

$x^2 - 4 = 0$

$x^2 = 4$

$x = \pm 2$

The solution set is $\{-2, 2\}$.

60. $\left|x^2-9\right| = 0$

$x^2 - 9 = 0$

$x^2 = 9$

$x = \pm 3$

The solution set is $\{-3, 3\}$.

61. $\left|x^2-2x\right| = 3$

$x^2 - 2x = 3$ or $x^2 - 2x = -3$

$x^2 - 2x - 3 = 0$ or $x^2 - 2x + 3 = 0$

$(x-3)(x+1) = 0$ or $x = \frac{2 \pm \sqrt{4-12}}{2}$

$= \frac{2 \pm \sqrt{-8}}{2}$ no real sol.

$x = 3$ or $x = -1$

The solution set is $\{-1, 3\}$.

62. $\left|x^2+x\right| = 12$

$x^2 + x = 12$ or $x^2 + x = -12$

$x^2 + x - 12 = 0$ or $x^2 + x + 12 = 0$

$(x-3)(x+4) = 0$ or $x = \frac{-1 \pm \sqrt{1-48}}{2}$

$= \frac{1 \pm \sqrt{-47}}{2}$ no real sol.

$x = 3$ or $x = -4$

The solution set is $\{-4, 3\}$.

63. $\left|x^2+x-1\right| = 1$

$x^2 + x - 1 = 1$ or $x^2 + x - 1 = -1$

$x^2 + x - 2 = 0$ or $x^2 + x = 0$

$(x-1)(x+2) = 0$ or $x(x+1) = 0$

$x = 1, x = -2$ or $x = 0, x = -1$

The solution set is $\{-2, -1, 0, 1\}$.

64. $\left|x^2+3x-2\right| = 2$

$x^2 + 3x - 2 = 2$ or $x^2 + 3x - 2 = -2$

$x^2 + 3x - 4 = 0$ or $x^2 + 3x = 0$

$(x+4)(x-1) = 0$ or $x(x+3) = 0$

$x = -4,\ x = 1$ or $x = 0, x = -3$

The solution set is $\{-4, -3, 0, 1\}$.

65. $x^2 = 4x$

$x^2 - 4x = 0$

$x(x-4) = 0$

$x = 0$ or $x - 4 = 0$

$x = 4$

The solution set is $\{0, 4\}$.

66. $x^2 = -8x$

$x^2 + 8x = 0$

$x(x+8) = 0$

$x = 0$ or $x + 8 = 0$

$x = -8$

The solution set is $\{-8, 0\}$.

67. $z^2 + 4z - 12 = 0$

$(z+6)(z-2) = 0$

$z + 6 = 0$ or $z - 2 = 0$

$z = -6$ $\quad z = 2$

The solution set is $\{-6, 2\}$.

68. $v^2 + 7v + 12 = 0$

$(v+4)(v+3) = 0$

$v + 4 = 0$ or $v + 3 = 0$

$v = -4$ $\quad v = -3$

The solution set is $\{-4, -3\}$.

69. $2x^2-5x-3=0$

$(2x+1)(x-3)=0$

$2x+1=0$ or $x-3=0$

$2x=-1$ $\quad x=3$

$x=-\frac{1}{2}$

The solution set is $\left\{-\frac{1}{2},3\right\}$.

70. $3x^2+5x+2=0$

$(3x+2)(x+1)=0$

$3x+2=0$ or $x+1=0$

$3x=-2$ $\quad x=-1$

$x=-\frac{2}{3}$

The solution set is $\left\{-1,-\frac{2}{3}\right\}$

71. $x(x-7)+12=0$

$x^2-7x+12=0$

$(x-4)(x-3)=0$

$x-4=0$ or $x-3=0$

$x=4$ $\quad x=3$

The solution set is $\{3,4\}$.

72. $x(x+1)=12$

$x^2+x=12$

$x^2+x-12=0$

$(x+4)(x-3)=0$

$x+4=0$ or $x-3=0$

$x=-4$ $\quad x=3$

The solution set is $\{-4,3\}$.

73. $4x^2+9=12x$

$4x^2-12x+9=0$

$(2x-3)^2=0$

$2x-3=0$

$2x=3$

$x=\frac{3}{2}$

The solution set is $\left\{\frac{3}{2}\right\}$.

74. $25x^2+16=40x$

$25x^2-40x+16=0$

$(5x-4)^2=0$

$5x-4=0$

$5x=4$

$x=\frac{4}{5}$

The solution set is $\left\{\frac{4}{5}\right\}$.

75. $6x-5=\frac{6}{x}$

$x(6x-5)=x\left(\frac{6}{x}\right)$

$6x^2-5x=6$

$6x^2-5x-6=0$

$(3x+2)(2x-3)=0$

$3x+2=0$ or $2x-3=0$

$3x=-2$ $\quad 2x=3$

$x=-\frac{2}{3}$ $\quad x=\frac{3}{2}$

The solution set is $\left\{-\frac{2}{3},\frac{3}{2}\right\}$.

76. $x+\frac{12}{x}=7$

$x\left(x+\frac{12}{x}\right)=x(7)$

$x^2+12=7x$

$x^2-7x+12=0$

$(x-3)(x-4)=0$

$x-3=0$ or $x-4=0$

$x=3$ $\quad x=4$

The solution set is $\{3,4\}$.

77. $\dfrac{4(x-2)}{x-3}+\dfrac{3}{x}=\dfrac{-3}{x(x-3)}$

LCD = $x(x-3)$

$$\frac{4x(x-2)}{x(x-3)}+\frac{3(x-3)}{x(x-3)}=\frac{-3}{x(x-3)}$$
$$4x(x-2)+3(x-3)=-3$$
$$4x^2-8x+3x-9=-3$$
$$4x^2-5x-9=-3$$
$$4x^2-5x-6=0$$
$$(4x+3)(x-2)=0$$
$$4x+3=0 \quad \text{or} \quad x-2=0$$
$$4x=-3 \qquad x=2$$
$$x=-\frac{3}{4}$$

The solution set is $\left\{-\dfrac{3}{4},2\right\}$.

78. $\dfrac{5}{x+4}=4+\dfrac{3}{x-2}$

LCD = $(x-2)(x+4)$

$$\frac{5(x-2)}{(x-2)(x+4)}=\frac{4(x-2)(x+4)}{(x-2)(x+4)}+\frac{3(x+4)}{(x-2)(x+4)}$$
$$5(x-2)=4(x-2)(x+4)+3(x+4)$$
$$5x-10=4\left(x^2+2x-8\right)+3x+12$$
$$5x-10=4x^2+8x-32+3x+12$$
$$5x-10=4x^2+11x-20$$
$$0=4x^2+6x-10$$
$$0=2x^2+3x-5$$
$$0=(2x+5)(x-1)$$
$$2x+5=0 \quad \text{or} \quad x-1=0$$
$$2x=-5 \qquad x=1$$
$$x=-\frac{5}{2}$$

The solution set is $\left\{-\dfrac{5}{2},1\right\}$.

79. $\dfrac{x}{x^2-1}-\dfrac{x+3}{x^2-x}=\dfrac{-3}{x^2+x}$

LCD = $x(x+1)(x-1)$

$$x(x+1)(x-1)\left(\frac{x}{x^2-1}-\frac{x+3}{x^2-x}=\frac{-3}{x^2+x}\right)$$
$$x^2-(x+1)(x+3)=-3(x-1)$$
$$x^2-\left(x^2+4x+3\right)=-3x+3$$
$$x^2-x^2-4x-3=-3x+3$$
$$-4x-3=-3x+3$$
$$-4x=-3x+6$$
$$-x=6$$
$$x=-6$$

The solution set is $\{-6\}$.

80. $\dfrac{x+1}{x^2+2x}-\dfrac{x+4}{x^2+x}=\dfrac{-3}{x^2+3x+2}$

LCD = $x(x+2)(x+1)$

$$x(x+2)(x+1)\left(\frac{x+1}{x^2+2x}-\frac{x+4}{x^2+x}=\frac{-3}{x^2+3x+2}\right)$$
$$(x+1)(x+1)-(x+4)(x+2)=-3(x)$$
$$x^2+2x+1-\left(x^2+6x+8\right)=-3x$$
$$x^2+2x+1-x^2-6x-8=-3x$$
$$-4x-7=-3x$$
$$-4x=-3x+7$$
$$-x=7$$
$$x=-7$$

The solution set is $\{-7\}$.

81. $x^4-5x^2+4=0$

$$\left(x^2-4\right)\left(x^2-1\right)=0$$
$$x^2-4=0 \ \text{or}\ x^2-1=0$$
$$x=\pm 2 \ \text{or}\ x=\pm 1$$

The solution set is $\{-2,-1,1,2\}$.

82. $x^4-10x^2+25=0$

$$\left(x^2-5\right)\left(x^2-5\right)=0$$
$$x^2-5=0 \to x=\pm\sqrt{5}$$

The solution set is $\left\{-\sqrt{5},\sqrt{5}\right\}$.

83. $(x+2)^2+7(x+2)+12=0$

$let\ \ p=x+2 \to p^2=(x+2)^2$

$p^2+7p+12=0$

$(p+3)(p+4)=0$

$p+3=0$ or $p+4=0$

$p=-3 \to x+2=-3 \to x=-5$

or $p=-4 \to x+2=-4 \to x=-6$

The solution set is $\{-6,-5\}$.

84. $(2x+5)^2-(2x+5)-6=0$

$let\ \ p=2x+5 \to p^2=(2x+5)^2$

$p^2-p-6=0$

$(p-3)(p+2)=0$

$p-3=0$ *or* $p+2=0$

$p=3 \to 2x+5=3 \to x=-1$

or $p=-2 \to 2x+5=-2 \to x=-\frac{7}{2}$

The solution set is $\left\{-\frac{7}{2},-1\right\}$.

85. $2(s+1)^2-5(s+1)=3$

$let\ \ p=s+1 \to p^2=(s+1)^2$

$2p^2-5p=3$

$2p^2-5p-3=0$

$(2p+1)(p-3)=0$

$2p+1=0$ or $p-3=0$

$p=-\frac{1}{2} \to s+1=-\frac{1}{2} \to s=-\frac{3}{2}$

or $p=3 \to s+1=3 \to s=2$

The solution set is $\left\{-\frac{3}{2},2\right\}$.

86. $3(1-y)^2+5(1-y)+2=0$

$let\ \ p=1-y \to p^2=(1-y)^2$

$3p^2+5p+2=0$

$(3p+2)(p+1)=0$

$3p+2=0$ or $p+1=0$

$p=-\frac{2}{3} \to 1-y=-\frac{2}{3} \to y=\frac{5}{3}$

or $p=-1 \to 1-y=-1 \to y=2$

The solution set is $\left\{\frac{5}{3},2\right\}$.

87. $x^3+x^2-20x=0$

$x\left(x^2+x-20\right)=0$

$x(x+5)(x-4)=0$

$x=0$ or $x+5=0$ or $x-4=0$

$x=-5$ $\quad x=4$

The solution set is $\{-5,0,4\}$.

88. $x^3+6x^2-7x=0$

$x\left(x^2+6x-7\right)=0$

$x(x+7)(x-1)=0$

$x=0$ or $x+7=0$ or $x-1=0$

$x=-7$ $\quad x=1$

The solution set is $\{-7,0,1\}$.

89. $x^3+x^2-x-1=0$

$x^2(x+1)-1(x+1)=0$

$(x+1)\left(x^2-1\right)=0$

$(x+1)(x-1)(x+1)=0$

$x+1=0$ or $x-1=0$

$x=-1$ $\quad x=1$

The solution set is $\{-1,1\}$.

90. $x^3+4x^2-x-4=0$

$x^2(x+4)-1(x+4)=0$

$(x+4)\left(x^2-1\right)=0$

$(x+4)(x-1)(x+1)=0$

$x+4=0$ or $x-1=0$ or $x+1=0$

$x=-4$ $\quad x=1$ $\quad x=-1$

The solution set is $\{-4,-1,1\}$.

91. $2x^3+4=x^2+8x$

$2x^3-x^2-8x+4=0$

$x^2(2x-1)-4(2x-1)=0$

$(2x-1)\left(x^2-4\right)=0$

$(2x-1)(x-2)(x+2)=0$

$2x-1=0$ or $x-2=0$ or $x+2=0$

$2x=1$ $\quad x=2$ $\quad x=-2$

$x=\frac{1}{2}$

The solution set is $\left\{-2,\frac{1}{2},2\right\}$.

92. $3x^3 + 4x^2 = 27x + 36$

$$3x^3 + 4x^2 - 27x - 36 = 0$$
$$x^2(3x+4) - 9(3x+4) = 0$$
$$(3x+4)(x^2 - 9) = 0$$
$$(3x+4)(x-3)(x+3) = 0$$
$$3x + 4 = 0 \quad \text{or} \quad x - 3 = 0 \quad \text{or} \quad x + 3 = 0$$
$$3x = -4 \qquad x = 3 \qquad x = -3$$
$$x = -\frac{4}{3}$$

The solution set is $\left\{-3, -\frac{4}{3}, 3\right\}$.

93. $x^2 = 25 \Rightarrow x = \pm\sqrt{25} \Rightarrow x = \pm 5$

The solution set is $\{-5, 5\}$.

94. $x^2 = 36 \Rightarrow x = \pm\sqrt{36} \Rightarrow x = \pm 6$

The solution set is $\{-6, 6\}$.

95. $(x-1)^2 = 4$

$$x - 1 = \pm\sqrt{4}$$
$$x - 1 = \pm 2$$
$$x - 1 = 2 \quad \text{or} \quad x - 1 = -2$$
$$\Rightarrow x = 3 \quad \text{or} \quad x = -1$$

The solution set is $\{-1, 3\}$.

96. $(x+2)^2 = 1$

$$x + 2 = \pm\sqrt{1}$$
$$x + 2 = \pm 1$$
$$x + 2 = 1 \quad \text{or} \quad x + 2 = -1$$
$$\Rightarrow x = -1 \quad \text{or} \quad x = -3$$

The solution set is $\{-3, -1\}$.

97. $(2x+3)^2 = 9$

$$2x + 3 = \pm\sqrt{9}$$
$$2x + 3 = \pm 3$$
$$2x + 3 = 3 \quad \text{or} \quad 2x + 3 = -3$$
$$\Rightarrow x = 0 \quad \text{or} \quad x = -3$$

The solution set is $\{-3, 0\}$.

98. $(3x-2)^2 = 4$

$$3x - 2 = \pm\sqrt{4}$$
$$3x - 2 = \pm 2$$
$$3x - 2 = 2 \quad \text{or} \quad 3x - 2 = -2$$
$$\Rightarrow x = \frac{4}{3} \quad \text{or} \quad x = 0$$

The solution set is $\left\{0, \frac{4}{3}\right\}$.

99. $\left(\frac{8}{2}\right)^2 = 4^2 = 16$

100. $\left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$

101. $\left(\frac{1/2}{2}\right)^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$

102. $\left(\frac{\left(-\frac{1}{3}\right)}{2}\right)^2 = \left(-\frac{1}{6}\right)^2 = \frac{1}{36}$

103. $\left(\frac{\left(-\frac{2}{3}\right)}{2}\right)^2 = \left(-\frac{1}{3}\right)^2 = \frac{1}{9}$

104. $\left(\frac{\left(-\frac{2}{5}\right)}{2}\right)^2 = \left(-\frac{1}{5}\right)^2 = \frac{1}{25}$

105. $x^2 + 4x = 21$

$$x^2 + 4x + 4 = 21 + 4$$
$$(x+2)^2 = 25$$
$$x + 2 = \pm\sqrt{25} \Rightarrow x + 2 = \pm 5$$
$$x = -2 \pm 5 \Rightarrow x = 3 \quad \text{or} \quad x = -7$$

The solution set is $\{-7, 3\}$.

106. $x^2 - 6x = 13$

$x^2 - 6x + 9 = 13 + 9$

$(x-3)^2 = 22 \Rightarrow x - 3 = \pm\sqrt{22}$

$x = 3 \pm \sqrt{22}$

The solution set is $\{3-\sqrt{22}, 3+\sqrt{22}\}$.

107. $x^2 - \frac{1}{2}x - \frac{3}{16} = 0$

$x^2 - \frac{1}{2}x = \frac{3}{16}$

$x^2 - \frac{1}{2}x + \frac{1}{16} = \frac{3}{16} + \frac{1}{16}$

$\left(x - \frac{1}{4}\right)^2 = \frac{1}{4}$

$x - \frac{1}{4} = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$

$x = \frac{1}{4} \pm \frac{1}{2} \Rightarrow x = \frac{3}{4}$ or $x = -\frac{1}{4}$

The solution set is $\left\{-\frac{1}{4}, \frac{3}{4}\right\}$.

108. $x^2 + \frac{2}{3}x - \frac{1}{3} = 0$

$x^2 + \frac{2}{3}x = \frac{1}{3}$

$x^2 + \frac{2}{3}x + \frac{1}{9} = \frac{1}{3} + \frac{1}{9}$

$\left(x + \frac{1}{3}\right)^2 = \frac{4}{9}$

$x + \frac{1}{3} = \pm\sqrt{\frac{4}{9}} = \pm\frac{2}{3}$

$x = -\frac{1}{3} \pm \frac{2}{3}$

$x = \frac{1}{3}$ or $x = -1$

The solution set is $\left\{-1, \frac{1}{3}\right\}$.

109. $3x^2 + x - \frac{1}{2} = 0$

$x^2 + \frac{1}{3}x - \frac{1}{6} = 0$

$x^2 + \frac{1}{3}x = \frac{1}{6}$

$x^2 + \frac{1}{3}x + \frac{1}{36} = \frac{1}{6} + \frac{1}{36}$

$\left(x + \frac{1}{6}\right)^2 = \frac{7}{36}$

$x + \frac{1}{6} = \pm\sqrt{\frac{7}{36}}$

$x + \frac{1}{6} = \pm\frac{\sqrt{7}}{6}$

$x = \frac{-1 \pm \sqrt{7}}{6}$

The solution set is $\left\{\frac{-1-\sqrt{7}}{6}, \frac{-1+\sqrt{7}}{6}\right\}$.

110. $2x^2 - 3x - 1 = 0$

$x^2 - \frac{3}{2}x - \frac{1}{2} = 0$

$x^2 - \frac{3}{2}x = \frac{1}{2}$

$x^2 - \frac{3}{2}x + \frac{9}{16} = \frac{1}{2} + \frac{9}{16}$

$\left(x - \frac{3}{4}\right)^2 = \frac{17}{16}$

$x - \frac{3}{4} = \pm\sqrt{\frac{17}{16}}$

$x - \frac{3}{4} = \pm\frac{\sqrt{17}}{4}$

$x = \frac{3 \pm \sqrt{17}}{4}$

The solution set is $\left\{\frac{3-\sqrt{17}}{4}, \frac{3+\sqrt{17}}{4}\right\}$.

111. $x^2 - 4x + 2 = 0$

$a = 1, \quad b = -4, \quad c = 2$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16-8}}{2} = \frac{4 \pm \sqrt{8}}{2}$$

$$= \frac{4 \pm 2\sqrt{2}}{2} = 2 \pm \sqrt{2}$$

The solution set is $\left\{2-\sqrt{2},\ 2+\sqrt{2}\right\}$.

112. $x^2 + 4x + 2 = 0$

$a = 1, \quad b = 4, \quad c = 2$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{16-8}}{2} = \frac{-4 \pm \sqrt{8}}{2}$$

$$= \frac{-4 \pm 2\sqrt{2}}{2} = -2 \pm \sqrt{2}$$

The solution set is $\left\{-2-\sqrt{2},\ -2+\sqrt{2}\right\}$.

113. $x^2 - 5x - 1 = 0$

$a = 1,\ b = -5,\ c = -1$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{5 \pm \sqrt{25+4}}{2} = \frac{5 \pm \sqrt{29}}{2}$$

The solution set is $\left\{\frac{5-\sqrt{29}}{2}, \frac{5+\sqrt{29}}{2}\right\}$.

114. $x^2 + 5x + 3 = 0$

$a = 1,\ b = 5,\ c = 3$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(1)(3)}}{2(1)}$$

$$= \frac{-5 \pm \sqrt{25-12}}{2} = \frac{-5 \pm \sqrt{13}}{2}$$

The solution set is $\left\{\frac{-5-\sqrt{13}}{2}, \frac{-5+\sqrt{13}}{2}\right\}$.

115. $2x^2 - 5x + 3 = 0$

$a = 2, \quad b = -5, \quad c = 3$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(3)}}{2(2)}$$

$$= \frac{5 \pm \sqrt{25-24}}{4} = \frac{5 \pm 1}{4}$$

The solution set is $\left\{1,\ \frac{3}{2}\right\}$.

116. $2x^2 + 5x + 3 = 0$

$a = 2, \quad b = 5, \quad c = 3$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(2)(3)}}{2(2)} = \frac{-5 \pm \sqrt{25-24}}{4} = \frac{-5 \pm 1}{4}$$

The solution set is $\left\{-\frac{3}{2}, -1\right\}$.

117. $4y^2 - y + 2 = 0$

$a = 4, \quad b = -1, \quad c = 2$

$$y = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(4)(2)}}{2(4)}$$

$$= \frac{1 \pm \sqrt{1-32}}{8} = \frac{1 \pm \sqrt{-31}}{8}$$

No real solution.

118. $4t^2 + t + 1 = 0$

$a = 4, \quad b = 1, \quad c = 1$

$$t = \frac{-1 \pm \sqrt{1^2 - 4(4)(1)}}{2(4)}$$

$$= \frac{-1 \pm \sqrt{1-16}}{8} = \frac{-1 \pm \sqrt{-15}}{8}$$

No real solution.

119. $4x^2 = 1 - 2x$

$4x^2 + 2x - 1 = 0$

$a = 4, \quad b = 2, \quad c = -1$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(4)(-1)}}{2(4)}$$

$$= \frac{-2 \pm \sqrt{4+16}}{8} = \frac{-2 \pm \sqrt{20}}{8}$$

$$= \frac{-2 \pm 2\sqrt{5}}{8} = \frac{-1 \pm \sqrt{5}}{4}$$

The solution set is $\left\{\frac{-1-\sqrt{5}}{4}, \frac{-1+\sqrt{5}}{4}\right\}$.

120. $2x^2 = 1 - 2x$

$2x^2 + 2x - 1 = 0$

$a = 2, \quad b = 2, \quad c = -1$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(2)(-1)}}{2(2)} = \frac{-2 \pm \sqrt{4+8}}{4}$$

$$= \frac{-2 \pm \sqrt{12}}{4} = \frac{-2 \pm 2\sqrt{3}}{4} = \frac{-1 \pm \sqrt{3}}{2}$$

The solution set is $\left\{\frac{-1-\sqrt{3}}{2}, \frac{-1+\sqrt{3}}{2}\right\}$.

121. $x^2 + \sqrt{3}\,x - 3 = 0$

$a = 1,\ b = \sqrt{3},\ c = -3$

$$x = \frac{-\sqrt{3} \pm \sqrt{\left(\sqrt{3}\right)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{-\sqrt{3} \pm \sqrt{3+12}}{2} = \frac{-\sqrt{3} \pm \sqrt{15}}{2}$$

The solution set is $\left\{\frac{-\sqrt{3}-\sqrt{15}}{2}, \frac{-\sqrt{3}+\sqrt{15}}{2}\right\}$.

122. $x^2 + \sqrt{2}\,x - 2 = 0$

$a = 1,\ b = \sqrt{2},\ c = -2$

$$x = \frac{-\sqrt{2} \pm \sqrt{\left(\sqrt{2}\right)^2 - 4(1)(-2)}}{2(1)}$$

$$= \frac{-\sqrt{2} \pm \sqrt{2+8}}{2} = \frac{-\sqrt{2} \pm \sqrt{10}}{2}$$

The solution set is $\left\{\frac{-\sqrt{2}-\sqrt{10}}{2}, \frac{-\sqrt{2}+\sqrt{10}}{2}\right\}$.

123. $x^2 - 5x + 7 = 0$

$a = 1, \quad b = -5, \quad c = 7$

$b^2 - 4ac = (-5)^2 - 4(1)(7)$

$= 25 - 28 = -3$

Since the discriminant < 0, we have no real solutions.

124. $x^2 + 5x + 7 = 0$

$a = 1, \quad b = 5, \quad c = 7$

$b^2 - 4ac = (5)^2 - 4(1)(7)$

$= 25 - 28 = -3$

Since the discriminant < 0, we have no real solutions.

125. $9x^2 - 30x + 25 = 0$

$a = 9, \quad b = -30, \quad c = 25$

$b^2 - 4ac = (-30)^2 - 4(9)(25)$

$= 900 - 900 = 0$

Since the discriminant = 0, we have one repeated real solution.

126. $25x^2 - 20x + 4 = 0$

$a = 25, \quad b = -20, \quad c = 4$

$b^2 - 4ac = (-20)^2 - 4(25)(4)$

$= 400 - 400 = 0$

Since the discriminant = 0, we have one repeated real solution.

127. $3x^2 + 5x - 8 = 0$

$a = 3, \quad b = 5, \quad c = -8$

$b^2 - 4ac = (5)^2 - 4(3)(-8)$

$= 25 + 96 = 121$

Since the discriminant > 0, we have two unequal real solutions.

128. $2x^2 - 3x - 4 = 0$

$a = 2, \quad b = -3, \quad c = -4$

$b^2 - 4ac = (-3)^2 - 4(2)(-4)$

$= 9 + 32 = 41$

Since the discriminant > 0, we have two unequal real solutions.

129. Solving for R:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$RR_1R_2\left(\frac{1}{R}\right) = RR_1R_2\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$R_1R_2 = RR_2 + RR_1$$

$$R_1R_2 = R(R_2 + R_1)$$

$$\frac{R_1R_2}{R_2 + R_1} = \frac{R(R_2 + R_1)}{R_2 + R_1}$$

$$\frac{R_1R_2}{R_2 + R_1} = R \quad \text{or} \quad R = \frac{R_1R_2}{R_1 + R_2}$$

130. Solving for r:

$$A = P(1 + rt)$$
$$A = P + Prt$$
$$A - P = Prt$$
$$\frac{A-P}{Pt} = \frac{Prt}{Pt}$$
$$r = \frac{A-P}{Pt}$$

131. Solving for R:

$$F = \frac{mv^2}{R}$$
$$RF = R\left(\frac{mv^2}{R}\right)$$
$$RF = mv^2$$
$$\frac{RF}{F} = \frac{mv^2}{F}$$
$$R = \frac{mv^2}{F}$$

132. Solving for T:

$$PV = nRT$$
$$\frac{PV}{nR} = \frac{nRT}{nR}$$
$$T = \frac{PV}{nR}$$

133. Solving for r:

$$S = \frac{a}{1-r}$$
$$(1-r)\cdot S = (1-r)\cdot\frac{a}{1-r}$$
$$(1-r)S = a$$
$$\frac{(1-r)S}{S} = \frac{a}{S}$$
$$1 - r = \frac{a}{S}$$
$$1 - r - 1 = \frac{a}{S} - 1$$
$$-r = \frac{a}{S} - 1$$
$$r = 1 - \frac{a}{S} \quad \text{or} \quad r = \frac{S-a}{S}$$

134. Solving for t:

$$v = -gt + v_0$$
$$v - v_0 = -gt + v_0 - v_0$$
$$v - v_0 = -gt$$
$$\frac{v-v_0}{-g} = \frac{-gt}{-g}$$
$$\frac{v_0 - v}{g} = t \quad \text{or} \quad t = \frac{v_0 - v}{g}$$

135. The roots of a quadratic equation are

$$x_1 = \frac{-b-\sqrt{b^2-4ac}}{2a} \text{ and } x_2 = \frac{-b+\sqrt{b^2-4ac}}{2a}$$
$$x_1 + x_2 = \frac{-b-\sqrt{b^2-4ac}}{2a} + \frac{-b+\sqrt{b^2-4ac}}{2a}$$
$$= \frac{-b-\sqrt{b^2-4ac}-b+\sqrt{b^2-4ac}}{2a}$$
$$= \frac{-2b}{2a}$$
$$= -\frac{b}{a}$$

136. The roots of a quadratic equation are

$$x_1 = \frac{-b-\sqrt{b^2-4ac}}{2a} \text{ and } x_2 = \frac{-b+\sqrt{b^2-4ac}}{2a}$$
$$x_1 \cdot x_2 = \left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right)\left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)$$
$$= \frac{(-b)^2 - \left(\sqrt{b^2-4ac}\right)^2}{(2a)^2}$$
$$= \frac{b^2 - b^2 + 4ac}{4a^2}$$
$$= \frac{4ac}{4a^2}$$
$$= \frac{c}{a}$$

137. In order to have one repeated solution, we need the discriminant to be 0.

$a = k$, $b = 1$, $c = k$

$$b^2 - 4ac = 0$$
$$1^2 - 4(k)(k) = 0$$
$$1 - 4k^2 = 0$$
$$4k^2 = 1$$
$$k^2 = \frac{1}{4}$$
$$k = \pm\sqrt{\frac{1}{4}}$$
$$k = \frac{1}{2} \quad \text{or} \quad k = -\frac{1}{2}$$

138. In order to have one repeated solution, we need the discriminant to be 0.

$a = 1$, $b = -k$, $c = 4$

$$b^2 - 4ac = 0$$
$$(-k)^2 - 4(1)(4) = 0$$
$$k^2 - 16 = 0$$
$$(k-4)(k+4) = 0$$
$$k = 4 \quad \text{or} \quad k = -4$$

139. For $ax^2 + bx + c = 0$:

$$x_1 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \text{ and } x_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

For $ax^2 - bx + c = 0$:

$$x_1^* = \frac{-(-b) - \sqrt{(-b)^2 - 4ac}}{2a} = -\left(\frac{-b + \sqrt{b^2 - 4ac}}{2a}\right) = -x_2$$

and

$$x_2^* = \frac{-(-b) + \sqrt{(-b)^2 - 4ac}}{2a} = -\left(\frac{-b - \sqrt{b^2 - 4ac}}{2a}\right) = -x_1$$

140. For $ax^2 + bx + c = 0$:

$$x_1 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \text{ and } x_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

For $cx^2 + bx + a = 0$:

$$x_1^* = \frac{-b - \sqrt{b^2 - 4(c)(a)}}{2c} = \frac{-b - \sqrt{b^2 - 4ac}}{2c}$$
$$= \frac{-b - \sqrt{b^2 - 4ac}}{2c} \cdot \frac{-b + \sqrt{b^2 - 4ac}}{-b + \sqrt{b^2 - 4ac}}$$
$$= \frac{b^2 - \left(b^2 - 4ac\right)}{2c\left(-b + \sqrt{b^2 - 4ac}\right)} = \frac{4ac}{2c\left(-b + \sqrt{b^2 - 4ac}\right)}$$
$$= \frac{2a}{-b + \sqrt{b^2 - 4ac}}$$
$$= \frac{1}{x_2}$$

and

$$x_2^* = \frac{-b + \sqrt{b^2 - 4(c)(a)}}{2c} = \frac{-b + \sqrt{b^2 - 4ac}}{2c}$$
$$= \frac{-b + \sqrt{b^2 - 4ac}}{2c} \cdot \frac{-b - \sqrt{b^2 - 4ac}}{-b - \sqrt{b^2 - 4ac}}$$
$$= \frac{b^2 - \left(b^2 - 4ac\right)}{2c\left(-b - \sqrt{b^2 - 4ac}\right)} = \frac{4ac}{2c\left(-b - \sqrt{b^2 - 4ac}\right)}$$
$$= \frac{2a}{-b - \sqrt{b^2 - 4ac}}$$
$$= \frac{1}{x_1}$$

141. a. $x^2 = 9$ and $x = 3$ are not equivalent because they do not have the same solution set. In the first equation we can also have $x = -3$.

b. $x = \sqrt{9}$ and $x = 3$ are equivalent because $\sqrt{9} = 3$.

c. $(x-1)(x-2) = (x-1)^2$ and $x - 2 = x - 1$ are not equivalent because they do not have the same solution set.
The first equation has the solution set $\{1\}$ while the second equation has no solutions.

142 – 143. Answers will vary.

144. Answers will vary. Methods may include the quadratic formula, factoring, completing the square, graphing, etc.

145. Answers will vary. Knowing the discriminant allows us to know how many real solutions the equation will have.

146. Answers will vary. One possibility:
Two distinct: $x^2-3x-18=0$
One repeated: $x^2-14x+49=0$
No real: $x^2+x+4=0$

147. Answers will vary.

148. Answers will vary. Since absolute value is never negative, the equation $|x|=-2$ has no real solution.

Section A.7

1. True; if the complex number contains an imaginary part, its square can be negative. For example, $(2i)^2=4i^2=4(-1)=-4$.

2. 5; $(2+i)(2-i)=2^2-i^2=4-(-1)=4+1=5$

3. False; in the complex number system, a quadratic equation has two solutions.

4. real; imaginary; imaginary unit

5. $\{-2i,2i\}$

6. False; the conjugate of $2+5i$ is $2-5i$.

7. True; the set of real numbers is a subset of the set of complex numbers.

8. False; if $2-3i$ is a solution of a quadratic equation with real coefficients, then its conjugate, $2+3i$, is also a solution.

9. $(2-3i)+(6+8i)=(2+6)+(-3+8)i=8+5i$

10. $$\begin{aligned}(4+5i)+(-8+2i)&=(4+(-8))+(5+2)i\\&=-4+7i\end{aligned}$$

11. $$\begin{aligned}(-3+2i)-(4-4i)&=(-3-4)+(2-(-4))i\\&=-7+6i\end{aligned}$$

12. $$\begin{aligned}(3-4i)-(-3-4i)&=(3-(-3))+(-4-(-4))i\\&=6+0i=6\end{aligned}$$

13. $$\begin{aligned}(2-5i)-(8+6i)&=(2-8)+(-5-6)i\\&=-6-11i\end{aligned}$$

14. $$\begin{aligned}(-8+4i)-(2-2i)&=(-8-2)+(4-(-2))i\\&=-10+6i\end{aligned}$$

15. $3(2-6i)=6-18i$

16. $-4(2+8i)=-8-32i$

17. $2i(2-3i)=4i-6i^2=4i-6(-1)=6+4i$

18. $3i(-3+4i)=-9i+12i^2=-9i+12(-1)=-12-9i$

19. $$\begin{aligned}(3-4i)(2+i)&=6+3i-8i-4i^2\\&=6-5i-4(-1)\\&=10-5i\end{aligned}$$

20. $$\begin{aligned}(5+3i)(2-i)&=10-5i+6i-3i^2\\&=10+i-3(-1)\\&=13+i\end{aligned}$$

21. $$\begin{aligned}(-6+i)(-6-i)&=36+6i-6i-i^2\\&=36-(-1)\\&=37\end{aligned}$$

22. $$\begin{aligned}(-3+i)(3+i)&=-9-3i+3i+i^2\\&=-9+(-1)\\&=-10\end{aligned}$$

23. $$\begin{aligned}\frac{10}{3-4i}&=\frac{10}{3-4i}\cdot\frac{3+4i}{3+4i}=\frac{30+40i}{9+12i-12i-16i^2}\\&=\frac{30+40i}{9-16(-1)}=\frac{30+40i}{25}\\&=\frac{30}{25}+\frac{40}{25}i\\&=\frac{6}{5}+\frac{8}{5}i\end{aligned}$$

24. $\dfrac{13}{5-12i}=\dfrac{13}{5-12i}\cdot\dfrac{5+12i}{5+12i}$
$=\dfrac{65+156i}{25+60i-60i-144i^2}$
$=\dfrac{65+156i}{25-144(-1)}=\dfrac{65+156i}{169}$
$=\dfrac{65}{169}+\dfrac{156}{169}i$
$=\dfrac{5}{13}+\dfrac{12}{13}i$

25. $\dfrac{2+i}{i}=\dfrac{2+i}{i}\cdot\dfrac{-i}{-i}=\dfrac{-2i-i^2}{-i^2}$
$=\dfrac{-2i-(-1)}{-(-1)}=\dfrac{1-2i}{1}=1-2i$

26. $\dfrac{2-i}{-2i}=\dfrac{2-i}{-2i}\cdot\dfrac{i}{i}=\dfrac{2i-i^2}{-2i^2}$
$=\dfrac{2i-(-1)}{-2(-1)}=\dfrac{1+2i}{2}=\dfrac{1}{2}+i$

27. $\dfrac{6-i}{1+i}=\dfrac{6-i}{1+i}\cdot\dfrac{1-i}{1-i}=\dfrac{6-6i-i+i^2}{1-i+i-i^2}$
$=\dfrac{6-7i+(-1)}{1-(-1)}=\dfrac{5-7i}{2}=\dfrac{5}{2}-\dfrac{7}{2}i$

28. $\dfrac{2+3i}{1-i}=\dfrac{2+3i}{1-i}\cdot\dfrac{1+i}{1+i}=\dfrac{2+2i+3i+3i^2}{1+i-i-i^2}$
$=\dfrac{2+5i+3(-1)}{1-(-1)}=\dfrac{-1+5i}{2}=-\dfrac{1}{2}+\dfrac{5}{2}i$

29. $\left(\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i\right)^2=\dfrac{1}{4}+2\left(\dfrac{1}{2}\right)\left(\dfrac{\sqrt{3}}{2}i\right)+\dfrac{3}{4}i^2$
$=\dfrac{1}{4}+\dfrac{\sqrt{3}}{2}i+\dfrac{3}{4}(-1)=-\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i$

30. $\left(\dfrac{\sqrt{3}}{2}-\dfrac{1}{2}i\right)^2=\dfrac{3}{4}-2\left(\dfrac{\sqrt{3}}{2}\right)\left(\dfrac{1}{2}i\right)+\dfrac{1}{4}i^2$
$=\dfrac{3}{4}-\dfrac{\sqrt{3}}{2}i+\dfrac{1}{4}(-1)=\dfrac{1}{2}-\dfrac{\sqrt{3}}{2}i$

31. $(1+i)^2=1+2i+i^2=1+2i+(-1)=2i$

32. $(1-i)^2=1-2i+i^2=1-2i+(-1)=-2i$

33. $i^{23}=i^{22+1}=i^{22}\cdot i=\left(i^2\right)^{11}\cdot i=(-1)^{11}i=-i$

34. $i^{14}=\left(i^2\right)^7=(-1)^7=-1$

35. $i^{-15}=\dfrac{1}{i^{15}}=\dfrac{1}{i^{14+1}}=\dfrac{1}{i^{14}\cdot i}=\dfrac{1}{(i^2)^7\cdot i}$
$=\dfrac{1}{(-1)^7 i}=\dfrac{1}{-i}=\dfrac{1}{-i}\cdot\dfrac{i}{i}=\dfrac{i}{-i^2}=\dfrac{i}{-(-1)}=i$

36. $i^{-23}=\dfrac{1}{i^{23}}=\dfrac{1}{i^{22+1}}=\dfrac{1}{i^{22}\cdot i}=\dfrac{1}{(i^2)^{11}\cdot i}$
$=\dfrac{1}{(-1)^{11}i}=\dfrac{1}{-i}=\dfrac{1}{-i}\cdot\dfrac{i}{i}=\dfrac{i}{-i^2}=\dfrac{i}{-(-1)}=i$

37. $i^6-5=\left(i^2\right)^3-5=(-1)^3-5=-1-5=-6$

38. $4+i^3=4+i^2\cdot i=4+(-1)i=4-i$

39. $6i^3-4i^5=i^3(6-4i^2)$
$=i^2\cdot i(6-4(-1))=-1\cdot i(10)=-10i$

40. $4i^3-2i^2+1=4i^2\cdot i-2i^2+1$
$=4(-1)i-2(-1)+1$
$=-4i+2+1$
$=3-4i$

41. $(1+i)^3=(1+i)(1+i)(1+i)=(1+2i+i^2)(1+i)$
$=(1+2i-1)(1+i)=2i(1+i)$
$=2i+2i^2=2i+2(-1)$
$=-2+2i$

42. $(3i)^4+1=81i^4+1=81(1)+1=82$

43. $i^7(1+i^2)=i^7(1+(-1))=i^7(0)=0$

44. $2i^4(1+i^2)=2(1)(1+(-1))=2(0)=0$

45. $i^6+i^4+i^2+1=\left(i^2\right)^3+\left(i^2\right)^2+i^2+1$
$=(-1)^3+(-1)^2+(-1)+1$
$=-1+1-1+1$
$=0$

46. $i^7+i^5+i^3+i=\left(i^2\right)^3\cdot i+\left(i^2\right)^2\cdot i+i^2\cdot i+i$
$=(-1)^3\cdot i+(-1)^2\cdot i+(-1)\cdot i+i$
$=-i+i-i+i$
$=0$

47. $\sqrt{-4}=2i$

48. $\sqrt{-9}=3i$

49. $\sqrt{-25}=5i$

50. $\sqrt{-64}=8i$

51. $\sqrt{(3+4i)(4i-3)}=\sqrt{12i-9+16i^2-12i}$
$=\sqrt{-9+16(-1)}$
$=\sqrt{-25}$
$=5i$

52. $\sqrt{(4+3i)(3i-4)}=\sqrt{12i-16+9i^2-12i}$
$=\sqrt{-16+9(-1)}$
$=\sqrt{-25}$
$=5i$

53. $x^2+4=0$
$x^2=-4$
$x=\pm\sqrt{-4}$
$x=\pm 2i$
The solution set is $\{-2i, 2i\}$.

54. $x^2-4=0$
$(x+2)(x-2)=0$
$x=-2$ or $x=2$
The solution set is $\{-2, 2\}$.

55. $x^2-16=0$
$(x+4)(x-4)=0$
$x=-4$ or $x=4$
The solution set is $\{-4, 4\}$.

56. $x^2+25=0$
$x^2=-25$
$x=\pm\sqrt{-25}=\pm 5i$
The solution set is $\{-5i, 5i\}$.

57. $x^2-6x+13=0$
$a=1, b=-6, c=13,$
$b^2-4ac=(-6)^2-4(1)(13)=36-52=-16$
$x=\frac{-(-6)\pm\sqrt{-16}}{2(1)}=\frac{6\pm 4i}{2}=3\pm 2i$
The solution set is $\{3-2i, 3+2i\}$.

58. $x^2+4x+8=0$
$a=1, b=4, c=8$
$b^2-4ac=4^2-4(1)(8)=16-32=-16$
$x=\frac{-4\pm\sqrt{-16}}{2(1)}=\frac{-4\pm 4i}{2}=-2\pm 2i$
The solution set is $\{-2-2i, -2+2i\}$.

59. $x^2-6x+10=0$
$a=1, b=-6, c=10$
$b^2-4ac=(-6)^2-4(1)(10)=36-40=-4$
$x=\frac{-(-6)\pm\sqrt{-4}}{2(1)}=\frac{6\pm 2i}{2}=3\pm i$
The solution set is $\{3-i, 3+i\}$.

60. $x^2-2x+5=0$
$a=1, b=-2, c=5$
$b^2-4ac=(-2)^2-4(1)(5)=4-20=-16$
$x=\frac{-(-2)\pm\sqrt{-16}}{2(1)}=\frac{2\pm 4i}{2}=1\pm 2i$
The solution set is $\{1-2i, 1+2i\}$.

61. $8x^2-4x+1=0$
$a=8, b=-4, c=1$
$b^2-4ac=(-4)^2-4(8)(1)=16-32=-16$
$x=\frac{-(-4)\pm\sqrt{-16}}{2(8)}=\frac{4\pm 4i}{16}=\frac{1}{4}\pm\frac{1}{4}i$
The solution set is $\left\{\frac{1}{4}-\frac{1}{4}i, \frac{1}{4}+\frac{1}{4}i\right\}$.

62. $10x^2+6x+1=0$

$a=10,\ b=6,\ c=1$

$b^2-4ac=6^2-4(10)(1)=36-40=-4$

$x=\dfrac{-6\pm\sqrt{-4}}{2(10)}=\dfrac{-6\pm 2i}{20}=-\dfrac{3}{10}\pm\dfrac{1}{10}i$

The solution set is $\left\{-\dfrac{3}{10}-\dfrac{1}{10}i,\ -\dfrac{3}{10}+\dfrac{1}{10}i\right\}$.

63. $5x^2+1=2x$

$5x^2-2x+1=0$

$a=5,\ b=-2,\ c=1$

$b^2-4ac=(-2)^2-4(5)(1)=4-20=-16$

$x=\dfrac{-(-2)\pm\sqrt{-16}}{2(5)}=\dfrac{2\pm 4i}{10}=\dfrac{1}{5}\pm\dfrac{2}{5}i$

The solution set is $\left\{\dfrac{1}{5}-\dfrac{2}{5}i,\ \dfrac{1}{5}+\dfrac{2}{5}i\right\}$.

64. $13x^2+1=6x$

$13x^2-6x+1=0$

$a=13,\ b=-6,\ c=1$

$b^2-4ac=(-6)^2-4(13)(1)=36-52=-16$

$x=\dfrac{-(-6)\pm\sqrt{-16}}{2(13)}=\dfrac{6\pm 4i}{26}=\dfrac{3}{13}\pm\dfrac{2}{13}i$

The solution set is $\left\{\dfrac{3}{13}-\dfrac{2}{13}i, \dfrac{3}{13}+\dfrac{2}{13}i\right\}$.

65. $x^2+x+1=0$

$a=1,\ b=1,\ c=1,$

$b^2-4ac=1^2-4(1)(1)=1-4=-3$

$x=\dfrac{-1\pm\sqrt{-3}}{2(1)}=\dfrac{-1\pm\sqrt{3}\,i}{2}=-\dfrac{1}{2}\pm\dfrac{\sqrt{3}}{2}i$

The solution set is $\left\{-\dfrac{1}{2}-\dfrac{\sqrt{3}}{2}i,\ -\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i\right\}$.

66. $x^2-x+1=0$

$a=1,\ b=-1,\ c=1$

$b^2-4ac=(-1)^2-4(1)(1)=1-4=-3$

$x=\dfrac{-(-1)\pm\sqrt{-3}}{2(1)}=\dfrac{1\pm\sqrt{3}\,i}{2}=\dfrac{1}{2}\pm\dfrac{\sqrt{3}}{2}i$

The solution set is $\left\{\dfrac{1}{2}-\dfrac{\sqrt{3}}{2}i,\ \dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i\right\}$.

67. $x^3-8=0$

$(x-2)\left(x^2+2x+4\right)=0$

$x-2=0\Rightarrow x=2$

or $x^2+2x+4=0$

$a=1,\ b=2,\ c=4$

$b^2-4ac=2^2-4(1)(4)=4-16=-12$

$x=\dfrac{-2\pm\sqrt{-12}}{2(1)}=\dfrac{-2\pm 2\sqrt{3}\,i}{2}=-1\pm\sqrt{3}i$

The solution set is $\left\{2,-1-\sqrt{3}i,-1+\sqrt{3}i\right\}$.

68. $x^3+27=0$

$(x+3)\left(x^2-3x+9\right)=0$

$x+3=0\Rightarrow x=-3$

or $x^2-3x+9=0$

$a=1,\ b=-3,\ c=9$

$b^2-4ac=(-3)^2-4(1)(9)=9-36=-27$

$x=\dfrac{-(-3)\pm\sqrt{-27}}{2(1)}=\dfrac{3\pm 3\sqrt{3}\,i}{2}=\dfrac{3}{2}\pm\dfrac{3\sqrt{3}}{2}i$

The solution set is $\left\{-3, \dfrac{3}{2}-\dfrac{3\sqrt{3}}{2}i, \dfrac{3}{2}+\dfrac{3\sqrt{3}}{2}i\right\}$.

69. $x^4=16$

$x^4-16=0$

$\left(x^2-4\right)\left(x^2+4\right)=0$

$(x-2)(x+2)\left(x^2+4\right)=0$

$x-2=0$ or $x+2=0$ or $x^2+4=0$

$x=2$ or $x=-2$ or $x^2=-4$

$x=2$ or $x=-2$ or $x=\pm\sqrt{-4}=\pm 2i$

The solution set is $\{-2, 2, -2i, 2i\}$.

70. $x^4=1$

$x^4-1=0$

$\left(x^2-1\right)\left(x^2+1\right)=0$

$(x-1)(x+1)\left(x^2+1\right)=0$

$x-1=0$ or $x+1=0$ or $x^2+1=0$

$x=1$ or $x=-1$ or $x^2=-1$

$x=1$ or $x=-1$ or $x=\pm\sqrt{-1}=\pm i$

The solution set is $\{-1, 1, -i, i\}$.

71.
$$x^4+13x^2+36=0$$
$$\left(x^2+9\right)\left(x^2+4\right)=0$$
$$x^2+9=0 \quad\text{or}\quad x^2+4=0$$
$$x^2=-9 \quad\text{or}\quad x^2=-4$$
$$x=\pm\sqrt{-9} \quad\text{or}\quad x=\pm\sqrt{-4}$$
$$x=\pm 3i \quad\text{or}\quad x=\pm 2i$$
The solution set is $\{-3i, 3i, -2i, 2i\}$.

72.
$$x^4+3x^2-4=0$$
$$\left(x^2-1\right)\left(x^2+4\right)=0$$
$$(x-1)(x+1)\left(x^2+4\right)=0$$
$$x-1=0 \quad\text{or}\quad x+1=0 \quad\text{or}\quad x^2+4=0$$
$$x=1 \quad\text{or}\quad x=-1 \quad\text{or}\quad x^2=-4$$
$$x=1 \quad\text{or}\quad x=-1 \quad\text{or}\quad x=\pm\sqrt{-4}=\pm 2i$$
The solution set is $\{-1, 1, -2i, 2i\}$.

73. $3x^2-3x+4=0$
$a=3, b=-3, c=4$
$b^2-4ac=(-3)^2-4(3)(4)=9-48=-39$
The equation has two complex solutions that are conjugates of each other.

74. $2x^2-4x+1=0$
$a=2, b=-4, c=1$
$b^2-4ac=(-4)^2-4(2)(1)=16-8=8$
The equation has two unequal real number solutions.

75.
$$2x^2+3x=4$$
$$2x^2+3x-4=0$$
$a=2, b=3, c=-4$
$b^2-4ac=3^2-4(2)(-4)=9+32=41$
The equation has two unequal real solutions.

76.
$$x^2+6=2x$$
$$x^2-2x+6=0$$
$a=1, b=-2, c=6$
$b^2-4ac=(-2)^2-4(1)(6)=4-24=-20$
The equation has two complex solutions that are conjugates of each other.

77. $9x^2-12x+4=0$
$a=9, b=-12, c=4$
$b^2-4ac=(-12)^2-4(9)(4)=144-144=0$
The equation has a repeated real solution.

78. $4x^2+12x+9=0$
$a=4, b=12, c=9$
$b^2-4ac=12^2-4(4)(9)=144-144=0$
The equation has a repeated real solution.

79. The other solution is $\overline{2+3i}=2-3i$.

80. The other solution is $\overline{4-i}=4+i$.

81. $z+\overline{z}=3-4i+\overline{3-4i}=3-4i+3+4i=6$

82.
$$\begin{aligned} w-\overline{w} &= 8+3i-\left(\overline{8+3i}\right)\\ &=8+3i-(8-3i)\\ &=8+3i-8+3i\\ &=0+6i\\ &=6i \end{aligned}$$

83.
$$\begin{aligned} z\cdot\overline{z} &= (3-4i)(\overline{3-4i})\\ &=(3-4i)(3+4i)\\ &=9+12i-12i-16i^2\\ &=9-16(-1)\\ &=25 \end{aligned}$$

84.
$$\begin{aligned} \overline{z-w} &= \overline{3-4i-(8+3i)}\\ &=\overline{3-4i-8-3i}\\ &=\overline{-5-7i}\\ &=-5+7i \end{aligned}$$

85.
$$\begin{aligned} Z &= \frac{V}{I}=\frac{18+i}{3-4i}=\frac{18+i}{3-4i}\cdot\frac{3+4i}{3+4i}\\ &=\frac{54+72i+3i+4i^2}{9+12i-12i-16i^2}=\frac{54+75i-4}{9+16}\\ &=\frac{50+75i}{25}=2+3i \end{aligned}$$
The impedance is $2+3i$ ohms.

86.
$$\begin{aligned} \frac{1}{Z} &= \frac{1}{Z_1}+\frac{1}{Z_2}=\frac{1}{2+i}+\frac{1}{4-3i}=\frac{(4-3i)+(2+i)}{(2+i)(4-3i)}\\ &=\frac{6-2i}{8-6i+4i-3i^2}=\frac{6-2i}{8-2i+3}=\frac{6-2i}{11-2i} \end{aligned}$$

So, $Z = \frac{11-2i}{6-2i} = \frac{11-2i}{6-2i} \cdot \frac{6+2i}{6+2i}$

$= \frac{66+22i-12i-4i^2}{36+12i-12i-4i^2} = \frac{66+10i+4}{36+4}$

$= \frac{70+10i}{40} = \frac{7}{4} + \frac{1}{4}i$

The total impedance is $\frac{7}{4} + \frac{1}{4}i$ ohms.

87. $z + \overline{z} = (a+bi) + \overline{(a+bi)}$
$= a+bi+a-bi$
$= 2a$

$z - \overline{z} = a+bi - \overline{(a+bi)}$
$= a+bi-(a-bi)$
$= a+bi-a+bi$
$= 2bi$

88. $\overline{\overline{z}} = \overline{\overline{a+bi}} = \overline{a-bi} = a+bi = z$

89. $\overline{z+w} = \overline{(a+bi)+(c+di)}$
$= \overline{(a+c)+(b+d)i}$
$= (a+c)-(b+d)i$
$= (a-bi)+(c-di)$
$= \overline{a+bi} + \overline{c+di}$
$= \overline{z} + \overline{w}$

90. $\overline{z \cdot w} = \overline{(a+bi)\cdot(c+di)}$
$= \overline{ac+adi+bci+bdi^2}$
$= \overline{(ac-bd)+(ad+bc)i}$
$= (ac-bd)-(ad+bc)i$

$\overline{z} \cdot \overline{w} = \overline{a+bi} \cdot \overline{c+di}$
$= (a-bi)(c-di)$
$= ac-adi-bci+bdi^2$
$= (ac-bd)-(ad+bc)i$

91 – 93. Answers will vary.

94. Although the set of real numbers is a subset of the set of complex numbers, not all rules that work in the real number system can be used in the larger complex number system. The rule that allows us to write the product of two square roots as the square root of the product only works in the real number system. That is, $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$ only when $\sqrt{a}$ and $\sqrt{b}$ are real numbers. In the complex number system we must first convert the radicals to complex form. In this case this means we need to write $\sqrt{-9}$ as $\sqrt{-1 \cdot 9} = \sqrt{9} \cdot \sqrt{-1} = 3i$. Then we can multiply to get $\sqrt{-9} \cdot \sqrt{-9} = 3i \cdot 3i = 9i^2 = 9(-1) = -9$.

Section A.8

1. mathematical modeling

2. interest

3. uniform motion

4. False; the amount charged for the use of principal is the interest.

5. True; this is the uniform motion formula.

6. If there are x pounds of coffee A, then there are $100 - x$ pounds of coffee B.

7. Let A represent the area of the circle and r the radius. The area of a circle is the product of π times the square of the radius: $A = \pi r^2$

8. Let C represent the circumference of a circle and r the radius. The circumference of a circle is the product of π times twice the radius: $C = 2\pi r$

9. Let A represent the area of the square and s the length of a side. The area of the square is the square of the length of a side: $A = s^2$

10. Let P represent the perimeter of a square and s the length of a side. The perimeter of a square is four times the length of a side: $P = 4s$

11. Let F represent the force, m the mass, and a the acceleration. Force equals the product of the mass times the acceleration: $F = ma$

12. Let P represent the pressure, F the force, and A the area. Pressure is the force per unit area: $P = \frac{F}{A}$

13. Let W represent the work, F the force, and d the distance. Work equals force times distance: $W = Fd$

14. Let K represent the kinetic energy, m the mass, and v the velocity. Kinetic energy is one-half the product of the mass and the square of the velocity: $K = \frac{1}{2}mv^2$

15. $C =$ total variable cost in dollars, $x =$ number of dishwashers manufactured: $C = 150x$

16. $R =$ total revenue in dollars, $x =$ number of dishwashers sold: $R = 250x$

17. Let x represent the amount of money invested in bonds. Then $50,000 - x$ represents the amount of money invested in CD's. Since the total interest is to be \$6,000, we have:

$$\begin{aligned} 0.15x + 0.07(50,000 - x) &= 6,000 \\ (100)(0.15x + 0.07(50,000 - x)) &= (6,000)(100) \\ 15x + 7(50,000 - x) &= 600,000 \\ 15x + 350,000 - 7x &= 600,000 \\ 8x + 350,000 &= 600,000 \\ 8x &= 250,000 \\ x &= 31,250 \end{aligned}$$

\$31,250 should be invested in bonds at 15% and \$18,750 should be invested in CD's at 7%.

18. Let x represent the amount of money invested in bonds. Then $50,000 - x$ represents the amount of money invested in CD's. Since the total interest is to be \$7,000, we have:

$$\begin{aligned} 0.15x + 0.07(50,000 - x) &= 7,000 \\ (100)(0.15x + 0.07(50,000 - x)) &= (7,000)(100) \\ 15x + 7(50,000 - x) &= 700,000 \\ 15x + 350,000 - 7x &= 700,000 \\ 8x + 350,000 &= 700,000 \\ 8x &= 350,000 \\ x &= 43,750 \end{aligned}$$

\$43,750 should be invested in bonds at 15% and \$6,250 should be invested in CD's at 7%.

19. Let x represent the amount of money loaned at 8%. Then $12,000 - x$ represents the amount of money loaned at 18%. Since the total interest is to be \$1,000, we have:

$$\begin{aligned} 0.08x + 0.18(12,000 - x) &= 1,000 \\ (100)(0.08x + 0.18(12,000 - x)) &= (1,000)(100) \\ 8x + 18(12,000 - x) &= 100,000 \\ 8x + 216,000 - 18x &= 100,000 \\ -10x + 216,000 &= 100,000 \\ -10x &= -116,000 \\ x &= 11,600 \end{aligned}$$

\$11,600 is loaned at 8% and \$400 is at 18%.

20. Let x represent the amount of money loaned at 16%. Then $1,000,000 - x$ represents the amount of money loaned at 19%. Since the total interest is to be \$1,000,000(0.18), we have:

$$\begin{aligned} 0.16x + 0.19(1,000,000 - x) &= 1,000,000(0.18) \\ 0.16x + 190,000 - 0.19x &= 180,000 \\ -0.03x + 190,000 &= 180,000 \\ -0.03x &= -10,000 \\ x &= \frac{-10,000}{-0.03} \\ x &= \$333,333.33 \end{aligned}$$

Wendy can lend \$333,333.33 at 16%.

21. Let x represent the number of pounds of Earl Grey tea. Then $100 - x$ represents the number of pounds of Orange Pekoe tea.

$$\begin{aligned} 5x + 3(100 - x) &= 4.50(100) \\ 5x + 300 - 3x &= 450 \\ 2x + 300 &= 450 \\ 2x &= 150 \\ x &= 75 \end{aligned}$$

75 pounds of Earl Grey tea must be blended with 25 pounds of Orange Pekoe.

22. Let x represent the number of pounds of the first kind of coffee. Then $100 - x$ represents the number of pounds of the second kind of coffee.

$$\begin{aligned} 2.75x + 5(100 - x) &= 3.90(100) \\ 2.75x + 500 - 5x &= 390 \\ -2.25x + 500 &= 390 \\ -2.25x &= -110 \\ x &\approx 48.9 \end{aligned}$$

Approximately 49 pounds of the first kind of coffee must be blended with approximately 51 pounds of the second kind of coffee.

23. Let x represent the number of pounds of cashews. Then $x + 60$ represents the number of pounds in the mixture.

$$9x+3.50(60)=7.50(x+60)$$
$$9x+210=7.50x+450$$
$$1.5x=240$$
$$x=160$$

160 pounds of cashews must be added to the 60 pounds of almonds.

24. Let x represent the number of caramels in the box. Then $30-x$ represents the number of cremes in the box.

$$\text{Revenue}-\text{Cost}=\text{Profit}$$
$$12.50-\left(0.25x+0.45(30-x)\right)=3.00$$
$$12.50-\left(0.25x+13.5-0.45x\right)=3.00$$
$$12.50-\left(13.5-0.20x\right)=3.00$$
$$12.50-13.50+0.20x=3.00$$
$$-1.00+0.20x=3.00$$
$$0.20x=4.00$$
$$x=20$$

The box should contain 20 caramels and 10 cremes.

25. Let r represent the speed of the current.

	Rate	Time	Distance
Upstream	$16-r$	$\frac{20}{60}=\frac{1}{3}$	$\frac{16-r}{3}$
Downstream	$16+r$	$\frac{15}{60}=\frac{1}{4}$	$\frac{16+r}{4}$

Since the distance is the same in each direction:

$$\frac{16-r}{3}=\frac{16+r}{4}$$
$$4(16-r)=3(16+r)$$
$$64-4r=48+3r$$
$$16=7r$$
$$r=\frac{16}{7}\approx 2.286$$

The speed of the current is approximately 2.286 miles per hour.

26. Let r represent the speed of the motorboat.

	Rate	Time	Distance
Upstream	$r-3$	5	$5(r-3)$
Downstream	$r+3$	2.5	$2.5(r+3)$

The distance is the same in each direction:

$$5(r-3)=2.5(r+3)$$
$$5r-15=2.5r+7.5$$
$$2.5r=22.5$$
$$r=9$$

The speed of the motorboat is 9 miles per hour.

27. Let r represent the speed of the current.

	Rate	Time	Distance
Upstream	$15-r$	$\frac{10}{15-r}$	10
Downstream	$15+r$	$\frac{10}{15+r}$	10

Since the total time is 1.5 hours, we have:

$$\frac{10}{15-r}+\frac{10}{15+r}=1.5$$
$$10(15+r)+10(15-r)=1.5(15-r)(15+r)$$
$$150+10r+150-10r=1.5(225-r^2)$$
$$300=1.5(225-r^2)$$
$$200=225-r^2$$
$$r^2-25=0$$
$$(r-5)(r+5)=0$$
$$r=5 \text{ or } r=-5$$

Speed must be positive, so disregard $r=-5$. The speed of the current is 5 miles per hour.

28. Let r represent the rate of the slower car. Then $r+10$ represents the rate of the faster car.

	Rate	Time	Distance
Slower car	r	3.5	$3.5r$
Faster car	$r+10$	3	$3(r+10)$

$$3.5r=3(r+10)$$
$$3.5r=3r+30$$
$$0.5r=30$$
$$r=60$$

The slower car travels at a rate of 60 miles per hour. The faster car travels at a rate of 70 miles per hour. The distance is (70)(3) = 210 miles.

29. Let r represent Karen's normal walking speed.

	Rate	Time	Distance
With walkway	$r+2.5$	$\frac{50}{r+2.5}$	50
Against walkway	$r-2.5$	$\frac{50}{r-2.5}$	50

Since the total time is 40 seconds:

$$\frac{50}{r+2.5}+\frac{50}{r-2.5}=40$$
$$50(r-2.5)+50(r+2.5)=40(r-2.5)(r+2.5)$$
$$50r-125+50r+125=40(r^2-6.25)$$
$$100r=40r^2-250$$
$$0=40r^2-100r-250$$
$$0=4r^2-10r-25$$

$$r = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(4)(-25)}}{2(4)}$$

$$= \frac{10 \pm \sqrt{500}}{8} = \frac{10 \pm 10\sqrt{5}}{8} = \frac{5 \pm 5\sqrt{5}}{4}$$

$r \approx 4.05$ or $r \approx -1.55$

Speed must be positive, so disregard $r \approx -1.55$. Karen' normal walking speed is approximately 4.05 feet per second.

30. Let r represent the speed of the Montparnasse walkway.

	Rate	Time	Distance
Walking with	$1.5 + r$	$\frac{200}{1.5+r}$	200
Standing still	r	$\frac{200}{r}$	200

Walking with the walkway takes 30 seconds less time than standing still on the walkway:

$$\frac{200}{1.5+r} = \frac{200}{r} - 30$$

$$200r = 200(1.5+r) - 30r(r+1.5)$$

$$200r = 300 + 200r - 30r^2 - 45r$$

$$30r^2 + 45r - 300 = 0$$

$$2r^2 + 3r - 20 = 0$$

$$(2r+5)(r+4) = 0$$

$2r - 5 = 0$ or $r + 4 = 0$

$r = \frac{5}{2} = 2.5$ or $r = -4$

Speed must be positive, so disregard $r = -4$. The speed of the Montparnasse walkways is 2.5 meters per second.

31. Let w represent the width of a regulation doubles tennis court. Then $2w + 6$ represents the length. The area is 2808 square feet:

$$w(2w+6) = 2808$$

$$2w^2 + 6w = 2808$$

$$2w^2 + 6w - 2808 = 0$$

$$w^2 + 3w - 1404 = 0$$

$$(w+39)(w-36) = 0$$

$w + 39 = 0$ or $w - 36 = 0$

$w = -39$ or $w = 36$

The width must be positive, so disregard $w = -39$. The width of a regulation doubles tennis court is 36 feet and the length is 2(36) + 6 = 78 feet.

32. Let t represent the time it takes the HP LaserJet 2420 to complete the print job alone. Then $t + 10$ represents the time it takes the HP LaserJet 1300 to complete the print job alone.

	Time to do job	Part of job done in one minute
HP LJ 2420	t	$\frac{1}{t}$
HP LJ 1300	$t + 10$	$\frac{1}{t+10}$
Together	12	$\frac{1}{12}$

$$\frac{1}{t} + \frac{1}{t+10} = \frac{1}{12}$$

$$12(t+10) + 12t = t(t+10)$$

$$12t + 120 + 12t = t^2 + 10t$$

$$0 = t^2 - 14t - 120$$

$$0 = (t-20)(t+6)$$

$t - 20 = 0$ or $t + 6 = 0$

$t = 20$ or $t = -6$

Time must be positive, so disregard $t = -6$. The HP LaserJet 2420 takes 20 minutes to complete the job alone, printing $\frac{600}{20} = 30$ pages per minute. The HP LaserJet 1300 takes 20 + 10 = 30 minutes to complete the job alone, printing $\frac{600}{30} = 20$ pages per minute.

33. Let t represent the time it takes to do the job together.

	Time to do job	Part of job done in one minute
Trent	30	$\frac{1}{30}$
Lois	20	$\frac{1}{20}$
Together	t	$\frac{1}{t}$

$$\frac{1}{30} + \frac{1}{20} = \frac{1}{t}$$

$$2t + 3t = 60$$

$$5t = 60$$

$$t = 12$$

Working together, the job can be done in 12 minutes.

34. Let t represent the time it takes April to do the job working alone.

	Time to do job	Part of job done in one hour
Patrice	10	$\frac{1}{10}$
April	t	$\frac{1}{t}$
Together	6	$\frac{1}{6}$

$$\frac{1}{10}+\frac{1}{t}=\frac{1}{6}$$
$$3t+30=5t$$
$$2t=30$$
$$t=15$$

April would take 15 hours to paint the rooms.

35. $l =$ length of the garden
$w =$ width of the garden

a. The length of the garden is to be twice its width. Thus, $l=2w$.
The dimensions of the fence are $l+4$ and $w+4$.
The perimeter is 46 feet, so:
$$2(l+4)+2(w+4)=46$$
$$2(2w+4)+2(w+4)=46$$
$$4w+8+2w+8=46$$
$$6w+16=46$$
$$6w=30$$
$$w=5$$
The dimensions of the garden are 5 feet by 10 feet.

b. Area $=l\cdot w=5\cdot 10=50$ square feet

c. If the dimensions of the garden are the same, then the length and width of the fence are also the same $(l+4)$. The perimeter is 46 feet, so:
$$2(l+4)+2(l+4)=46$$
$$2l+8+2l+8=46$$
$$4l+16=46$$
$$4l=30$$
$$l=7.5$$
The dimensions of the garden are 7.5 feet by 7.5 feet.

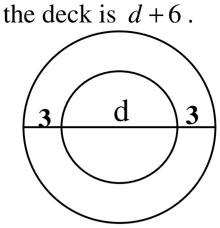
d. Area $=l\cdot w=7.5(7.5)=56.25$ square feet.

36. $l =$ length of the pond
$w =$ width of the pond

a. The pond is to be a square. Thus, $l=w$.
The dimensions of the fenced area are $w+6$ on each side. The perimeter is 100 feet, so:
$$4(w+6)=100$$
$$4w+24=100$$
$$4w=76$$
$$w=19$$
The dimensions of the pond are 19 feet by 19 feet.

b. The length of the pond is to be three times the width. Thus, $l=3w$. The dimensions of the fenced area are $w+6$ and $l+6$. The perimeter is 100 feet, so:
$$2(w+6)+2(l+6)=100$$
$$2(w+6)+2(3w+6)=100$$
$$2w+12+6w+12=100$$
$$8w+24=100$$
$$8w=76$$
$$w=9.5$$
$$l=3(9.5)=28.5$$
The dimensions of the pond are 9.5 feet by 28.5 feet.

c. If the pond is circular, the diameter is d and the diameter of the circle with the pond and the deck is $d+6$.



The perimeter is 100 feet, so:
$$\pi(d+6)=100$$
$$\pi d+6\pi=100$$
$$\pi d=100-6\pi$$
$$d=\frac{100}{\pi}-6\approx 25.83$$
The diameter of the pond is 25.83 feet.

d. $\text{Area}_{\text{square}}=l\cdot w=19(19)=361\text{ ft}^2$.

$\text{Area}_{\text{rectangle}}=l\cdot w=28.5(9.5)=270.75\text{ ft}^2$.

$\text{Area}_{\text{circle}}=\pi r^2=\pi\left(\frac{25.83}{2}\right)^2\approx 524\text{ ft}^2$.

The circular pond has the largest area.

37. Let t represent the time it takes for the defensive back to catch the tight end.

	Time to run 100 yards	Time	Rate	Distance
Tight End	12 sec	t	$\frac{100}{12}=\frac{25}{3}$	$\frac{25}{3}t$
Def. Back	10 sec	t	$\frac{100}{10}=10$	$10t$

Since the defensive back has to run 5 yards farther, we have:

$$\frac{25}{3}t+5=10t$$
$$25t+15=30t$$
$$15=5t$$
$$t=3 \quad \rightarrow \quad 10t=30$$

The defensive back will catch the tight end at the 45 yard line (15 + 30 = 45).

38. Let x represent the number of highway miles traveled. Then $30,000-x$ represents the number of city miles traveled.

$$\frac{x}{40}+\frac{30,000-x}{25}=900$$
$$200\left(\frac{x}{40}+\frac{30,000-x}{25}\right)=200(900)$$
$$5x+240,000-8x=180,000$$
$$-3x+240,000=180,000$$
$$-3x=-60,000$$
$$x=20,000$$

Therese is allowed to claim 20,000 miles as a business expense.

39. Let x represent the number of gallons of pure water. Then $x+1$ represents the number of gallons in the 60% solution.

$$(\%)(\text{gallons})+(\%)(\text{gallons})=(\%)(\text{gallons})$$
$$0(x)+1(1)=0.60(x+1)$$
$$1=0.6x+0.6$$
$$0.4=0.6x$$
$$x=\frac{4}{6}=\frac{2}{3}$$

$\frac{2}{3}$ gallon of pure water should be added.

40. Let x represent the number of liters to be drained and replaced with pure antifreeze.

$$(\%)(\text{liters})+(\%)(\text{liters})=(\%)(\text{liters})$$
$$1(x)+0.40(15-x)=0.60(15)$$
$$x+6-0.40x=9$$
$$0.60x=3$$
$$x=5$$

5 liters should be drained and replaced with pure antifreeze.

41. Let x represent the number of ounces of water to be evaporated; the amount of salt remains the same. Therefore, we get

$$0.04(32)=0.06(32-x)$$
$$1.28=1.92-0.06x$$
$$0.06x=0.64$$
$$x=\frac{0.64}{0.06}=\frac{64}{6}=\frac{32}{3}=10\tfrac{2}{3}$$

$10\frac{2}{3}\approx 10.67$ ounces of water need to be evaporated.

42. Let x represent the number of gallons of water to be evaporated; the amount of salt remains the same.

$$0.03(240)=0.05(240-x)$$
$$7.2=12-0.05x$$
$$0.05x=4.8$$
$$x=\frac{4.8}{0.05}=96$$

96 gallons of water need to be evaporated.

43. Let x represent the number of grams of pure gold. Then $60-x$ represents the number of grams of 12 karat gold to be used.

$$x+\frac{1}{2}(60-x)=\frac{2}{3}(60)$$
$$x+30-0.5x=40$$
$$0.5x=10$$
$$x=20$$

20 grams of pure gold should be mixed with 40 grams of 12 karat gold.

44. Let x represent the number of atoms of oxygen. $2x$ represents the number of atoms of hydrogen. $x+1$ represents the number of atoms of carbon.

$$x+2x+x+1=45$$
$$4x=44$$
$$x=11$$

There are 11 atoms of oxygen and 22 atoms of hydrogen in the sugar molecule.

45. Let t represent the time it takes for Mike to catch up with Dan. Since the distances are the same, we have:

$$\frac{1}{6}t = \frac{1}{9}(t+1)$$
$$3t = 2t + 2$$
$$t = 2$$

Mike will pass Dan after 2 minutes, which is a distance of $\frac{1}{3}$ mile.

46. Let t represent the time of flight with the wind. The distance is the same in each direction:

$$330t = 270(5-t)$$
$$330t = 1350 - 270t$$
$$600t = 1350$$
$$t = 2.25$$

The distance the plane can fly and still return safely is 330(2.25) = 742.5 miles.

47. Let t represent the time the auxiliary pump needs to run. Since the two pumps are emptying one tanker, we have:

$$\frac{3}{4} + \frac{t}{9} = 1$$
$$27 + 4t = 36$$
$$4t = 9$$
$$t = \frac{9}{4} = 2.25$$

The auxiliary pump must run for 2.25 hours. It must be started at 9:45 a.m. for the tanker to be emptied by noon.

48. Let x represent the number of pounds of pure cement. Then $x + 20$ represents the number of pounds in the 40% mixture.

$$x + 0.25(20) = 0.40(x+20)$$
$$x + 5 = 0.4x + 8$$
$$0.6x = 3$$
$$x = \frac{30}{6} = 5$$

5 pounds of pure cement should be added.

49. Let t represent the time for the tub to fill with the faucets on and the stopper removed. Since one tub is being filled, we have:

$$\frac{t}{15} + \left(-\frac{t}{20}\right) = 1$$
$$4t - 3t = 60$$
$$t = 60$$

60 minutes is required to fill the tub.

50. Let t be the time the 5 horsepower pump needs to run to finish emptying the pool. Since the two pumps are emptying one pool, we have:

$$\frac{t+2}{5} + \frac{2}{8} = 1$$
$$4(2+t) + 5 = 20$$
$$8 + 4t + 5 = 20$$
$$4t = 7$$
$$t = 1.75$$

The 5 horsepower pump must run for an additional 1.75 hours or 1 hour and 45 minutes to empty the pool.

51. Let t represent the time spent running. Then $5-t$ represents the time spent biking.

	Rate	Time	Distance
Run	6	t	$6t$
Bike	25	$5-t$	$25(5-t)$

The total distance is 87 miles:

$$6t + 25(5-t) = 87$$
$$6t + 125 - 25t = 87$$
$$-19t + 125 = 87$$
$$-19t = -38$$
$$t = 2$$

The time spent running is 2 hours, so the distance of the run is $6(2) = 12$ miles. The distance of the bicycle race is $25(5-2) = 75$ miles.

52. Let r represent the speed of the eastbound cyclist. Then $r+5$ represents the speed of the westbound cyclist.

	Rate	Time	Distance
Eastbound	r	6	$6r$
Westbound	$r+5$	6	$6(r+5)$

The total distance is 246 miles:

$$6r + 6(r+5) = 246$$
$$6r + 6r + 30 = 246$$
$$12r + 30 = 246$$
$$12r = 216$$
$$r = 18$$

The speed of the eastbound cyclist is 18 miles per hour, and the speed of the westbound cyclist is $18+5=23$ miles per hour.

53. Burke's rate is $\frac{100}{12}$ meters/sec. In 9.99 seconds, Burke will run $\frac{100}{12}(9.99)=83.25$ meters. Lewis would win by 16.75 meters.

54. $A=2\pi r^2+2\pi rh$. Since $A=188.5$ square inches and $h=7$ inches,

$$2\pi r^2+2\pi r(7)=188.5$$

$$2\pi r^2+14\pi r-188.5=0$$

$$r=\frac{-14\pi\pm\sqrt{(14\pi)^2-4(2\pi)(-188.5)}}{2(2\pi)}.$$

$$=\frac{-14\pi\pm\sqrt{6671.9642}}{4\pi}$$

$r\approx 3$ or ~~$r\approx -10$~~

The radius of the coffee can is approximately 3 inches.

55. Let x = length of side of original sheet in feet.
Length of box: $x-2$ feet
Width of box: $x-2$ feet
Height of box: 1 foot

$$V=l\cdot w\cdot h$$

$$4=(x-2)(x-2)(1)$$

$$4=x^2-4x+4$$

$$0=x^2-4x$$

$$0=x(x-4)$$

$x=0$ or $x=4$

Discard $x=0$ since that is not a feasible length for the original sheet. Therefore, the original sheet should measure 4 feet on each side.

56. Let x = width of original sheet in feet.
Length of sheet: $2x$
Length of box: $2x-2$ feet
Width of box: $x-2$ feet
Height of box: 1 foot

$$V=l\cdot w\cdot h$$

$$4=(2x-2)(x-2)(1)$$

$$4=2x^2-6x+4$$

$$0=2x^2-6x$$

$$0=x^2-3x$$

$$0=x(x-3)$$

$x=0$ or $x=3$

Discard $x=0$ since that is not a feasible length for the original sheet. Therefore, the original sheet is 3 feet wide and 6 feet long.

57. Let x be the original selling price of the shirt.
Profit = Revenue − Cost

$4=x-0.40x-20\rightarrow 24=0.60x\rightarrow x=40$

The original price should be \$40 to ensure a profit of \$4 after the sale.

If the sale is 50% off, the profit is:

$40-0.50(40)-20=40-20-20=0$

At 50% off there will be no profit.

58. Answers will vary.

59. It is impossible to mix two solutions with a lower concentration and end up with a new solution with a higher concentration.

<u>Algebraic Solution:</u>
Let x = the number of liters of 25% solution.

$$(\%)(\text{liters})+(\%)(\text{liters})=(\%)(\text{liters})$$

$$0.25x+0.48(20)=0.58(20+x)$$

$$0.25x+9.6=10.6+0.58x$$

$$-0.33x=1$$

$$x\approx -3.03 \text{ liters}$$

(not possible)

60. Let t_1 and t_2 represent the times for the two segments of the trip. Since Atlanta is halfway between Chicago and Miami, the distances are equal.

$$45t_1=55t_2$$

$$t_1=\frac{55}{45}t_2$$

$$t_1=\frac{11}{9}t_2$$

Computing the average speed:

$$\text{Avg Speed} = \frac{\text{Distance}}{\text{Time}} = \frac{45t_1 + 55t_2}{t_1 + t_2}$$
$$= \frac{45\left(\frac{11}{9}t_2\right) + 55t_2}{\frac{11}{9}t_2 + t_2} = \frac{55t_2 + 55t_2}{\left(\frac{11t_2 + 9t_2}{9}\right)}$$
$$= \frac{110t_2}{\left(\frac{20t_2}{9}\right)} = \frac{990t_2}{20t_2}$$
$$= \frac{99}{2} = 49.5 \text{ miles per hour}$$

The average speed for the trip from Chicago to Miami is 49.5 miles per hour.

61. The time traveled with the tail wind was:
$t = \frac{919}{550} \approx 1.67091$ hours .

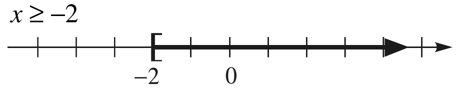
Since they were 20 minutes $\left(\frac{1}{3}\text{ hour}\right)$ early, the time in still air would have been:

$$1.67091 \text{ hrs } + 20 \text{ min} = (1.67091 + 0.33333) \text{ hrs}$$
$$\approx 2.00424 \text{ hrs}$$

Thus, with no wind, the ground speed is $\frac{919}{2.00424} \approx 458.53$. Therefore, the tail wind is $550 - 458.53 = 91.47$ knots .

Section A.9

1. $x \geq -2$



2. False. -5 is to the left of -3 on the number line, so $-5 > -3$.

3. $|-2| = 2$

4. True

5. negative

6. closed interval

7. $-5, 5$

8. $-5 < x < 5$

9. True

10. True

11. Interval: $[0, 2]$
Inequality: $0 \leq x \leq 2$

12. Interval: $(-1, 2)$
Inequality: $-1 < x < 2$

13. Interval: $[2, \infty)$
Inequality: $x \geq 2$

14. Interval: $(-\infty, 0]$
Inequality: $x \leq 0$

15. Interval: $[0, 3)$
Inequality: $0 \leq x < 3$

16. Interval: $(-1, 1]$
Inequality: $-1 < x \leq 1$

17. **a.**
$$3 < 5$$
$$3 + 3 < 5 + 3$$
$$6 < 8$$

b.
$$3 < 5$$
$$3 - 5 < 5 - 5$$
$$-2 < 0$$

c.
$$3 < 5$$
$$3(3) < 3(5)$$
$$9 < 15$$

d.
$$3 < 5$$
$$-2(3) > -2(5)$$
$$-6 > -10$$

18. **a.**
$$2 > 1$$
$$2 + 3 > 1 + 3$$
$$5 > 4$$

b.
$$2 > 1$$
$$2 - 5 > 1 - 5$$
$$-3 > -4$$

c.
$$2 > 1$$
$$3(2) > 3(1)$$
$$6 > 3$$

d. $2 > 1$
$-2(2) < -2(1)$
$-4 < -2$

19. a. $4 > -3$
$4 + 3 > -3 + 3$
$7 > 0$

b. $4 > -3$
$4 - 5 > -3 - 5$
$-1 > -8$

c. $4 > -3$
$3(4) > 3(-3)$
$12 > -9$

d. $4 > -3$
$-2(4) < -2(-3)$
$-8 < 6$

20. a. $-3 > -5$
$-3 + 3 > -5 + 3$
$0 > -2$

b. $-3 > -5$
$-3 - 5 > -5 - 5$
$-8 > -10$

c. $-3 > -5$
$3(-3) > 3(-5)$
$-9 > -15$

d. $-3 > -5$
$-2(-3) < -2(-5)$
$6 < 10$

21. a. $2x + 1 < 2$
$2x + 1 + 3 < 2 + 3$
$2x + 4 < 5$

b. $2x + 1 < 2$
$2x + 1 - 5 < 2 - 5$
$2x - 4 < -3$

c. $2x + 1 < 2$
$3(2x + 1) < 3(2)$
$6x + 3 < 6$

d. $2x + 1 < 2$
$-2(2x + 1) > -2(2)$
$-4x - 2 > -4$

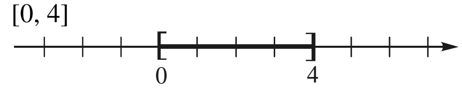
22. a. $1 - 2x > 5$
$1 - 2x + 3 > 5 + 3$
$4 - 2x > 8$

b. $1 - 2x > 5$
$1 - 2x - 5 > 5 - 5$
$-4 - 2x > 0$

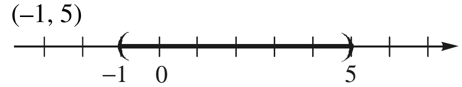
c. $1 - 2x > 5$
$3(1 - 2x) > 3(5)$
$3 - 6x > 15$

d. $1 - 2x > 5$
$-2(1 - 2x) < -2(5)$
$-2 + 4x < -10$

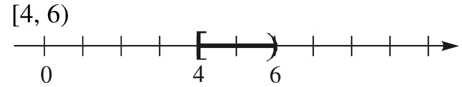
23. $[0, 4]$



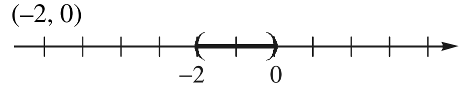
24. $(-1, 5)$



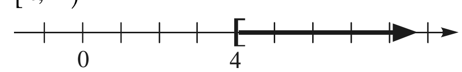
25. $[4, 6)$



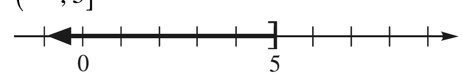
26. $(-2, 0)$



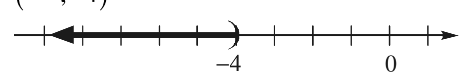
27. $[4, \infty)$



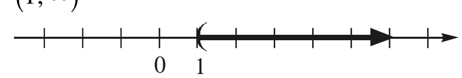
28. $(-\infty, 5]$



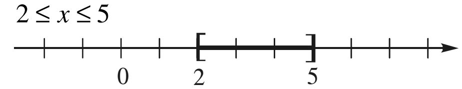
29. $(-\infty, -4)$



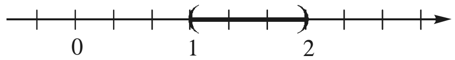
30. $(1, \infty)$



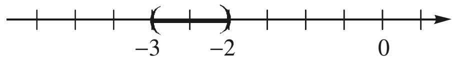
31. $2 \le x \le 5$



32. $1 < x < 2$

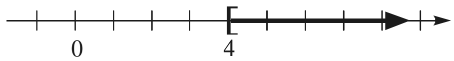


33. $-3 < x < -2$

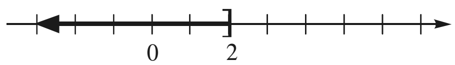


34. $0 \le x < 1$

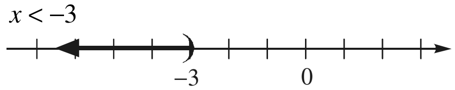
35. $x \ge 4$



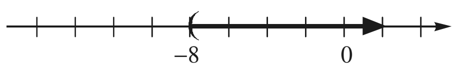
36. $x \le 2$



37. $x < -3$



38. $x > -8$



39. If $x < 5$, then $x - 5 < 0$.

40. If $x < -4$, then $x + 4 < 0$.

41. If $x > -4$, then $x + 4 > 0$.

42. If $x > 6$, then $x - 6 > 0$.

43. If $x \ge -4$, then $3x \ge -12$.

44. If $x \le 3$, then $2x \le 6$.

45. If $x > 6$, then $-2x < -12$.

46. If $x > -2$, then $-4x < 8$.

47. If $x \ge 5$, then $-4x \le -20$.

48. If $x \le -4$, then $-3x \ge 12$.

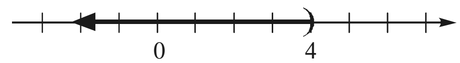
49. If $2x > 6$, then $x > 3$.

50. If $3x \le 12$, then $x \le 4$.

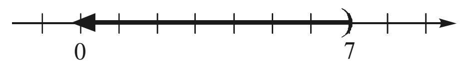
51. If $-\frac{1}{2}x \le 3$, then $x \ge -6$.

52. If $-\frac{1}{4}x > 1$, then $x < -4$.

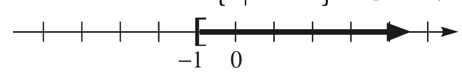
53.
$$x + 1 < 5$$
$$x + 1 - 1 < 5 - 1$$
$$x < 4$$
$\{x \mid x < 4\}$ or $(-\infty, 4)$



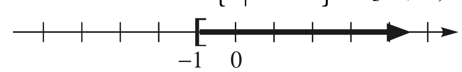
54.
$$x - 6 < 1$$
$$x - 6 + 6 < 1 + 6$$
$$x < 7$$
The solution set is $\{x \mid x < 7\}$ or $(-\infty, 7)$.



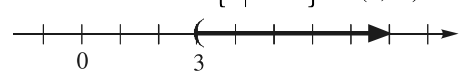
55.
$$1 - 2x \le 3$$
$$-2x \le 2$$
$$x \ge -1$$
The solution set is $\{x \mid x \ge -1\}$ or $[-1, \infty)$.



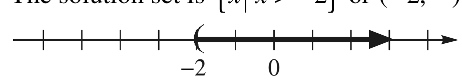
56.
$$2 - 3x \le 5$$
$$-3x \le 3$$
$$x \ge -1$$
The solution set is $\{x \mid x \ge -1\}$ or $[-1, \infty)$.



57.
$$3x - 7 > 2$$
$$3x > 9$$
$$x > 3$$
The solution set is $\{x \mid x > 3\}$ or $(3, \infty)$.



58.
$$2x + 5 > 1$$
$$2x > -4$$
$$x > -2$$
The solution set is $\{x \mid x > -2\}$ or $(-2, \infty)$.

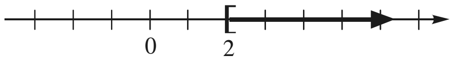


59. $3x-1\geq 3+x$

$2x\geq 4$

$x\geq 2$

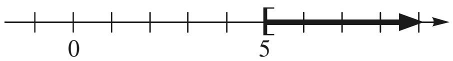
The solution set is $\{x|\, x\geq 2\}$ or $[2,\infty)$.



60. $2x-2\geq 3+x$

$x\geq 5$

The solution set is $\{x|\, x\geq 5\}$ or $[5,\infty)$.



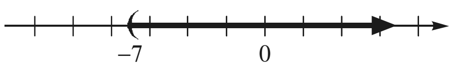
61. $-2(x+3)<8$

$-2x-6<8$

$-2x<14$

$x>-7$

The solution set is $\{x|\, x>-7\}$ or $(-7,\infty)$.



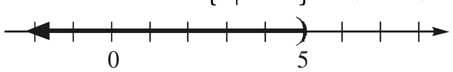
62. $-3(1-x)<12$

$-3+3x<12$

$3x<15$

$x<5$

The solution set is $\{x|\, x<5\}$ or $(-\infty,5)$.



63. $4-3(1-x)\leq 3$

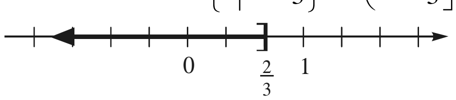
$4-3+3x\leq 3$

$3x+1\leq 3$

$3x\leq 2$

$x\leq \frac{2}{3}$

The solution set is $\left\{x \middle| x\leq \frac{2}{3}\right\}$ or $\left(-\infty,\frac{2}{3}\right]$.



64. $8-4(2-x)\leq -2x$

$8-8+4x\leq -2x$

$4x\leq -2x$

$6x\leq 0$

$x\leq 0$

The solution set is $\{x|\, x\leq 0\}$ or $(-\infty,0]$.

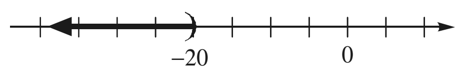
65. $\frac{1}{2}(x-4)>x+8$

$\frac{1}{2}x-2>x+8$

$-\frac{1}{2}x>10$

$x<-20$

The solution set is $\{x|\, x<-20\}$ or $(-\infty,-20)$.



66. $3x+4>\frac{1}{3}(x-2)$

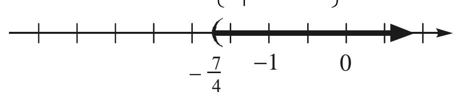
$3x+4>\frac{1}{3}x-\frac{2}{3}$

$9x+12>x-2$

$8x>-14$

$x>-\frac{7}{4}$

The solution set is $\left\{x \middle| x>-\frac{7}{4}\right\}$ or $(-\frac{7}{4},\infty)$.



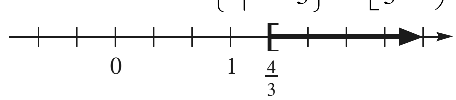
67. $\frac{x}{2}\geq 1-\frac{x}{4}$

$2x\geq 4-x$

$3x\geq 4$

$x\geq \frac{4}{3}$

The solution set is $\left\{x \middle| x\geq \frac{4}{3}\right\}$ or $\left[\frac{4}{3},\infty\right)$.

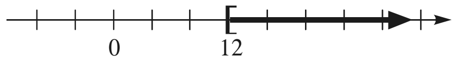


68. $\frac{x}{3} \ge 2 + \frac{x}{6}$

$2x \ge 12 + x$

$x \ge 12$

The solution set is $\{x \mid x \ge 12\}$ or $[12, \infty)$.

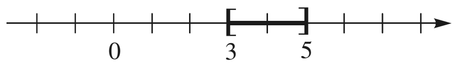


69. $0 \le 2x - 6 \le 4$

$6 \le 2x \le 10$

$3 \le x \le 5$

The solution set is $\{x \mid 3 \le x \le 5\}$ or $[3, 5]$.

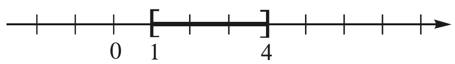


70. $4 \le 2x + 2 \le 10$

$2 \le 2x \le 8$

$1 \le x \le 4$

The solution set is $\{x \mid 1 \le x \le 4\}$ or $[1, 4]$.

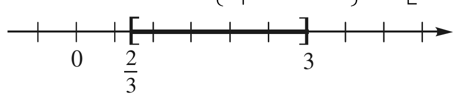


71. $-5 \le 4 - 3x \le 2$

$-9 \le -3x \le -2$

$3 \ge x \ge \frac{2}{3}$

The solution set is $\left\{x \,\middle|\, \frac{2}{3} \le x \le 3\right\}$ or $\left[\frac{2}{3}, 3\right]$.

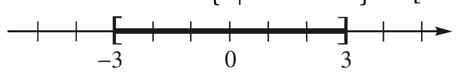


72. $-3 \le 3 - 2x \le 9$

$-6 \le -2x \le 6$

$3 \ge x \ge -3$

The solution set is $\{x \mid -3 \le x \le 3\}$ or $[-3, 3]$.



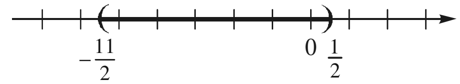
73. $-3 < \frac{2x-1}{4} < 0$

$-12 < 2x - 1 < 0$

$-11 < 2x < 1$

$-\frac{11}{2} < x < \frac{1}{2}$

The solution set is $\left\{x \,\middle|\, -\frac{11}{2} < x < \frac{1}{2}\right\}$ or $\left(-\frac{11}{2}, \frac{1}{2}\right)$.



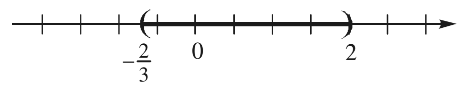
74. $0 < \frac{3x+2}{2} < 4$

$0 < 3x + 2 < 8$

$-2 < 3x < 6$

$-\frac{2}{3} < x < 2$

The solution set is $\left\{x \,\middle|\, -\frac{2}{3} < x < 2\right\}$ or $\left(-\frac{2}{3}, 2\right)$.

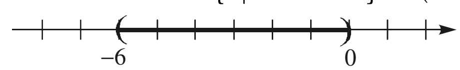


75. $1 < 1 - \frac{1}{2}x < 4$

$0 < -\frac{1}{2}x < 3$

$0 > x > -6$ or $-6 < x < 0$

The solution set is $\{x \mid -6 < x < 0\}$ or $(-6, 0)$.

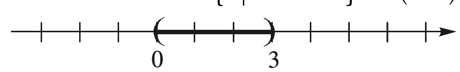


76. $0 < 1 - \frac{1}{3}x < 1$

$-1 < -\frac{1}{3}x < 0$

$3 > x > 0$ or $0 < x < 3$

The solution set is $\{x \mid 0 < x < 3\}$ or $(0, 3)$.



77. $(x+2)(x-3) > (x-1)(x+1)$

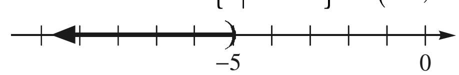
$x^2 - x - 6 > x^2 - 1$

$-x - 6 > -1$

$-x > 5$

$x < -5$

The solution set is $\{x \mid x < -5\}$ or $(-\infty, -5)$.



78. $(x-1)(x+1) > (x-3)(x+4)$

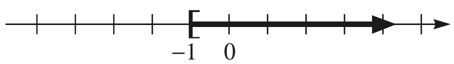
$$x^2-1 > x^2+x-12$$
$$-1 > x-12$$
$$-x > -11$$
$$x < 11$$

The solution set is $\{x \mid x < 11\}$ or $(-\infty, 11)$.

79. $x(4x+3) \le (2x+1)^2$

$$4x^2+3x \le 4x^2+4x+1$$
$$3x \le 4x+1$$
$$-x \le 1$$
$$x \ge -1$$

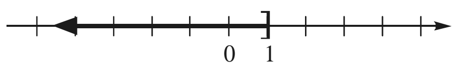
The solution set is $\{x \mid x \ge -1\}$ or $[-1, \infty)$.



80. $x(9x-5) \le (3x-1)^2$

$$9x^2-5x \le 9x^2-6x+1$$
$$-5x \le -6x+1$$
$$x \le 1$$

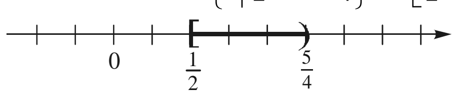
The solution set is $\{x \mid x \le 1\}$ or $(-\infty, 1]$.



81. $\frac{1}{2} \le \frac{x+1}{3} < \frac{3}{4}$

$$6 \le 4x+4 < 9$$
$$2 \le 4x < 5$$
$$\frac{1}{2} \le x < \frac{5}{4}$$

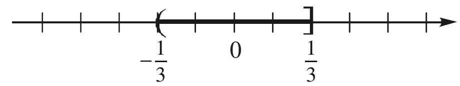
The solution set is $\left\{x \middle| \frac{1}{2} \le x < \frac{5}{4}\right\}$ or $\left[\frac{1}{2}, \frac{5}{4}\right)$.



82. $\frac{1}{3} < \frac{x+1}{2} \le \frac{2}{3}$

$$2 < 3x+3 \le 4$$
$$-1 < 3x \le 1$$
$$-\frac{1}{3} < x \le \frac{1}{3}$$

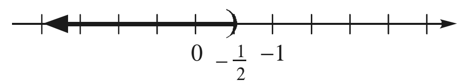
The solution set is $\left\{x \middle| -\frac{1}{3} < x \le \frac{1}{3}\right\}$ or $\left(-\frac{1}{3}, \frac{1}{3}\right]$.



83. $(4x+2)^{-1} < 0$

$$\frac{1}{4x+2} < 0$$
$$4x+2 < 0$$
$$x < -\frac{1}{2}$$

The solution set is $\left\{x \middle| x < -\frac{1}{2}\right\}$ or $\left(-\infty, -\frac{1}{2}\right)$.



84. $(2x-1)^{-1} > 0$

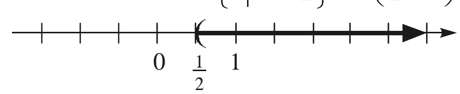
$$\frac{1}{2x-1} > 0$$

Since $\frac{1}{2x-1} > 0$, this means $2x-1 > 0$.

Therefore,

$$2x-1 > 0$$
$$x > \frac{1}{2}$$

The solution set is $\left\{x \middle| x > \frac{1}{2}\right\}$ or $\left(\frac{1}{2}, \infty\right)$.



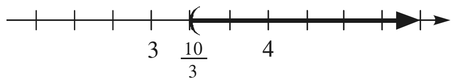
85. $0 < \frac{2}{x} < \frac{3}{5}$

$0 < \frac{2}{x}$ and $\frac{2}{x} < \frac{3}{5}$

Since $\frac{2}{x} > 0$, this means that $x > 0$. Therefore,

$$\frac{2}{x}<\frac{3}{5}$$
$$5x\left(\frac{2}{x}\right)<5x\left(\frac{3}{5}\right)$$
$$10<3x$$
$$\frac{10}{3}<x$$

The solution set is $\left\{x \middle| x>\frac{10}{3}\right\}$ or $\left(\frac{10}{3},\infty\right)$.



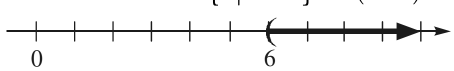
86. $0<\frac{4}{x}<\frac{2}{3}$

$0<\frac{4}{x}$ and $\frac{4}{x}<\frac{2}{3}$

Since $\frac{4}{x}>0$, this means that $x>0$. Therefore,

$$\frac{4}{x}<\frac{2}{3}$$
$$3x\left(\frac{4}{x}\right)<3x\left(\frac{2}{3}\right)$$
$$12<2x$$
$$6<x$$

The solution set is $\{x|x>6\}$ or $(6,\infty)$.



87. $0<(2x-4)^{-1}<\frac{1}{2}$

$0<\frac{1}{2x-4}<\frac{1}{2}$

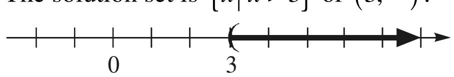
$0<\frac{1}{2x-4}$ and $\frac{1}{2x-4}<\frac{1}{2}$

Since $\frac{1}{2x-4}>0$, this means that $2x-4>0$.

Therefore,

$$\frac{1}{2x-4}<\frac{1}{2}$$
$$\frac{1}{2(x-2)}<\frac{1}{2}$$
$$2(x-2)\left(\frac{1}{2(x-2)}\right)<2(x-2)\left(\frac{1}{2}\right)$$
$$1<x-2$$
$$3<x$$

The solution set is $\{x|x>3\}$ or $(3,\infty)$.



88. $0<(3x+6)^{-1}<\frac{1}{3}$

$0<\frac{1}{3x+6}<\frac{1}{3}$

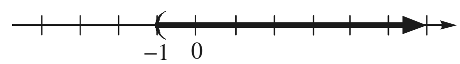
$0<\frac{1}{3x+6}$ and $\frac{1}{3x+6}<\frac{1}{3}$

Since $\frac{1}{3x+6}>0$, this means that $3x+6>0$.

Therefore,

$$\frac{1}{3x+6}<\frac{1}{3}$$
$$\frac{1}{3(x+2)}<\frac{1}{3}$$
$$3(x+2)\left(\frac{1}{3(x+2)}\right)<3(x+2)\left(\frac{1}{3}\right)$$
$$1<x+2$$
$$-1<x$$

The solution set is $\{x|x>-1\}$ or $(-1,\infty)$.

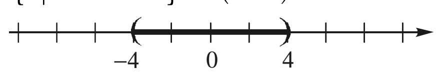


89. $|2x|<8$

$-8<2x<8$

$-4<x<4$

$\{x|-4<x<4\}$ or $(-4,4)$

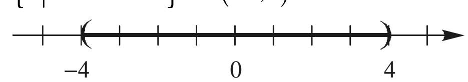


90. $|3x|<12$

$-12<3x<12$

$-4<x<4$

$\{x|-4<x<4\}$ or $(-4,4)$

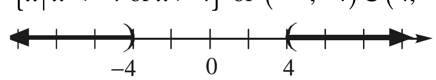


91. $|3x|>12$

$3x<-12$ or $3x>12$

$x<-4$ or $x>4$

$\{x|x<-4 \text{ or } x>4\}$ or $(-\infty,-4)\cup(4,\infty)$

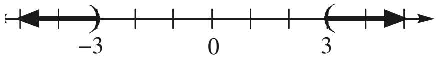


92. $|2x|>6$

$2x<-6$ or $2x>6$

$x<-3$ or $x>3$

$\{x|x<-3 \text{ or } x>3\}$ or $(-\infty,-3)\cup(3,\infty)$



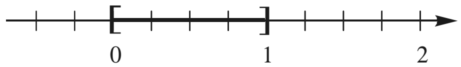
93. $|2x-1| \le 1$

$-1 \le 2x-1 \le 1$

$0 \le 2x \le 2$

$0 \le x \le 1$

$\{x \mid 0 \le x \le 1\}$ or $[0,1]$



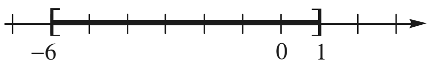
94. $|2x+5| \le 7$

$-7 \le 2x+5 \le 7$

$-12 \le 2x \le 2$

$-6 \le x \le 1$

$\{x \mid -6 \le x \le 1\}$ or $[-6,1]$



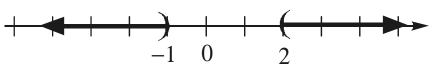
95. $|1-2x| > 3$

$1-2x < -3$ or $1-2x > 3$

$-2x < -4$ or $-2x > 2$

$x > 2$ or $x < -1$

$\{x \mid x < -1 \text{ or } x > 2\}$ or $(-\infty,-1)\cup(2,\infty)$



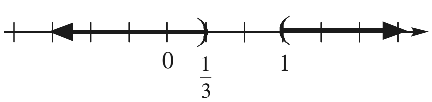
96. $|2-3x| > 1$

$2-3x < -1$ or $2-3x > 1$

$-3x < -3$ or $-3x > -1$

$x > 1$ or $x < \frac{1}{3}$

$\left\{x \,\middle|\, x < \frac{1}{3} \text{ or } x > 1\right\}$ or $\left(-\infty,\frac{1}{3}\right)\cup(1,\infty)$



97. $|-4x|+|-5| \le 9$

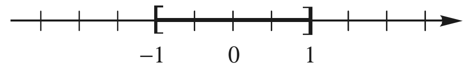
$|-4x|+5 \le 9$

$|-4x| \le 4$

$-4 \le 4x \le 4$

$-1 \le x \le 1$

$\{x \mid -1 \le x \le 1\}$ or $[-1,1]$



98. $|-x|-|4| \le 2$

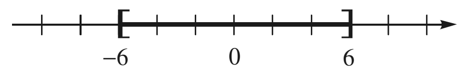
$|-x|-4 \le 2$

$|-x| \le 6$

$-6 \le -x \le 6$

$6 \ge x \ge -6$

$\{x \mid -6 \le x \le 6\}$ or $[-6, 6]$



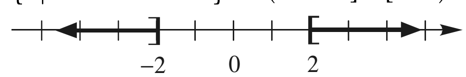
99. $|-2x| \ge |-4|$

$|2x| \ge 4$

$2x \le -4$ or $2x \ge 4$

$x \le -2$ or $x \ge 2$

$\{x \mid x \le -2 \text{ or } x \ge 2\}$ or $(-\infty,-2]\cup[2,\infty)$



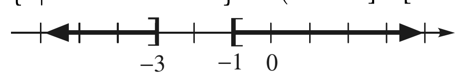
100. $|-x-2| \ge 1$

$-x-2 \le -1$ or $-x-2 \ge 1$

$-x \le 1$ or $-x \ge 3$

$x \ge -1$ or $x \le -3$

$\{x \mid x \le -3 \text{ or } x \ge -1\}$ or $(-\infty,-3]\cup[-1,\infty)$



101. x differs from 2 by less than $\frac{1}{2}$.

$|x-2| < \frac{1}{2}$

$-\frac{1}{2} < x-2 < \frac{1}{2}$

$\frac{3}{2} < x < \frac{5}{2}$

$\left\{x \,\middle|\, \frac{3}{2} < x < \frac{5}{2}\right\}$

102. x differs from -1 by less than 1

$$\left|x-(-1)\right|<1$$
$$\left|x+1\right|<1$$
$$-1<x+1<1$$
$$-2<x<0$$
$$\{x \mid -2<x<0\}$$

103. x differs from -3 by more than 2.

$$\left|x-(-3)\right|>2$$
$$\left|x+3\right|>2$$
$$x+3<-2 \text{ or } x+3>2$$
$$x<-5 \text{ or } x>-1$$
$$\{x \mid x<-5 \text{ or } x>-1\}$$

104. x differs from 2 by more than 3.

$$\left|x-2\right|>3$$
$$x-2<-3 \text{ or } x-2>3$$
$$x<-1 \text{ or } x>5$$
$$\{x \mid x<-1 \text{ or } x>5\}$$

105. $\sqrt{3x+6}$

We need $3x+6\geq 0$

$$3x\geq -6$$
$$x\geq -2$$

To the domain is $\{x \mid x\geq -2\}$ or $[-2,\infty)$.

106. $\sqrt{8+2x}$

We need $8+2x\geq 0$

$$2x\geq -8$$
$$x\geq -4$$

To the domain is $\{x \mid x\geq -4\}$ or $[-4,\infty)$.

107. 21 < young adult's age < 30

108. 40 ≤ middle-aged < 60

109. **a.** Let x = age at death.

$$x-30\geq 46.60$$
$$x\geq 76.60$$

Therefore, the average life expectancy for a 30-year-old male in 2005 will be greater than or equal to 76.60 years.

b. Let x = age at death.

$$x-30\geq 51.03$$
$$x\geq 81.03$$

Therefore, the average life expectancy for a 30-year-old female in 2005 will be greater than or equal to 81.03 years.

c. By the given information, a female can expect to live $81.03-76.60=4.43$ years longer.

110. $V=20T$

$$80°\leq T\leq 120°$$
$$80°\leq \frac{V}{20}\leq 120°$$
$$1600\leq V\leq 2400$$

The volume ranges from 1600 to 2400 cubic centimeters, inclusive.

111. Let P represent the selling price and C represent the commission.

Calculating the commission:

$$C=45,000+0.25(P-900,000)$$
$$=45,000+0.25P-225,000$$
$$=0.25P-180,000$$

Calculate the commission range, given the price range:

$$900,000\leq P\leq 1,100,000$$
$$0.25(900,000)\leq 0.25P\leq 0.25(1,100,000)$$
$$225,000\leq 0.25P\leq 275,000$$
$$225,000-180,000\leq 0.25P-180,000\leq 275,000-180,000$$
$$45,000\leq C\leq 95,000$$

The agent's commission ranges from \$45,000 to \$95,000, inclusive.

$$\frac{45,000}{900,000}=0.05=5\% \text{ to } \frac{95,000}{1,100,000}=0.086=8.6\%,$$

inclusive.

As a percent of selling price, the commission ranges from 5% to 8.6%, inclusive.

112. Let C represent the commission.
Calculate the commission range:
$$25+0.4(200)\le C\le 25+0.4(3000)$$
$$105\le C\le 1225$$
The commissions are at least \$105 and at most \$1225.

113. Let W = weekly wages and T = tax withheld.
Calculating the withholding tax range, given the range of weekly wages:
$$700\le W\le 900$$
$$700-693\le W-693\le 900-693$$
$$7\le W-693\le 207$$
$$0.25(7)\le 0.25(W-693)\le 0.25(207)$$
$$1.75\le 0.25(W-693)\le 51.75$$
$$1.75+82.35\le 0.25(W-693)+82.35\le 51.75+82.35$$
$$84.10\le T\le 134.10$$
The amount withheld varies from \$84.10 to \$134.10, inclusive.

114. Let x represent the length of time Sue should exercise on the seventh day.
$$200\le 40+45+0+50+25+35+x\le 300$$
$$200\le 195+x\le 300$$
$$5\le x\le 105$$
Sue will stay within the ACSM guidelines by exercising from 5 to 105 minutes.

115. Let K represent the monthly usage in kilowatt-hours and let C represent the monthly customer bill.
Calculating the bill: $C=0.0944K+12.55$
Calculating the range of kilowatt-hours, given the range of bills:
$$76.27\le C\le 248.55$$
$$76.27\le 0.0944K+12.55\le 248.55$$
$$63.72\le 0.0944K\le 236.00$$
$$675.00\le K\le 2500.00$$
The usage varies from 675.00 kilowatt-hours to 2500.00 kilowatt-hours, inclusive.

116. Let W represent the amount of water used (in thousands of gallons). Let C represent the customer charge (in dollars).
Calculating the charge:
$$C=37.62+3.86(W-12)$$
$$=37.62+3.86W-46.32$$
$$=3.86W-8.70$$
Calculating the range of water usage, given the range of charges:
$$68.50\le C\le 122.54$$
$$68.50\le 3.86W-8.70\le 122.54$$
$$77.20\le 3.86W\le 131.24$$
$$20\le W\le 34$$
The range of water usage ranged from 20,000 to 34,000 gallons.

117. Let C represent the dealer's cost and M represent the markup over dealer's cost.
If the price is \$18,000, then
$$18,000=C+MC=C(1+M)$$
Solving for C yields: $C=\dfrac{18,000}{1+M}$
Calculating the range of dealer costs, given the range of markups:
$$0.12\le M\le 0.18$$
$$1.12\le 1+M\le 1.18$$
$$\frac{1}{1.12}\ge\frac{1}{1+M}\ge\frac{1}{1.18}$$
$$\frac{18,000}{1.12}\ge\frac{18,000}{1+M}\ge\frac{18,000}{1.18}$$
$$16,071.43\ge C\ge 15,254.24$$
The dealer's cost varies from \$15,254.24 to \$16,071.43, inclusive.

118. Let T represent the test scores of the people in the top 2.5%.
$$T>1.96(12)+100=123.52$$
People in the top 2.5% will have test scores greater than 123.52. That is, $T>123.52$ or $(123.52,\infty)$.

119. **a.** Let T represent the score on the last test and G represent the course grade.
Calculating the course grade and solving for the last test:

$$G = \frac{68+82+87+89+T}{5}$$
$$G = \frac{326+T}{5}$$
$$5G = 326+T$$
$$T = 5G-326$$

Calculating the range of scores on the last test, given the grade range:

$$80 \le G < 90$$
$$400 \le 5G < 450$$
$$74 \le 5G-326 < 124$$
$$74 \le T < 124$$

To get a grade of B, you need at least a 74 on the fifth test.

b. Let T represent the score on the last test and G represent the course grade.
Calculating the course grade and solving for the last test:

$$G = \frac{68+82+87+89+2T}{6}$$
$$G = \frac{326+2T}{6}$$
$$G = \frac{163+T}{3}$$
$$T = 3G-163$$

Calculating the range of scores on the last test, given the grade range:

$$80 \le G < 90$$
$$240 \le 3G < 270$$
$$77 \le 3G-163 < 107$$
$$77 \le T < 107$$

To get a grade of B, you need at least a 77 on the fifth test.

120. Let C represent the number of calories in a serving of regular Miracle Whip®, and let F represent the grams of fat in a serving of regular Miracle Whip®.

One possibility for a "light" classification is that the 20 calories in a serving of Miracle Whip® Light is less than or equal to one-third the calories in regular Miracle Whip®. That is, $20 \le \frac{1}{3}C$.

The second possibility for a "light" classification is that the 1.5 grams of fat in a serving of Miracle Whip® Light is less than or equal to one-half the grams of fat in regular Miracle Whip®. That is, $1.5 \le \frac{1}{2}F$.

We have:

$$20 \le \frac{1}{3}C \quad \text{or} \quad 1.5 \le \frac{1}{2}F$$
$$60 \le C \quad \text{or} \quad 3 \le F$$

A serving of regular Miracle Whip® either contains at least 60 calories or at least 3 grams of fat, or both.

121. Since $a < b$,

$$\frac{a}{2} < \frac{b}{2} \quad \text{and} \quad \frac{a}{2} < \frac{b}{2}$$
$$\frac{a}{2}+\frac{a}{2} < \frac{a}{2}+\frac{b}{2} \quad \text{and} \quad \frac{a}{2}+\frac{b}{2} < \frac{b}{2}+\frac{b}{2}$$
$$a < \frac{a+b}{2} \quad \text{and} \quad \frac{a+b}{2} < b$$

Thus, $a < \frac{a+b}{2} < b$.

122. From problem 121, $a < \frac{a+b}{2} < b$, so

$$d\left(a, \frac{a+b}{2}\right) = \frac{a+b}{2} - a = \frac{a+b-2a}{2} = \frac{b-a}{2} \text{ and}$$
$$d\left(b, \frac{a+b}{2}\right) = b - \frac{a+b}{2} = \frac{2b-a-b}{2} = \frac{b-a}{2}.$$

Therefore, $\frac{a+b}{2}$ is equidistant from a and b.

123. If $0 < a < b$, then

$$ab > a^2 > 0 \quad \text{and} \quad b^2 > ab > 0$$
$$\left(\sqrt{ab}\right)^2 > a^2 \quad \text{and} \quad b^2 > \left(\sqrt{ab}\right)^2$$
$$\sqrt{ab} > a \quad \text{and} \quad b > \sqrt{ab}$$

Thus, $a < \sqrt{ab} < b$.

124. Show that $\sqrt{ab} < \dfrac{a+b}{2}$.

$$\frac{a+b}{2} - \sqrt{ab} = \frac{1}{2}\left(a - 2\sqrt{ab} + b\right)$$
$$= \frac{1}{2}\left(\sqrt{a} - \sqrt{b}\right)^2 > 0, \text{ since } a \neq b.$$

Therefore, $\sqrt{ab} < \dfrac{a+b}{2}$.

125. For $0 < a < b$, $\dfrac{1}{h} = \dfrac{1}{2}\left(\dfrac{1}{a} + \dfrac{1}{b}\right)$

$$h \cdot \frac{1}{h} = \frac{1}{2}\left(\frac{b+a}{ab}\right) \cdot h$$
$$1 = \frac{1}{2}\left(\frac{b+a}{ab}\right) \cdot h$$
$$h = \frac{2ab}{a+b}$$
$$h - a = \frac{2ab}{a+b} - a = \frac{2ab - a(a+b)}{a+b}$$
$$= \frac{2ab - a^2 - ab}{a+b} = \frac{ab - a^2}{a+b}$$
$$= \frac{a(b-a)}{a+b} > 0$$

Therefore, $h > a$.

$$b - h = b - \frac{2ab}{a+b} = \frac{b(a+b) - 2ab}{a+b}$$
$$= \frac{ab + b^2 - 2ab}{a+b} = \frac{b^2 - ab}{a+b}$$
$$= \frac{b(b-a)}{a+b} > 0$$

Therefore, $h < b$, and we have $a < h < b$.

126. Show that $h = \dfrac{(\text{geometric mean})^2}{\text{arithmetic mean}} = \dfrac{\left(\sqrt{ab}\right)^2}{\left(\dfrac{1}{2}(a+b)\right)}$

From Problem 125, we know:

$$\frac{1}{h} = \frac{1}{2}\left(\frac{1}{a} + \frac{1}{b}\right)$$
$$\frac{2}{h} = \frac{1}{a} + \frac{1}{b} = \frac{b+a}{ab}$$
$$\frac{h}{2} = \frac{ab}{a+b}$$
$$h = 2 \cdot \frac{ab}{a+b} = \frac{\left(\sqrt{ab}\right)^2}{\left(\dfrac{1}{2}(a+b)\right)}$$

127. Since $0 < a < b$, then $a - b < 0$ and $ab > 0$.

Therefore, $\dfrac{a-b}{ab} < 0$. So,

$$\frac{a}{ab} - \frac{b}{ab} < 0$$
$$\frac{1}{b} - \frac{1}{a} < 0$$
$$\frac{1}{b} < \frac{1}{a}$$

Now, since $b > 0$, then $\dfrac{1}{b} > 0$, so we have

$$0 < \frac{1}{b} < \frac{1}{a}.$$

128. Answers will vary. One possibility:
No solution: $4x + 6 \leq 2(x-5) + 2x$
One solution: $3x + 5 \leq 2(x+3) + 1 \leq 3(x+2) - 1$

129. Since $x^2 \geq 0$, we have

$$x^2 + 1 \geq 0 + 1$$
$$x^2 + 1 \geq 1$$

Therefore, the expression $x^2 + 1$ can never be less than -5.

130 – 131. Answers will vary.

Section A.10

1. 9; -9

2. 4; $|-4| = 4$

3. index

4. True

5. cube root

6. False; $\sqrt[4]{(-3)^4} = |-3| = 3$

7. $\sqrt[3]{27} = \sqrt[3]{3^3} = 3$

8. $\sqrt[4]{16} = \sqrt[4]{2^4} = 2$

9. $\sqrt[3]{-8} = \sqrt[3]{(-2)^3} = -2$

10. $\sqrt[3]{-1}=\sqrt[3]{(-1)^3}=-1$

11. $\sqrt{8}=\sqrt{4\cdot 2}=2\sqrt{2}$

12. $\sqrt[3]{54}=\sqrt[3]{27\cdot 2}=3\sqrt[3]{2}$

13. $\sqrt[3]{-8x^4}=\sqrt[3]{-8x^3\cdot x}=-2x\sqrt[3]{x}$

14. $\sqrt[4]{48x^5}=\sqrt[4]{16x^4\cdot 3x}=2x\sqrt[4]{3x}$

15. $\sqrt[4]{x^{12}y^8}=\sqrt[4]{\left(x^3\right)^4\left(y^2\right)^4}=x^3y^2$

16. $\sqrt[5]{x^{10}y^5}=\sqrt[5]{\left(x^2\right)^5y^5}=x^2y$

17. $\sqrt[4]{\dfrac{x^9y^7}{xy^3}}=\sqrt[4]{x^8y^4}=x^2y$

18. $\sqrt[3]{\dfrac{3xy^2}{81x^4y^2}}=\sqrt[3]{\dfrac{1}{27x^3}}=\dfrac{\sqrt[3]{1}}{\sqrt[3]{27x^3}}=\dfrac{1}{3x}$

19. $\sqrt{36x}=6\sqrt{x}$

20. $\sqrt{9x^5}=3\sqrt{x^4\cdot x}=3x^2\sqrt{x}$

21. $\sqrt{3x^2}\sqrt{12x}=\sqrt{36x^2\cdot x}=6x\sqrt{x}$

22. $\sqrt{5x}\sqrt{20x^3}=\sqrt{100x^4}=10x^2$

23. $\left(\sqrt{5}\sqrt[3]{9}\right)^2=\left(\sqrt{5}\right)^2\left(\sqrt[3]{9}\right)^2$
$=5\cdot\sqrt[3]{9^2}=5\sqrt[3]{81}=5\cdot 3\sqrt[3]{3}=15\sqrt[3]{3}$

24. $\left(\sqrt[3]{3}\sqrt{10}\right)^4=\left(\sqrt[3]{3}\right)^4\left(\sqrt{10}\right)^4$
$=\sqrt[3]{3^4}\cdot 10^2=3\sqrt[3]{3}\cdot 100=300\sqrt[3]{3}$

25. $\left(3\sqrt{6}\right)\left(2\sqrt{2}\right)=6\sqrt{12}=6\sqrt{4\cdot 3}=12\sqrt{3}$

26. $\left(5\sqrt{8}\right)\left(-3\sqrt{3}\right)=-15\sqrt{24}=-30\sqrt{6}$

27. $3\sqrt{2}+4\sqrt{2}=(3+4)\sqrt{2}=7\sqrt{2}$

28. $6\sqrt{5}-4\sqrt{5}=(6-4)\sqrt{5}=2\sqrt{5}$

29. $-\sqrt{18}+2\sqrt{8}=-\sqrt{9\cdot 2}+2\sqrt{4\cdot 2}$
$=-3\sqrt{2}+4\sqrt{2}$
$=(-3+4)\sqrt{2}$
$=\sqrt{2}$

30. $2\sqrt{12}-3\sqrt{27}=2\sqrt{4\cdot 3}-3\sqrt{9\cdot 3}$
$=4\sqrt{3}-9\sqrt{3}$
$=(4-9)\sqrt{3}$
$=-5\sqrt{3}$

31. $\left(\sqrt{3}+3\right)\left(\sqrt{3}-1\right)=\left(\sqrt{3}\right)^2+3\sqrt{3}-\sqrt{3}-3$
$=3+2\sqrt{3}-3$
$=2\sqrt{3}$

32. $\left(\sqrt{5}-2\right)\left(\sqrt{5}+3\right)=\left(\sqrt{5}\right)^2-2\sqrt{5}+3\sqrt{5}-6$
$=5+\sqrt{5}-6$
$=\sqrt{5}-1$

33. $5\sqrt[3]{2}-2\sqrt[3]{54}=5\sqrt[3]{2}-2\cdot 3\sqrt[3]{2}$
$=5\sqrt[3]{2}-6\sqrt[3]{2}$
$=(5-6)\sqrt[3]{2}$
$=-\sqrt[3]{2}$

34. $9\sqrt[3]{24}-\sqrt[3]{81}=9\cdot 2\sqrt[3]{3}-3\sqrt[3]{3}$
$=18\sqrt[3]{3}-3\sqrt[3]{3}$
$=(18-3)\sqrt[3]{3}$
$=15\sqrt[3]{3}$

35. $\left(\sqrt{x}-1\right)^2=\left(\sqrt{x}\right)^2-2\sqrt{x}+1$
$=x-2\sqrt{x}+1$

36. $\left(\sqrt{x}+\sqrt{5}\right)^2=\left(\sqrt{x}\right)^2+2\left(\sqrt{x}\right)\left(\sqrt{5}\right)+\left(\sqrt{5}\right)^2$
$=x+2\sqrt{5x}+5$

37. $\sqrt[3]{16x^4}-\sqrt[3]{2x}=\sqrt[3]{8x^3\cdot 2x}-\sqrt[3]{2x}$
$=2x\sqrt[3]{2x}-\sqrt[3]{2x}$
$=(2x-1)\sqrt[3]{2x}$

38. $\sqrt[4]{32x}+\sqrt[4]{2x^5}=\sqrt[4]{16\cdot 2x}+\sqrt[4]{x^4\cdot 2x}$
$=2\sqrt[4]{2x}+x\sqrt[4]{2x}$
$=(2+x)\sqrt[4]{2x}$ or $(x+2)\sqrt[4]{2x}$

39. $\sqrt{8x^3}-3\sqrt{50x}=\sqrt{4x^2\cdot 2x}-3\sqrt{25\cdot 2x}$
$=2x\sqrt{2x}-15\sqrt{2x}$
$=(2x-15)\sqrt{2x}$

40. $3x\sqrt{9y}+4\sqrt{25y}=9x\sqrt{y}+20\sqrt{y}$
$=(9x+20)\sqrt{y}$

41. $\sqrt[3]{16x^4y}-3x\sqrt[3]{2xy}+5\sqrt[3]{-2xy^4}$
$=\sqrt[3]{8x^3\cdot 2xy}-3x\sqrt[3]{2xy}+5\sqrt[3]{-y^3\cdot 2xy}$
$=2x\sqrt[3]{2xy}-3x\sqrt[3]{2xy}-5y\sqrt[3]{2xy}$
$=(2x-3x-5y)\sqrt[3]{2xy}$
$=(-x-5y)\sqrt[3]{2xy}$ or $-(x+5y)\sqrt[3]{2xy}$

42. $8xy-\sqrt{25x^2y^2}+\sqrt[3]{8x^3y^3}=8xy-5xy+2xy$
$=(8-5+2)xy$
$=5xy$

43. $\dfrac{1}{\sqrt{2}}=\dfrac{1}{\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}}=\dfrac{\sqrt{2}}{2}$

44. $\dfrac{2}{\sqrt{3}}=\dfrac{2}{\sqrt{3}}\cdot\dfrac{\sqrt{3}}{\sqrt{3}}=\dfrac{2\sqrt{3}}{3}$

45. $\dfrac{-\sqrt{3}}{\sqrt{5}}=\dfrac{-\sqrt{3}}{\sqrt{5}}\cdot\dfrac{\sqrt{5}}{\sqrt{5}}=\dfrac{-\sqrt{15}}{5}$

46. $\dfrac{-\sqrt{3}}{\sqrt{8}}=\dfrac{-\sqrt{3}}{2\sqrt{2}}=\dfrac{-\sqrt{3}}{2\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}}=\dfrac{-\sqrt{6}}{2\cdot 2}=\dfrac{-\sqrt{6}}{4}$

47. $\dfrac{\sqrt{3}}{5-\sqrt{2}}=\dfrac{\sqrt{3}}{5-\sqrt{2}}\cdot\dfrac{5+\sqrt{2}}{5+\sqrt{2}}$
$=\dfrac{\sqrt{3}\left(5+\sqrt{2}\right)}{25-2}$
$=\dfrac{\sqrt{3}\left(5+\sqrt{2}\right)}{23}$ or $\dfrac{5\sqrt{3}+\sqrt{6}}{23}$

48. $\dfrac{\sqrt{2}}{\sqrt{7}+2}=\dfrac{\sqrt{2}}{\sqrt{7}+2}\cdot\dfrac{\sqrt{7}-2}{\sqrt{7}-2}$
$=\dfrac{\sqrt{2}\left(\sqrt{7}-2\right)}{7-4}$
$=\dfrac{\sqrt{2}\left(\sqrt{7}-2\right)}{3}$ or $\dfrac{\sqrt{14}-2\sqrt{2}}{3}$

49. $\dfrac{2-\sqrt{5}}{2+3\sqrt{5}}=\dfrac{2-\sqrt{5}}{2+3\sqrt{5}}\cdot\dfrac{2-3\sqrt{5}}{2-3\sqrt{5}}$
$=\dfrac{4-2\sqrt{5}-6\sqrt{5}+15}{4-45}$
$=\dfrac{19-8\sqrt{5}}{-41}=\dfrac{8\sqrt{5}-19}{41}$

50. $\dfrac{\sqrt{3}-1}{2\sqrt{3}+3}=\dfrac{\sqrt{3}-1}{2\sqrt{3}+3}\cdot\dfrac{2\sqrt{3}-3}{2\sqrt{3}-3}$
$=\dfrac{6-2\sqrt{3}-3\sqrt{3}+3}{12-9}=\dfrac{9-5\sqrt{3}}{3}$

51. $\dfrac{5}{\sqrt[3]{2}}=\dfrac{5}{\sqrt[3]{2}}\cdot\dfrac{\sqrt[3]{4}}{\sqrt[3]{4}}=\dfrac{5\sqrt[3]{4}}{2}$

52. $\dfrac{-2}{\sqrt[3]{9}}=\dfrac{-2}{\sqrt[3]{9}}\cdot\dfrac{\sqrt[3]{3}}{\sqrt[3]{3}}=\dfrac{-2\sqrt[3]{3}}{3}$

53. $\dfrac{\sqrt{x+h}-\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}=\dfrac{\sqrt{x+h}-\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}\cdot\dfrac{\sqrt{x+h}-\sqrt{x}}{\sqrt{x+h}-\sqrt{x}}$
$=\dfrac{(x+h)-2\sqrt{x(x+h)}+x}{(x+h)-x}$
$=\dfrac{x+h-2\sqrt{x^2+xh}+x}{x+h-x}$
$=\dfrac{2x+h-2\sqrt{x^2+xh}}{h}$

54. $\dfrac{\sqrt{x+h}+\sqrt{x-h}}{\sqrt{x+h}-\sqrt{x-h}}$

$=\dfrac{\sqrt{x+h}+\sqrt{x-h}}{\sqrt{x+h}-\sqrt{x-h}}\cdot\dfrac{\sqrt{x+h}+\sqrt{x-h}}{\sqrt{x+h}+\sqrt{x-h}}$

$=\dfrac{(x+h)+2\sqrt{(x-h)(x+h)}+(x-h)}{(x+h)-(x-h)}$

$=\dfrac{x+h+2\sqrt{x^2-h^2}+x-h}{x+h-x+h}$

$=\dfrac{2x+2\sqrt{x^2-h^2}}{2h}$

$=\dfrac{x+\sqrt{x^2-h^2}}{h}$

55. $\sqrt[3]{2t-1}=2$

$\left(\sqrt[3]{2t-1}\right)^3=2^3$

$2t-1=8$

$2t=9$

$t=\dfrac{9}{2}$

Check: $\sqrt[3]{2\left(\dfrac{9}{2}\right)-1}=\sqrt[3]{9-1}=\sqrt[3]{8}=2$

The solution set is $\left\{\dfrac{9}{2}\right\}$.

56. $\sqrt[3]{3t+1}=-2$

$\left(\sqrt[3]{3t+1}\right)^3=(-2)^3$

$3t+1=-8$

$3t=-9$

$t=-3$

Check: $\sqrt[3]{3(-3)+1}=\sqrt[3]{-9+1}=\sqrt[3]{-8}=-2$

The solution set is $\{-3\}$.

57. $\sqrt{15-2x}=x$

$\left(\sqrt{15-2x}\right)^2=x^2$

$15-2x=x^2$

$x^2+2x-15=0$

$(x+5)(x-3)=0$

$x=-5$ or $x=3$

Check –5: $\sqrt{15-2(-5)}=\sqrt{25}=5\neq-5$

Check 3: $\sqrt{15-2(3)}=\sqrt{9}=3=3$

Disregard $x=-5$ as extraneous.

The solution set is $\{3\}$.

58. $\sqrt{12-x}=x$

$\left(\sqrt{12-x}\right)^2=x^2$

$12-x=x^2$

$x^2+x-12=0$

$(x+4)(x-3)=0$

$x=-4$ or $x=3$

Check –4: $\sqrt{12-(-4)}=\sqrt{16}=4\neq-4$

Check 3: $\sqrt{12-3}=\sqrt{9}=3=3$

Disregard $x=-4$ as extraneous.

The solution set is $\{3\}$.

59. $8^{2/3}=\left(\sqrt[3]{8}\right)^2=2^2=4$

60. $4^{3/2}=\left(\sqrt{4}\right)^3=2^3=8$

61. $(-27)^{1/3}=\sqrt[3]{-27}=-3$

62. $16^{3/4}=\left(\sqrt[4]{16}\right)^3=2^3=8$

63. $16^{3/2}=\left(\sqrt{16}\right)^3=4^3=64$

64. $25^{3/2}=\left(\sqrt{25}\right)^3=5^3=125$

65. $9^{-3/2}=\dfrac{1}{9^{3/2}}=\dfrac{1}{\left(\sqrt{9}\right)^3}=\dfrac{1}{3^3}=\dfrac{1}{27}$

66. $16^{-3/2}=\dfrac{1}{16^{3/2}}=\dfrac{1}{\left(\sqrt{16}\right)^3}=\dfrac{1}{4^3}=\dfrac{1}{64}$

67. $\left(\frac{9}{8}\right)^{3/2} = \left(\sqrt{\frac{9}{8}}\right)^3 = \left(\frac{3}{2\sqrt{2}}\right)^3 = \frac{3^3}{2^3\left(\sqrt{2}\right)^3}$

$= \frac{27}{8\cdot 2\sqrt{2}} = \frac{27}{16\sqrt{2}} = \frac{27}{16\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{27\sqrt{2}}{32}$

68. $\left(\frac{27}{8}\right)^{2/3} = \left(\sqrt[3]{\frac{27}{8}}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

69. $\left(\frac{8}{9}\right)^{-3/2} = \left(\frac{9}{8}\right)^{3/2} = \left(\sqrt{\frac{9}{8}}\right)^3 = \left(\frac{3}{2\sqrt{2}}\right)^3$

$= \frac{3^3}{2^3\left(\sqrt{2}\right)^3} = \frac{27}{8\cdot 2\sqrt{2}} = \frac{27}{16\sqrt{2}}$

$= \frac{27}{16\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{27\sqrt{2}}{32}$

70. $\left(\frac{8}{27}\right)^{-2/3} = \left(\frac{27}{8}\right)^{2/3} = \left(\sqrt[3]{\frac{27}{8}}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

71. $x^{3/4}x^{1/3}x^{-1/2} = x^{3/4+1/3-1/2} = x^{7/12}$

72. $x^{2/3}x^{1/2}x^{-1/4} = x^{2/3+1/2-1/4} = x^{11/12}$

73. $\left(x^3y^6\right)^{1/3} = \left(x^3\right)^{1/3}\left(y^6\right)^{1/3} = xy^2$

74. $\left(x^4y^8\right)^{3/4} = \left(x^4\right)^{3/4}\left(y^8\right)^{3/4} = x^3y^6$

75. $\frac{\left(x^2y\right)^{1/3}\left(xy^2\right)^{2/3}}{x^{2/3}y^{2/3}} = \frac{\left(x^2\right)^{1/3}\left(y\right)^{1/3}\left(x\right)^{2/3}\left(y^2\right)^{2/3}}{x^{2/3}y^{2/3}}$

$= \frac{x^{2/3}y^{1/3}x^{2/3}y^{4/3}}{x^{2/3}y^{2/3}}$

$= x^{2/3+2/3-2/3}y^{1/3+4/3-2/3}$

$= x^{2/3}y^1 = x^{2/3}y$

76. $\frac{\left(xy\right)^{1/4}\left(x^2y^2\right)^{1/2}}{\left(x^2y\right)^{3/4}} = \frac{x^{1/4}y^{1/4}\left(x^2\right)^{1/2}\left(y^2\right)^{1/2}}{\left(x^2\right)^{3/4}y^{3/4}}$

$= \frac{x^{1/4}y^{1/4}xy}{x^{3/2}y^{3/4}}$

$= x^{1/4+1-3/2}y^{1/4+1-3/4}$

$= x^{-1/4}y^{1/2} = \frac{y^{1/2}}{x^{1/4}}$

77. $\frac{\left(16x^2y^{-1/3}\right)^{3/4}}{\left(xy^2\right)^{1/4}} = \frac{16^{3/4}\left(x^2\right)^{3/4}\left(y^{-1/3}\right)^{3/4}}{x^{1/4}\left(y^2\right)^{1/4}}$

$= \frac{\left(\sqrt[4]{16}\right)^3x^{3/2}y^{-1/4}}{x^{1/4}y^{1/2}}$

$= 2^3x^{3/2-1/4}y^{-1/4-1/2}$

$= 8x^{5/4}y^{-3/4}$

$= \frac{8x^{5/4}}{y^{3/4}}$

78. $\frac{\left(4x^{-1}y^{1/3}\right)^{3/2}}{\left(xy\right)^{3/2}} = \frac{4^{3/2}\left(x^{-1}\right)^{3/2}\left(y^{1/3}\right)^{3/2}}{x^{3/2}y^{3/2}}$

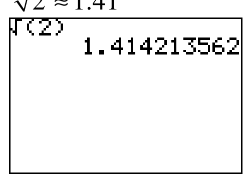
$= \frac{\left(\sqrt{4}\right)^3x^{-3/2}y^{1/2}}{x^{3/2}y^{3/2}}$

$= 2^3x^{-3/2-3/2}y^{1/2-3/2}$

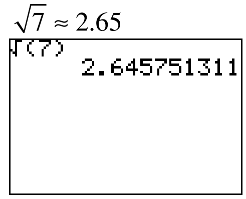
$= 8x^{-3}y^{-1}$

$= \frac{8}{x^3y}$

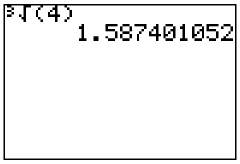
79. $\sqrt{2} \approx 1.41$



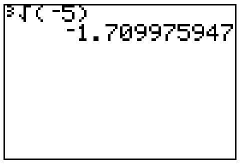
80. $\sqrt{7} \approx 2.65$



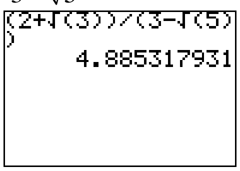
81. $\sqrt[3]{4} \approx 1.59$



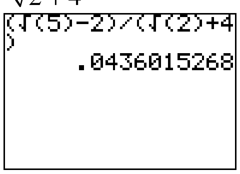
82. $\sqrt[3]{-5} \approx -1.71$



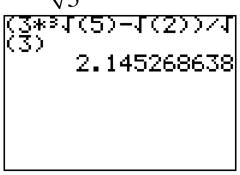
83. $\dfrac{2+\sqrt{3}}{3-\sqrt{5}} \approx 4.89$



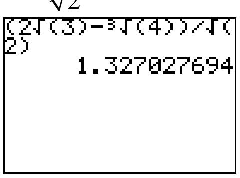
84. $\dfrac{\sqrt{5}-2}{\sqrt{2}+4} \approx 0.04$



85. $\dfrac{3\sqrt[3]{5}-\sqrt{2}}{\sqrt{3}} \approx 2.15$



86. $\dfrac{2\sqrt{3}-\sqrt[3]{4}}{\sqrt{2}} \approx 1.33$



87. $\dfrac{x}{(1+x)^{1/2}} + 2(1+x)^{1/2} = \dfrac{x+2(1+x)^{1/2}(1+x)^{1/2}}{(1+x)^{1/2}}$

$= \dfrac{x+2(1+x)}{(1+x)^{1/2}}$

$= \dfrac{x+2+2x}{(1+x)^{1/2}}$

$= \dfrac{3x+2}{(1+x)^{1/2}}$

88. $\dfrac{1+x}{2x^{1/2}} + x^{1/2} = \dfrac{1+x+x^{1/2}\cdot 2x^{1/2}}{2x^{1/2}}$

$= \dfrac{1+x+2x}{2x^{1/2}} = \dfrac{3x+1}{2x^{1/2}}$

89. $2x\left(x^2+1\right)^{1/2} + x^2 \cdot \dfrac{1}{2}\left(x^2+1\right)^{-1/2} \cdot 2x$

$= 2x\left(x^2+1\right)^{1/2} + \dfrac{x^3}{\left(x^2+1\right)^{1/2}}$

$= \dfrac{2x\left(x^2+1\right)^{1/2}\cdot\left(x^2+1\right)^{1/2} + x^3}{\left(x^2+1\right)^{1/2}}$

$= \dfrac{2x\left(x^2+1\right)^{1/2+1/2} + x^3}{\left(x^2+1\right)^{1/2}} = \dfrac{2x\left(x^2+1\right)^{1} + x^3}{\left(x^2+1\right)^{1/2}}$

$= \dfrac{2x^3+2x+x^3}{\left(x^2+1\right)^{1/2}} = \dfrac{3x^3+2x}{\left(x^2+1\right)^{1/2}}$

$= \dfrac{x\left(3x^2+2\right)}{\left(x^2+1\right)^{1/2}}$

90. $(x+1)^{1/3} + x\cdot\dfrac{1}{3}(x+1)^{-2/3},\ x \neq -1$

$= (x+1)^{1/3} + \dfrac{x}{3(x+1)^{2/3}}$

$= \dfrac{3(x+1)^{2/3}(x+1)^{1/3} + x}{3(x+1)^{2/3}}$

$= \dfrac{3(x+1)^{2/3+1/3} + x}{3(x+1)^{2/3}} = \dfrac{3(x+1)^{1} + x}{3(x+1)^{2/3}}$

$= \dfrac{3x+3+x}{3(x+1)^{2/3}} = \dfrac{4x+3}{3(x+1)^{2/3}}$

91. $\sqrt{4x+3}\cdot\frac{1}{2\sqrt{x-5}}+\sqrt{x-5}\cdot\frac{1}{5\sqrt{4x+3}}, x>5$

$=\frac{\sqrt{4x+3}}{2\sqrt{x-5}}+\frac{\sqrt{x-5}}{5\sqrt{4x+3}}$

$=\frac{\sqrt{4x+3}\cdot 5\cdot\sqrt{4x+3}+\sqrt{x-5}\cdot 2\cdot\sqrt{x-5}}{10\sqrt{x-5}\sqrt{4x+3}}$

$=\frac{5(4x+3)+2(x-5)}{10\sqrt{(x-5)(4x+3)}}$

$=\frac{20x+15+2x-10}{10\sqrt{(x-5)(4x+3)}}$

$=\frac{22x+5}{10\sqrt{(x-5)(4x+3)}}$

92. $\frac{\sqrt[3]{8x+1}}{3\sqrt[3]{(x-2)^2}}+\frac{\sqrt[3]{x-2}}{24\sqrt[3]{(8x+1)^2}}, x\neq 2, x\neq -\frac{1}{8}$

$=\frac{8\sqrt[3]{8x+1}\cdot\sqrt[3]{(8x+1)^2}+\sqrt[3]{x-2}\cdot\sqrt[3]{(x-2)^2}}{24\sqrt[3]{(x-2)^2}\cdot\sqrt[3]{(8x+1)^2}}$

$=\frac{8\sqrt[3]{(8x+1)^3}+\sqrt[3]{(x-2)^3}}{24\sqrt[3]{(x-2)^2}\cdot\sqrt[3]{(8x+1)^2}}$

$=\frac{8(8x+1)+x-2}{24\sqrt[3]{(x-2)^2(8x+1)^2}}$

$=\frac{64x+8+x-2}{24\sqrt[3]{(x-2)^2(8x+1)^2}}$

$=\frac{65x+6}{24\sqrt[3]{(x-2)^2(8x+1)^2}}$

93. $\frac{\left(\sqrt{1+x}-x\cdot\frac{1}{2\sqrt{1+x}}\right)}{1+x}=\frac{\left(\sqrt{1+x}-\frac{x}{2\sqrt{1+x}}\right)}{1+x}$

$=\frac{\left(\frac{2\sqrt{1+x}\sqrt{1+x}-x}{2\sqrt{1+x}}\right)}{1+x}$

$=\frac{2(1+x)-x}{2(1+x)^{1/2}}\cdot\frac{1}{1+x}$

$=\frac{2+x}{2(1+x)^{3/2}}$

94. $\frac{\left(\sqrt{x^2+1}-x\cdot\frac{2x}{2\sqrt{x^2+1}}\right)}{x^2+1}$

$=\frac{\left(\sqrt{x^2+1}-\frac{x^2}{\sqrt{x^2+1}}\right)}{x^2+1}$

$=\frac{\left(\sqrt{x^2+1}\cdot\frac{\sqrt{x^2+1}}{\sqrt{x^2+1}}-\frac{x^2}{\sqrt{x^2+1}}\right)}{x^2+1}$

$=\frac{\left(\frac{x^2+1-x^2}{\sqrt{x^2+1}}\right)}{x^2+1}=\frac{1}{\sqrt{x^2+1}}\cdot\frac{1}{x^2+1}$

$=\frac{1}{\left(x^2+1\right)^{3/2}}$

95. $\frac{(x+4)^{1/2}-2x(x+4)^{-1/2}}{x+4}$

$=\frac{\left((x+4)^{1/2}-\frac{2x}{(x+4)^{1/2}}\right)}{x+4}$

$=\frac{\left((x+4)^{1/2}\cdot\frac{(x+4)^{1/2}}{(x+4)^{1/2}}-\frac{2x}{(x+4)^{1/2}}\right)}{x+4}$

$=\frac{\left(\frac{x+4-2x}{(x+4)^{1/2}}\right)}{x+4}$

$=\frac{-x+4}{(x+4)^{1/2}}\cdot\frac{1}{x+4}$

$=\frac{-x+4}{(x+4)^{3/2}}$

$=\frac{4-x}{(x+4)^{3/2}}$

96. $\dfrac{\left(9-x^2\right)^{1/2}+x^2\left(9-x^2\right)^{-1/2}}{9-x^2}, -3<x<3$

$$\frac{\left(\left(9-x^2\right)^{1/2}+\dfrac{x^2}{\left(9-x^2\right)^{1/2}}\right)}{9-x^2}$$

$$=\frac{\left(\dfrac{\left(9-x^2\right)^{1/2}\cdot\left(9-x^2\right)^{1/2}+x^2}{\left(9-x^2\right)^{1/2}}\right)}{9-x^2}$$

$$=\frac{\left(9-x^2\right)^{1/2}\cdot\left(9-x^2\right)^{1/2}+x^2}{\left(9-x^2\right)^{1/2}}\cdot\frac{1}{9-x^2}$$

$$=\frac{9-x^2+x^2}{\left(9-x^2\right)^{1/2}}\cdot\frac{1}{9-x^2}$$

$$=\frac{9}{\left(9-x^2\right)^{3/2}}$$

97. $\dfrac{\dfrac{x^2}{\left(x^2-1\right)^{1/2}}-\left(x^2-1\right)^{1/2}}{x^2}, x<-1 \text{ or } x>1$

$$=\frac{\left(\dfrac{x^2-\left(x^2-1\right)^{1/2}\cdot\left(x^2-1\right)^{1/2}}{\left(x^2-1\right)^{1/2}}\right)}{x^2}$$

$$=\frac{x^2-\left(x^2-1\right)^{1/2}\cdot\left(x^2-1\right)^{1/2}}{\left(x^2-1\right)^{1/2}}\cdot\frac{1}{x^2}$$

$$=\frac{x^2-\left(x^2-1\right)}{\left(x^2-1\right)^{1/2}}\cdot\frac{1}{x^2}$$

$$=\frac{x^2-x^2+1}{\left(x^2-1\right)^{1/2}}\cdot\frac{1}{x^2}$$

$$=\frac{1}{x^2\left(x^2-1\right)^{1/2}}$$

98. $\dfrac{\left(x^2+4\right)^{1/2}-x^2\left(x^2+4\right)^{-1/2}}{x^2+4}$

$$=\frac{\left(\left(x^2+4\right)^{1/2}-\dfrac{x^2}{\left(x^2+4\right)^{1/2}}\right)}{x^2+4}$$

$$=\frac{\left(\dfrac{\left(x^2+4\right)^{1/2}\cdot\left(x^2+4\right)^{1/2}-x^2}{\left(x^2+4\right)^{1/2}}\right)}{x^2+4}$$

$$=\frac{\left(x^2+4\right)^{1/2}\cdot\left(x^2+4\right)^{1/2}-x^2}{\left(x^2+4\right)^{1/2}}\cdot\frac{1}{x^2+4}$$

$$=\frac{x^2+4-x^2}{\left(x^2+4\right)^{1/2}}\cdot\frac{1}{x^2+4}=\frac{4}{\left(x^2+4\right)^{3/2}}$$

99. $\dfrac{\dfrac{1+x^2}{2\sqrt{x}}-2x\sqrt{x}}{\left(1+x^2\right)^2}, x>0$

$$=\frac{\left(\dfrac{1+x^2-\left(2\sqrt{x}\right)\left(2x\sqrt{x}\right)}{2\sqrt{x}}\right)}{\left(1+x^2\right)^2}$$

$$=\frac{1+x^2-\left(2\sqrt{x}\right)\left(2x\sqrt{x}\right)}{2\sqrt{x}}\cdot\frac{1}{\left(1+x^2\right)^2}$$

$$=\frac{1+x^2-4x^2}{2\sqrt{x}}\cdot\frac{1}{\left(1+x^2\right)^2}=\frac{1-3x^2}{2\sqrt{x}\left(1+x^2\right)^2}$$

100. $\dfrac{2x\left(1-x^2\right)^{1/3}+\dfrac{2}{3}x^3\left(1-x^2\right)^{-2/3}}{\left(1-x^2\right)^{2/3}}, x\neq-1, x\neq1$

$$=\frac{\left(2x\left(1-x^2\right)^{1/3}+\dfrac{2x^3}{3\left(1-x^2\right)^{2/3}}\right)}{\left(1-x^2\right)^{2/3}}$$

$$=\frac{\left(\dfrac{2x\left(1-x^2\right)^{1/3}3\left(1-x^2\right)^{2/3}+2x^3}{3\left(1-x^2\right)^{2/3}}\right)}{\left(1-x^2\right)^{2/3}}$$

$$= \frac{6x\left(1-x^2\right)^{1/3+2/3}+2x^3}{3\left(1-x^2\right)^{2/3}} \cdot \frac{1}{\left(1-x^2\right)^{2/3}}$$

$$= \frac{6x\left(1-x^2\right)+2x^3}{3\left(1-x^2\right)^{2/3+2/3}} = \frac{6x-6x^3+2x^3}{3\left(1-x^2\right)^{4/3}}$$

$$= \frac{6x-4x^3}{3\left(1-x^2\right)^{4/3}} = \frac{2x\left(3-2x^2\right)}{3\left(1-x^2\right)^{4/3}}$$

101. $(x+1)^{3/2} + x \cdot \frac{3}{2}(x+1)^{1/2}$

$= (x+1)^{1/2}\left(x+1+\frac{3}{2}x\right)$

$= (x+1)^{1/2}\left(\frac{5}{2}x+1\right)$

$= \frac{1}{2}(x+1)^{1/2}(5x+2)$

102. $(x^2+4)^{4/3} + x \cdot \frac{4}{3}(x^2+4)^{1/3} \cdot 2x$

$= (x^2+4)^{1/3}\left(x^2+4+\frac{8}{3}x^2\right)$

$= (x^2+4)^{1/3}\left(\frac{11}{3}x^2+4\right)$

$= \frac{1}{3}\left(x^2+4\right)^{1/3}\left(11x^2+12\right)$

103. $6x^{1/2}\left(x^2+x\right) - 8x^{3/2} - 8x^{1/2}$

$= 2x^{1/2}\left(3(x^2+x)-4x-4\right)$

$= 2x^{1/2}\left(3x^2-x-4\right)$

$= 2x^{1/2}(3x-4)(x+1)$

104. $6x^{1/2}(2x+3) + x^{3/2} \cdot 8$

$= 2x^{1/2}(3(2x+3)+4x)$

$= 2x^{1/2}(10x+9)$

105. $3\left(x^2+4\right)^{4/3} + x \cdot 4\left(x^2+4\right)^{1/3} \cdot 2x$

$= \left(x^2+4\right)^{1/3}\left[3\left(x^2+4\right)+8x^2\right]$

$= \left(x^2+4\right)^{1/3}\left[3x^2+12+8x^2\right]$

$= \left(x^2+4\right)^{1/3}\left(11x^2+12\right)$

106. $2x(3x+4)^{4/3} + x^2 \cdot 4(3x+4)^{1/3}$

$= 2x(3x+4)^{1/3}\left[(3x+4)+2x\right]$

$= 2x(3x+4)^{1/3}(5x+4)$

107. $4(3x+5)^{1/3}(2x+3)^{3/2} + 3(3x+5)^{4/3}(2x+3)^{1/2}$

$= (3x+5)^{1/3}(2x+3)^{1/2}\left[4(2x+3)+3(3x+5)\right]$

$= (3x+5)^{1/3}(2x+3)^{1/2}(8x+12+9x+15)$

$= (3x+5)^{1/3}(2x+3)^{1/2}(17x+27)$

where $x \ge -\frac{3}{2}$.

108. $6(6x+1)^{1/3}(4x-3)^{3/2} + 6(6x+1)^{4/3}(4x-3)^{1/2}$

$= 6(6x+1)^{1/3}(4x-3)^{1/2}\left[(4x-3)+(6x+1)\right]$

$= 6(6x+1)^{1/3}(4x-3)^{1/2}(10x-2)$

$= 6(6x+1)^{1/3}(4x-3)^{1/2}(2)(5x-1)$

$= 12(6x+1)^{1/3}(4x-3)^{1/2}(5x-1)$

where $x \ge \frac{3}{4}$.

109. $3x^{-1/2} + \frac{3}{2}x^{1/2}, x > 0$

$= \frac{3}{x^{1/2}} + \frac{3}{2}x^{1/2}$

$= \frac{3 \cdot 2 + 3x^{1/2} \cdot x^{1/2}}{2x^{1/2}} = \frac{6+3x}{2x^{1/2}} = \frac{3(x+2)}{2x^{1/2}}$

110. $8x^{1/3} - 4x^{-2/3}, x \ne 0$

$= 8x^{1/3} - \frac{4}{x^{2/3}}$

$= \frac{8x^{1/3} \cdot x^{2/3} - 4}{x^{2/3}} = \frac{8x-4}{x^{2/3}} = \frac{4(2x-1)}{x^{2/3}}$

111. **a.** $V = 40(12)^2\sqrt{\frac{96}{12}-0.608}$

$\approx 15,660.4$ gallons

b. $V = 40(1)^2\sqrt{\frac{96}{1}-0.608} \approx 390.7$ gallons

112. **a.** $v = \sqrt{64 \cdot 4 + 0^2} = \sqrt{256}$

$= 16$ feet per second

b. $v = \sqrt{64 \cdot 16 + 0^2} = \sqrt{1024}$

$= 32$ feet per second

c. $v = \sqrt{64 \cdot 2 + 4^2} = \sqrt{144}$
$= 12$ feet per second

113. $T = 2\pi\sqrt{\frac{64}{32}} = 2\pi\sqrt{2} \approx 8.89$ seconds

114. $T = 2\pi\sqrt{\frac{16}{32}} = 2\pi\sqrt{\frac{1}{2}} = \frac{2\pi}{\sqrt{2}}$
$= \pi\sqrt{2} \approx 4.44$ seconds

115. 8 inches = 8/12 = 2/3 feet

$$T = 2\pi\sqrt{\frac{\left(\frac{2}{3}\right)}{32}} = 2\pi\sqrt{\frac{1}{48}} = 2\pi\left(\frac{1}{4\sqrt{3}}\right)$$
$$= \frac{\pi}{2\sqrt{3}} = \frac{\pi\sqrt{3}}{6} \approx 0.91 \text{ seconds}$$

116. 4 inches = 4/12 = 1/3 feet

$$T = 2\pi\sqrt{\frac{\left(\frac{1}{3}\right)}{32}} = 2\pi\sqrt{\frac{1}{96}} = 2\pi\left(\frac{1}{4\sqrt{6}}\right)$$
$$= \frac{\pi}{2\sqrt{6}} = \frac{\pi\sqrt{6}}{12} \approx 0.64 \text{ seconds}$$

117. Answers may vary. One possibility follows: If $a = -5$, then $\sqrt{a^2} = \sqrt{(-5)^2} = \sqrt{25} = 5 \neq a$.
Since we use the principal square root, which is always non-negative,

$$\sqrt{a^2} = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

which is the definition of $|a|$, so $\sqrt{a^2} = |a|$.