Chapter 10
Analytic Geometry

Section 10.1

Not applicable

Section 10.2

1. $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

2. $\left(\frac{-4}{2}\right)^2=4$

3. $(x+4)^2=9$

$x+4=\pm 3$

$x+4=3$ or $x+4=-3$

$x=-1$ or $x=-7$

The solution set is $\{-7,-1\}$.

4. $(-2,-5)$

5. 3, up

6. parabola

7. c

8. $(3,2)$

9. $(3,6)$

10. $y=-2$

11. (b); the graph has a vertex $(h,k)=(0,0)$ and opens up. Therefore, the equation of the graph has the form $x^2=4ay$. The graph passes through the point $(2,1)$ so we have

$(2)^2=4a(1)$

$4=4a$

$1=a$

Thus, the equation of the graph is $x^2=4y$.

12. (g); the graph has vertex $(h,k)=(1,1)$ and opens to the left. Therefore, the equation of the graph has the form $(y-1)^2=-4a(x-1)$.

13. (e); the graph has vertex $(h,k)=(1,1)$ and opens to the right. Therefore, the equation of the graph has the form $(y-1)^2=4a(x-1)$.

14. (d); the graph has vertex $(h,k)=(0,0)$ and opens down. Therefore, the equation of the graph has the form $x^2=-4ay$. The graph passes through the point $(-2,-1)$ so we have

$(-2)^2=-4a(-1)$

$4=4a$

$a=1$

Thus, the equation of the graph is $x^2=-4y$.

15. (h); the graph has vertex $(h,k)=(-1,-1)$ and opens down. Therefore, the equation of the graph has the form $(x+1)^2=-4a(y+1)$.

16. (a); the graph has vertex $(h,k)=(0,0)$ and opens to the right. Therefore, the equation of the graph has the form $y^2=4ax$. The graph passes through the point $(1,2)$ so we have

$(2)^2=4a(1)$

$4=4a$

$1=a$

Thus, the equation of the graph is $y^2=4x$.

17. (c); the graph has vertex $(h,k)=(0,0)$ and opens to the left. Therefore, the equation of the graph has the form $y^2=-4ax$. The graph passes through the point $(-1,-2)$ so we have

$(-2)^2=-4a(-1)$

$4=4a$

$1=a$

Thus, the equation of the graph is $y^2=-4x$.

18. (f); the graph has vertex $(h,k)=(-1,-1)$ and opens up; further, $a=1$. Therefore, the equation of the graph has the form $(x+1)^2=4(y+1)$.

19. The focus is (4, 0) and the vertex is (0, 0). Both lie on the horizontal line $y=0$. $a=4$ and since (4, 0) is to the right of (0, 0), the parabola opens to the right. The equation of the parabola is:

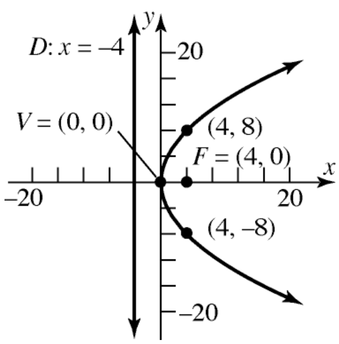
$y^2 = 4ax$

$y^2 = 4 \cdot 4 \cdot x$

$y^2 = 16x$

Letting $x=4$, we find $y^2 = 64$ or $y = \pm 8$.

The points (4, 8) and (4, –8) define the latus rectum.



20. The focus is (0, 2) and the vertex is (0, 0). Both lie on the vertical line $x=0$. $a=2$ and since (0, 2) is above (0, 0), the parabola opens up. The equation of the parabola is:

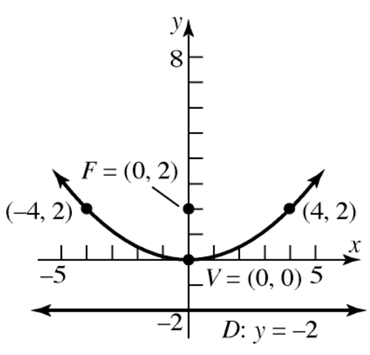
$x^2 = 4ay$

$x^2 = 4 \cdot 2 \cdot y$

$x^2 = 8y$

Letting $y=2$, we find $x^2 = 16$ or $x = \pm 4$.

The points (–4, 2) and (4, 2) define the latus rectum.



21. The focus is (0, –3) and the vertex is (0, 0). Both lie on the vertical line $x=0$. $a=3$ and since (0, –3) is below (0, 0), the parabola opens down. The equation of the parabola is:

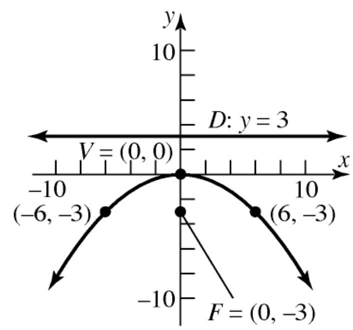
$x^2 = -4ay$

$x^2 = -4 \cdot 3 \cdot y$

$x^2 = -12y$

Letting $y=-3$, we find $x^2 = 36$ or $x = \pm 6$.

The points $(-6,-3)$ and $(6,-3)$ define the latus rectum.



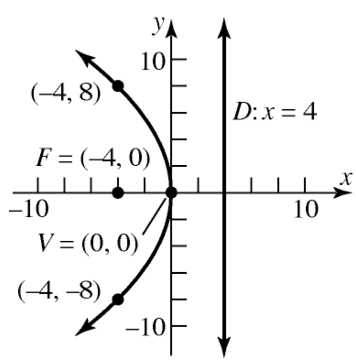
22. The focus is (–4, 0) and the vertex is (0, 0). Both lie on the horizontal line $y=0$. $a=4$ and since (–4, 0) is to the left of (0, 0), the parabola opens to the left. The equation of the parabola is:

$y^2 = -4ax$

$y^2 = -4 \cdot 4 \cdot x$

$y^2 = -16x$

Letting $x=-4$, we find $y^2 = 64$ or $y = \pm 8$. The points (–4, 8) and (–4, –8) define the latus rectum.



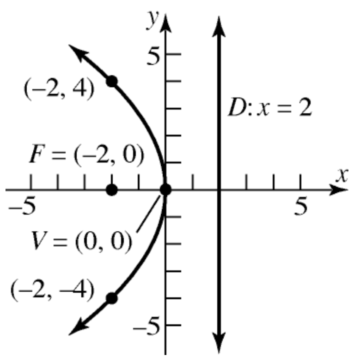
23. The focus is (–2, 0) and the directrix is $x = 2$. The vertex is (0, 0). $a = 2$ and since (–2, 0) is to the left of (0, 0), the parabola opens to the left. The equation of the parabola is:

$y^2 = -4ax$

$y^2 = -4 \cdot 2 \cdot x$

$y^2 = -8x$

Letting $x = -2$, we find $y^2 = 16$ or $y = \pm 4$. The points (–2, 4) and (–2, –4) define the latus rectum.



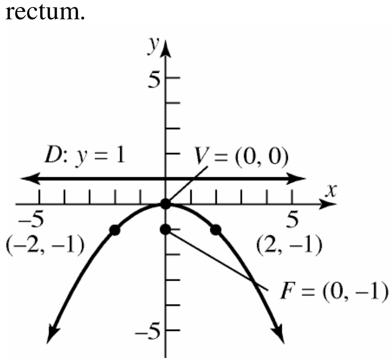
24. The focus is (0, –1) and the directrix is $y = 1$. The vertex is (0, 0). $a = 1$ and since (0, –1) is below (0, 0), the parabola opens down. The equation of the parabola is:

$x^2 = -4ay$

$x^2 = -4 \cdot 1 \cdot y$

$x^2 = -4y$

Letting $y = -1$, we find $x^2 = 4$ or $x = \pm 2$. The points (–2, –1) and (2, –1) define the latus rectum.



25. The directrix is $y = -\frac{1}{2}$ and the vertex is (0, 0). The focus is $\left(0, \frac{1}{2}\right)$. $a = \frac{1}{2}$ and since $\left(0, \frac{1}{2}\right)$ is above (0, 0), the parabola opens up. The equation of the parabola is:

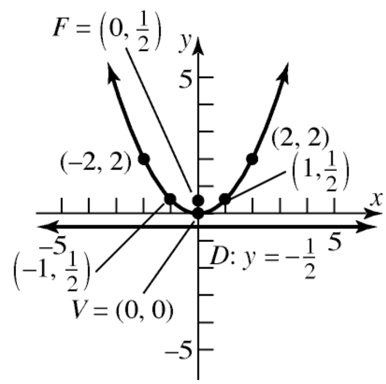
$x^2 = 4ay$

$x^2 = 4 \cdot \frac{1}{2} \cdot y$

$x^2 = 2y$

Letting $y = \frac{1}{2}$, we find $x^2 = 1$ or $x = \pm 1$.

The points $\left(1, \frac{1}{2}\right)$ and $\left(-1, \frac{1}{2}\right)$ define the latus rectum.



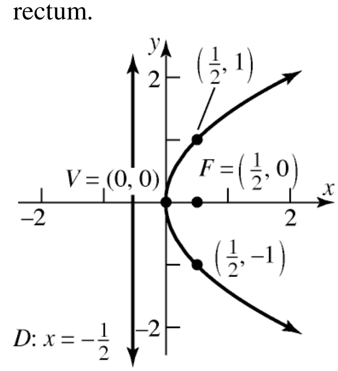
26. The directrix is $x = -\frac{1}{2}$ and the vertex is (0, 0). The focus is $\left(\frac{1}{2}, 0\right)$. $a = \frac{1}{2}$ and since $\left(\frac{1}{2}, 0\right)$ is to the right of (0, 0), the parabola opens to the right. The equation of the parabola is:

$y^2 = 4ax$

$y^2 = 4 \cdot \frac{1}{2} \cdot x$

$y^2 = 2x$

Letting $x = \frac{1}{2}$, we find $y^2 = 1$ or $y = \pm 1$. The points $\left(\frac{1}{2}, -1\right)$ and $\left(\frac{1}{2}, 1\right)$ define the latus rectum.



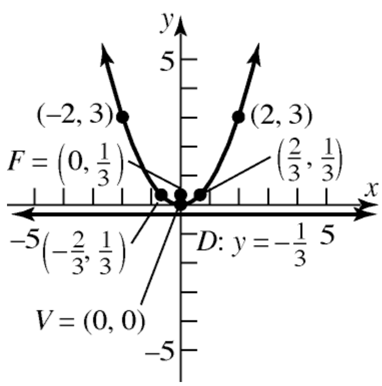
27. Vertex: (0, 0). Since the axis of symmetry is vertical, the parabola opens up or down. Since (2, 3) is above (0, 0), the parabola opens up. The equation has the form $x^2 = 4ay$. Substitute the coordinates of (2, 3) into the equation to find a:

$2^2 = 4a \cdot 3$

$4 = 12a$

$a = \frac{1}{3}$

The equation of the parabola is: $x^2 = \frac{4}{3}y$. The focus is $\left(0, \frac{1}{3}\right)$. Letting $y = \frac{1}{3}$, we find $x^2 = \frac{4}{9}$ or $x = \pm\frac{2}{3}$. The points $\left(\frac{2}{3}, \frac{1}{3}\right)$ and $\left(-\frac{2}{3}, \frac{1}{3}\right)$ define the latus rectum.



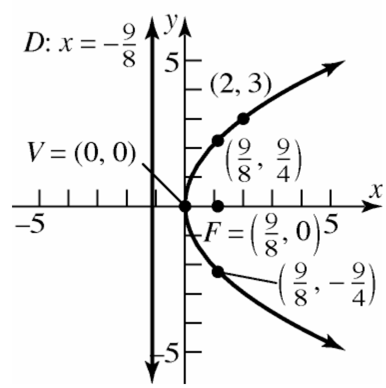
28. Vertex: (0, 0). Since the axis of symmetry is horizontal, the parabola opens left or right. Since (2, 3) is to the right of (0, 0), the parabola opens to the right. The equation has the form $y^2 = 4ax$. Substitute the coordinates of (2, 3) into the equation to find a:

$3^2 = 4a \cdot 2$

$9 = 8a$

$a = \frac{9}{8}$

The equation of the parabola is: $y^2 = \frac{9}{2}x$. The focus is $\left(\frac{9}{8}, 0\right)$. Letting $x = \frac{9}{8}$, we find $y^2 = \frac{81}{16}$ or $y = \pm\frac{9}{4}$. The points $\left(\frac{9}{8}, \frac{9}{4}\right)$ and $\left(\frac{9}{8}, -\frac{9}{4}\right)$ define the latus rectum.



29. The vertex is (2, –3) and the focus is (2, –5). Both lie on the vertical line $x = 2$.

$a = |-5-(-3)| = 2$ and since (2, –5) is below (2, –3), the parabola opens down. The equation of the parabola is:

$(x-h)^2 = -4a(y-k)$

$(x-2)^2 = -4(2)(y-(-3))$

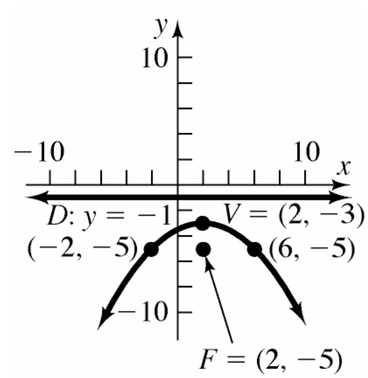
$(x-2)^2 = -8(y+3)$

Letting $y = -5$, we find

$(x-2)^2 = 16$

$x - 2 = \pm 4 \Rightarrow x = -2$ or $x = 6$

The points (–2, –5) and (6, –5) define the latus rectum.



30. The vertex is (4, –2) and the focus is (6, –2). Both lie on the horizontal line $y = -2$.

$a = |4-6| = 2$ and since (6, –2) is to the right of (4, –2), the parabola opens to the right. The equation of the parabola is:

$(y-k)^2 = 4a(x-h)$

$(y-(-2))^2 = 4(2)(x-4)$

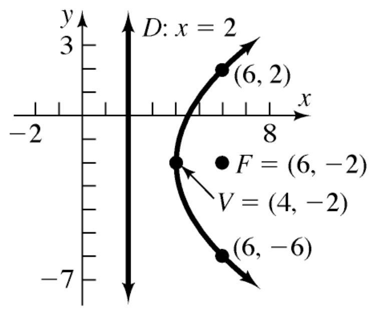
$(y+2)^2 = 8(x-4)$

Letting $x = 6$, we find

$(y+2)^2 = 16$

$y+2 = \pm 4 \Rightarrow y = -6$ or $y = 2$

The points (6, –6) and (6, 2) define the latus rectum.



31. The vertex is (–1, –2) and the focus is (0, –2). Both lie on the horizontal line $y = -2$.

$a = |-1-0| = 1$ and since (0, –2) is to the right of (–1, –2), the parabola opens to the right. The equation of the parabola is:

$(y-k)^2 = 4a(x-h)$

$(y-(-2))^2 = 4(1)(x-(-1))$

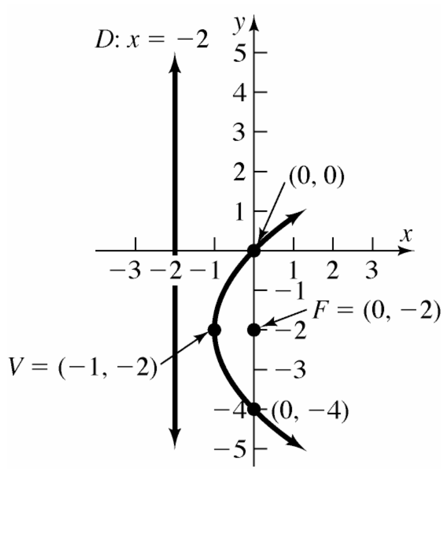
$(y+2)^2 = 4(x+1)$

Letting $x = 0$, we find

$(y+2)^2 = 4$

$y+2 = \pm 2 \Rightarrow y = -4$ or $y = 0$

The points (0, –4) and (0, 0) define the latus rectum.



32. The vertex is (3, 0) and the focus is (3, –2). Both lie on the horizontal line $x = 3$. $a = |-2-0| = 2$ and since (3, –2) is below of (3, 0), the parabola opens down. The equation of the parabola is:

$(x-h)^2 = -4a(y-k)$

$(x-3)^2 = -4(2)(y-0)$

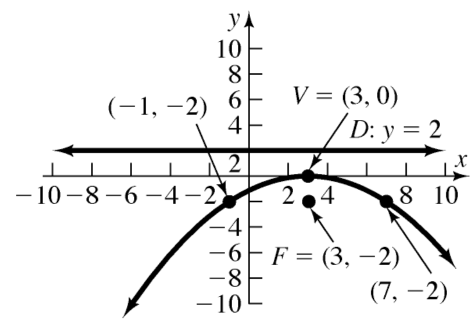
$(x-3)^2 = -8y$

Letting $y = -2$, we find

$(x-3)^2 = 16$

$x-3 = \pm 4 \Rightarrow x = -1$ or $x = 7$

The points (–1, –2) and (7, –2) define the latus rectum.

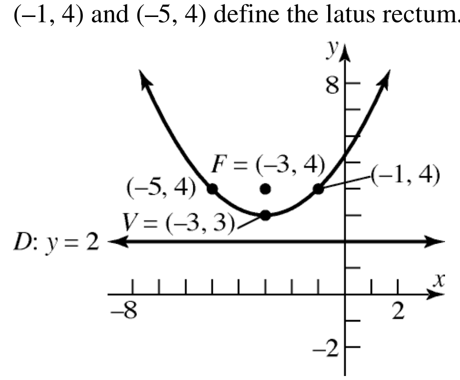


33. The directrix is $y = 2$ and the focus is (–3, 4). This is a vertical case, so the vertex is (–3, 3). $a = 1$ and since (–3, 4) is above $y = 2$, the parabola opens up. The equation of the parabola is: $(x-h)^2 = 4a(y-k)$

$(x-(-3))^2 = 4 \cdot 1 \cdot (y-3)$

$(x+3)^2 = 4(y-3)$

Letting $y = 4$, we find $(x+3)^2 = 4$ or $x+3 = \pm 2$. So, $x = -1$ or $x = -5$. The points (–1, 4) and (–5, 4) define the latus rectum.

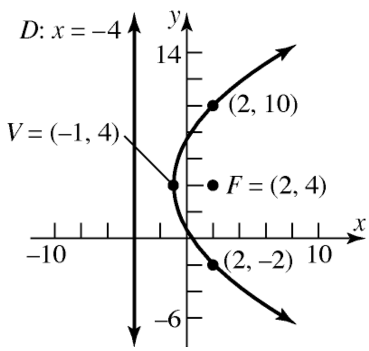


34. The directrix is $x=-4$ and the focus is (2, 4). This is a horizontal case, so the vertex is (–1, 4). $a=3$ and since (2, 4) is to the right of $x=-4$, the parabola opens to the right. The equation of the parabola is: $(y-k)^2=4a(x-h)$

$(y-4)^2=4\cdot 3\cdot(x-(-1))$

$(y-4)^2=12(x+1)$

Letting $x=2$, we find $(y-4)^2=36$ or $y-4=\pm 6$. So, $y=-2$ or $y=10$. The points (2, –2) and (2, 10) define the latus rectum.



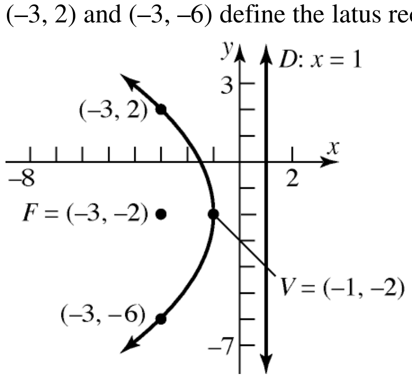
35. The directrix is $x=1$ and the focus is (–3, –2). This is a horizontal case, so the vertex is (–1, –2). $a=2$ and since (–3, –2) is to the left of $x=1$, the parabola opens to the left. The equation of the parabola is:

$(y-k)^2=-4a(x-h)$

$(y-(-2))^2=-4\cdot 2\cdot(x-(-1))$

$(y+2)^2=-8(x+1)$

Letting $x=-3$, we find $(y+2)^2=16$ or $y+2=\pm 4$. So, $y=2$ or $y=-6$. The points (–3, 2) and (–3, –6) define the latus rectum.



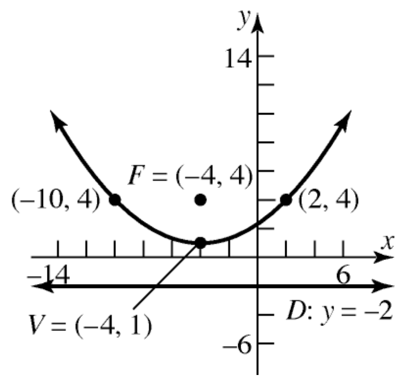
36. The directrix is $y=-2$ and the focus is (–4, 4). This is a vertical case, so the vertex is (–4, 1). $a=3$ and since (–4, 4) is above $y=-2$, the parabola opens up. The equation of the parabola is:

$(x-h)^2=4a(y-k)$

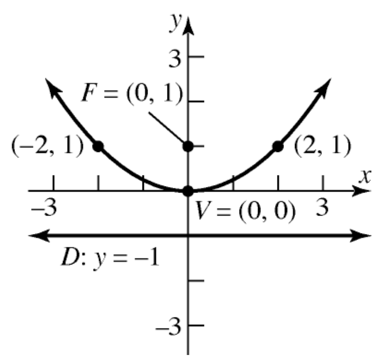
$(x-(-4))^2=4\cdot 3\cdot(y-1)$

$(x+4)^2=12(y-1)$

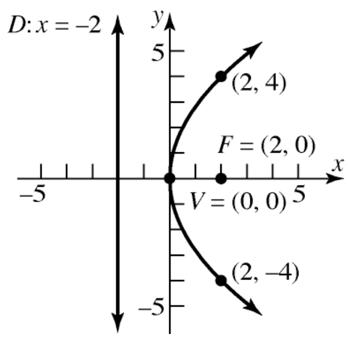
Letting $y=4$, we find $(x+4)^2=36$ or $x+4=\pm 6$. So, $x=-10$ or $x=2$. The points (–10, 4) and (2, 4) define the latus rectum.



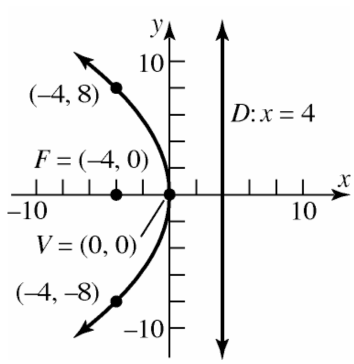
37. The equation $x^2=4y$ is in the form $x^2=4ay$ where $4a=4$ or $a=1$.
Thus, we have:
Vertex: (0, 0)
Focus: (0, 1)
Directrix: $y=-1$



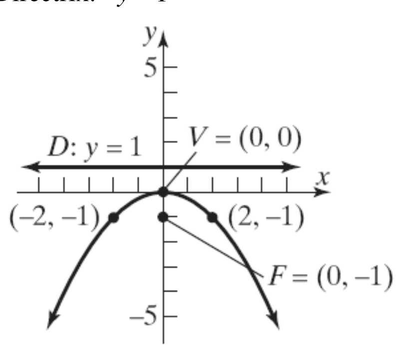
38. The equation $y^2 = 8x$ is in the form $y^2 = 4ax$ where $4a = 8$ or $a = 2$. Thus, we have:
Vertex: (0, 0)
Focus: (2, 0)
Directrix: $x = -2$



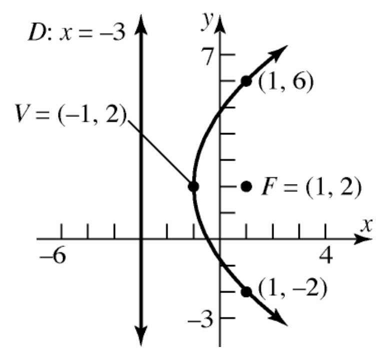
39. The equation $y^2 = -16x$ is in the form $y^2 = -4ax$ where $-4a = -16$ or $a = 4$. Thus, we have:
Vertex: (0, 0)
Focus: (–4, 0)
Directrix: $x = 4$



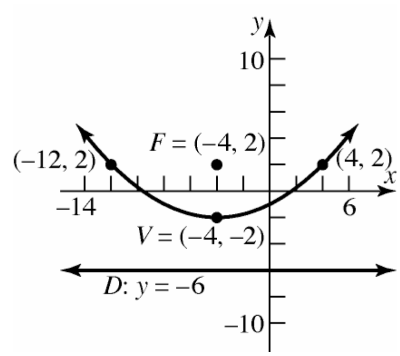
40. The equation $x^2 = -4y$ is in the form $x^2 = -4ay$ where $-4a = -4$ or $a = 1$. Thus, we have:
Vertex: (0, 0)
Focus: (0, –1)
Directrix: $y = 1$



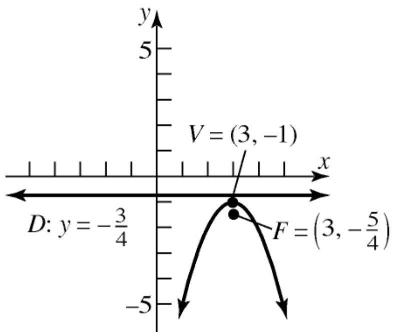
41. The equation $(y-2)^2 = 8(x+1)$ is in the form $(y-k)^2 = 4a(x-h)$ where $4a = 8$ or $a = 2$, $h = -1$, and $k = 2$. Thus, we have:
Vertex: (–1, 2);
Focus: (1, 2);
Directrix: $x = -3$



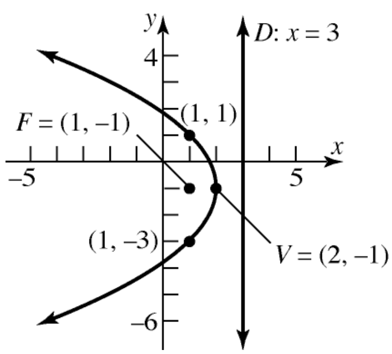
42. The equation $(x+4)^2 = 16(y+2)$ is in the form $(x-h)^2 = 4a(y-k)$ where $4a = 16$ or $a = 4$, $h = -4$, and $k = -2$. Thus, we have:
Vertex: (–4, –2);
Focus: (–4, 2)
Directrix: $y = -6$



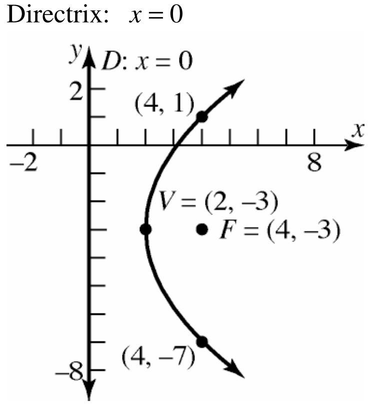
43. **a.** The equation $(x-3)^2 = -(y+1)$ is in the form $(x-h)^2 = -4a(y-k)$ where $-4a = -1$ or $a = \frac{1}{4}$, $h = 3$, and $k = -1$. Thus, we have:
Vertex: (3, –1);
Focus: $\left(3, -\frac{5}{4}\right)$;
Directrix: $y = -\frac{3}{4}$



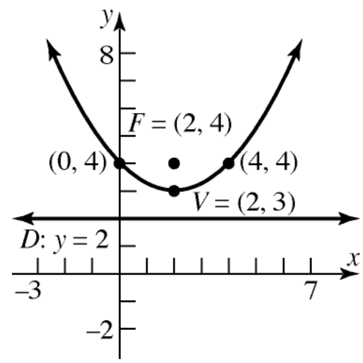
44. The equation $(y+1)^2 = -4(x-2)$ is in the form $(y-k)^2 = -4a(x-h)$ where $-4a = -4$ or $a = 1$, $h = 2,$ and $k = -1$. Thus, we have:
Vertex: (2, –1);
Focus: $(1, -1)$
Directrix: $x = 3$



45. The equation $(y+3)^2 = 8(x-2)$ is in the form $(y-k)^2 = 4a(x-h)$ where $4a = 8$ or $a = 2$, $h = 2,$ and $k = -3$. Thus, we have:
Vertex: (2, –3);
Focus: (4, –3)
Directrix: $x = 0$



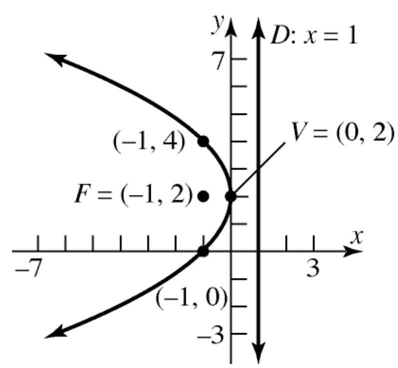
46. The equation $(x-2)^2 = 4(y-3)$ is in the form $(x-h)^2 = 4a(y-k)$ where $4a = 4$ or $a = 1$, $h = 2,$ and $k = 3$. Thus, we have:
Vertex: (2, 3); Focus: (2, 4); Directrix: $y = 2$



47. Complete the square to put in standard form:

$$y^2 - 4y + 4x + 4 = 0$$
$$y^2 - 4y + 4 = -4x$$
$$(y-2)^2 = -4x$$

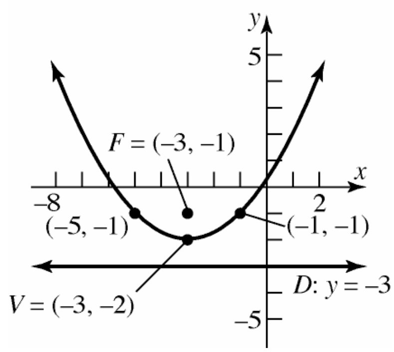
The equation is in the form $(y-k)^2 = -4a(x-h)$ where $-4a = -4$ or $a = 1$, $h = 0,$ and $k = 2$. Thus, we have:
Vertex: (0, 2); Focus: (–1, 2); Directrix: $x = 1$



48. Complete the square to put in standard form:

$$x^2 + 6x - 4y + 1 = 0$$
$$x^2 + 6x + 9 = 4y - 1 + 9$$
$$(x+3)^2 = 4(y+2)$$

The equation is in the form $(x-h)^2 = 4a(y-k)$ where $4a = 4$ or $a = 1$, $h = -3,$ and $k = -2$.
Thus, we have:
Vertex: (–3, –2); Focus: (–3, –1)
Directrix: $y = -3$

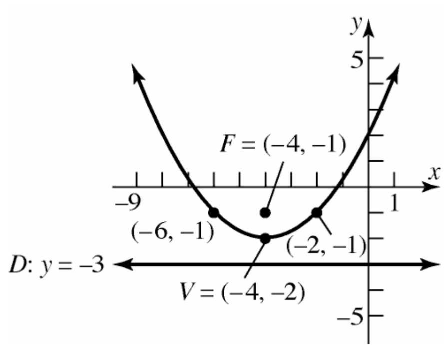


49. Complete the square to put in standard form:

$$x^2+8x=4y-8$$
$$x^2+8x+16=4y-8+16$$
$$(x+4)^2=4(y+2)$$

The equation is in the form $(x-h)^2=4a(y-k)$ where $4a=4$ or $a=1$, $h=-4$, and $k=-2$.

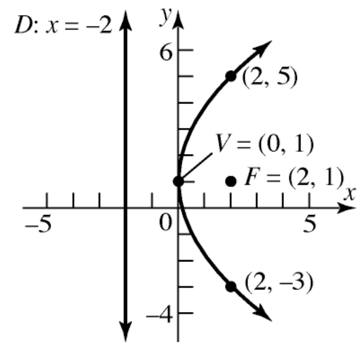
Thus, we have:
Vertex: (–4, –2);
Focus: (–4, –1)
Directrix: $y=-3$



50. Complete the square to put in standard form:

$$y^2-2y=8x-1$$
$$y^2-2y+1=8x-1+1$$
$$\left(y-1\right)^2=8x$$

The equation is in the form $(y-k)^2=4a(x-h)$ where $4a=8$ or $a=2$, $h=0$, and $k=1$. Thus, we have:
Vertex: (0, 1);
Focus: (2, 1)
Directrix: $x=-2$



51. Complete the square to put in standard form:

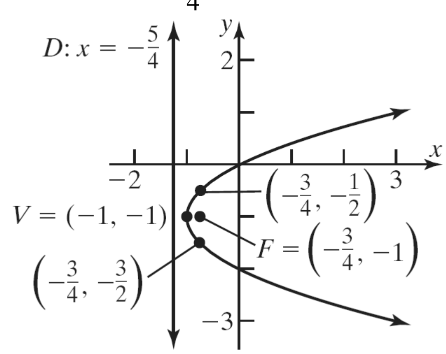
$$y^2+2y-x=0$$
$$y^2+2y+1=x+1$$
$$\left(y+1\right)^2=x+1$$

The equation is in the form $(y-k)^2=4a(x-h)$ where $4a=1$ or $a=\frac{1}{4}$, $h=-1$, and $k=-1$.

Thus, we have:

Vertex: (–1, –1); Focus: $\left(-\frac{3}{4},-1\right)$

Directrix: $x=-\frac{5}{4}$



52. Complete the square to put in standard form:

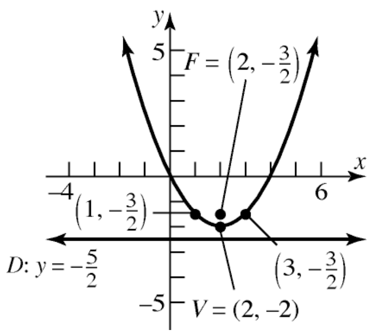
$$x^2-4x=2y$$
$$x^2-4x+4=2y+4$$
$$\left(x-2\right)^2=2\left(y+2\right)$$

The equation is in the form $(x-h)^2=4a(y-k)$ where $4a=2$ or $a=\frac{1}{2}$, $h=2$, and $k=-2$.

Thus, we have:

Vertex: (2, –2); Focus: $\left(2,-\frac{3}{2}\right)$

Directrix: $y=-\frac{5}{2}$



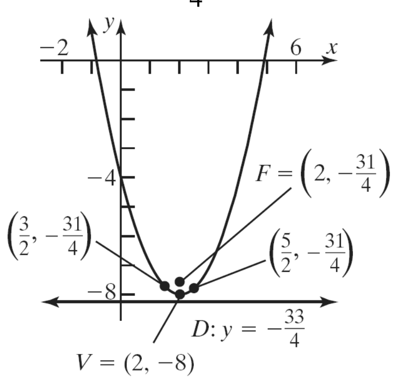
53. Complete the square to put in standard form:

$$x^2 - 4x = y + 4$$
$$x^2 - 4x + 4 = y + 4 + 4$$
$$(x-2)^2 = y + 8$$

The equation is in the form $(x-h)^2 = 4a(y-k)$ where $4a = 1$ or $a = \frac{1}{4}$, $h = 2$, and $k = -8$. Thus, we have:

Vertex: (2, –8); Focus: $\left(2, -\frac{31}{4}\right)$

Directrix: $y = -\frac{33}{4}$



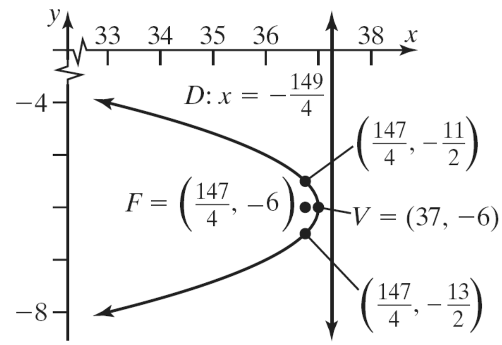
54. Complete the square to put in standard form:

$$y^2 + 12y = -x + 1$$
$$y^2 + 12y + 36 = -x + 1 + 36$$
$$(y+6)^2 = -(x-37)$$

The equation is in the form $(y-k)^2 = -4a(x-h)$ where $-4a = -1$ or $a = \frac{1}{4}$, $h = 37$, and $k = -6$. Thus, we have:

Vertex: (37, –6); Focus: $\left(\frac{147}{4}, -6\right)$;

Directrix: $x = \frac{149}{4}$



55. $(y-1)^2 = c(x-0)$

$(y-1)^2 = cx$

$(2-1)^2 = c(1) \Rightarrow 1 = c$

$(y-1)^2 = x$

56. $(x-1)^2 = c(y-2)$

$(2-1)^2 = c(1-2)$

$1 = -c \Rightarrow c = -1$

$(x-1)^2 = -(y-2)$

57. $(y-1)^2 = c(x-2)$

$(0-1)^2 = c(1-2)$

$1 = -c \Rightarrow c = -1$

$(y-1)^2 = -(x-2)$

58. $(x-0)^2 = c(y-(-1))$

$x^2 = c(y+1)$

$2^2 = c(0+1) \Rightarrow 4 = c$

$x^2 = 4(y+1)$

59. $(x-0)^2 = c(y-1)$

$x^2 = c(y-1)$

$2^2 = c(2-1)$

$4 = c$

$x^2 = 4(y-1)$

60. $(x-1)^2 = c(y-(-1))$

$(x-1)^2 = c(y+1)$

$(0-1)^2 = c(1+1) \Rightarrow 1 = 2c \Rightarrow c = \frac{1}{2}$

$(x-1)^2 = \frac{1}{2}(y+1)$

61. $(y-0)^2 = c(x-(-2))$

$y^2 = c(x+2)$

$1^2 = c(0+2) \Rightarrow 1 = 2c \Rightarrow c = \frac{1}{2}$

$y^2 = \frac{1}{2}(x+2)$

62. $(y-0)^2 = c(x-1)$

$y^2 = c(x-1)$

$1^2 = c(0-1)$

$1 = -c$

$c = -1$

$y^2 = -(x-1)$

63. Set up the problem so that the vertex of the parabola is at (0, 0) and it opens up. Then the equation of the parabola has the form: $x^2 = 4ay$. Since the parabola is 10 feet across and 4 feet deep, the points (5, 4) and (–5, 4) are on the parabola. Substitute and solve for a:

$5^2 = 4a(4) \Rightarrow 25 = 16a \Rightarrow a = \frac{25}{16}$

a is the distance from the vertex to the focus. Thus, the receiver (located at the focus) is $\frac{25}{16} = 1.5625$ feet, or 18.75 inches from the base of the dish, along the axis of the parabola.

64. Set up the problem so that the vertex of the parabola is at (0, 0) and it opens up. Then the equation of the parabola has the form: $x^2 = 4ay$. Since the parabola is 6 feet across and 2 feet deep, the points (3, 2) and (–3, 2) are on the parabola. Substitute and solve for a:

$3^2 = 4a(2) \Rightarrow 9 = 8a \Rightarrow a = \frac{9}{8}$

a is the distance from the vertex to the focus. Thus, the receiver (located at the focus) is $\frac{9}{8} = 1.125$ feet, or 13.5 inches from the base of the dish, along the axis of the parabola.

65. Set up the problem so that the vertex of the parabola is at (0, 0) and it opens up. Then the equation of the parabola has the form: $x^2 = 4ay$. Since the parabola is 4 inches across and 1 inch deep, the points (2, 1) and (–2, 1) are on the parabola. Substitute and solve for a:

$2^2 = 4a(1) \quad \Rightarrow \quad 4 = 4a \quad \Rightarrow \quad a = 1$

a is the distance from the vertex to the focus. Thus, the bulb (located at the focus) should be 1 inch from the vertex.

66. Set up the problem so that the vertex of the parabola is at (0, 0) and it opens up. Then the equation of the parabola has the form: $x^2 = 4ay$. Since the focus is 1 inch from the vertex and the depth is 2 inches, $a = 1$ and the points $(x, 2)$ and $(-x, 2)$ are on the parabola. Substitute and solve for x:

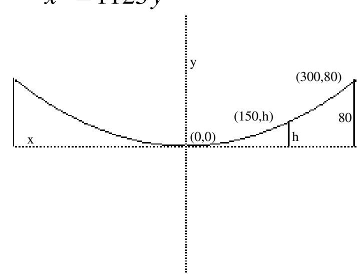
$x^2 = 4(1)(2) \Rightarrow x^2 = 8 \Rightarrow x = \pm 2\sqrt{2}$

The diameter of the headlight is $4\sqrt{2} \approx 5.66$ inches.

67. Set up the problem so that the vertex of the parabola is at (0, 0) and it opens up. Then the equation of the parabola has the form: $x^2 = cy$. The point (300, 80) is a point on the parabola. Solve for *c* and find the equation:

$300^2 = c(80) \Rightarrow c = 1125$

$x^2 = 1125y$



Since the height of the cable 150 feet from the center is to be found, the point (150, *h*) is a point on the parabola. Solve for *h*:

$150^2 = 1125h$

$22{,}500 = 1125h$

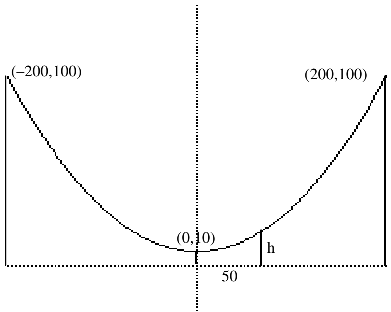
$20 = h$

The height of the cable 150 feet from the center is 20 feet.

68. Set up the problem so that the vertex of the parabola is at (0, 10) and it opens up. Then the equation of the parabola has the form: $x^2 = c(y-10)$.

The point (200, 100) is a point on the parabola. Solve for *c* and find the equation:

$200^2 = c(100-10)$
$40,000 = 90c$
$444.44 \approx c$
$x^2 = 444.44(y-10)$



Since the height of the cable 50 feet from the center is to be found, the point (50, *h*) is a point on the parabola. Solve for *h*:

$50^2 = 444.44(h-10)$
$2500 = 444.44h - 4444.4$
$6944.4 = 444.44h$
$15.625 \approx h$

The height of the cable 50 feet from the center is about 15.625 feet.

69. Set up the problem so that the vertex of the parabola is at (0, 0) and it opens up. Then the equation of the parabola has the form: $x^2 = 4ay$. a is the distance from the vertex to the focus (where the source is located), so $a = 2$. Since the opening is 5 feet across, there is a point (2.5, *y*) on the parabola.

Solve for *y*: $x^2 = 8y$
$2.5^2 = 8y$
$6.25 = 8y$
$y = 0.78125$ feet

The depth of the searchlight should be 0.78125 feet.

70. Set up the problem so that the vertex of the parabola is at (0, 0) and it opens up. Then the equation of the parabola has the form: $x^2 = 4ay$. a is the distance from the vertex to the focus (where the source is located), so $a = 2$. Since the depth is 4 feet, there is a point (*x*, 4) on the parabola. Solve for *x*:

$x^2 = 8y \Rightarrow x^2 = 8\cdot 4 \Rightarrow x^2 = 32 \Rightarrow x = \pm 4\sqrt{2}$

The width of the opening of the searchlight should be $8\sqrt{2} \approx 11.31$ feet.

71. Set up the problem so that the vertex of the parabola is at (0, 0) and it opens up. Then the equation of the parabola has the form: $x^2 = 4ay$. Since the parabola is 20 feet across and 6 feet deep, the points (10, 6) and (–10, 6) are on the parabola. Substitute and solve for a:

$10^2 = 4a(6)$
$100 = 24a$
$a \approx 4.17$ feet

The heat will be concentrated about 4.17 feet from the base, along the axis of symmetry.

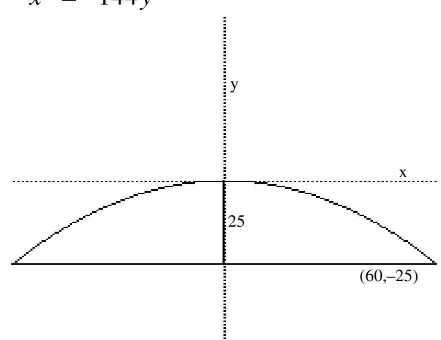
72. Set up the problem so that the vertex of the parabola is at (0, 0) and it opens up. Then the equation of the parabola has the form: $x^2 = 4ay$. Since the parabola is 4 inches across and 3 inches deep, the points (2, 3) and (–2, 3) are on the parabola. Substitute and solve for a:

$2^2 = 4a(3)$
$4 = 12a$
$a = \frac{1}{3}$ inch

The collected light will be concentrated 1/3 inch from the base of the mirror along the axis of symmetry.

73. Set up the problem so that the vertex of the parabola is at (0, 0) and it opens down. Then the equation of the parabola has the form: $x^2 = cy$. The point (60, –25) is a point on the parabola. Solve for *c* and find the equation:

$60^2 = c(-25) \Rightarrow c = -144$
$x^2 = -144y$



To find the height of the bridge 10 feet from the center the point (10, *y*) is a point on the parabola. Solve for *y*:

$10^2 = -144y$
$100 = -144y$
$-0.69 \approx y$

The height of the bridge 10 feet from the center is about 25 – 0.69 = 24.31 feet. To find the height of the bridge 30 feet from the center the point (30, y) is a point on the parabola.
Solve for y:

$$30^2 = -144y$$
$$900 = -144y$$
$$-6.25 = y$$

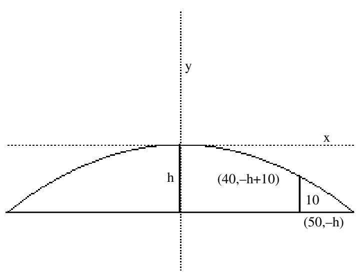
The height of the bridge 30 feet from the center is 25 – 6.25 = 18.75 feet. To find the height of the bridge, 50 feet from the center, the point (50, y) is a point on the parabola. Solve for y:

$$50^2 = -144y$$
$$2500 = -144y$$
$$y = -17.36$$

The height of the bridge 50 feet from the center is about 25 – 17.36 = 7.64 feet.

74. Set up the problem so that the vertex of the parabola is at (0, 0) and it opens down. Then the equation of the parabola has the form: $x^2 = cy$.
The points (50, –h) and (40, –h+10) are points on the parabola. Substitute and solve for c and h:

$$50^2 = c(-h) \qquad 40^2 = c(-h+10)$$
$$ch = -2500 \qquad 1600 = -ch + 10c$$



$$1600 = -(-2500) + 10c$$
$$1600 = 2500 + 0c$$
$$-900 = 10c$$
$$-90 = c$$
$$-90h = -2500$$
$$h \approx 27.78$$

The height of the bridge at the center is about 27.78 feet.

75. a. Imagine placing the Arch along the x-axis with the peak along the y-axis. Since the Arch is 625 feet high and is 598 feet wide at its base, we would have the points $(-299,0)$, $(0,625)$, and $(299,0)$. The equation of the parabola would have the form $y = ax^2 + c$. Using the point $(0,625)$ we have

$$625 = a(0)^2 + c$$
$$625 = c$$

The model then becomes $y = ax^2 + 625$.
Next, using the point $(299,0)$ we get

$$0 = a(299)^2 + 625$$
$$-625 = (299)^2 a$$
$$a = -\frac{625}{(299)^2}$$

Thus, the equation of the parabola with the same given dimensions is

$$y = -\frac{625}{(299)^2}x^2 + 625.$$

b. Using $y = -\frac{625}{(299)^2}x^2 + 625$, we get

Width (ft)	x	Height (ft), model
567	283.5	63.12
478	239	225.67
308	154	459.2

c. No; the heights computed by using the model do not fit the actual heights.

76. $Ax^2 + Ey = 0 \quad A \neq 0, \ E \neq 0$

$$Ax^2 = -Ey \Rightarrow x^2 = -\frac{E}{A}y$$

This is the equation of a parabola with vertex at (0, 0) and axis of symmetry being the y-axis. The focus is $\left(0, -\frac{E}{4A}\right)$. The directrix is $y = \frac{E}{4A}$. The parabola opens up if $-\frac{E}{A} > 0$ and down if $-\frac{E}{A} < 0$.

77. $Cy^2 + Dx = 0 \quad C \neq 0,\ D \neq 0$

$Cy^2 = -Dx$

$y^2 = -\frac{D}{C}x$

This is the equation of a parabola with vertex at (0, 0) and whose axis of symmetry is the *x*-axis. The focus is $\left(-\frac{D}{4C}, 0\right)$. The directrix is $x = \frac{D}{4C}$. The parabola opens to the right if $-\frac{D}{C} > 0$ and to the left if $-\frac{D}{C} < 0$.

78. $Ax^2 + Dx + Ey + F = 0 \quad A \neq 0$

a. If $E \neq 0$, then:

$$Ax^2 + Dx = -Ey - F$$

$$A\left(x^2 + \frac{D}{A}x + \frac{D^2}{4A^2}\right) = -Ey - F + \frac{D^2}{4A}$$

$$\left(x + \frac{D}{2A}\right)^2 = \frac{1}{A}\left(-Ey - F + \frac{D^2}{4A}\right)$$

$$\left(x + \frac{D}{2A}\right)^2 = \frac{-E}{A}\left(y + \frac{F}{E} - \frac{D^2}{4AE}\right)$$

$$\left(x + \frac{D}{2A}\right)^2 = \frac{-E}{A}\left(y - \frac{D^2 - 4AF}{4AE}\right)$$

This is the equation of a parabola whose vertex is $\left(-\frac{D}{2A}, \frac{D^2 - 4AF}{4AE}\right)$ and whose axis of symmetry is parallel to the *y*-axis.

b. If $E = 0$, then

$Ax^2 + Dx + F = 0 \Rightarrow x = \frac{-D \pm \sqrt{D^2 - 4AF}}{2A}$

If $D^2 - 4AF = 0$, then $x = -\frac{D}{2A}$ is a single vertical line.

c. If $E = 0$, then

$Ax^2 + Dx + F = 0 \Rightarrow x = \frac{-D \pm \sqrt{D^2 - 4AF}}{2A}$

If $D^2 - 4AF > 0$, then

$x = \frac{-D + \sqrt{D^2 - 4AF}}{2A}$ and

$x = \frac{-D - \sqrt{D^2 - 4AF}}{2A}$ are two vertical lines.

d. If $E = 0$, then

$Ax^2 + Dx + F = 0 \Rightarrow x = \frac{-D \pm \sqrt{D^2 - 4AF}}{2A}$

If $D^2 - 4AF < 0$, there is no real solution. The graph contains no points.

79. $Cy^2 + Dx + Ey + F = 0 \quad C \neq 0$

a. If $D \neq 0$, then:

$$Cy^2 + Ey = -Dx - F$$

$$C\left(y^2 + \frac{E}{C}y + \frac{E^2}{4C^2}\right) = -Dx - F + \frac{E^2}{4C}$$

$$\left(y + \frac{E}{2C}\right)^2 = \frac{1}{C}\left(-Dx - F + \frac{E^2}{4C}\right)$$

$$\left(y + \frac{E}{2C}\right)^2 = \frac{-D}{C}\left(x + \frac{F}{D} - \frac{E^2}{4CD}\right)$$

$$\left(y + \frac{E}{2C}\right)^2 = \frac{-D}{C}\left(x - \frac{E^2 - 4CF}{4CD}\right)$$

This is the equation of a parabola whose vertex is $\left(\frac{E^2 - 4CF}{4CD}, -\frac{E}{2C}\right)$, and whose axis of symmetry is parallel to the *x*-axis.

b. If $D = 0$, then

$Cy^2 + Ey + F = 0 \Rightarrow y = \frac{-E \pm \sqrt{E^2 - 4CF}}{2C}$

If $E^2 - 4CF = 0$, then $y = -\frac{E}{2C}$ is a single horizontal line.

c. If $D = 0$, then

$Cy^2 + Ey + F = 0 \Rightarrow y = \frac{-E \pm \sqrt{E^2 - 4CF}}{2C}$

If $E^2 - 4CF > 0$, then

$y = \frac{-E + \sqrt{E^2 - 4CF}}{2C}$ and

$y = \frac{-E - \sqrt{E^2 - 4CF}}{2C}$ are two horizontal lines.

d. If $D = 0$, then

$Cy^2 + Ey + F = 0 \Rightarrow y = \frac{-E \pm \sqrt{E^2 - 4CF}}{2C}$

If $E^2 - 4CF < 0$, then there is no real solution. The graph contains no points.

Section 10.3

1. $d=\sqrt{(4-2)^2+(-2-(-5))^2}=\sqrt{2^2+3^2}=\sqrt{13}$

2. $\left(-\frac{3}{2}\right)^2=\frac{9}{4}$

3. x-intercepts: $0^2=16-4x^2$
$4x^2=16$
$x^2=4$
$x=\pm 2 \rightarrow (-2,0),(2,0)$

y-intercepts: $y^2=16-4(0)^2$
$y^2=16$
$y=\pm 4 \rightarrow (0,-4),(0,4)$

The intercepts are $(-2,0)$, $(2,0)$, $(0,-4)$, and $(0,4)$.

4. $(2,5)$; change x to $-x$: $-2 \rightarrow -(-2)=2$

5. left 1; down 4

6. $(x-2)^2+(y-(-3))^2=1^2$
$(x-2)^2+(y+3)^2=1$

7. ellipse

8. major

9. $(0,-5);(0,5)$

10. 5; 3; x

11. $(-2,-3)$ $(6,-3)$

12. $(1,4)$

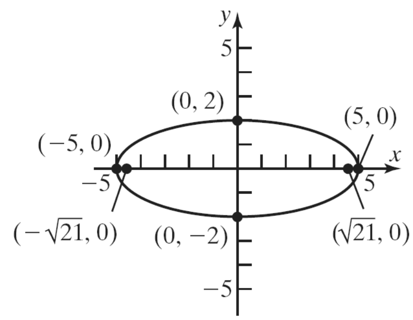
13. (c); the major axis is along the x-axis and the vertices are at $(-4,0)$ and $(4,0)$.

14. (d); the major axis is along the y-axis and the vertices are at $(0,-4)$ and $(0,4)$.

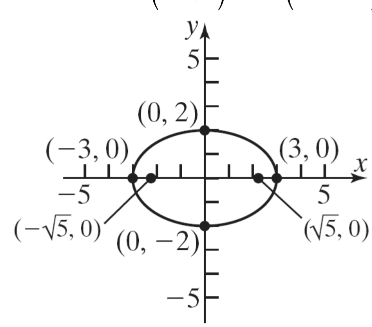
15. (b); the major axis is along the y-axis and the vertices are at $(0,-2)$ and $(0,2)$.

16. (a); the major axis is along the x-axis and the vertices are at $(-2,0)$ and $(2,0)$.

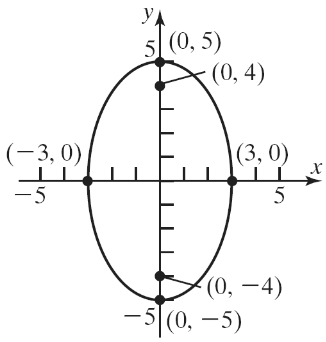
17. $\frac{x^2}{25}+\frac{y^2}{4}=1$
The center of the ellipse is at the origin.
$a=5,\ b=2$. The vertices are (5, 0) and (–5, 0).
Find the value of c:
$c^2=a^2-b^2=25-4=21 \rightarrow c=\sqrt{21}$
The foci are $(\sqrt{21},0)$ and $(-\sqrt{21},0)$.



18. $\frac{x^2}{9}+\frac{y^2}{4}=1$
The center of the ellipse is at the origin.
$a=3,\ b=2$. The vertices are (3, 0) and (–3, 0).
Find the value of c:
$c^2=a^2-b^2=9-4=5 \rightarrow c=\sqrt{5}$
The foci are $(\sqrt{5},0)$ and $(-\sqrt{5},0)$.



19. $\frac{x^2}{9}+\frac{y^2}{25}=1$
The center of the ellipse is at the origin.
$a=5,\ b=3$. The vertices are (0, 5) and (0, –5).
Find the value of c:
$c^2=a^2-b^2=25-9=16$
$c=4$
The foci are (0, 4) and (0, –4).



20. $x^2+\dfrac{y^2}{16}=1$

The center of the ellipse is at the origin.

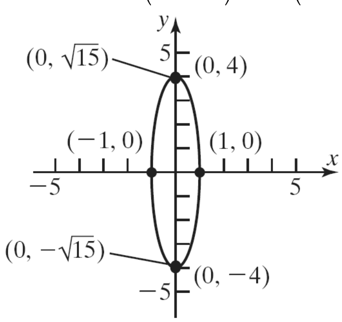
$a=4,\ b=1$. The vertices are (0, 4) and (0, –4).

Find the value of c:

$c^2=a^2-b^2=16-1=15$

$c=\sqrt{15}$

The foci are $\left(0,\sqrt{15}\right)$ and $\left(0,-\sqrt{15}\right)$



21. $4x^2+y^2=16$

Divide by 16 to put in standard form:

$\dfrac{4x^2}{16}+\dfrac{y^2}{16}=\dfrac{16}{16}$

$\dfrac{x^2}{4}+\dfrac{y^2}{16}=1$

The center of the ellipse is at the origin.

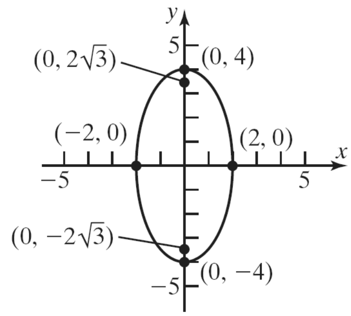
$a=4,\ b=2$.

The vertices are (0, 4) and (0, –4). Find the value of c:

$c^2=a^2-b^2=16-4=12$

$c=\sqrt{12}=2\sqrt{3}$

The foci are $\left(0,2\sqrt{3}\right)$ and $\left(0,-2\sqrt{3}\right)$.



22. $x^2+9y^2=18$

Divide by 18 to put in standard form:

$\dfrac{x^2}{18}+\dfrac{9y^2}{18}=\dfrac{18}{18}$

$\dfrac{x^2}{18}+\dfrac{y^2}{2}=1$

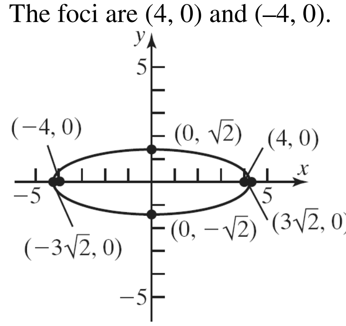
The center of the ellipse is at the origin.

$a=3\sqrt{2},\ b=\sqrt{2}$. The vertices are $\left(3\sqrt{2},0\right)$ and $\left(-3\sqrt{2},0\right)$. Find the value of c:

$c^2=a^2-b^2=18-2=16$

$c=4$

The foci are (4, 0) and (–4, 0).



23. $4y^2+x^2=8$

Divide by 8 to put in standard form:

$\dfrac{4y^2}{8}+\dfrac{x^2}{8}=\dfrac{8}{8}$

$\dfrac{x^2}{8}+\dfrac{y^2}{2}=1$

The center of the ellipse is at the origin.

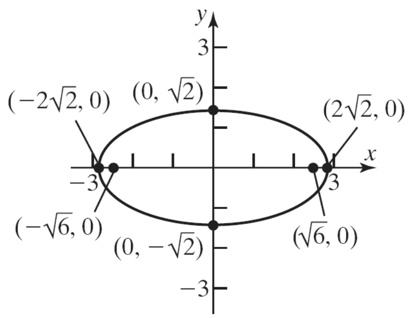
$a=\sqrt{8}=2\sqrt{2},\ b=\sqrt{2}$.

The vertices are $\left(2\sqrt{2},0\right)$ and $\left(-2\sqrt{2},0\right)$. Find the value of c:

$c^2=a^2-b^2=8-2=6$

$c=\sqrt{6}$

The foci are $\left(\sqrt{6},0\right)$ and $\left(-\sqrt{6},0\right)$.



24. $4y^2 + 9x^2 = 36$

Divide by 36 to put in standard form:

$$\frac{4y^2}{36} + \frac{9x^2}{36} = \frac{36}{36}$$

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

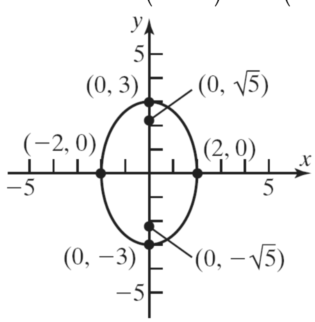
The center of the ellipse is at the origin.

$a = 3,\ b = 2$. The vertices are (0, 3) and (0, –3).

Find the value of c:

$$c^2 = a^2 - b^2 = 9 - 4 = 5 \quad \rightarrow \quad c = \sqrt{5}$$

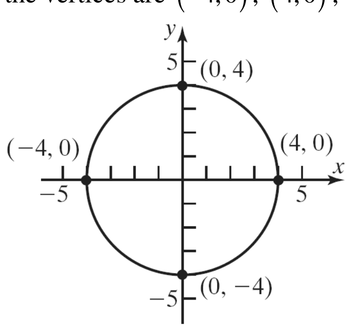
The foci are $\left(0, \sqrt{5}\right)$ and $\left(0, -\sqrt{5}\right)$.



25. $x^2 + y^2 = 16$

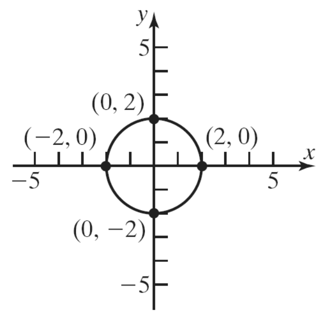
$$\frac{x^2}{16} + \frac{y^2}{16} = 1$$

This is a circle whose center is at (0, 0) and radius = 4. The focus of the ellipse is $\left(0,0\right)$ and the vertices are $\left(-4,0\right)$, $\left(4,0\right)$, $\left(0,-4\right)$, $\left(0,4\right)$.



26. $x^2 + y^2 = 4 \quad \rightarrow \quad \frac{x^2}{4} + \frac{y^2}{4} = 1$

This is a circle whose center is at (0, 0) and radius = 2. The focus of the ellipse is $\left(0,0\right)$ and the vertices are $\left(-2,0\right)$, $\left(2,0\right)$, $\left(0,-2\right)$, $\left(0,2\right)$.

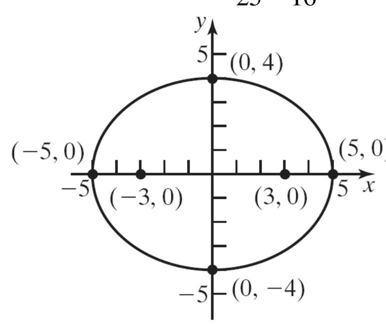


27. Center: (0, 0); Focus: (3, 0); Vertex: (5, 0); Major axis is the x-axis; $a = 5$; $c = 3$. Find b:

$$b^2 = a^2 - c^2 = 25 - 9 = 16$$

$$b = 4$$

Write the equation: $\frac{x^2}{25} + \frac{y^2}{16} = 1$

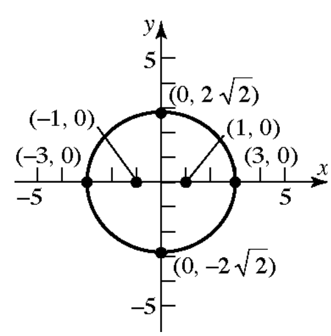


28. Center: (0, 0); Focus: (–1, 0); Vertex: (3, 0); Major axis is the x-axis; $a = 3$; $c = 1$. Find b:

$$b^2 = a^2 - c^2 = 9 - 1 = 8$$

$$b = 2\sqrt{2}$$

Write the equation: $\frac{x^2}{9} + \frac{y^2}{8} = 1$

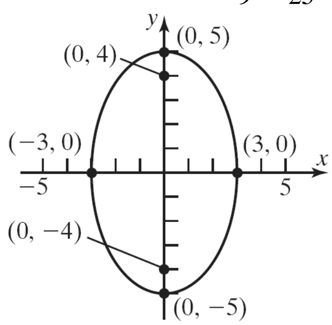


29. Center: (0, 0); Focus: (0, –4); Vertex: (0, 5); Major axis is the y-axis; $a = 5$; $c = 4$. Find b:

$b^2 = a^2 - c^2 = 25 - 16 = 9$

$b = 3$

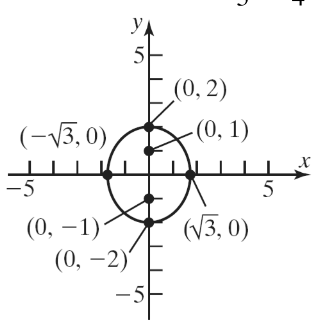
Write the equation: $\dfrac{x^2}{9} + \dfrac{y^2}{25} = 1$



30. Center: (0, 0); Focus: (0, 1); Vertex: (0, –2); Major axis is the y-axis; $a = 2$; $c = 1$. Find b:

$b^2 = a^2 - c^2 = 4 - 1 = 3 \;\;\rightarrow\;\; b = \sqrt{3}$

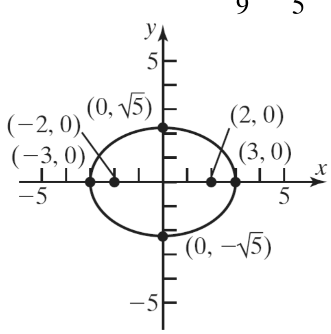
Write the equation: $\dfrac{x^2}{3} + \dfrac{y^2}{4} = 1$



31. Foci: (±2, 0); Length of major axis is 6. Center: (0, 0); Major axis is the x-axis; $a = 3$; $c = 2$. Find b:

$b^2 = a^2 - c^2 = 9 - 4 = 5 \;\;\rightarrow\;\; b = \sqrt{5}$

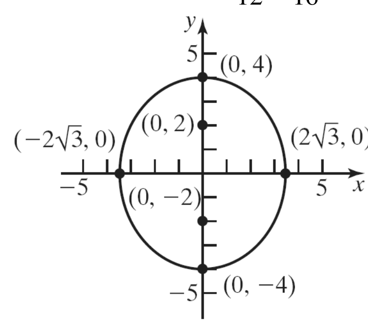
Write the equation: $\dfrac{x^2}{9} + \dfrac{y^2}{5} = 1$



32. Foci: (0, ±2); length of the major axis is 8. Center: (0, 0); Major axis is the y-axis; $a = 4$; $c = 2$. Find b :

$b^2 = a^2 - c^2 = 16 - 4 = 12 \;\;\rightarrow\;\; b = 2\sqrt{3}$

Write the equation: $\dfrac{x^2}{12} + \dfrac{y^2}{16} = 1$



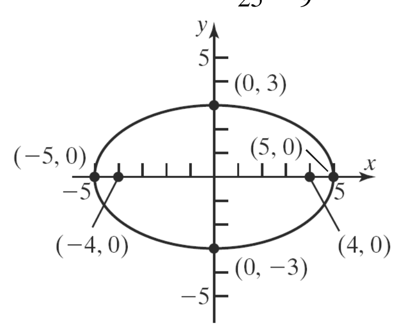
33. Focus: $(-4, 0)$; Vertices: $(-5, 0)$ and $(5, 0)$;

Center: $(0, 0)$; Major axis is the x-axis.

$a = 5$; $c = 4$. Find b:

$b^2 = a^2 - c^2 = 25 - 16 = 9 \;\;\rightarrow\;\; b = 3$

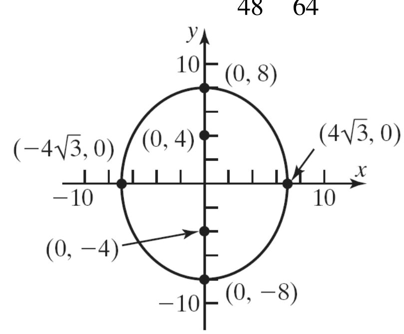
Write the equation: $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1$



34. Focus: (0, –4); Vertices: (0, ±8). Center: (0, 0); Major axis is the y-axis; $a = 8$; $c = 4$. Find b:

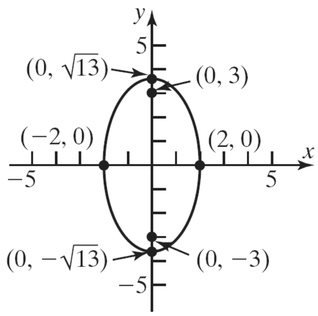
$b^2 = a^2 - c^2 = 64 - 16 = 48 \;\;\rightarrow\;\; b = 4\sqrt{3}$

Write the equation: $\dfrac{x^2}{48} + \dfrac{y^2}{64} = 1$



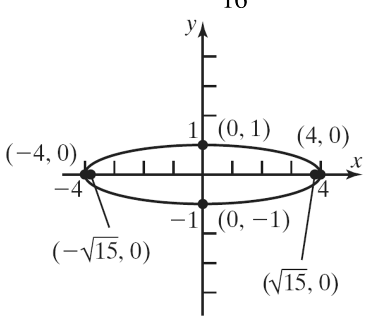
35. Foci: (0, ±3); x-intercepts are ±2.
Center: (0, 0); Major axis is the y-axis;
$c = 3$; $b = 2$. Find a:
$a^2 = b^2 + c^2 = 4 + 9 = 13 \quad \rightarrow \quad a = \sqrt{13}$

Write the equation: $\dfrac{x^2}{4} + \dfrac{y^2}{13} = 1$



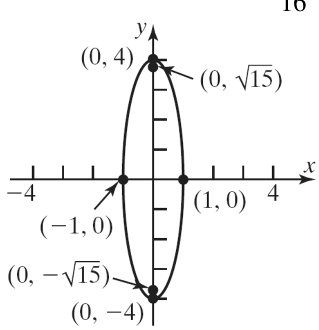
36. Vertices: (±4, 0); y-intercepts are ±1. Center: (0, 0); Major axis is the x-axis; $a = 4$; $b = 1$.
Find c:
$c^2 = a^2 - b^2 = 16 - 1 = 15$
$a = \sqrt{15}$

Write the equation: $\dfrac{x^2}{16} + y^2 = 1$



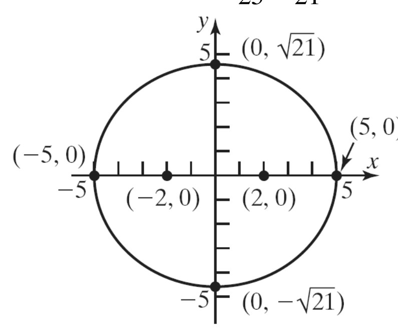
37. Center: (0, 0); Vertex: (0, 4); $b = 1$; Major axis is the y-axis; $a = 4$; $b = 1$.

Write the equation: $x^2 + \dfrac{y^2}{16} = 1$



38. Vertices: (±5, 0); $c = 2$; Major axis is the x-axis; $a = 5$; Find b:
$b^2 = a^2 - c^2 = 25 - 4 = 21$
$b = \sqrt{21}$

Write the equation: $\dfrac{x^2}{25} + \dfrac{y^2}{21} = 1$



39. Center: $(-1, 1)$
Major axis: parallel to x-axis
Length of major axis: $4 = 2a \rightarrow a = 2$
Length of minor axis: $2 = 2b \rightarrow b = 1$
$$\frac{(x+1)^2}{4} + (y-1)^2 = 1$$

40. Center: $(-1, -1)$
Major axis: parallel to y-axis
Length of major axis: $4 = 2a \rightarrow a = 2$
Length of minor axis: $2 = 2b \rightarrow b = 1$
$$(x+1)^2 + \frac{(y+1)^2}{4} = 1$$

41. Center: $(1, 0)$
Major axis: parallel to y-axis
Length of major axis: $4 = 2a \rightarrow a = 2$
Length of minor axis: $2 = 2b \rightarrow b = 1$
$$(x-1)^2 + \frac{y^2}{4} = 1$$

42. Center: $(0, 1)$
Major axis: parallel to x-axis
Length of major axis: $4 = 2a \rightarrow a = 2$
Length of minor axis: $2 = 2b \rightarrow b = 1$
$$\frac{x^2}{4} + (y-1)^2 = 1$$

43. The equation $\frac{(x-3)^2}{4}+\frac{(y+1)^2}{9}=1$ is in the form $\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$ (major axis parallel to the y-axis) where $a=3,\ b=2,$ $h=3,$ and $k=-1$. Solving for c:

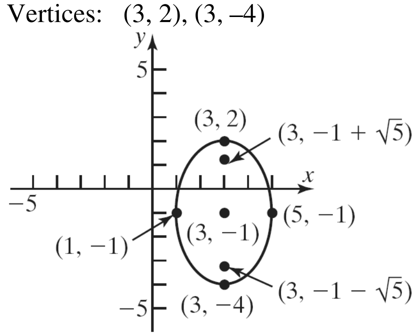
$c^2=a^2-b^2=9-4=5\rightarrow c=\sqrt{5}$

Thus, we have:

Center: (3, –1)

Foci: $\left(3,-1+\sqrt{5}\right),\ \left(3,-1-\sqrt{5}\right)$

Vertices: (3, 2), (3, –4)



44. The equation $\frac{(x+4)^2}{9}+\frac{(y+2)^2}{4}=1$ is in the form $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$

(major axis parallel to the x-axis) where $a=3,\ b=2,\ h=-4,$ and $k=-2$.

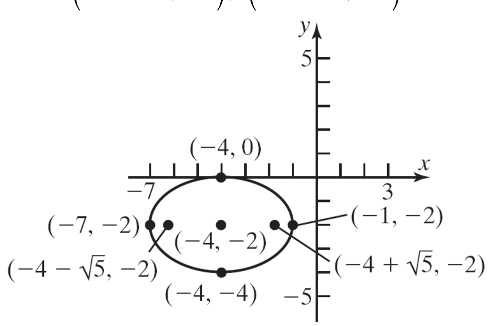
Solving for c:

$c^2=a^2-b^2=9-4=5\rightarrow c=\sqrt{5}$

Thus, we have:

Center: (–4, –2); Vertices: (–7, –2), (–1, –2)

Foci: $\left(-4+\sqrt{5},-2\right),\ \left(-4-\sqrt{5},-2\right)$



45. Divide by 16 to put the equation in standard form:

$$(x+5)^2+4(y-4)^2=16$$

$$\frac{(x+5)^2}{16}+\frac{4(y-4)^2}{16}=\frac{16}{16}$$

$$\frac{(x+5)^2}{16}+\frac{(y-4)^2}{4}=1$$

The equation is in the form

$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ (major axis parallel to the x-axis) where $a=4,\ b=2,\ h=-5,$ and $k=4$.

Solving for c:

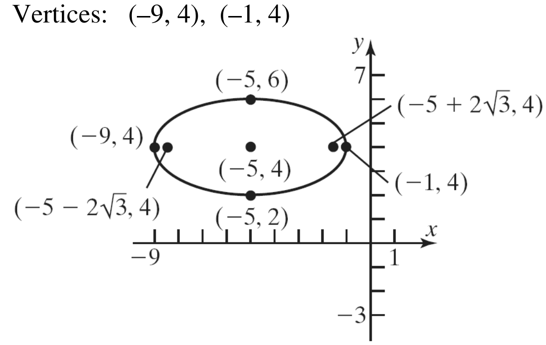
$c^2=a^2-b^2=16-4=12\rightarrow c=\sqrt{12}=2\sqrt{3}$

Thus, we have:

Center: (–5, 4)

Foci: $\left(-5-2\sqrt{3},4\right),\ \left(-5+2\sqrt{3},4\right)$

Vertices: (–9, 4), (–1, 4)



46. Divide by 18 to put the equation in standard form:

$$9(x-3)^2+(y+2)^2=18$$

$$\frac{9(x-3)^2}{18}+\frac{(y+2)^2}{18}=\frac{18}{18}$$

$$\frac{(x-3)^2}{2}+\frac{(y+2)^2}{18}=1$$

The equation is in the form

$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$ (major axis parallel to the y-axis) where $a=3\sqrt{2},\ b=\sqrt{2},$ $h=3,$ and $k=-2$.

Solving for c:

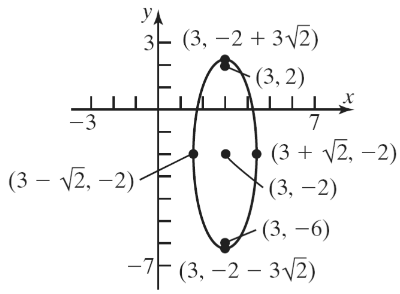
$c^2=a^2-b^2=18-2=16\rightarrow c=4$

Thus, we have:

Center: (3, –2)

Foci: (3, 2), (3, –6)

Vertices: $\left(3,-2+3\sqrt{2}\right),\ \left(3,-2-3\sqrt{2}\right)$



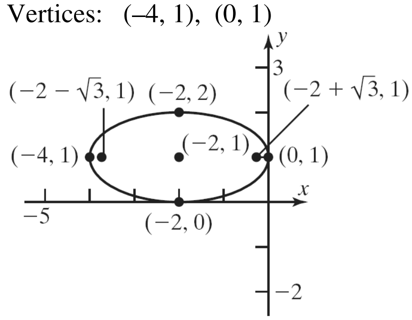
47. Complete the square to put the equation in standard form:

$$x^2+4x+4y^2-8y+4=0$$
$$(x^2+4x+4)+4(y^2-2y+1)=-4+4+4$$
$$(x+2)^2+4(y-1)^2=4$$
$$\frac{(x+2)^2}{4}+\frac{4(y-1)^2}{4}=\frac{4}{4}$$
$$\frac{(x+2)^2}{4}+(y-1)^2=1$$

The equation is in the form $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ (major axis parallel to the x-axis) where $a=2,\ b=1,\ h=-2,$ and $k=1$.

Solving for c: $c^2=a^2-b^2=4-1=3 \rightarrow c=\sqrt{3}$

Thus, we have:
Center: (–2, 1)
Foci: $\left(-2-\sqrt{3},1\right),\ \left(-2+\sqrt{3},1\right)$
Vertices: (–4, 1), (0, 1)



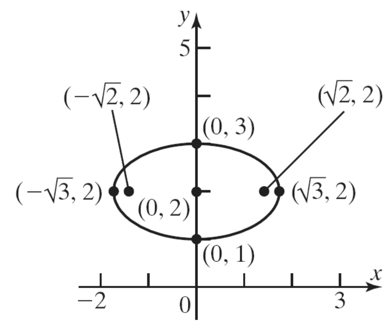
48. Complete the square to put the equation in standard form:

$$x^2+3y^2-12y+9=0$$
$$x^2+3(y^2-4y+4)=-9+12$$
$$x^2+3(y-2)^2=3$$
$$\frac{x^2}{3}+\frac{3(y-2)^2}{3}=\frac{3}{3}$$
$$\frac{x^2}{3}+(y-2)^2=1$$

The equation is in the form $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ (major axis parallel to the x-axis) where $a=\sqrt{3},\ b=1,\ h=0,$ and $k=2$.

Solving for c:
$c^2=a^2-b^2=3-1=2 \rightarrow c=\sqrt{2}$

Thus, we have:
Center: (0, 2)
Foci: $\left(-\sqrt{2},2\right),\ \left(\sqrt{2},2\right)$
Vertices: $\left(-\sqrt{3},2\right),\ \left(\sqrt{3},2\right)$



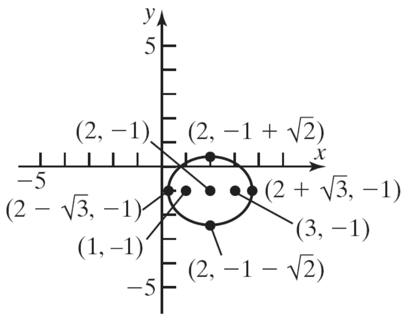
49. Complete the square to put the equation in standard form:

$$2x^2+3y^2-8x+6y+5=0$$
$$2(x^2-4x)+3(y^2+2y)=-5$$
$$2(x^2-4x+4)+3(y^2+2y+1)=-5+8+3$$
$$2(x-2)^2+3(y+1)^2=6$$
$$\frac{2(x-2)^2}{6}+\frac{3(y+1)^2}{6}=\frac{6}{6}$$
$$\frac{(x-2)^2}{3}+\frac{(y+1)^2}{2}=1$$

The equation is in the form $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ (major axis parallel to the x-axis) where
$a=\sqrt{3},\ b=\sqrt{2},\ h=2,$ and $k=-1$.

Solving for c: $c^2=a^2-b^2=3-2=1 \quad\rightarrow\quad c=1$

Thus, we have:
Center: (2, –1)
Foci: (1, –1), (3, –1)
Vertices: $\left(2-\sqrt{3},-1\right),\ \left(2+\sqrt{3},-1\right)$



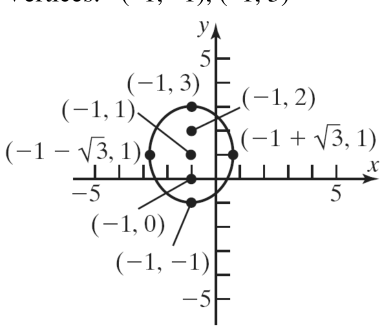
50. Complete the square to put the equation in standard form:

$$4x^2+3y^2+8x-6y=5$$
$$4(x^2+2x)+3(y^2-2y)=5$$
$$4(x^2+2x+1)+3(y^2-2y+1)=5+4+3$$
$$4(x+1)^2+3(y-1)^2=12$$
$$\frac{4(x+1)^2}{12}+\frac{3(y-1)^2}{12}=\frac{12}{12}$$
$$\frac{(x+1)^2}{3}+\frac{(y-1)^2}{4}=1$$

The equation is in the form
$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$ (major axis parallel to the y-axis) where $a=2,\ b=\sqrt{3},\ h=-1,$ and $k=1$.

Solving for c: $c^2=a^2-b^2=4-3=1 \quad \rightarrow \quad c=1$

Thus, we have:
Center: (–1, 1)
Foci: (–1, 0), (–1, 2)
Vertices: (–1, –1), (–1, 3)



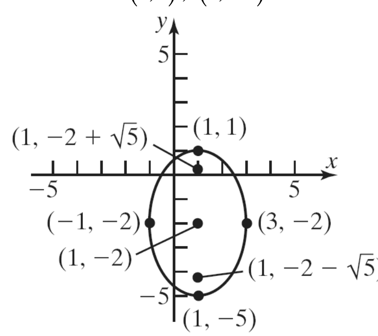
51. Complete the square to put the equation in standard form:

$$9x^2+4y^2-18x+16y-11=0$$
$$9(x^2-2x)+4(y^2+4y)=11$$
$$9(x^2-2x+1)+4(y^2+4y+4)=11+9+16$$
$$9(x-1)^2+4(y+2)^2=36$$
$$\frac{9(x-1)^2}{36}+\frac{4(y+2)^2}{36}=\frac{36}{36}$$
$$\frac{(x-1)^2}{4}+\frac{(y+2)^2}{9}=1$$

The equation is in the form
$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$ (major axis parallel to the y-axis) where $a=3,\ b=2,\ h=1,$ and $k=-2$.

Solving for c:
$c^2=a^2-b^2=9-4=5 \rightarrow c=\sqrt{5}$

Thus, we have:
Center: $(1,-2)$
Foci: $\left(1,-2+\sqrt{5}\right),\ \left(1,-2-\sqrt{5}\right)$
Vertices: $(1,1),\ (1,-5)$



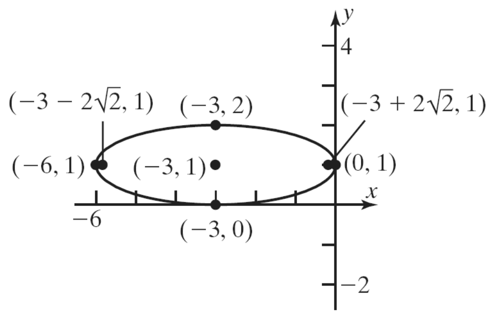
52. Complete the square to put the equation in standard form:

$$x^2+9y^2+6x-18y+9=0$$
$$(x^2+6x)+9(y^2-2y)=-9$$
$$(x^2+6x+9)+9(y^2-2y+1)=-9+9+9$$
$$(x+3)^2+9(y-1)^2=9$$
$$\frac{(x+3)^2}{9}+\frac{9(y-1)^2}{9}=\frac{9}{9}$$
$$\frac{(x+3)^2}{9}+(y-1)^2=1$$

The equation is in the form
$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ (major axis parallel to the x-axis) where $a=3,\ b=1,\ h=-3,$ and $k=1$.

Solving for c: $c^2=a^2-b^2=9-1=8 \rightarrow c=2\sqrt{2}$

Thus, we have:
Center: (–3, 1)
Foci: $\left(-3+2\sqrt{2},1\right),\ \left(-3-2\sqrt{2},1\right)$
Vertices: (0, 1), (–6, 1)



53. Complete the square to put the equation in standard form:

$$4x^2 + y^2 + 4y = 0$$
$$4x^2 + y^2 + 4y + 4 = 4$$
$$4x^2 + (y+2)^2 = 4$$
$$\frac{4x^2}{4} + \frac{(y+2)^2}{4} = \frac{4}{4}$$
$$x^2 + \frac{(y+2)^2}{4} = 1$$

The equation is in the form

$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$ (major axis parallel to the y-axis) where $a = 2,\ b = 1,\ h = 0,$ and $k = -2$.

Solving for c:

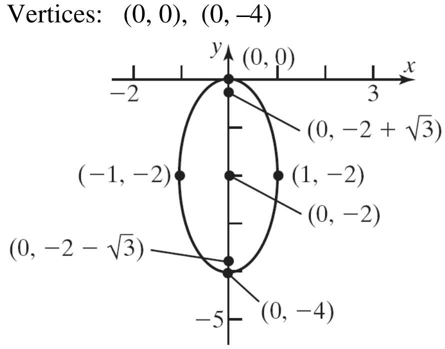
$c^2 = a^2 - b^2 = 4 - 1 = 3 \quad \rightarrow \quad c = \sqrt{3}$

Thus, we have:

Center: (0,–2)

Foci: $\left(0, -2+\sqrt{3}\right),\ \left(0, -2-\sqrt{3}\right)$

Vertices: (0, 0), (0, –4)



54. Complete the square to put the equation in standard form:

$$9x^2 + y^2 - 18x = 0$$
$$9(x^2 - 2x + 1) + y^2 = 9$$
$$9(x-1)^2 + y^2 = 9$$
$$\frac{9(x-1)^2}{9} + \frac{y^2}{9} = \frac{9}{9}$$
$$(x-1)^2 + \frac{y^2}{9} = 1$$

The equation is in the form

$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$ (major axis parallel to the y-axis) where $a = 3,\ b = 1,\ h = 1,$ and $k = 0$.

Solving for c:

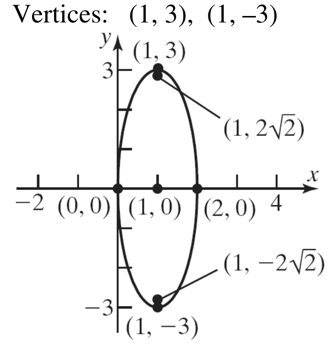
$c^2 = a^2 - b^2 = 9 - 1 = 8 \quad \rightarrow \quad c = 2\sqrt{2}$

Thus, we have:

Center: (1, 0)

Foci: $\left(1, 2\sqrt{2}\right),\ \left(1, -2\sqrt{2}\right)$

Vertices: (1, 3), (1, –3)

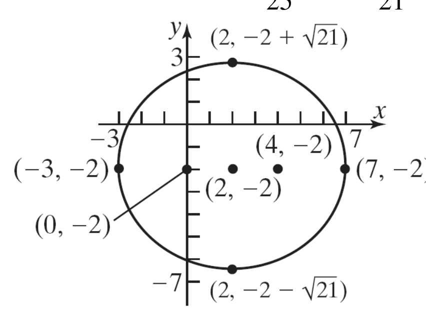


55. Center: (2, –2); Vertex: (7, –2); Focus: (4, –2); Major axis parallel to the x-axis; $a = 5;\ c = 2$.

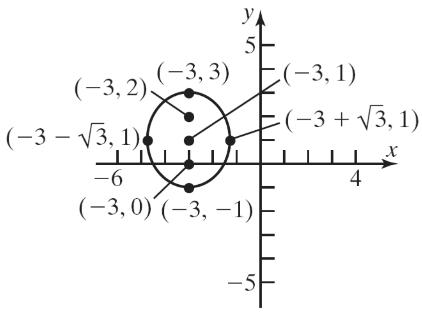
Find b:

$b^2 = a^2 - c^2 = 25 - 4 = 21 \quad \rightarrow \quad b = \sqrt{21}$

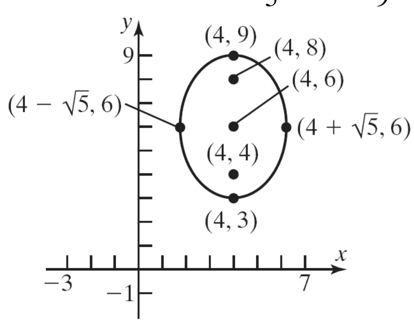
Write the equation: $\frac{(x-2)^2}{25} + \frac{(y+2)^2}{21} = 1$



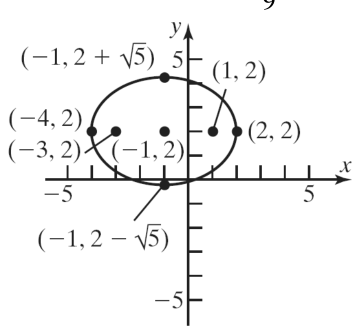
56. Center: (–3, 1); Vertex: (–3, 3); Focus: (–3, 0); Major axis parallel to the *y*-axis; $a = 2$; $c = 1$.
Find b:
$b^2 = a^2 - c^2 = 4 - 1 = 3 \rightarrow b = \sqrt{3}$
Write the equation: $\dfrac{(x+3)^2}{3} + \dfrac{(y-1)^2}{4} = 1$



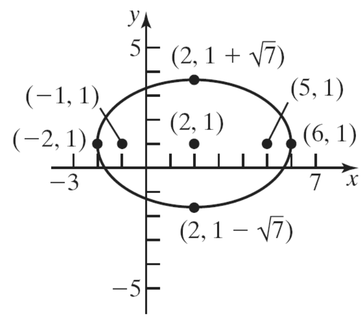
57. Vertices: (4, 3), (4, 9); Focus: (4, 8); Center: (4, 6); Major axis parallel to the *y*-axis; $a = 3$; $c = 2$. Find b:
$b^2 = a^2 - c^2 = 9 - 4 = 5 \rightarrow b = \sqrt{5}$
Write the equation: $\dfrac{(x-4)^2}{5} + \dfrac{(y-6)^2}{9} = 1$



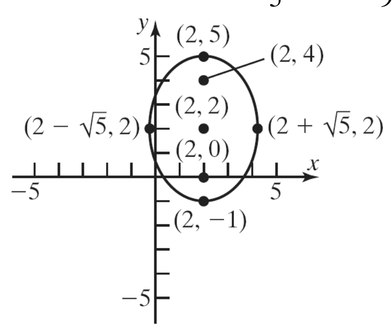
58. Foci: (1, 2), (–3, 2); Vertex: (–4, 2); Center: (–1, 2); Major axis parallel to the *x*-axis; $a = 3$; $c = 2$. Find b:
$b^2 = a^2 - c^2 = 9 - 4 = 5 \rightarrow b = \sqrt{5}$
Write the equation: $\dfrac{(x+1)^2}{9} + \dfrac{(y-2)^2}{5} = 1$



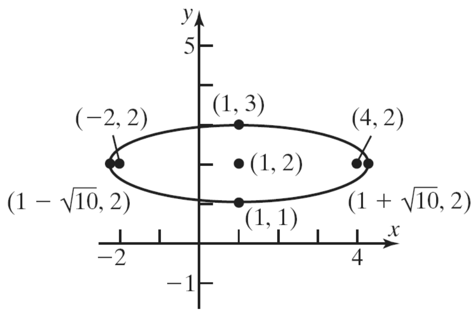
59. Foci: (5, 1), (–1, 1);
Length of the major axis = 8; Center: (2, 1); Major axis parallel to the *x*-axis; $a = 4$; $c = 3$.
Find b:
$b^2 = a^2 - c^2 = 16 - 9 = 7 \rightarrow b = \sqrt{7}$
Write the equation: $\dfrac{(x-2)^2}{16} + \dfrac{(y-1)^2}{7} = 1$



60. Vertices: (2, 5), (2, –1); $c = 2$;
Center: (2, 2); Major axis parallel to the *y*-axis; $a = 3$; $c = 2$. Find b:
$b^2 = a^2 - c^2 = 9 - 4 = 5 \rightarrow b = \sqrt{5}$
Write the equation: $\dfrac{(x-2)^2}{5} + \dfrac{(y-2)^2}{9} = 1$



61. Center: (1, 2); Focus: (4, 2); contains the point (1, 3); Major axis parallel to the *x*-axis; $c = 3$.
The equation has the form:
$$\frac{(x-1)^2}{a^2} + \frac{(y-2)^2}{b^2} = 1$$
Since the point (1, 3) is on the curve:
$$\frac{0}{a^2} + \frac{1}{b^2} = 1$$
$$\frac{1}{b^2} = 1 \rightarrow b^2 = 1 \rightarrow b = 1$$
Find a:
$a^2 = b^2 + c^2 = 1 + 9 = 10 \rightarrow a = \sqrt{10}$
Write the equation: $\dfrac{(x-1)^2}{10} + (y-2)^2 = 1$



62. Center: (1, 2); Focus: (1, 4); contains the point (2, 2); Major axis parallel to the y-axis; $c = 2$.
The equation has the form:

$$\frac{(x-1)^2}{b^2} + \frac{(y-2)^2}{a^2} = 1$$

Since the point (2, 2) is on the curve:

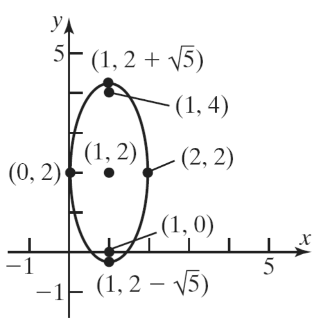
$$\frac{1}{b^2} + \frac{0}{a^2} = 1$$

$$\frac{1}{b^2} = 1 \rightarrow b^2 = 1 \rightarrow b = 1$$

Find a:

$$a^2 = b^2 + c^2 = 1 + 4 = 5 \rightarrow a = \sqrt{5}$$

Write the equation: $(x-1)^2 + \frac{(y-2)^2}{5} = 1$



63. Center: (1, 2); Vertex: (4, 2); contains the point (1, 5); Major axis parallel to the x-axis; $a = 3$.
The equation has the form:

$$\frac{(x-1)^2}{a^2} + \frac{(y-2)^2}{b^2} = 1$$

Since the point (1, 5) is on the curve:

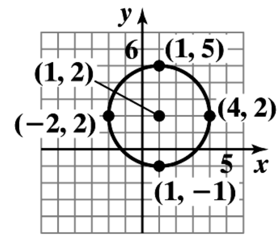
$$\frac{0}{9} + \frac{3^2}{b^2} = 1$$

$$\frac{9}{b^2} = 1 \rightarrow b^2 = 9 \rightarrow b = 3$$

Solve for c:

$c^2 = a^2 - b^2 = 9 - 9 = 0$. Thus, $c = 0$.

Write the equation: $\frac{(x-1)^2}{9} + \frac{(y-2)^2}{9} = 1$



64. Center: (1, 2); Vertex: (1, 4); contains the point $(1+\sqrt{3}, 3)$; Major axis parallel to the y-axis; $a = 2$.

The equation has the form: $\frac{(x-1)^2}{b^2} + \frac{(y-2)^2}{a^2} = 1$

Since the point $(1+\sqrt{3}, 3)$ is on the curve:

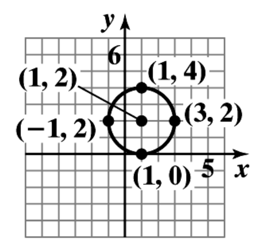
$$\frac{3}{4} + \frac{1}{b^2} = 1$$

$$\frac{1}{b^2} = \frac{1}{4} \rightarrow b^2 = 4 \rightarrow b = 2$$

Write the equation: $\frac{(x-1)^2}{4} + \frac{(y-2)^2}{4} = 1$

Solve for c:

$c^2 = a^2 - b^2 = 4 - 4 = 0$. Thus, $c = 0$.



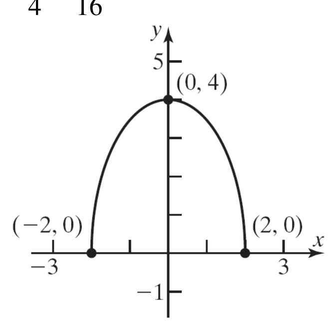
65. Rewrite the equation:

$$y = \sqrt{16 - 4x^2}$$

$$y^2 = 16 - 4x^2, \quad y \ge 0$$

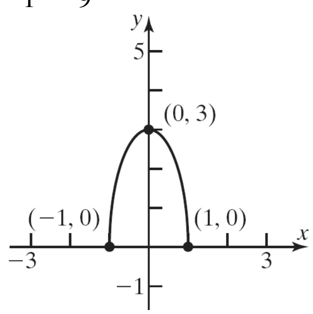
$$4x^2 + y^2 = 16, \quad y \ge 0$$

$$\frac{x^2}{4} + \frac{y^2}{16} = 1, \quad y \ge 0$$



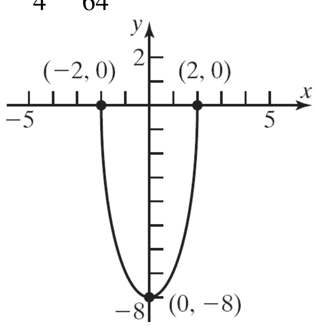
66. Rewrite the equation:

$$y = \sqrt{9 - 9x^2}$$
$$y^2 = 9 - 9x^2, \quad y \ge 0$$
$$9x^2 + y^2 = 9, \quad y \ge 0$$
$$\frac{x^2}{1} + \frac{y^2}{9} = 1, \quad y \ge 0$$



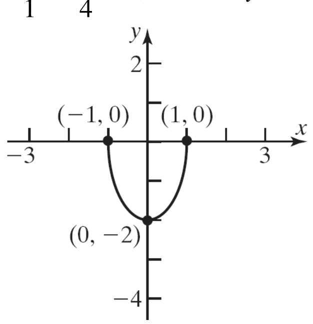
67. Rewrite the equation:

$$y = -\sqrt{64 - 16x^2}$$
$$y^2 = 64 - 16x^2, \quad y \le 0$$
$$16x^2 + y^2 = 64, \quad y \le 0$$
$$\frac{x^2}{4} + \frac{y^2}{64} = 1, \quad y \le 0$$



68. Rewrite the equation:

$$y = -\sqrt{4 - 4x^2}$$
$$y^2 = 4 - 4x^2, \quad y \le 0$$
$$4x^2 + y^2 = 4, \quad y \le 0$$
$$\frac{x^2}{1} + \frac{y^2}{4} = 1, \quad y \le 0$$



69. The center of the ellipse is (0, 0). The length of the major axis is 20, so $a = 10$. The length of half the minor axis is 6, so $b = 6$. The ellipse is situated with its major axis on the x-axis. The equation is: $\frac{x^2}{100} + \frac{y^2}{36} = 1$.

70. The center of the ellipse is (0, 0). The length of the major axis is 30, so $a = 15$. The length of half the minor axis is 10, so $b = 10$. The ellipse is situated with its major axis on the x-axis. The equation is: $\frac{x^2}{225} + \frac{y^2}{100} = 1$.

The roadway is 12 feet above the axis of the ellipse. At the center ($x = 0$), the roadway is 2 feet above the arch.

At a point 5 feet either side of the center, evaluate the equation at $x = 5$:

$$\frac{5^2}{225} + \frac{y^2}{100} = 1$$
$$\frac{y^2}{100} = 1 - \frac{25}{225} = \frac{200}{225}$$
$$y = 10\sqrt{\frac{200}{225}} \approx 9.43$$

The vertical distance from the roadway to the arch is $12 - 9.43 \approx 2.57$ feet.

At a point 10 feet either side of the center, evaluate the equation at $x = 10$:

$$\frac{10^2}{225} + \frac{y^2}{100} = 1$$
$$\frac{y^2}{100} = 1 - \frac{100}{225} = \frac{125}{225}$$
$$y = 10\sqrt{\frac{125}{225}} \approx 7.45$$

The vertical distance from the roadway to the arch is $12 - 7.45 \approx 4.55$ feet.

At a point 15 feet either side of the center, the roadway is 12 feet above the arch.

71. Assume that the half ellipse formed by the gallery is centered at (0, 0). Since the hall is 100 feet long, $2a = 100$ or $a = 50$. The distance from the center to the foci is 25 feet, so $c = 25$. Find the height of the gallery which is b:

$$b^2 = a^2 - c^2 = 2500 - 625 = 1875$$
$$b = \sqrt{1875} \approx 43.3$$

The ceiling will be 43.3 feet high in the center.

72. Assume that the half ellipse formed by the gallery is centered at (0, 0). Since the distance between the foci is 100 feet and Jim is 6 feet from the nearest wall, the length of the gallery is 112 feet. $2a = 112$ or $a = 56$. The distance from the center to the foci is 50 feet, so $c = 50$. Find the height of the gallery which is b:

$$b^2 = a^2 - c^2 = 3136 - 2500 = 636$$
$$b = \sqrt{636} \approx 25.2$$

The ceiling will be 25.2 feet high in the center.

73. Place the semi-elliptical arch so that the x-axis coincides with the water and the y-axis passes through the center of the arch. Since the bridge has a span of 120 feet, the length of the major axis is 120, or $2a = 120$ or $a = 60$. The maximum height of the bridge is 25 feet, so $b = 25$. The equation is: $\frac{x^2}{3600} + \frac{y^2}{625} = 1$.

The height 10 feet from the center:

$$\frac{10^2}{3600} + \frac{y^2}{625} = 1$$
$$\frac{y^2}{625} = 1 - \frac{100}{3600}$$
$$y^2 = 625 \cdot \frac{3500}{3600}$$
$$y \approx 24.65 \text{ feet}$$

The height 30 feet from the center:

$$\frac{30^2}{3600} + \frac{y^2}{625} = 1$$
$$\frac{y^2}{625} = 1 - \frac{900}{3600}$$
$$y^2 = 625 \cdot \frac{2700}{3600}$$
$$y \approx 21.65 \text{ feet}$$

The height 50 feet from the center:

$$\frac{50^2}{3600} + \frac{y^2}{625} = 1$$
$$\frac{y^2}{625} = 1 - \frac{2500}{3600}$$
$$y^2 = 625 \cdot \frac{1100}{3600}$$
$$y \approx 13.82 \text{ feet}$$

74. Place the semi-elliptical arch so that the x-axis coincides with the water and the y-axis passes through the center of the arch. Since the bridge has a span of 100 feet, the length of the major axis is 100, or $2a = 100$ or $a = 50$. Let h be the maximum height of the bridge. The equation is:

$$\frac{x^2}{2500} + \frac{y^2}{h^2} = 1.$$

The height of the arch 40 feet from the center is 10 feet. So (40, 10) is a point on the ellipse. Substitute and solve for h:

$$\frac{40^2}{2500} + \frac{10^2}{h^2} = 1$$
$$\frac{10^2}{h^2} = 1 - \frac{1600}{2500} = \frac{9}{25}$$
$$9h^2 = 2500$$
$$h = \frac{50}{3} \approx 16.67$$

The height of the arch at its center is 16.67 feet.

75. If the x-axis is placed along the 100 foot length and the y-axis is placed along the 50 foot length, the equation for the ellipse is: $\frac{x^2}{50^2} + \frac{y^2}{25^2} = 1$.

Find y when $x = 40$:

$$\frac{40^2}{50^2} + \frac{y^2}{25^2} = 1$$
$$\frac{y^2}{625} = 1 - \frac{1600}{2500}$$
$$y^2 = 625 \cdot \frac{9}{25}$$
$$y = 15 \text{ feet}$$

To get the width of the ellipse at $x = 40$, we need to double the y value. Thus, the width 10 feet from a vertex is 30 feet.

76. Place the semi-elliptical arch so that the x-axis coincides with the major axis and the y-axis passes through the center of the arch. Since the height of the arch at the center is 20 feet, $b = 20$. The length of the major axis is to be found, so it is necessary to solve for a. The equation is:

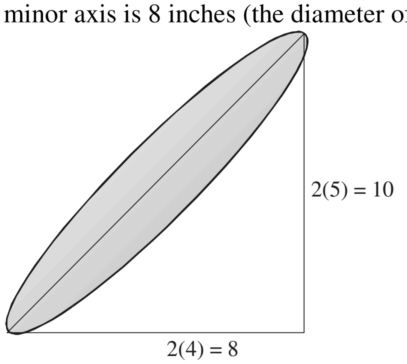
$$\frac{x^2}{a^2} + \frac{y^2}{400} = 1.$$

The height of the arch 28 feet from the center is to be 13 feet, so the point (28, 13) is on the ellipse. Substitute and solve for a:

$$\frac{28^2}{a^2}+\frac{13^2}{400}=1$$
$$\frac{784}{a^2}=1-\frac{169}{400}=\frac{231}{400}$$
$$231a^2=313600$$
$$a^2=1357.576$$
$$a=36.845$$

The span of the bridge is 73.69 feet.

77. Because of the pitch of the roof, the major axis will run parallel to the direction of the pitch and the minor axis will run perpendicular to the direction of the pitch. The length of the major axis can be determined from the pitch by using the Pythagorean Theorem. The length of the minor axis is 8 inches (the diameter of the pipe).



The length of the major axis is
$\sqrt{(8)^2+(10)^2}=\sqrt{164}=2\sqrt{41}$ inches.

78. The length of the football gives the length of the major axis so we have $2a=11.125$ or $a=5.5625$. At its center, the prolate spheroid is a circle of radius *b*. This means
$$2\pi b=28.25$$
$$b=\frac{28.25}{2\pi}$$
Thus, $\frac{4}{3}\pi ab^2=\frac{4}{3}\pi(5.5625)\left(\frac{28.25}{2\pi}\right)^2\approx 471$.

The football contains approximately 471 cubic inches of air.

79. Since the mean distance is 93 million miles, $a=93$ million. The length of the major axis is 186 million. The perihelion is
186 million – 94.5 million = 91.5 million miles.

The distance from the center of the ellipse to the sun (focus) is
93 million – 91.5 million = 1.5 million miles.

Therefore, $c=1.5$ million. Find b:
$$b^2=a^2-c^2$$
$$=\left(93\times10^6\right)^2-\left(1.5\times10^6\right)^2$$
$$=8.64675\times10^{15}$$
$$=8646.75\times10^{12}$$
$$b=92.99\times10^6$$
The equation of the orbit is:
$$\frac{x^2}{\left(93\times10^6\right)^2}+\frac{y^2}{\left(92.99\times10^6\right)^2}=1$$
We can simplify the equation by letting our units for *x* and *y* be millions of miles. The equation then becomes:
$$\frac{x^2}{8649}+\frac{y^2}{8646.75}=1$$

80. Since the mean distance is 142 million miles, $a=142$ million. The length of the major axis is 284 million. The aphelion is
284 million – 128.5 million = 155.5 million miles.
The distance from the center of the ellipse to the sun (focus) is
142 million – 128.5 million = 13.5 million miles.
Therefore, $c=13.5$ million. Find b:
$$b^2=a^2-c^2$$
$$=\left(142\times10^6\right)^2-\left(13.5\times10^6\right)^2$$
$$=1.998175\times10^{16}$$
$$b=141.36\times10^6$$
The equation of the orbit is:
$$\frac{x^2}{\left(142\times10^6\right)^2}+\frac{y^2}{\left(141.36\times10^6\right)^2}=1$$
We can simplify the equation by letting our units for *x* and *y* be millions of miles. The equation then becomes:
$$\frac{x^2}{20{,}164}+\frac{y^2}{19{,}981.75}=1$$

81. The mean distance is
507 million – 23.2 million = 483.8 million miles.

The perihelion is
483.8 million – 23.2 million = 460.6 million miles.

Since $a=483.8\times10^6$ and $c=23.2\times10^6$, we can find b:

$$b^2 = a^2 - c^2$$
$$= \left(483.8\times10^6\right)^2 - \left(23.2\times10^6\right)^2$$
$$= 2.335242\times10^{17}$$
$$b = 483.2\times10^6$$

The equation of the orbit of Jupiter is:

$$\frac{x^2}{\left(483.8\times10^6\right)^2} + \frac{y^2}{\left(483.2\times10^6\right)^2} = 1$$

We can simplify the equation by letting our units for x and y be millions of miles. The equation then becomes:

$$\frac{x^2}{234{,}062.44} + \frac{y^2}{233{,}524.2} = 1$$

82. The mean distance is
4551 million + 897.5 million = 5448.5 million miles.
The aphelion is
5448.5 million + 897.5 million = 6346 million miles.
Since $a = 5448.5\times10^6$ and $c = 897.5\times10^6$, we can find b:

$$b^2 = a^2 - c^2$$
$$= \left(5448.5\times10^6\right)^2 - \left(897.5\times10^6\right)^2$$
$$= 2.8880646\times10^{19}$$
$$b = 5374.07\times10^6$$

The equation of the orbit of Pluto is:

$$\frac{x^2}{\left(5448.5\times10^6\right)^2} + \frac{y^2}{\left(5374.07\times10^6\right)^2} = 1$$

We can simplify the equation by letting our units for x and y be millions of miles. The equation then becomes:

$$\frac{x^2}{29{,}686{,}152.25} + \frac{y^2}{28{,}880{,}646} = 1$$

83. a. Put the equation in standard ellipse form:

$$Ax^2 + Cy^2 + F = 0$$
$$Ax^2 + Cy^2 = -F$$
$$\frac{Ax^2}{-F} + \frac{Cy^2}{-F} = 1$$
$$\frac{x^2}{(-F/A)} + \frac{y^2}{(-F/C)} = 1$$

where $A \neq 0, C \neq 0, F \neq 0$, and $-F/A$ and $-F/C$ are positive.

If $A \neq C$, then $-\frac{F}{A} \neq -\frac{F}{C}$. So, this is the equation of an ellipse with center at (0, 0).

b. If $A = C$, the equation becomes:

$$Ax^2 + Ay^2 = -F \;\rightarrow\; x^2 + y^2 = \frac{-F}{A}$$

This is the equation of a circle with center at (0, 0) and radius of $\sqrt{\frac{-F}{A}}$.

84. Complete the square on the given equation:

$$Ax^2 + Cy^2 + Dx + Ey + F = 0, \quad A \neq 0, C \neq 0$$
$$Ax^2 + Cy^2 + Dx + Ey = -F$$
$$A\left(x^2 + \frac{D}{A}x\right) + C\left(y^2 + \frac{E}{C}y\right) = -F$$
$$A\left(x + \frac{D}{2A}\right)^2 + C\left(y + \frac{E}{2C}\right)^2 = \frac{D^2}{4A} + \frac{E^2}{4C} - F$$

where $A \cdot C > 0$.

Let $U = \frac{D^2}{4A} + \frac{E^2}{4C} - F$.

a. If U is of the same sign as A (and C), then

$$\frac{\left(x + \frac{D}{2A}\right)^2}{\frac{U}{A}} + \frac{\left(y + \frac{E}{2C}\right)^2}{\frac{U}{C}} = 1$$

This is the equation of an ellipse whose center is $\left(\frac{-D}{2A}, \frac{-E}{2C}\right)$.

b. If $U = 0$, the graph is the single point $\left(\frac{-D}{2A}, \frac{-E}{2C}\right)$.

c. If U is of the opposite sign as A (and C), this graph contains no points since the left side always has the opposite sign of the right side.

85. Answers will vary.

Section 10.4

1. $d=\sqrt{(-2-3)^2+(1-(-4))^2}$
$=\sqrt{(-5)^2+(5)^2}=\sqrt{25+25}=\sqrt{50}=5\sqrt{2}$

2. $\left(\frac{5}{2}\right)^2=\frac{25}{4}$

3. x-intercepts: $0^2=9+4x^2$
$4x^2=-9$
$x^2=-\frac{9}{4}$ (no real solution)
y-intercepts: $y^2=9+4(0)^2$
$y^2=9$
$y=\pm 3 \rightarrow (0,-3),(0,3)$
The intercepts are $(0,-3)$ and $(0,3)$.

4. True; the graph of $y^2=9+x^2$ is a hyperbola with its center at the origin.

5. right 5 units; down 4 units

6. $y=\frac{x^2-9}{x^2-4}$; $p(x)=x^2-9$, $q(x)=x^2-4$
The vertical asymptotes are the zeros of q.
$q(x)=0$
$x^2-4=0$
$x^2=4$
$x=-2, x=2$
The lines $x=-2$, and $x=2$ are the vertical asymptotes. The degree of the numerator, $p(x)=x^2-9$, is $n=2$. The degree of the denominator, $q(x)=x^2-4$, is $m=2$. Since $n=m$, the line $y=\frac{1}{1}=1$ is a horizontal asymptote. Since this is a rational function and there is a horizontal asymptote, there are no oblique asymptotes.

7. hyperbola

8. transverse axis

9. b

10. $(2,4)$; $(2,-2)$

11. $(2,6)$; $(2,-4)$

12. 4

13. 2; 3; x

14. $y=-\frac{4}{9}x$; $y=\frac{4}{9}x$

15. (b); the hyperbola opens to the left and right, and has vertices at $(\pm 1,0)$. Thus, the graph has an equation of the form $x^2-\frac{y^2}{b^2}=1$.

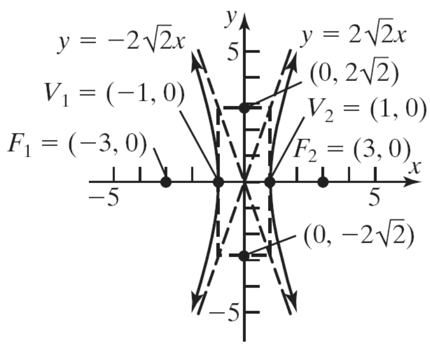
16. (c); the hyperbola opens up and down, and has vertices at $(0,\pm 2)$. Thus, the graph has an equation of the form $\frac{y^2}{4}-\frac{x^2}{b^2}=1$.

17. (a); the hyperbola opens to the left and right, and has vertices at $(\pm 2,0)$. Thus, the graph has an equation of the form $\frac{x^2}{4}-\frac{y^2}{b^2}=1$.

18. (d); the hyperbola opens up and down, and has vertices at $(0,\pm 1)$. Thus, the graph has an equation of the form $y^2-\frac{x^2}{b^2}=1$.

19. Center: (0, 0); Focus: (3, 0); Vertex: (1, 0); Transverse axis is the x-axis; $a=1$; $c=3$. Find the value of b:
$b^2=c^2-a^2=9-1=8$
$b=\sqrt{8}=2\sqrt{2}$

Write the equation: $x^2 - \dfrac{y^2}{8} = 1$.



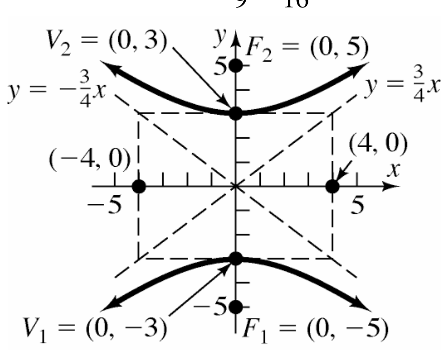
20. Center: (0, 0); Focus: (0, 5); Vertex: (0, 3); Transverse axis is the y-axis; $a = 3$; $c = 5$.

Find the value of b:

$b^2 = c^2 - a^2 = 25 - 9 = 16$

$b = 4$

Write the equation: $\dfrac{y^2}{9} - \dfrac{x^2}{16} = 1$.



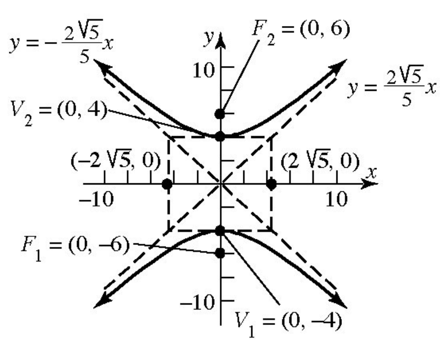
21. Center: (0, 0); Focus: (0, –6); Vertex: (0, 4) Transverse axis is the y-axis; $a = 4$; $c = 6$.

Find the value of b:

$b^2 = c^2 - a^2 = 36 - 16 = 20$

$b = \sqrt{20} = 2\sqrt{5}$

Write the equation: $\dfrac{y^2}{16} - \dfrac{x^2}{20} = 1$.



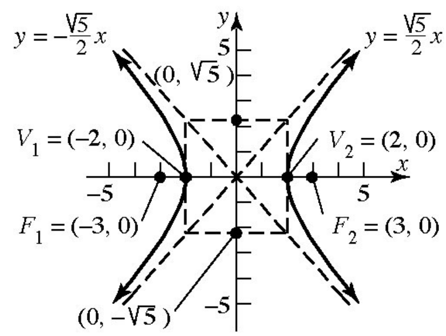
22. Center: (0, 0); Focus: (–3, 0); Vertex: (2, 0) Transverse axis is the x-axis; $a = 2$; $c = 3$.

Find the value of b:

$b^2 = c^2 - a^2 = 9 - 4 = 5$

$b = \sqrt{5}$

Write the equation: $\dfrac{x^2}{4} - \dfrac{y^2}{5} = 1$.

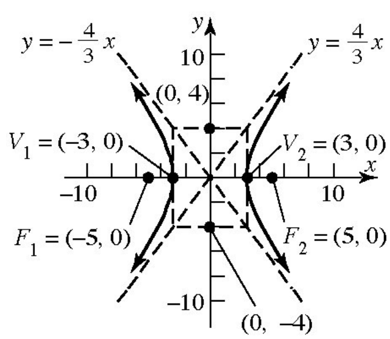


23. Foci: (–5, 0), (5, 0); Vertex: (3, 0) Center: (0, 0); Transverse axis is the x-axis; $a = 3$; $c = 5$.

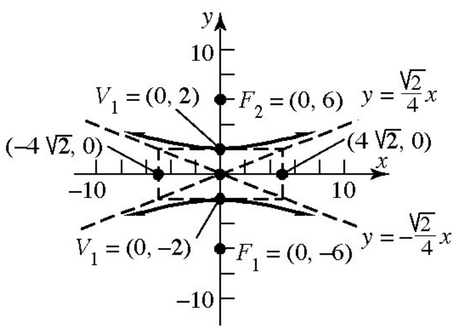
Find the value of b:

$b^2 = c^2 - a^2 = 25 - 9 = 16 \Rightarrow b = 4$

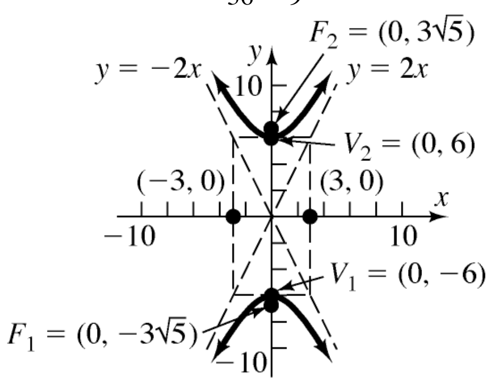
Write the equation: $\dfrac{x^2}{9} - \dfrac{y^2}{16} = 1$.



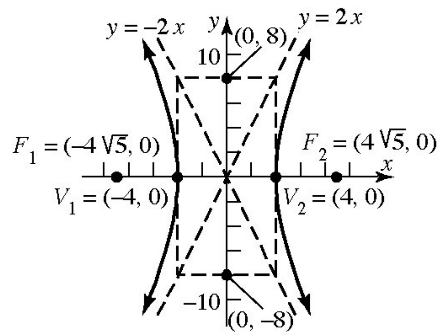
24. Focus: (0, 6); Vertices: (0, –2), (0, 2)
Center: (0, 0); Transverse axis is the y-axis; $a = 2$; $c = 6$.
Find the value of b:
$b^2 = c^2 - a^2 = 36 - 4 = 32 \Rightarrow b = 4\sqrt{2}$
Write the equation: $\dfrac{y^2}{4} - \dfrac{x^2}{32} = 1$.



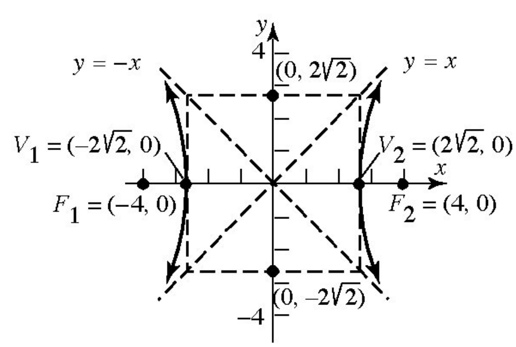
25. Vertices: (0, –6), (0, 6); asymptote: $y = 2x$;
Center: (0, 0); Transverse axis is the y-axis; $a = 6$. Find the value of b using the slope of the asymptote: $\dfrac{a}{b} = \dfrac{6}{b} = 2 \Rightarrow 2b = 6 \Rightarrow b = 3$
Find the value of c:
$c^2 = a^2 + b^2 = 36 + 9 = 45$
$c = 3\sqrt{5}$
Write the equation: $\dfrac{y^2}{36} - \dfrac{x^2}{9} = 1$.



26. Vertices: (–4, 0), (4, 0); asymptote: $y = 2x$;
Center: (0, 0); Transverse axis is the x-axis; $a = 4$. Find the value of b using the slope of the asymptote: $\dfrac{b}{a} = \dfrac{b}{4} = 2 \Rightarrow b = 8$
Find the value of c:
$c^2 = a^2 + b^2 = 16 + 64 = 80$
$c = 4\sqrt{5}$
Write the equation: $\dfrac{x^2}{16} - \dfrac{y^2}{64} = 1$.



27. Foci: (–4, 0), (4, 0); asymptote: $y = -x$;
Center: (0, 0); Transverse axis is the x-axis; $c = 4$. Using the slope of the asymptote:
$-\dfrac{b}{a} = -1 \Rightarrow -b = -a \Rightarrow b = a$.
Find the value of b:
$b^2 = c^2 - a^2 \Rightarrow a^2 + b^2 = c^2 \quad (c = 4)$
$b^2 + b^2 = 16 \Rightarrow 2b^2 = 16 \Rightarrow b^2 = 8$
$b = \sqrt{8} = 2\sqrt{2}$
$a = \sqrt{8} = 2\sqrt{2} \quad (a = b)$
Write the equation: $\dfrac{x^2}{8} - \dfrac{y^2}{8} = 1$.



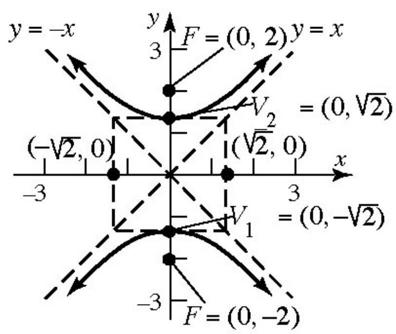
28. Foci: (0, –2), (0, 2); asymptote: $y = -x$;
Center: (0, 0); Transverse axis is the y-axis; $c = 2$. Using the slope of the asymptote:

$$-\frac{a}{b} = -1 \Rightarrow -b = -a \Rightarrow b = a$$

Find the value of b:

$$b^2 = c^2 - a^2$$
$$a^2 + b^2 = c^2 \qquad (c = 2)$$
$$b^2 + b^2 = 4 \Rightarrow 2b^2 = 4$$
$$b^2 = 2 \quad \Rightarrow \quad b = \sqrt{2}$$
$$a = \sqrt{2} \qquad (a = b)$$

Write the equation: $\frac{y^2}{2} - \frac{x^2}{2} = 1$.



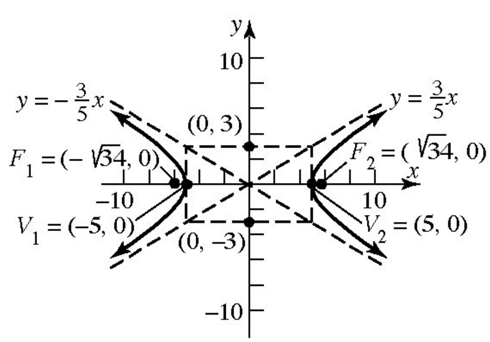
29. $\frac{x^2}{25} - \frac{y^2}{9} = 1$

The center of the hyperbola is at (0, 0).
$a = 5$, $b = 3$. The vertices are $(5, 0)$ and $(-5, 0)$. Find the value of c:

$$c^2 = a^2 + b^2 = 25 + 9 = 34 \Rightarrow c = \sqrt{34}$$

The foci are $(\sqrt{34}, 0)$ and $(-\sqrt{34}, 0)$.

The transverse axis is the x-axis. The asymptotes are $y = \frac{3}{5}x$; $y = -\frac{3}{5}x$.



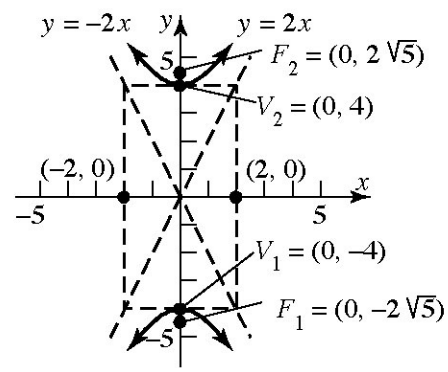
30. $\frac{y^2}{16} - \frac{x^2}{4} = 1$

The center of the hyperbola is at (0, 0).
$a = 4$, $b = 2$. The vertices are (0, 4) and (0, –4). Find the value of c:

$$c^2 = a^2 + b^2 = 16 + 4 = 20$$
$$c = \sqrt{20} = 2\sqrt{5}$$

The foci are $(0, 2\sqrt{5})$ and $(0, -2\sqrt{5})$.

The transverse axis is the y-axis. The asymptotes are $y = 2x$; $y = -2x$.



31. $4x^2 - y^2 = 16$

Divide both sides by 16 to put in standard form:

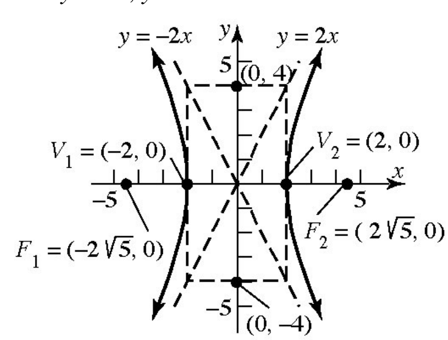
$$\frac{4x^2}{16} - \frac{y^2}{16} = \frac{16}{16} \Rightarrow \frac{x^2}{4} - \frac{y^2}{16} = 1$$

The center of the hyperbola is at (0, 0).
$a = 2$, $b = 4$.
The vertices are (2, 0) and (–2, 0).
Find the value of c:

$$c^2 = a^2 + b^2 = 4 + 16 = 20$$
$$c = \sqrt{20} = 2\sqrt{5}$$

The foci are $(2\sqrt{5}, 0)$ and $(-2\sqrt{5}, 0)$.

The transverse axis is the x-axis. The asymptotes are $y = 2x$; $y = -2x$.



32. $4y^2 - x^2 = 16$

Divide both sides by 16 to put in standard form:

$$\frac{4y^2}{16} - \frac{x^2}{16} = \frac{16}{16} \Rightarrow \frac{y^2}{4} - \frac{x^2}{16} = 1$$

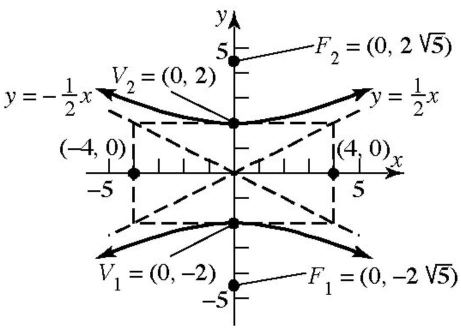
The center of the hyperbola is at (0, 0).

$a = 2,\ b = 4$. The vertices are $(0, 2)$ and $(0, -2)$. Find the value of c:

$$c^2 = a^2 + b^2 = 4 + 16 = 20 \ \rightarrow \ c = \sqrt{20} = 2\sqrt{5}$$

The foci are $\left(0, 2\sqrt{5}\right)$ and $\left(0, -2\sqrt{5}\right)$.

The transverse axis is the y-axis. The asymptotes are $y = \frac{1}{2}x$ and $y = -\frac{1}{2}x$.



33. $y^2 - 9x^2 = 9$

Divide both sides by 9 to put in standard form:

$$\frac{y^2}{9} - \frac{9x^2}{9} = \frac{9}{9} \Rightarrow \frac{y^2}{9} - x^2 = 1$$

The center of the hyperbola is at (0, 0).

$a = 3,\ b = 1$.

The vertices are (0, 3) and (0, –3).

Find the value of c:

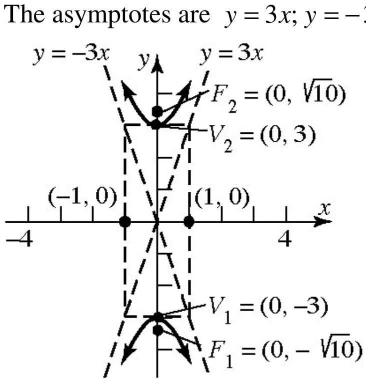
$$c^2 = a^2 + b^2 = 9 + 1 = 10$$

$$c = \sqrt{10}$$

The foci are $\left(0, \sqrt{10}\right)$ and $\left(0, -\sqrt{10}\right)$.

The transverse axis is the y-axis.

The asymptotes are $y = 3x;\ y = -3x$.



34. $x^2 - y^2 = 4$

Divide both sides by 4 to put in standard form:

$$\frac{x^2}{4} - \frac{y^2}{4} = \frac{4}{4} \Rightarrow \frac{x^2}{4} - \frac{y^2}{4} = 1.$$

The center of the hyperbola is at (0, 0).

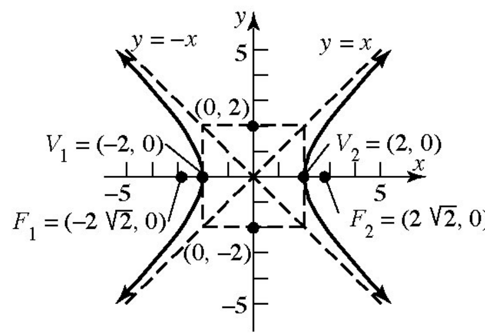
$a = 2,\ b = 2$. The vertices are $(2, 0)$ and $(-2, 0)$. Find the value of c:

$$c^2 = a^2 + b^2 = 4 + 4 = 8 \ \rightarrow \ c = \sqrt{8} = 2\sqrt{2}$$

The foci are $\left(2\sqrt{2}, 0\right)$ and $\left(-2\sqrt{2}, 0\right)$.

The transverse axis is the x-axis.

The asymptotes are $y = x;\ y = -x$.



35. $y^2 - x^2 = 25$

Divide both sides by 25 to put in standard form:

$$\frac{y^2}{25} - \frac{x^2}{25} = 1.$$

The center of the hyperbola is at (0, 0).

$a = 5,\ b = 5$. The vertices are $(0, 5)$ and $(0, -5)$. Find the value of c:

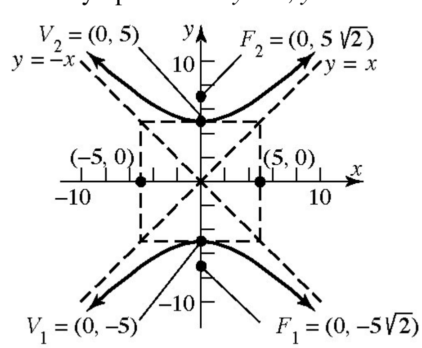
$$c^2 = a^2 + b^2 = 25 + 25 = 50$$

$$c = \sqrt{50} = 5\sqrt{2}$$

The foci are $\left(0, 5\sqrt{2}\right)$ and $\left(0, -5\sqrt{2}\right)$.

The transverse axis is the y-axis.

The asymptotes are $y = x;\ y = -x$.



36. $2x^2 - y^2 = 4$

Divide both sides by 4 to put in standard form:

$\frac{x^2}{2} - \frac{y^2}{4} = 1$.

The center of the hyperbola is at (0, 0).

$a = \sqrt{2},\ b = 2$.

The vertices are $\left(\sqrt{2}, 0\right)$ and $\left(-\sqrt{2}, 0\right)$.

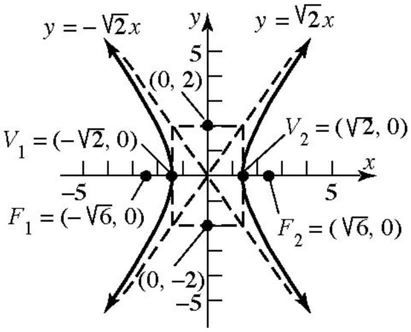
Find the value of c:

$c^2 = a^2 + b^2 = 2 + 4 = 6 \quad \rightarrow \quad c = \sqrt{6}$

The foci are $\left(\sqrt{6}, 0\right)$ and $\left(-\sqrt{6}, 0\right)$.

The transverse axis is the x-axis.

The asymptotes are $y = \sqrt{2}x;\ y = -\sqrt{2}\,x$.



37. The center of the hyperbola is at (0, 0).

$a = 1,\ b = 1$. The vertices are $(1, 0)$ and $(-1, 0)$.

Find the value of c:

$c^2 = a^2 + b^2 = 1 + 1 = 2$

$c = \sqrt{2}$

The foci are $\left(\sqrt{2}, 0\right)$ and $\left(-\sqrt{2}, 0\right)$.

The transverse axis is the x-axis.

The asymptotes are $y = x;\ y = -x$.

The equation is: $x^2 - y^2 = 1$.

38. The center of the hyperbola is at (0, 0).

$a = 1,\ b = 1$. The vertices are $(0, -1)$ and $(0, 1)$.

Find the value of c:

$c^2 = a^2 + b^2 = 1 + 1 = 2$

$c = \sqrt{2}$

The foci are $\left(0, -\sqrt{2}\right)$ and $\left(0, \sqrt{2}\right)$.

The transverse axis is the y-axis.

The asymptotes are $y = x;\ y = -x$.

The equation is: $y^2 - x^2 = 1$.

39. The center of the hyperbola is at (0, 0).

$a = 6,\ b = 3$.

The vertices are $(0, -6)$ and $(0, 6)$. Find the value of c:

$c^2 = a^2 + b^2 = 36 + 9 = 45$

$c = \sqrt{45} = 3\sqrt{5}$

The foci are $\left(0, -3\sqrt{5}\right)$ and $\left(0, 3\sqrt{5}\right)$.

The transverse axis is the y-axis.

The asymptotes are $y = 2x;\ y = -2x$. The equation is: $\frac{y^2}{36} - \frac{x^2}{9} = 1$.

40. The center of the hyperbola is at (0, 0).

$a = 2,\ b = 4$.

The vertices are $(-2, 0)$ and $(2, 0)$.

Find the value of c:

$c^2 = a^2 + b^2 = 4 + 16 = 20$

$c = \sqrt{20} = 2\sqrt{5}$

The foci are $\left(-2\sqrt{5}, 0\right)$ and $\left(2\sqrt{5}, 0\right)$.

The transverse axis is the x-axis.

The asymptotes are $y = 2x;\ y = -2x$.

The equation is: $\frac{x^2}{4} - \frac{y^2}{16} = 1$.

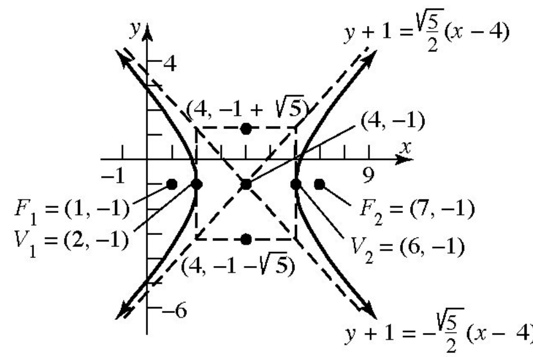
41. Center: (4, –1); Focus: (7, –1); Vertex: (6, –1);
Transverse axis is parallel to the x-axis;

$a = 2;\ c = 3$.

Find the value of b:

$b^2 = c^2 - a^2 = 9 - 4 = 5 \Rightarrow b = \sqrt{5}$

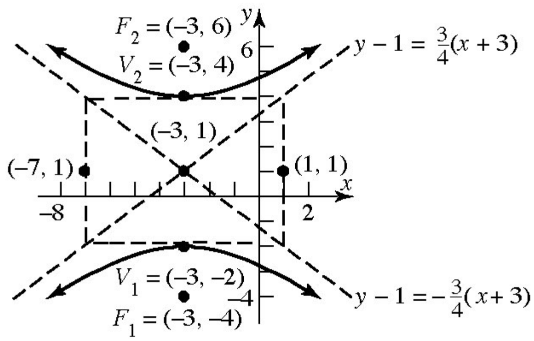
Write the equation: $\frac{(x-4)^2}{4} - \frac{(y+1)^2}{5} = 1$.



42. Center: (–3, 1); Focus: (–3, 6); Vertex: (–3, 4); Transverse axis is parallel to the y-axis; $a = 3$; $c = 5$. Find the value of b:

$b^2 = c^2 - a^2 = 25 - 9 = 16 \Rightarrow b = 4$

Write the equation: $\dfrac{(y-1)^2}{9} - \dfrac{(x+3)^2}{16} = 1$.



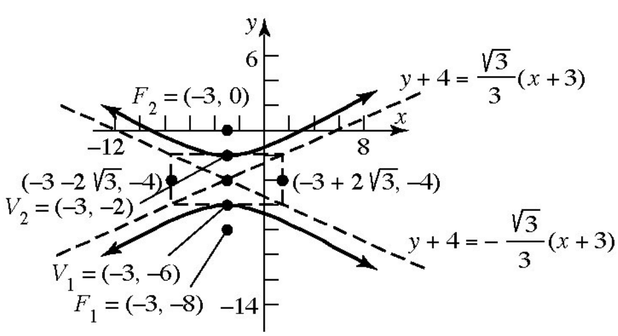
43. Center: (–3, –4); Focus: (–3, –8); Vertex: (–3, –2); Transverse axis is parallel to the y-axis; $a = 2$; $c = 4$.

Find the value of b:

$b^2 = c^2 - a^2 = 16 - 4 = 12$

$b = \sqrt{12} = 2\sqrt{3}$

Write the equation: $\dfrac{(y+4)^2}{4} - \dfrac{(x+3)^2}{12} = 1$.

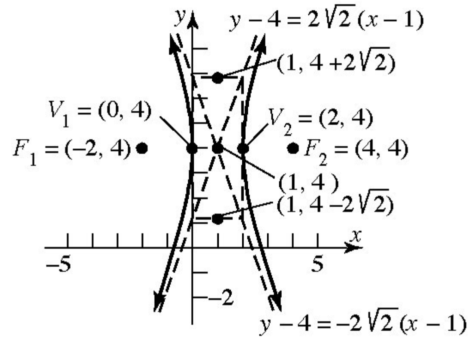


44. Center: (1, 4); Focus: (–2, 4); Vertex: (0, 4); Transverse axis is parallel to the x-axis; $a = 1$; $c = 3$. Find the value of b:

$b^2 = c^2 - a^2 = 9 - 1 = 8$

$b = \sqrt{8} = 2\sqrt{2}$

Write the equation: $(x-1)^2 - \dfrac{(y-4)^2}{8} = 1$.



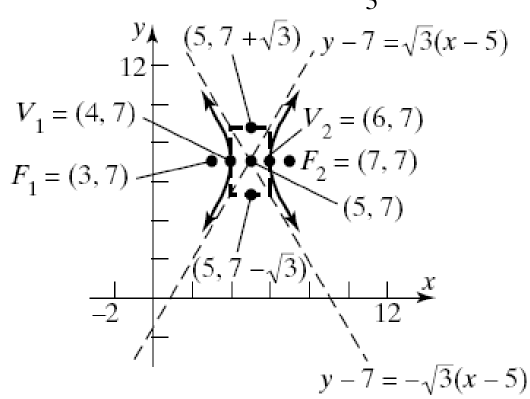
45. Foci: (3, 7), (7, 7); Vertex: (6, 7); Center: (5, 7); Transverse axis is parallel to the x-axis; $a = 1$; $c = 2$.

Find the value of b:

$b^2 = c^2 - a^2 = 4 - 1 = 3$

$b = \sqrt{3}$

Write the equation: $(x-5)^2 - \dfrac{(y-7)^2}{3} = 1$.



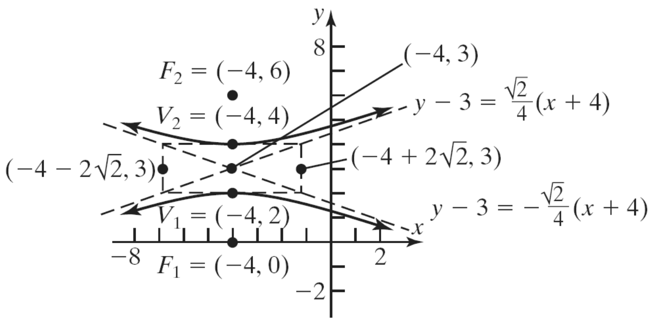
46. Focus: (–4, 0); Vertices: (–4, 4), (–4, 2); Center: (–4, 3); Transverse axis is parallel to the y-axis; $a=1$; $c=3$.

ind the value of b:

$b^2=c^2-a^2=9-1=8$

$b=\sqrt{8}=2\sqrt{2}$

Write the equation: $(y-3)^2-\dfrac{(x+4)^2}{8}=1$.



47. Vertices: (–1, –1), (3, –1); Center: (1, –1); Transverse axis is parallel to the x-axis; $a=2$.

Asymptote: $y+1=\dfrac{3}{2}(x-1)$

Using the slope of the asymptote, find the value of b:

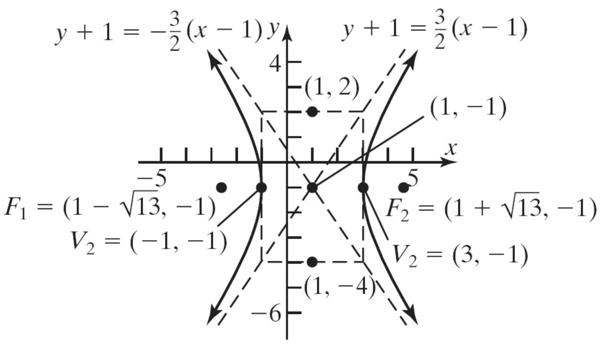
$\dfrac{b}{a}=\dfrac{b}{2}=\dfrac{3}{2}\Rightarrow b=3$

Find the value of c:

$c^2=a^2+b^2=4+9=13$

$c=\sqrt{13}$

Write the equation: $\dfrac{(x-1)^2}{4}-\dfrac{(y+1)^2}{9}=1$.



48. Vertices: (1, –3), (1, 1); Center: (1, –1); Transverse axis is parallel to the y-axis; $a=2$.

Asymptote: $y+1=\dfrac{3}{2}(x-1)$

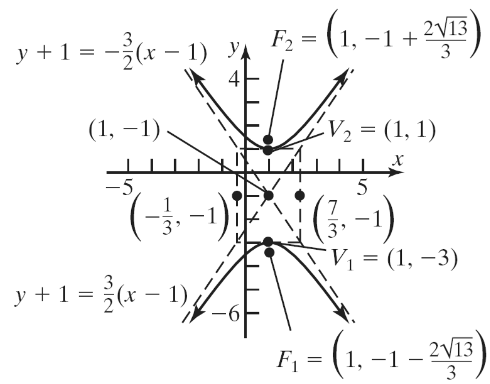
Using the slope of the asymptote, find the value of b:

$\dfrac{a}{b}=\dfrac{2}{b}=\dfrac{3}{2}\Rightarrow 3b=4\Rightarrow b=\dfrac{4}{3}$

Find the value of c:

$c^2=a^2+b^2=4+\dfrac{16}{9}=\dfrac{52}{9}\Rightarrow c=\sqrt{\dfrac{52}{9}}=\dfrac{2\sqrt{13}}{3}$

Write the equation: $\dfrac{(y+1)^2}{4}-\dfrac{9(x-1)^2}{16}=1$.



49. $\dfrac{(x-2)^2}{4}-\dfrac{(y+3)^2}{9}=1$

The center of the hyperbola is at (2, –3).

$a=2,\ b=3$.

The vertices are (0, –3) and (4, –3).

Find the value of c:

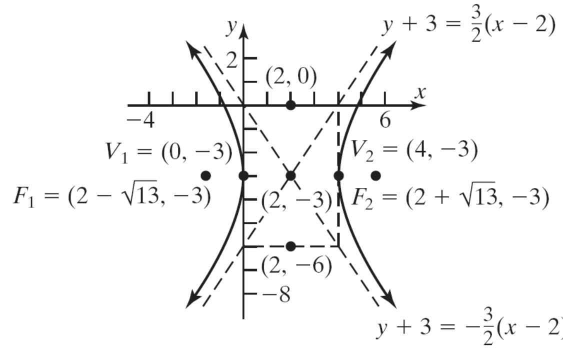
$c^2=a^2+b^2=4+9=13\Rightarrow c=\sqrt{13}$

Foci: $\left(2-\sqrt{13},-3\right)$ and $\left(2+\sqrt{13},-3\right)$.

Transverse axis: $y=-3$, parallel to x-axis.

Asymptotes: $y+3=\dfrac{3}{2}(x-2)$;

$$y+3=-\frac{3}{2}(x-2)$$



50. $\dfrac{(y+3)^2}{4}-\dfrac{(x-2)^2}{9}=1$

The center of the hyperbola is at $(2, -3)$.

$a=2,\ b=3$. The vertices are (2, –1) and (2, –5).

Find the value of c:

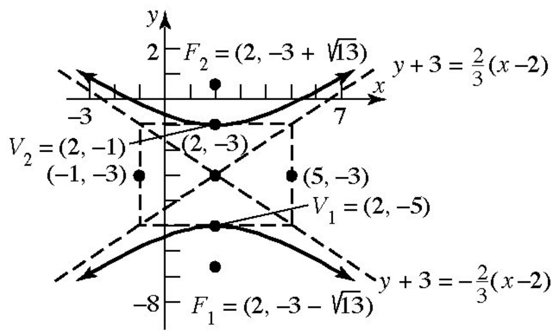
$c^2=a^2+b^2=4+9=13 \Rightarrow c=\sqrt{13}$

Foci: $\left(2,-3-\sqrt{13}\right)$ and $\left(2,-3+\sqrt{13}\right)$

Transverse axis: $x=2$, parallel to the y-axis

Asymptotes: $y+3=\dfrac{2}{3}(x-2)$;

$$y+3=-\dfrac{2}{3}(x-2)$$



51. $(y-2)^2-4(x+2)^2=4$

Divide both sides by 4 to put in standard form:

$\dfrac{(y-2)^2}{4}-(x+2)^2=1$.

The center of the hyperbola is at (–2, 2).

$a=2,\ b=1$.

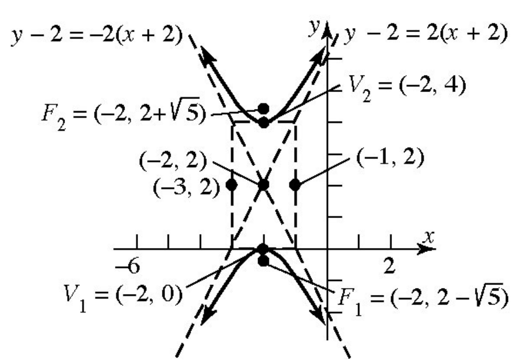
The vertices are (–2, 4) and (–2, 0). Find the value of c:

$c^2=a^2+b^2=4+1=5 \Rightarrow c=\sqrt{5}$

Foci: $\left(-2, 2-\sqrt{5}\right)$ and $\left(-2, 2+\sqrt{5}\right)$.

Transverse axis: $x=-2$, parallel to the y-axis.

Asymptotes: $y-2=2(x+2)$; $y-2=-2(x+2)$.



52. $(x+4)^2-9(y-3)^2=9$

Divide both sides by 9 to put in standard form:

$\dfrac{(x+4)^2}{9}-(y-3)^2=1$.

The center of the hyperbola is (–4, 3).

$a=3,\ b=1$.

The vertices are (–7, 3) and (–1, 3).

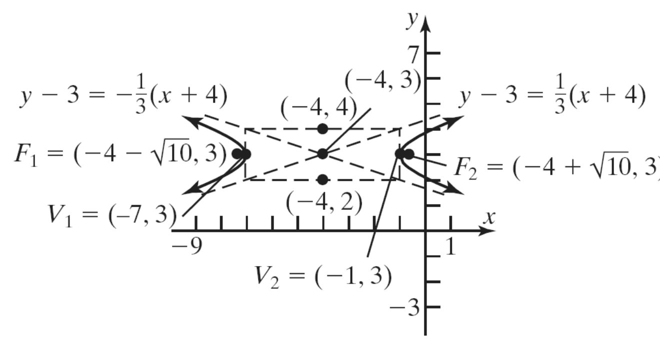
Find the value of c:

$c^2=a^2+b^2=9+1=10 \Rightarrow c=\sqrt{10}$

Foci: $\left(-4-\sqrt{10}, 3\right)$ and $\left(-4+\sqrt{10}, 3\right)$

Transverse axis: $y=3$, parallel to the x-axis.

Asymptotes: $y-3=\dfrac{1}{3}(x+4)$, $y-3=-\dfrac{1}{3}(x+4)$



53. $(x+1)^2-(y+2)^2=4$

Divide both sides by 4 to put in standard form:

$\dfrac{(x+1)^2}{4}-\dfrac{(y+2)^2}{4}=1$.

The center of the hyperbola is (–1, –2).

$a=2,\ b=2$.

The vertices are (–3, –2) and (1, –2).

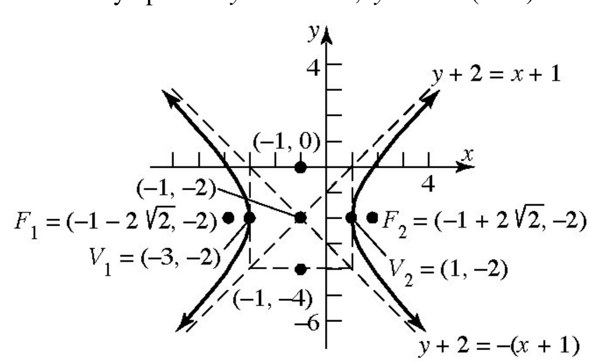
Find the value of c:

$c^2=a^2+b^2=4+4=8 \Rightarrow c=\sqrt{8}=2\sqrt{2}$

Foci: $\left(-1-2\sqrt{2}, -2\right)$ and $\left(-1+2\sqrt{2}, -2\right)$

Transverse axis: $y=-2$, parallel to the x-axis.

Asymptotes: $y+2=x+1$; $y+2=-(x+1)$



54. $(y-3)^2-(x+2)^2=4$

Divide both sides by 4 to put in standard form:

$\frac{(y-3)^2}{4}-\frac{(x+2)^2}{4}=1$. The center of the hyperbola is at (–2, 3). $a=2,\ b=2$.

The vertices are (–2, 5) and (–2, 1).

Find the value of c:

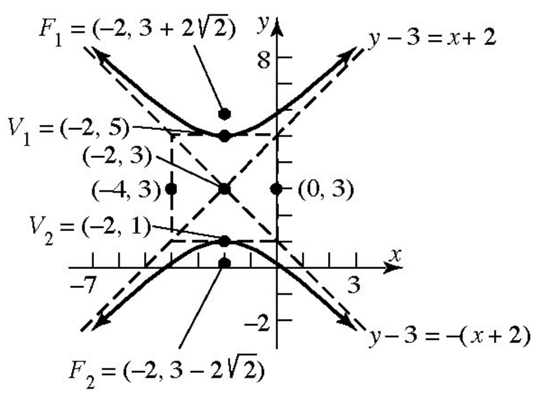
$c^2=a^2+b^2=4+4=8\Rightarrow c=\sqrt{8}=2\sqrt{2}$

Foci: $\left(-2,3-2\sqrt{2}\right)$ and $\left(-2,3+2\sqrt{2}\right)$

Transverse axis: $x=-2$, parallel to the y-axis.

Asymptotes: $y-3=x+2$;

$y-3=-(x+2)$



55. Complete the squares to put in standard form:

$$x^2-y^2-2x-2y-1=0$$
$$(x^2-2x+1)-(y^2+2y+1)=1+1-1$$
$$(x-1)^2-(y+1)^2=1$$

The center of the hyperbola is (1, –1).

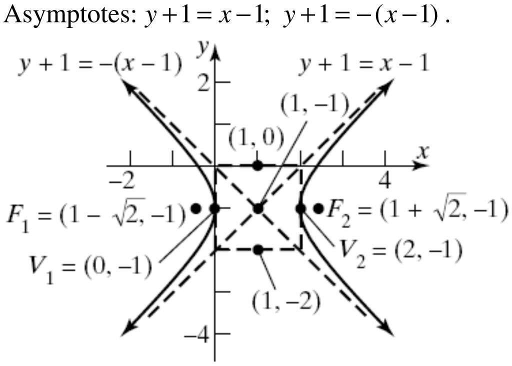
$a=1,\ b=1$. The vertices are (0, –1) and (2, –1). Find the value of c:

$c^2=a^2+b^2=1+1=2\Rightarrow c=\sqrt{2}$

Foci: $\left(1-\sqrt{2},-1\right)$ and $\left(1+\sqrt{2},-1\right)$.

Transverse axis: $y=-1$, parallel to x-axis.

Asymptotes: $y+1=x-1$; $y+1=-(x-1)$.



56. Complete the squares to put in standard form:

$$y^2-x^2-4y+4x-1=0$$
$$(y^2-4y+4)-(x^2-4x+4)=1+4-4$$
$$(y-2)^2-(x-2)^2=1$$

The center of the hyperbola is (2, 2).

$a=1,\ b=1$. The vertices are $(2,1)$ and $(2,3)$.

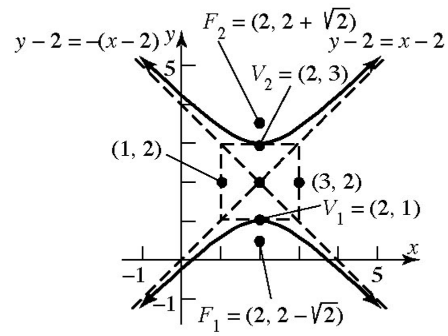
Find the value of c:

$c^2=a^2+b^2=1+1=2\Rightarrow c=\sqrt{2}$

Foci: $\left(2,2-\sqrt{2}\right)$ and $\left(2,2+\sqrt{2}\right)$.

Transverse axis: $x=2$, parallel to the y-axis.

Asymptotes: $y-2=x-2$; $y-2=-(x-2)$.



57. Complete the squares to put in standard form:

$$y^2-4x^2-4y-8x-4=0$$
$$(y^2-4y+4)-4(x^2+2x+1)=4+4-4$$
$$(y-2)^2-4(x+1)^2=4$$
$$\frac{(y-2)^2}{4}-(x+1)^2=1$$

The center of the hyperbola is (–1, 2).

$a=2,\ b=1$.

The vertices are (–1, 4) and (–1, 0).

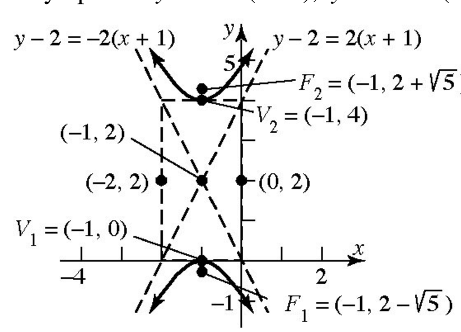
Find the value of c:

$c^2=a^2+b^2=4+1=5\Rightarrow c=\sqrt{5}$

Foci: $\left(-1,2-\sqrt{5}\right)$ and $\left(-1,2+\sqrt{5}\right)$.

Transverse axis: $x=-1$, parallel to the y-axis.

Asymptotes: $y-2=2(x+1)$; $y-2=-2(x+1)$.



58. Complete the squares to put in standard form:

$$2x^2 - y^2 + 4x + 4y - 4 = 0$$
$$2(x^2 + 2x + 1) - (y^2 - 4y + 4) = 4 + 2 - 4$$
$$2(x+1)^2 - (y-2)^2 = 2$$
$$(x+1)^2 - \frac{(y-2)^2}{2} = 1$$

The center of the hyperbola is (–1, 2).

$a = 1,\ b = \sqrt{2}$.

The vertices are (–2, 2) and (0, 2). Find the value of c:

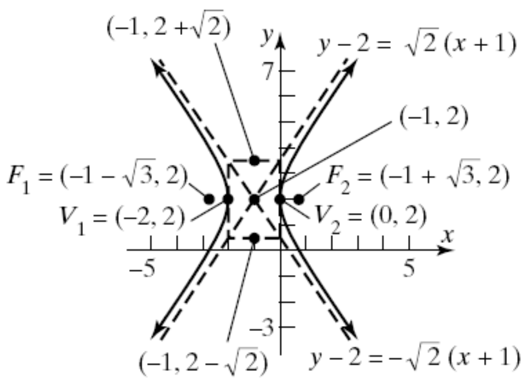
$c^2 = a^2 + b^2 = 1 + 2 = 3 \Rightarrow c = \sqrt{3}$

Foci: $\left(-1-\sqrt{3}, 2\right)$ and $\left(-1+\sqrt{3}, 2\right)$.

Transverse axis: $y = 2$, parallel to the x-axis.

Asymptotes:

$y - 2 = \sqrt{2}(x+1);\ y - 2 = -\sqrt{2}(x+1)$.



59. Complete the squares to put in standard form:

$$4x^2 - y^2 - 24x - 4y + 16 = 0$$
$$4(x^2 - 6x + 9) - (y^2 + 4y + 4) = -16 + 36 - 4$$
$$4(x-3)^2 - (y+2)^2 = 16$$
$$\frac{(x-3)^2}{4} - \frac{(y+2)^2}{16} = 1$$

The center of the hyperbola is (3, –2).

$a = 2,\ b = 4$.

The vertices are (1, –2) and (5, –2). Find the value of c:

$c^2 = a^2 + b^2 = 4 + 16 = 20$

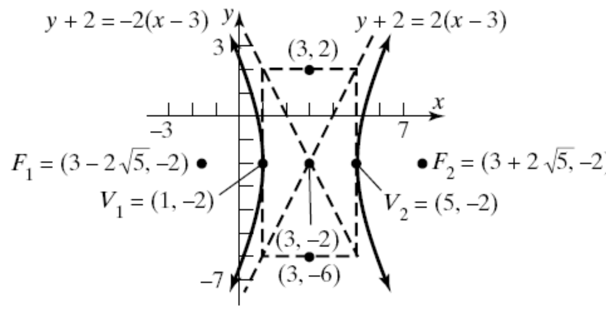
$c = \sqrt{20} = 2\sqrt{5}$

Foci: $\left(3 - 2\sqrt{5}, -2\right)$ and $\left(3 + 2\sqrt{5}, -2\right)$.

Transverse axis: $y = -2$, parallel to x-axis.

Asymptotes: $y + 2 = 2(x - 3)$;

$y + 2 = -2(x - 3)$



60. Complete the squares to put in standard form:

$$2y^2 - x^2 + 2x + 8y + 3 = 0$$
$$2(y^2 + 4y + 4) - (x^2 - 2x + 1) = -3 + 8 - 1$$
$$2(y+2)^2 - (x-1)^2 = 4$$
$$\frac{(y+2)^2}{2} - \frac{(x-1)^2}{4} = 1$$

The center of the hyperbola is (1,–2).

$a = \sqrt{2},\ b = 2$.

Vertices: $\left(1, -2-\sqrt{2}\right)$ and $\left(1, -2+\sqrt{2}\right)$

Find the value of c:

$c^2 = a^2 + b^2 = 2 + 4 = 6$

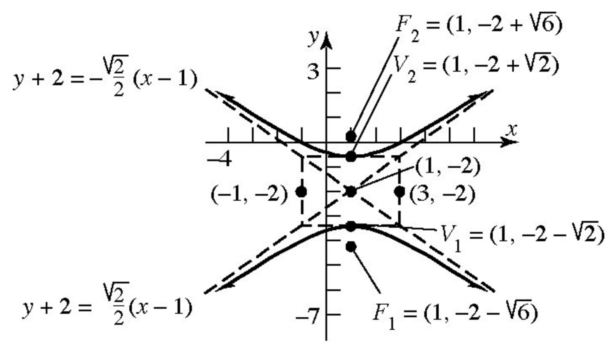
$c = \sqrt{6}$

Foci: $\left(1, -2-\sqrt{6}\right)$ and $\left(1, -2+\sqrt{6}\right)$.

Transverse axis: $x = 1$, parallel to the y-axis.

Asymptotes: $y + 2 = \frac{\sqrt{2}}{2}(x-1)$;

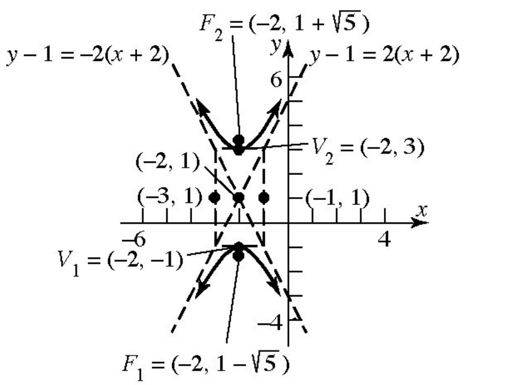
$$y + 2 = -\frac{\sqrt{2}}{2}(x-1)$$



61. Complete the squares to put in standard form:

$$y^2 - 4x^2 - 16x - 2y - 19 = 0$$
$$(y^2 - 2y + 1) - 4(x^2 + 4x + 4) = 19 + 1 - 16$$
$$(y-1)^2 - 4(x+2)^2 = 4$$
$$\frac{(y-1)^2}{4} - (x+2)^2 = 1$$

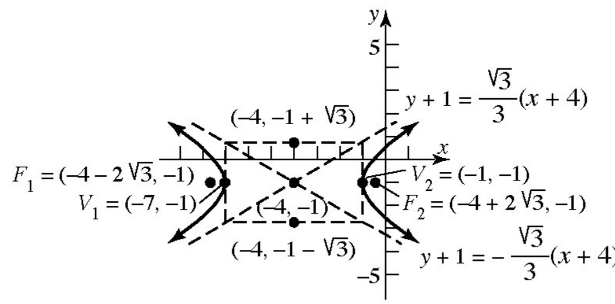
The center of the hyperbola is (–2, 1).
$a = 2,\ b = 1$.
The vertices are (–2, 3) and (–2, –1). Find the value of c:
$c^2 = a^2 + b^2 = 4 + 1 = 5$
$c = \sqrt{5}$
Foci: $\left(-2, 1-\sqrt{5}\right)$ and $\left(-2, 1+\sqrt{5}\right)$.
Transverse axis: $x = -2$, parallel to the y-axis.
Asymptotes: $y - 1 = 2(x+2);\ y - 1 = -2(x+2)$.



62. Complete the squares to put in standard form:

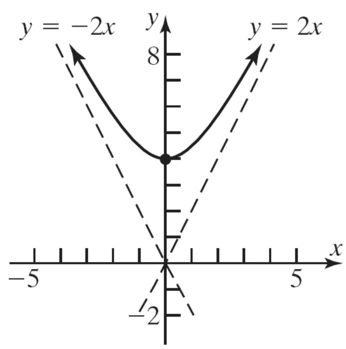
$$x^2 - 3y^2 + 8x - 6y + 4 = 0$$
$$(x^2 + 8x + 16) - 3(y^2 + 2y + 1) = -4 + 16 - 3$$
$$(x+4)^2 - 3(y+1)^2 = 9$$
$$\frac{(x+4)^2}{9} - \frac{(y+1)^2}{3} = 1$$

The center of the hyperbola is (–4, –1).
$a = 3,\ b = \sqrt{3}$. The vertices are (–7, –1) and (–1, –1). Find the value of c:
$c^2 = a^2 + b^2 = 9 + 3 = 12$
$c = \sqrt{12} = 2\sqrt{3}$
Foci: $\left(-4-2\sqrt{3}, -1\right)$ and $\left(-4+2\sqrt{3}, -1\right)$.
Transverse axis: $y = -1$, parallel to x-axis.
Asymptotes:
$$y + 1 = \frac{\sqrt{3}}{3}(x+4);\quad y + 1 = -\frac{\sqrt{3}}{3}(x+4)$$



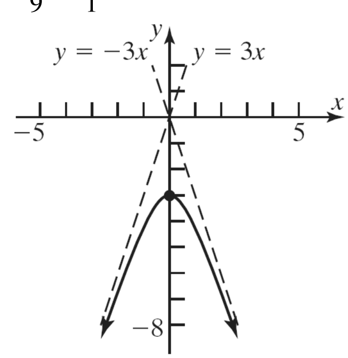
63. Rewrite the equation:

$$y = \sqrt{16 + 4x^2}$$
$$y^2 = 16 + 4x^2, \quad y \ge 0$$
$$y^2 - 4x^2 = 16, \quad y \ge 0$$
$$\frac{y^2}{16} - \frac{x^2}{4} = 1, \quad y \ge 0$$



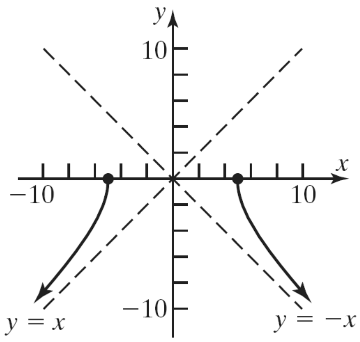
64. Rewrite the equation:

$$y = -\sqrt{9 + 9x^2}$$
$$y^2 = 9 + 9x^2, \quad y \le 0$$
$$y^2 - 9x^2 = 9, \quad y \le 0$$
$$\frac{y^2}{9} - \frac{x^2}{1} = 1, \quad y \le 0$$



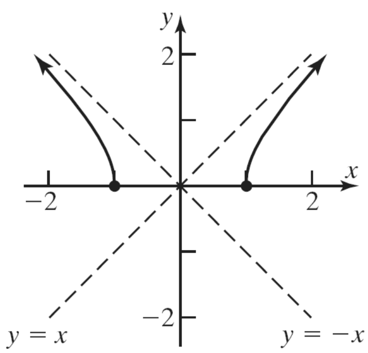
65. Rewrite the equation:

$$y = -\sqrt{-25 + x^2}$$
$$y^2 = -25 + x^2, \quad y \le 0$$
$$x^2 - y^2 = 25, \quad y \le 0$$
$$\frac{x^2}{25} - \frac{y^2}{25} = 1, \quad y \le 0$$



66. Rewrite the equation:

$$y = \sqrt{-1 + x^2}$$
$$y^2 = -1 + x^2, \quad y \ge 0$$
$$x^2 - y^2 = 1, \quad y \ge 0$$



67. $\dfrac{(x-3)^2}{4} - \dfrac{y^2}{25} = 1$

The graph will be a hyperbola. The center of the hyperbola is at (3, 0). $a = 2,\ b = 5$.

The vertices are (5, 0) and (1, 0).

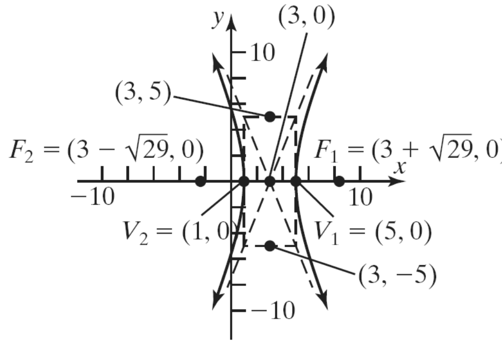
Find the value of c:

$c^2 = a^2 + b^2 = 4 + 25 = 29 \Rightarrow c = \sqrt{29}$

Foci: $\left(3 - \sqrt{29}, 0\right)$ and $\left(3 + \sqrt{29}, 0\right)$

Transverse axis is the x-axis.

Asymptotes: $y = \frac{5}{2}(x-3)$; $y = -\frac{5}{2}(x-3)$



68. $\dfrac{(y+2)^2}{16} - \dfrac{(x-2)^2}{4} = 1$

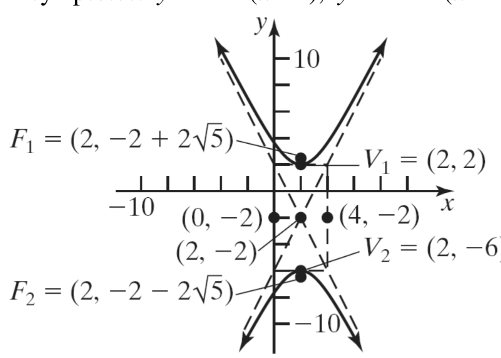
The graph will be a hyperbola. The center of the hyperbola is at (2, –2). $a = 4,\ b = 2$. The vertices are (2, 2) and (2, –6). Find the value of c:

$c^2 = a^2 + b^2 = 16 + 4 = 20 \Rightarrow c = \sqrt{20} = 2\sqrt{5}$

Foci: $\left(2, -2 - 2\sqrt{5}\right)$ and $\left(2, -2 + 2\sqrt{5}\right)$

Transverse axis: $x = 2$, parallel to the y-axis

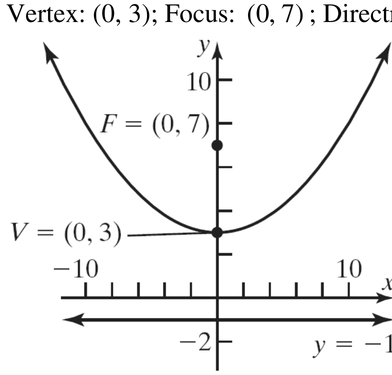
Asymptotes: $y + 2 = 2(x-2)$; $y + 2 = -2(x-2)$



69. $x^2 = 16(y-3)$

The graph will be a parabola. The equation is in the form $(x-h)^2 = 4a(y-k)$ where $4a = 16$ or $a = 4$, $h = 0$, and $k = 3$. Thus, we have:

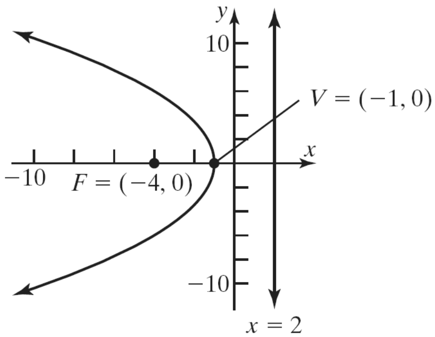
Vertex: (0, 3); Focus: $(0, 7)$; Directrix: $y = -1$



70. $y^2 = -12(x+1)$

The graph will be a parabola. The equation is in the form $(y-k)^2 = -4a(x-h)$ where $-4a = -12$ or $a = 3$, $h = -1$, and $k = 0$. Thus, we have:

Vertex: $(-1, 0)$; Focus: $(-4, 0)$; Directrix: $x = 2$



71. The graph will be an ellipse. Complete the square to put the equation in standard form:

$$25x^2 + 9y^2 - 250x + 400 = 0$$
$$(25x^2 - 250x) + 9y^2 = -400$$
$$25(x^2 - 10x) + 9y^2 = -400$$
$$25(x^2 - 10x + 25) + 9y^2 = -400 + 625$$
$$25(x-5)^2 + 9y^2 = 225$$
$$\frac{25(x-5)^2}{225} + \frac{9y^2}{225} = \frac{225}{225}$$
$$\frac{(x-5)^2}{9} + \frac{y^2}{25} = 1$$

The equation is in the form

$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$ (major axis parallel to the y-axis) where $a = 5$, $b = 3$, $h = 5$, and $k = 0$.

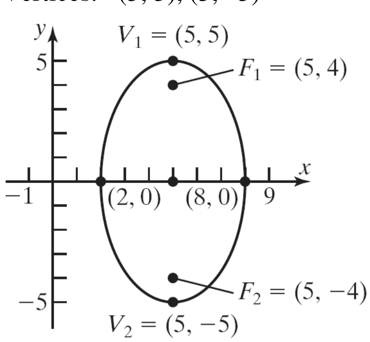
Solving for c: $c^2 = a^2 - b^2 = 25 - 9 = 16 \to c = 4$

Thus, we have:

Center: $(5, 0)$

Foci: $(5, 4)$, $(5, -4)$

Vertices: $(5, 5)$, $(5, -5)$



72. The graph will be an ellipse. Complete the square to put the equation in standard form:

$$x^2 + 36y^2 - 2x + 288y + 541 = 0$$
$$(x^2 - 2x) + (36y^2 + 288y) = -541$$
$$(x^2 - 2x) + 36(y^2 + 8y) = -541$$
$$(x^2 - 2x + 1) + 36(y^2 + 8y + 16) = -541 + 1 + 576$$
$$(x-1)^2 + 36(y+4)^2 = 36$$
$$\frac{(x-1)^2}{36} + \frac{36(y+4)^2}{36} = \frac{36}{36}$$
$$\frac{(x-1)^2}{36} + (y+4)^2 = 1$$

The equation is in the form

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ (major axis parallel to the x-axis) where $a = 6$, $b = 1$, $h = 1$, and $k = -4$.

Solving for c:

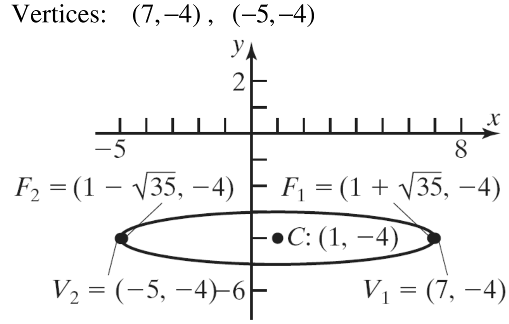
$c^2 = a^2 - b^2 = 36 - 1 = 35 \to c = \sqrt{35}$

Thus, we have:

Center: $(1, -4)$

Foci: $\left(1-\sqrt{35}, -4\right)$, $\left(1+\sqrt{35}, -4\right)$

Vertices: $(7, -4)$, $(-5, -4)$



73. The graph will be a parabola. Complete the square to put the equation in standard form:

$$x^2 - 6x - 8y - 31 = 0$$
$$x^2 - 6x = 8y + 31$$
$$x^2 - 6x + 9 = 8y + 31 + 9$$
$$(x-3)^2 = 8y + 40$$
$$(x-3)^2 = 8(y+5)$$

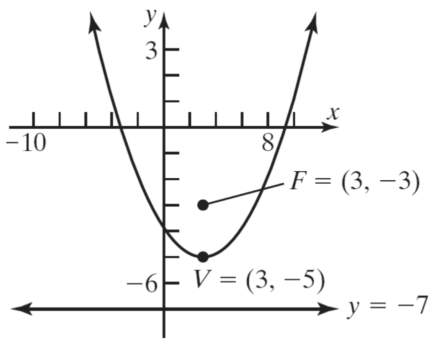
The equation is in the form

$(x-h)^2 = 4a(y-k)$ where $4a = 8$ or $a = 2$, $h = 3$, and $k = -5$. Thus, we have:

Vertex: $(3, -5)$;

Focus: $(3, -3)$;

Directrix: $y = -7$



74. The graph will be a hyperbola. Complete the squares to put in standard form:

$$9x^2 - y^2 - 18x - 8y - 88 = 0$$
$$9(x^2 - 2x + 1) - (y^2 + 8y + 16) = 88 + 9 - 16$$
$$9(x-1)^2 - (y+4)^2 = 81$$
$$\frac{(x-1)^2}{9} - \frac{(y+4)^2}{81} = 1$$

The center of the hyperbola is $(1, -4)$.

$a = 9,\ b = 81$.

The vertices are $(-2, -4)$ and $(4, -4)$.

Find the value of c:

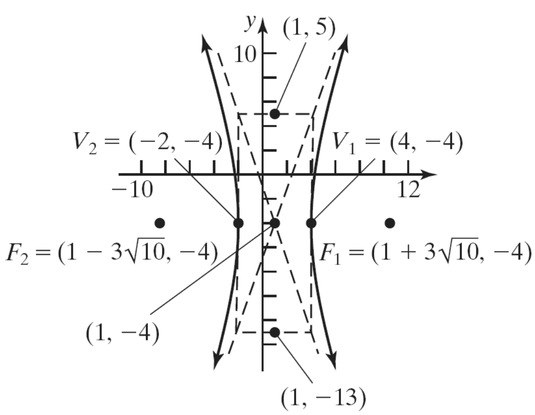
$c^2 = a^2 + b^2 = 9 + 81 = 90$

$c = \sqrt{90} = 3\sqrt{10}$

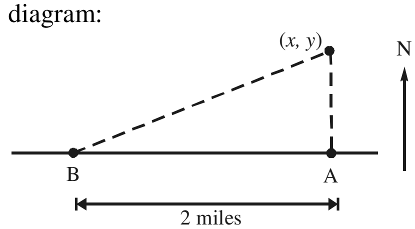
Foci: $\left(1 - 3\sqrt{10}, -4\right)$ and $\left(1 + 3\sqrt{10}, -4\right)$.

Transverse axis: $x = -2$, parallel to the y-axis.

Asymptotes: $y + 4 = 3(x-1);\ y + 4 = -3(x-1)$.



75. First note that all points where a burst could take place, such that the time difference would be the same as that for the first burst, would form a hyperbola with A and B as the foci. Start with a diagram:



Assume a coordinate system with the x-axis containing $\overline{BA}$ and the origin at the midpoint of $\overline{BA}$.

The ordered pair (x, y) represents the location of the fireworks. We know that sound travels at 1100 feet per second, so the person at point A is 1100 feet closer to the fireworks display than the person at point B. Since the difference of the distance from (x, y) to A and from (x, y) to B is the constant 1100, the point (x, y) lies on a hyperbola whose foci are at A and B. The hyperbola has the equation

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

where $2a = 1100$, so $a = 550$. Because the distance between the two people is 2 miles (10,560 feet) and each person is at a focus of the hyperbola, we have

$2c = 10{,}560$

$c = 5280$

$b^2 = c^2 - a^2 = 5280^2 - 550^2 = 27{,}575{,}900$

The equation of the hyperbola that describes the location of the fireworks display is

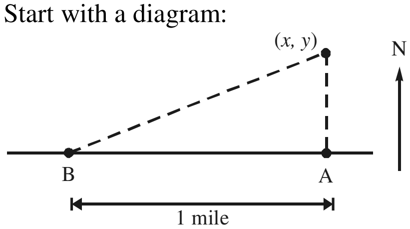
$$\frac{x^2}{550^2} - \frac{y^2}{27{,}575{,}900} = 1$$

Since the fireworks display is due north of the individual at A, we let $x = 5280$ and solve the equation for y.

$$\frac{5280^2}{550^2} - \frac{y^2}{27{,}575{,}900} = 1$$
$$-\frac{y^2}{27{,}575{,}900} = -91.16$$
$$y^2 = 2{,}513{,}819{,}044$$
$$y = 50{,}138$$

Therefore, the fireworks display was 50,138 feet (approximately 9.5 miles) due north of the person at point A.

76. First note that all points where the strike could take place, such that the time difference would be the same as that for the first strike, would form a hyperbola with A and B as the foci. Start with a diagram:



Assume a coordinate system with the x-axis containing $\overline{BA}$ and the origin at the midpoint of $\overline{BA}$.

The ordered pair (x, y) represents the location of the lightning strike. We know that sound travels at 1100 feet per second, so the person at point A is 2200 feet closer to the lightning strike than the person at point B. Since the difference of the distance from (x, y) to A and from (x, y) to B is the constant 2200, the point (x, y) lies on a hyperbola whose foci are at A and B. The hyperbola has the equation

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

where $2a = 2200$, so $a = 1100$. Because the distance between the two people is 1 mile (5,280 feet) and each person is at a focus of the hyperbola, we have

$$2c = 5,280$$
$$c = 2,640$$
$$b^2 = c^2 - a^2 = 2640^2 - 1100^2 = 5,759,600$$

The equation of the hyperbola that describes the location of the lightning strike is

$$\frac{x^2}{1100^2}-\frac{y^2}{5,759,600}=1$$

Since the lightning strike is due north of the individual at A, we let $x = 2640$ and solve for y.

$$\frac{2640^2}{1100^2}-\frac{y^2}{5,759,600}=1$$
$$-\frac{y^2}{5,759,600}=-4.76$$
$$y^2 = 27,415,696$$
$$y = 5236$$

The lightning strikes 5236 feet (approximately 0.99 miles) due north of the person at point A.

77. To determine the height, we first need to obtain the equation of the hyperbola used to generate the hyperboloid. Placing the center of the hyperbola at the origin, the equation of the hyperbola will have the form $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

The center diameter is 200 feet so we have $a=\frac{200}{2}=100$. We also know that the base diameter is 400 feet. Since the center of the hyperbola is at the origin, the points $(200,-360)$ and $(-200,-360)$ must be on the graph of our hyperbola (recall the center is 360 feet above ground). Therefore,

$$\frac{(200)^2}{(100)^2}-\frac{(-360)^2}{b^2}=1$$
$$4-\frac{360^2}{b^2}=1$$
$$3=\frac{360^2}{b^2}$$
$$b^2 = 43,200$$
$$b=\sqrt{43,200}=120\sqrt{3}$$

The equation of the hyperbola is

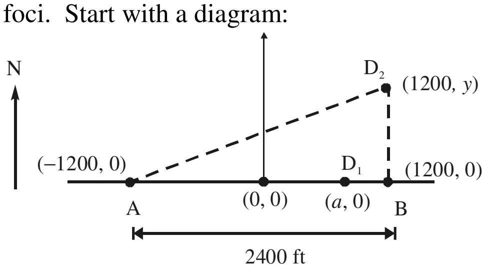
$$\frac{x^2}{10,000}-\frac{y^2}{43,200}=1$$

At the top of the tower we have $x=\frac{300}{2}=150$.

$$\frac{150^2}{10,000}-\frac{y^2}{43,200}=1$$
$$\frac{y^2}{43,200}=1.25$$
$$y^2 = 54000$$
$$y \approx 232.4$$

The height of the tower is approximately $232.4+360=592.4$ feet.

78. First note that all points where an explosion could take place, such that the time difference would be the same as that for the first detonation, would form a hyperbola with A and B as the foci. Start with a diagram:



Since A and B are the foci, we have

$2c = 2400 \quad \Rightarrow \quad c = 1200$

Since D_1 is on the transverse axis and is on the hyperbola, then it must be a vertex of the hyperbola. Since it is 300 feet from B, we have $a = 900$. Finally,

$b^2 = c^2 - a^2 = 1200^2 - 900^2 = 630,000$

Thus, the equation of the hyperbola is

$$\frac{x^2}{810,000} - \frac{y^2}{630,000} = 1$$

The point $(1200, y)$ needs to lie on the graph of the hyperbola. Thus, we have

$$\frac{(1200)^2}{810,000} - \frac{y^2}{630,000} = 1$$

$$-\frac{y^2}{630,000} = -\frac{7}{9}$$

$$y^2 = 490,000$$

$$y = 700$$

The second explosion should be set off 700 feet due north of point B.

79. **a.** Since the particles are deflected at a $45°$ angle, the asymptotes will be $y = \pm x$.

b. Since the vertex is 10 cm from the center of the hyperbola, we know that $a = 10$. The slope of the asymptotes is given by $\pm\frac{b}{a}$.

Therefore, we have

$\frac{b}{a} = 1 \quad \Rightarrow \quad \frac{b}{10} = 1 \quad \Rightarrow b = 10$

Using the origin as the center of the hyperbola, the equation of the particle path would be

$$\frac{x^2}{100} - \frac{y^2}{100} = 1, \quad x \geq 0$$

80. Assume the origin lies at the center of the hyperbola. From the equation we know that the hyperbola has a transverse axis that is parallel to the *y*-axis. The foci of the hyperbola are located at $(0, \pm c)$

$c^2 = a^2 + b^2 = 9 + 16 = 25 \quad \text{or} \quad c = 5$

Therefore, the foci of the hyperbola are at $(0, -5)$ and $(0, 5)$.

If we assume the parabola opens up, the common focus is at $(0, 5)$. The equation of our parabola will be $x^2 = 4a(y - k)$. The focal length of the parabola is given as $a = 6$. We also know that the distance focus of the parabola is located at $(0, k + a) = (0, 5)$. Thus,

$$k + a = 5$$
$$k + 6 = 5$$
$$k = -1$$

and the equation of our parabola becomes

$$x^2 = 4(6)(y - (-1))$$
$$x^2 = 24(y + 1)$$

or

$$y = \frac{1}{24}x^2 - 1.$$

81. Assume $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

If the eccentricity is close to 1, then $c \approx a$ and $b \approx 0$. When *b* is close to 0, the hyperbola is very narrow, because the slopes of the asymptotes are close to 0.

If the eccentricity is very large, then c is much larger than *a* and *b* is very large. The result is a hyperbola that is very wide, because the slopes of the asymptotes are very large.

For $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$, the opposite is true. When the eccentricity is close to 1, the hyperbola is very wide because the slopes of the asymptotes are close to 0. When the eccentricity is very large, the hyperbola is very narrow because the slopes of the asymptotes are very large.

82. If $a = b$, then $c^2 = a^2 + a^2 = 2a^2$. Thus, $\frac{c^2}{a^2} = 2$ or $\frac{c}{a} = \sqrt{2}$. The eccentricity of an equilateral hyperbola is $\sqrt{2}$.

83. $\frac{x^2}{4} - y^2 = 1 \quad (a = 2,\ b = 1)$

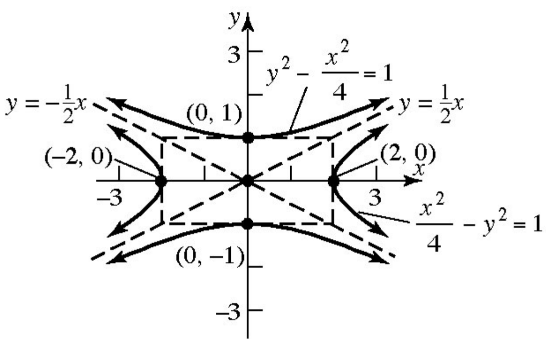
This is a hyperbola with horizontal transverse axis, centered at (0, 0) and has asymptotes:

$y = \pm\frac{1}{2}x$

$y^2 - \frac{x^2}{4} = 1 \quad (a = 1,\ b = 2)$

This is a hyperbola with vertical transverse axis, centered at (0, 0) and has asymptotes: $y = \pm\frac{1}{2}x$.

Since the two hyperbolas have the same asymptotes, they are conjugates.



84. $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Solve for y:

$$\frac{y^2}{a^2} = 1 + \frac{x^2}{b^2}$$

$$y^2 = a^2\left(1 + \frac{x^2}{b^2}\right)$$

$$y^2 = \frac{a^2x^2}{b^2}\left(\frac{b^2}{x^2} + 1\right)$$

$$y = \pm\frac{ax}{b}\sqrt{\frac{b^2}{x^2} + 1}$$

As $x \to -\infty$ or as $x \to \infty$, the term $\frac{b^2}{x^2}$ gets close to 0, so the expression under the radical gets closer to 1. Thus, the graph of the hyperbola gets closer to the lines $y = -\frac{a}{b}x$ and $y = \frac{a}{b}x$. These lines are the asymptotes of the hyperbola.

85. Put the equation in standard hyperbola form:

$$Ax^2 + Cy^2 + F = 0 \qquad A \cdot C < 0,\ F \neq 0$$

$$Ax^2 + Cy^2 = -F$$

$$\frac{Ax^2}{-F} + \frac{Cy^2}{-F} = 1$$

$$\frac{x^2}{\left(-\frac{F}{A}\right)} + \frac{y^2}{\left(-\frac{F}{C}\right)} = 1$$

Since $-F/A$ and $-F/C$ have opposite signs, this is a hyperbola with center (0, 0).

The transverse axis is the x-axis if $-F/A > 0$.

The transverse axis is the y-axis if $-F/A < 0$.

86. Complete the squares on the given equation:

$Ax^2 + Cy^2 + Dx + Ey + F = 0$, where $A \cdot C < 0$.

$$A\left(x^2 + \frac{D}{A}x\right) + C\left(y^2 + \frac{E}{C}y\right) = -F$$

To complete the square in x, we add $\left(\frac{D}{2A}\right)^2 = \frac{D^2}{4A^2}$ inside the first set of parentheses. Since we are multiplying by A, we need to add $A \cdot \frac{D^2}{4A^2} = \frac{D^2}{4A}$ to the right side of the equation.

To complete the square in y, we add $\left(\frac{E}{2C}\right)^2 = \frac{E^2}{4C^2}$ inside the second set of parentheses. Since we are multiplying by C, we add $C \cdot \frac{E^2}{4C^2} = \frac{E^2}{4C}$ to the right side of the equation. Therefore, we obtain the following:

$$A\left(x + \frac{D}{2A}\right)^2 + C\left(y + \frac{E}{2C}\right)^2 = \frac{D^2}{4A} + \frac{E^2}{4C} - F$$

Let $U = \frac{D^2}{4A} + \frac{E^2}{4C} - F$.

a. If $U \neq 0$, then

$$\frac{\left(x + \frac{D}{2A}\right)^2}{\frac{U}{A}} + \frac{\left(y + \frac{E}{2C}\right)^2}{\frac{U}{C}} = 1$$

with $\frac{U}{A}$ and $\frac{U}{C}$ having opposite signs. This is the equation of a hyperbola whose center is $\left(-\frac{D}{2A}, -\frac{E}{2C}\right)$.

b. If $U=0$, then

$$A\left(x+\frac{D}{2A}\right)^2+C\left(y+\frac{E}{2C}\right)^2=0$$

$$\left(y+\frac{E}{2C}\right)^2=\frac{-A}{C}\left(x+\frac{D}{2A}\right)^2$$

$$y+\frac{E}{2C}=\pm\sqrt{\frac{-A}{C}}\left(x+\frac{D}{2A}\right)$$

which is the graph of two intersecting lines, through the point $\left(-\frac{D}{2A},-\frac{E}{2C}\right)$, with slopes $\pm\sqrt{\left|\frac{A}{C}\right|}$.

Section 10.5

1. $\sin A\cos B+\cos A\sin B$

2. $2\sin\theta\cos\theta$

3. $\sqrt{\frac{1-\cos\theta}{2}}$

4. $\sqrt{\frac{1+\cos\theta}{2}}$

5. $\cot(2\theta)=\frac{A-C}{B}$

6. parabola

7. $B^2-4AC<0$

8. True

9. True

10. False; $\cot(2\theta)=\frac{A-C}{B}$

11. $x^2+4x+y+3=0$
$A=1$ and $C=0$; $AC=(1)(0)=0$. Since $AC=0$, the equation defines a parabola.

12. $2y^2-3y+3x=0$
$A=0$ and $C=2$; $AC=(0)(2)=0$. Since $AC=0$, the equation defines a parabola.

13. $6x^2+3y^2-12x+6y=0$
$A=6$ and $C=3$; $AC=(6)(3)=18$. Since $AC>0$ and $A\neq C$, the equation defines an ellipse.

14. $2x^2+y^2-8x+4y+2=0$
$A=2$ and $C=1$; $AC=(2)(1)=2$. Since $AC>0$ and $A\neq C$, the equation defines an ellipse.

15. $3x^2-2y^2+6x+4=0$
$A=3$ and $C=-2$; $AC=(3)(-2)=-6$. Since $AC<0$, the equation defines a hyperbola.

16. $4x^2-3y^2-8x+6y+1=0$
$A=4$ and $C=-3$; $AC=(4)(-3)=-12$. Since $AC<0$, the equation defines a hyperbola.

17. $2y^2-x^2-y+x=0$
$A=-1$ and $C=2$; $AC=(-1)(2)=-2$. Since $AC<0$, the equation defines a hyperbola.

18. $y^2-8x^2-2x-y=0$
$A=-8$ and $C=1$; $AC=(-8)(1)=-8$. Since $AC<0$, the equation defines a hyperbola.

19. $x^2+y^2-8x+4y=0$
$A=1$ and $C=1$; $AC=(1)(1)=1$. Since $AC>0$ and $A=C$, the equation defines a circle.

20. $2x^2+2y^2-8x+8y=0$
$A=2$ and $C=2$; $AC=(2)(2)=4$. Since $AC>0$ and $A=C$, the equation defines a circle.

21. $x^2+4xy+y^2-3=0$

$A=1$, $B=4$, and $C=1$;

$$\cot(2\theta)=\frac{A-C}{B}=\frac{1-1}{4}=\frac{0}{4}=0$$

$$2\theta=\frac{\pi}{2}\Rightarrow\theta=\frac{\pi}{4}$$

$$x=x'\cos\frac{\pi}{4}-y'\sin\frac{\pi}{4}$$

$$=\frac{\sqrt{2}}{2}x'-\frac{\sqrt{2}}{2}y'=\frac{\sqrt{2}}{2}(x'-y')$$

$$y=x'\sin\frac{\pi}{4}+y'\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}x'+\frac{\sqrt{2}}{2}y'$$

$$=\frac{\sqrt{2}}{2}(x'+y')$$

22. $x^2 - 4xy + y^2 - 3 = 0$

$A = 1,\ B = -4,$ and $C = 1;$

$$\cot(2\theta) = \frac{A-C}{B} = \frac{1-1}{-4} = \frac{0}{-4} = 0$$

$$2\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{4}$$

$$x = x'\cos\frac{\pi}{4} - y'\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}x' - \frac{\sqrt{2}}{2}y' = \frac{\sqrt{2}}{2}(x' - y')$$

$$y = x'\sin\frac{\pi}{4} + y'\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}x' + \frac{\sqrt{2}}{2}y' = \frac{\sqrt{2}}{2}(x' + y')$$

23. $5x^2 + 6xy + 5y^2 - 8 = 0$

$A = 5,\ B = 6,$ and $C = 5;$

$$\cot(2\theta) = \frac{A-C}{B} = \frac{5-5}{6} = \frac{0}{6} = 0$$

$$2\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{4}$$

$$x = x'\cos\frac{\pi}{4} - y'\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}x' - \frac{\sqrt{2}}{2}y' = \frac{\sqrt{2}}{2}(x' - y')$$

$$y = x'\sin\frac{\pi}{4} + y'\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}x' + \frac{\sqrt{2}}{2}y' = \frac{\sqrt{2}}{2}(x' + y')$$

24. $3x^2 - 10xy + 3y^2 - 32 = 0$

$A = 3,\ B = -10,$ and $C = 3;$

$$\cot(2\theta) = \frac{A-C}{B} = \frac{3-3}{-10} = \frac{0}{-10} = 0 \Rightarrow$$

$$2\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{4}$$

$$x = x'\cos\frac{\pi}{4} - y'\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}x' - \frac{\sqrt{2}}{2}y' = \frac{\sqrt{2}}{2}(x' - y')$$

$$y = x'\sin\frac{\pi}{4} + y'\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}x' + \frac{\sqrt{2}}{2}y' = \frac{\sqrt{2}}{2}(x' + y')$$

25. $13x^2 - 6\sqrt{3}xy + 7y^2 - 16 = 0$

$A = 13,\ B = -6\sqrt{3},$ and $C = 7;$

$$\cot(2\theta) = \frac{A-C}{B} = \frac{13-7}{-6\sqrt{3}} = \frac{6}{-6\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

$$2\theta = \frac{2\pi}{3} \Rightarrow \theta = \frac{\pi}{3}$$

$$x = x'\cos\frac{\pi}{3} - y'\sin\frac{\pi}{3} = \frac{1}{2}x' - \frac{\sqrt{3}}{2}y' = \frac{1}{2}\left(x' - \sqrt{3}y'\right)$$

$$y = x'\sin\frac{\pi}{3} + y'\cos\frac{\pi}{3} = \frac{\sqrt{3}}{2}x' + \frac{1}{2}y' = \frac{1}{2}\left(\sqrt{3}x' + y'\right)$$

26. $11x^2 + 10\sqrt{3}xy + y^2 - 4 = 0$

$A = 11,\ B = 10\sqrt{3},$ and $C = 1;$

$$\cot(2\theta) = \frac{A-C}{B} = \frac{11-1}{10\sqrt{3}} = \frac{10}{10\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$2\theta = \frac{\pi}{3} \Rightarrow \theta = \frac{\pi}{6}$$

$$x = x'\cos\frac{\pi}{6} - y'\sin\frac{\pi}{6} = \frac{\sqrt{3}}{2}x' - \frac{1}{2}y' = \frac{1}{2}\left(\sqrt{3}x' - y'\right)$$

$$y = x'\sin\frac{\pi}{6} + y'\cos\frac{\pi}{6} = \frac{1}{2}x' + \frac{\sqrt{3}}{2}y' = \frac{1}{2}\left(x' + \sqrt{3}y'\right)$$

27. $4x^2-4xy+y^2-8\sqrt{5}\,x-16\sqrt{5}\,y=0$

$A=4$, $B=-4$, and $C=1$;

$$\cot(2\theta)=\frac{A-C}{B}=\frac{4-1}{-4}=-\frac{3}{4};\quad \cos 2\theta=-\frac{3}{5}$$

$$\sin\theta=\sqrt{\frac{1-\left(-\frac{3}{5}\right)}{2}}=\sqrt{\frac{4}{5}}=\frac{2}{\sqrt{5}}=\frac{2\sqrt{5}}{5};$$

$$\cos\theta=\sqrt{\frac{1+\left(-\frac{3}{5}\right)}{2}}=\sqrt{\frac{1}{5}}=\frac{1}{\sqrt{5}}=\frac{\sqrt{5}}{5}$$

$$x=x'\cos\theta-y'\sin\theta=\frac{\sqrt{5}}{5}x'-\frac{2\sqrt{5}}{5}y'$$
$$=\frac{\sqrt{5}}{5}(x'-2y')$$

$$y=x'\sin\theta+y'\cos\theta=\frac{2\sqrt{5}}{5}x'+\frac{\sqrt{5}}{5}y'$$
$$=\frac{\sqrt{5}}{5}(2x'+y')$$

28. $x^2+4xy+4y^2+5\sqrt{5}\,y+5=0$

$A=1$, $B=4$, and $C=4$;

$$\cot(2\theta)=\frac{A-C}{B}=\frac{1-4}{4}=-\frac{3}{4};\quad \cos 2\theta=-\frac{3}{5}$$

$$\sin\theta=\sqrt{\frac{1-\left(-\frac{3}{5}\right)}{2}}=\sqrt{\frac{4}{5}}=\frac{2}{\sqrt{5}}=\frac{2\sqrt{5}}{5};$$

$$\cos\theta=\sqrt{\frac{1+\left(-\frac{3}{5}\right)}{2}}=\sqrt{\frac{1}{5}}=\frac{1}{\sqrt{5}}=\frac{\sqrt{5}}{5}$$

$$x=x'\cos\theta-y'\sin\theta=\frac{\sqrt{5}}{5}x'-\frac{2\sqrt{5}}{5}y'$$
$$=\frac{\sqrt{5}}{5}(x'-2y')$$

$$y=x'\sin\theta+y'\cos\theta=\frac{2\sqrt{5}}{5}x'+\frac{\sqrt{5}}{5}y'$$
$$=\frac{\sqrt{5}}{5}(2x'+y')$$

29. $25x^2-36xy+40y^2-12\sqrt{13}\,x-8\sqrt{13}\,y=0$

$A=25$, $B=-36$, and $C=40$;

$$\cot(2\theta)=\frac{A-C}{B}=\frac{25-40}{-36}=\frac{5}{12};\quad \cos 2\theta=\frac{5}{13}$$

$$\sin\theta=\sqrt{\frac{1-\frac{5}{13}}{2}}=\sqrt{\frac{4}{13}}=\frac{2}{\sqrt{13}}=\frac{2\sqrt{13}}{13};$$

$$\cos\theta=\sqrt{\frac{1+\frac{5}{13}}{2}}=\sqrt{\frac{9}{13}}=\frac{3}{\sqrt{13}}=\frac{3\sqrt{13}}{13}$$

$$x=x'\cos\theta-y'\sin\theta=\frac{3\sqrt{13}}{13}x'-\frac{2\sqrt{13}}{13}y'$$
$$=\frac{\sqrt{13}}{13}(3x'-2y')$$

$$y=x'\sin\theta+y'\cos\theta=\frac{2\sqrt{13}}{13}x'+\frac{3\sqrt{13}}{13}y'$$
$$=\frac{\sqrt{13}}{13}(2x'+3y')$$

30. $34x^2 - 24xy + 41y^2 - 25 = 0$

$A = 34$, $B = -24$, and $C = 41$;

$$\cot(2\theta) = \frac{A-C}{B} = \frac{34-41}{-24} = \frac{7}{24}; \quad \cos(2\theta) = \frac{7}{25}$$

$$\sin\theta = \sqrt{\frac{1-\frac{7}{25}}{2}} = \sqrt{\frac{9}{25}} = \frac{3}{5}; \quad \cos\theta = \sqrt{\frac{1+\frac{7}{25}}{2}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

$$x = x'\cos\theta - y'\sin\theta = \frac{4}{5}x' - \frac{3}{5}y' = \frac{1}{5}(4x' - 3y')$$

$$y = x'\sin\theta + y'\cos\theta = \frac{3}{5}x' + \frac{4}{5}y' = \frac{1}{5}(3x' + 4y')$$

31. $x^2 + 4xy + y^2 - 3 = 0$; $\theta = 45°$

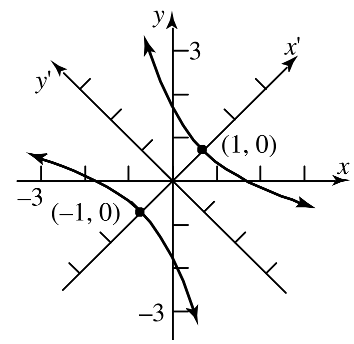
$$\left(\frac{\sqrt{2}}{2}(x'-y')\right)^2 + 4\left(\frac{\sqrt{2}}{2}(x'-y')\right)\left(\frac{\sqrt{2}}{2}(x'+y')\right) + \left(\frac{\sqrt{2}}{2}(x'+y')\right)^2 - 3 = 0$$

$$\frac{1}{2}\left(x'^2 - 2x'y' + y'^2\right) + 2\left(x'^2 - y'^2\right) + \frac{1}{2}\left(x'^2 + 2x'y' + y'^2\right) - 3 = 0$$

$$\frac{1}{2}x'^2 - x'y' + \frac{1}{2}y'^2 + 2x'^2 - 2y'^2 + \frac{1}{2}x'^2 + x'y' + \frac{1}{2}y'^2 = 3$$

$$3x'^2 - y'^2 = 3$$

$$\frac{x'^2}{1} - \frac{y'^2}{3} = 1$$



Hyperbola; center at the origin, transverse axis is the x'-axis, vertices (±1, 0).

32. $x^2 - 4xy + y^2 - 3 = 0$; $\theta = 45°$

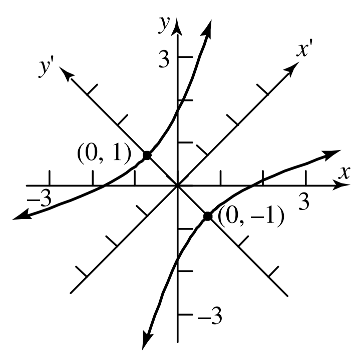
$$\left(\frac{\sqrt{2}}{2}(x'-y')\right)^2 - 4\left(\frac{\sqrt{2}}{2}(x'-y')\right)\left(\frac{\sqrt{2}}{2}(x'+y')\right) + \left(\frac{\sqrt{2}}{2}(x'+y')\right)^2 - 3 = 0$$

$$\frac{1}{2}\left(x'^2 - 2x'y' + y'^2\right) - 2\left(x'^2 - y'^2\right) + \frac{1}{2}\left(x'^2 + 2x'y' + y'^2\right) - 3 = 0$$

$$\frac{1}{2}x'^2 - x'y' + \frac{1}{2}y'^2 - 2x'^2 + 2y'^2 + \frac{1}{2}x'^2 + x'y' + \frac{1}{2}y'^2 = 3$$

$$-x'^2 + 3y'^2 = 3$$

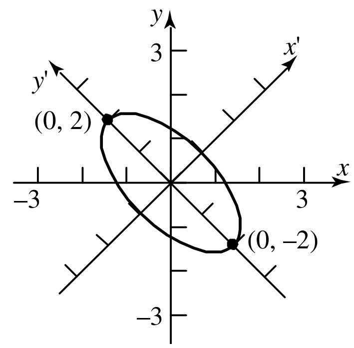
$$\frac{y'^2}{1} - \frac{x'^2}{3} = 1$$



Hyperbola; center at the origin, transverse axis is the y'-axis, vertices (0, ±1).

33. $5x^2+6xy+5y^2-8=0$; $\theta=45°$

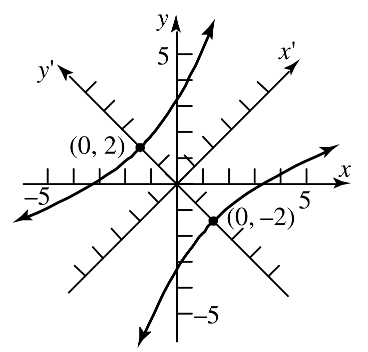
$$5\left(\frac{\sqrt{2}}{2}(x'-y')\right)^2+6\left(\frac{\sqrt{2}}{2}(x'-y')\right)\left(\frac{\sqrt{2}}{2}(x'+y')\right)+5\left(\frac{\sqrt{2}}{2}(x'+y')\right)^2-8=0$$
$$\frac{5}{2}\left(x'^2-2x'y'+y'^2\right)+3\left(x'^2-y'^2\right)+\frac{5}{2}\left(x'^2+2x'y'+y'^2\right)-8=0$$
$$\frac{5}{2}x'^2-5x'y'+\frac{5}{2}y'^2+3x'^2-3y'^2+\frac{5}{2}x'^2+5x'y'+\frac{5}{2}y'^2=8$$
$$8x'^2+2y'^2=8$$
$$\frac{x'^2}{1}+\frac{y'^2}{4}=1$$



Ellipse; center at the origin, major axis is the y'-axis, vertices (0, ±2).

34. $3x^2-10xy+3y^2-32=0$; $\theta=45°$

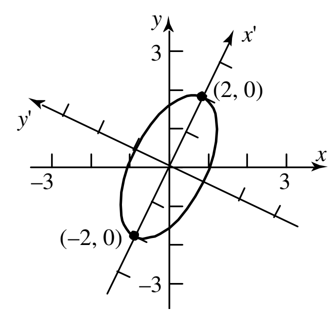
$$3\left(\frac{\sqrt{2}}{2}(x'-y')\right)^2-10\left(\frac{\sqrt{2}}{2}(x'-y')\right)\left(\frac{\sqrt{2}}{2}(x'+y')\right)+3\left(\frac{\sqrt{2}}{2}(x'+y')\right)^2-32=0$$
$$\frac{3}{2}\left(x'^2-2x'y'+y'^2\right)-5\left(x'^2-y'^2\right)+\frac{3}{2}\left(x'^2+2x'y'+y'^2\right)-32=0$$
$$\frac{3}{2}x'^2-3x'y'+\frac{3}{2}y'^2-5x'^2+5y'^2+\frac{3}{2}x'^2+3x'y'+\frac{3}{2}y'^2=32$$
$$-2x'^2+8y'^2=32$$
$$\frac{y'^2}{4}-\frac{x'^2}{16}=1$$



Hyperbola; center at the origin, transverse axis is the y'-axis, vertices (0, ±2).

35. $13x^2-6\sqrt{3}\,xy+7y^2-16=0$; $\theta=60°$

$$13\left(\frac{1}{2}\left(x'-\sqrt{3}y'\right)\right)^2-6\sqrt{3}\left(\frac{1}{2}\left(x'-\sqrt{3}y'\right)\right)\left(\frac{1}{2}\left(\sqrt{3}x'+y'\right)\right)+7\left(\frac{1}{2}\left(\sqrt{3}x'+y'\right)\right)^2-16=0$$
$$\frac{13}{4}\left(x'^2-2\sqrt{3}x'y'+3y'^2\right)-\frac{3\sqrt{3}}{2}\left(\sqrt{3}x'^2-2x'y'-\sqrt{3}y'^2\right)+\frac{7}{4}\left(3x'^2+2\sqrt{3}x'y'+y'^2\right)=16$$
$$\frac{13}{4}x'^2-\frac{13\sqrt{3}}{2}x'y'+\frac{39}{4}y'^2-\frac{9}{2}x'^2+3\sqrt{3}x'y'+\frac{9}{2}y'^2+\frac{21}{4}x'^2+\frac{7\sqrt{3}}{2}x'y'+\frac{7}{4}y'^2=16$$
$$4x'^2+16y'^2=16$$
$$\frac{x'^2}{4}+\frac{y'^2}{1}=1$$



Ellipse; center at the origin, major axis is the x'-axis, vertices (±2, 0).

36. $11x^2+10\sqrt{3}\,xy+y^2-4=0$; $\theta=30°$

$$11\left(\frac{1}{2}\left(\sqrt{3}x'-y'\right)\right)^2+10\sqrt{3}\left(\frac{1}{2}\left(\sqrt{3}x'-y'\right)\right)\left(\frac{1}{2}\left(x'+\sqrt{3}\,y'\right)\right)+\left(\frac{1}{2}\left(x'+\sqrt{3}y'\right)\right)^2-4=0$$

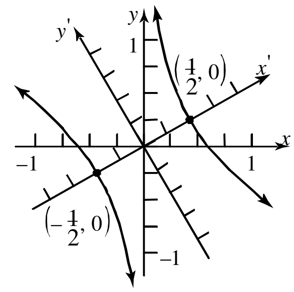
$$\frac{11}{4}\left(3x'^2-2\sqrt{3}x'y'+y'^2\right)+\frac{5\sqrt{3}}{2}\left(\sqrt{3}x'^2+2x'y'-\sqrt{3}y'^2\right)+\frac{1}{4}\left(x'^2+2\sqrt{3}x'y'+3y'^2\right)=4$$

$$\frac{33}{4}x'^2-\frac{11\sqrt{3}}{2}x'y'+\frac{11}{4}y'^2+\frac{15}{2}x'^2+5\sqrt{3}x'y'-\frac{15}{2}y'^2+\frac{1}{4}x'^2+\frac{\sqrt{3}}{2}x'y'+\frac{3}{4}y'^2=4$$

$$16x'^2-4y'^2=4$$

$$4x'^2-y'^2=1$$

$$\frac{x'^2}{\frac{1}{4}}-\frac{y'^2}{1}=1$$



Hyperbola; center at the origin, transverse axis is the x'-axis, vertices (±0.5, 0).

37. $4x^2-4xy+y^2-8\sqrt{5}\,x-16\sqrt{5}\,y=0$; $\theta\approx 63.4°$

$$4\left(\frac{\sqrt{5}}{5}(x'-2y')\right)^2-4\left(\frac{\sqrt{5}}{5}(x'-2y')\right)\left(\frac{\sqrt{5}}{5}(2x'+y')\right)+\left(\frac{\sqrt{5}}{5}(2x'+y')\right)^2$$

$$-8\sqrt{5}\left(\frac{\sqrt{5}}{5}(x'-2y')\right)-16\sqrt{5}\left(\frac{\sqrt{5}}{5}(2x'+y')\right)=0$$

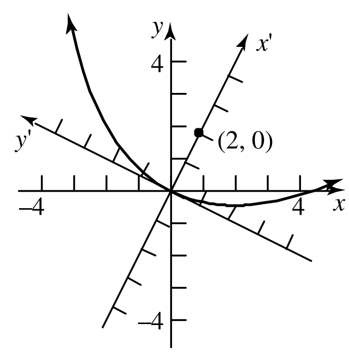
$$\frac{4}{5}\left(x'^2-4x'y'+4y'^2\right)-\frac{4}{5}\left(2x'^2-3x'y'-2y'^2\right)+\frac{1}{5}\left(4x'^2+4x'y'+y'^2\right)$$

$$-8x'+16y'-32x'-16y'=0$$

$$\frac{4}{5}x'^2-\frac{16}{5}x'y'+\frac{16}{5}y'^2-\frac{8}{5}x'^2+\frac{12}{5}x'y'+\frac{8}{5}y'^2+\frac{4}{5}x'^2+\frac{4}{5}x'y'+\frac{1}{5}y'^2-40x'=0$$

$$5y'^2-40x'=0$$

$$y'^2=8x'$$



Parabola; vertex at the origin, axis of symmetry is the x' axis, focus at (2, 0).

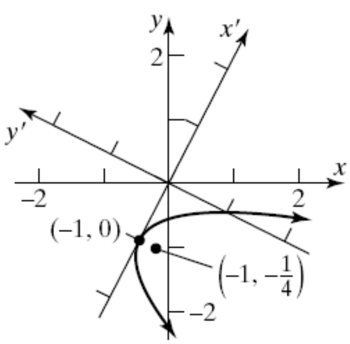
38. $x^2+4xy+4y^2+5\sqrt{5}\,y+5=0$; $\theta \approx 63.4°$

$$\left(\frac{\sqrt{5}}{5}(x'-2y')\right)^2+4\left(\frac{\sqrt{5}}{5}(x'-2y')\right)\left(\frac{\sqrt{5}}{5}(2x'+y')\right)+4\left(\frac{\sqrt{5}}{5}(2x'+y')\right)^2$$
$$+5\sqrt{5}\left(\frac{\sqrt{5}}{5}(2x'+y')\right)+5=0$$

$$\frac{1}{5}\left(x'^2-4x'y'+4y'^2\right)+\frac{4}{5}\left(2x'^2-3x'y'-2y'^2\right)+\frac{4}{5}\left(4x'^2+4x'y'+y'^2\right)$$
$$+10x'+5y'+5=0$$

$$\frac{1}{5}x'^2-\frac{4}{5}x'y'+\frac{4}{5}y'^2+\frac{8}{5}x'^2-\frac{12}{5}x'y'-\frac{8}{5}y'^2+\frac{16}{5}x'^2+\frac{16}{5}x'y'+\frac{4}{5}y'^2$$
$$+10x'+5y'+5=0$$
$$5x'^2+10x'+5y'+5=0$$
$$x'^2+2x'+1=-y'-1+1$$
$$(x'+1)^2=-y'$$

Parabola; vertex at (–1, 0), axis of symmetry parallel to the y'-axis; focus at $(x',y')=\left(-1,-\frac{1}{4}\right)$.



39. $25x^2-36xy+40y^2-12\sqrt{13}x-8\sqrt{13}y=0$; $\theta \approx 33.7°$

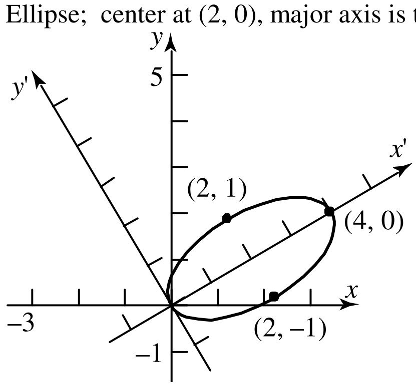
$$25\left(\frac{\sqrt{13}}{13}(3x'-2y')\right)^2-36\left(\frac{\sqrt{13}}{13}(3x'-2y')\right)\left(\frac{\sqrt{13}}{13}(2x'+3y')\right)+40\left(\frac{\sqrt{13}}{13}(2x'+3y')\right)^2$$
$$-12\sqrt{13}\left(\frac{\sqrt{13}}{13}(3x'-2y')\right)-8\sqrt{13}\left(\frac{\sqrt{13}}{13}(2x'+3y')\right)=0$$

$$\frac{25}{13}\left(9x'^2-12x'y'+4y'^2\right)-\frac{36}{13}\left(6x'^2+5x'y'-6y'^2\right)+\frac{40}{13}\left(4x'^2+12x'y'+9y'^2\right)$$
$$-36x'+24y'-16x'-24y'=0$$

$$\frac{225}{13}x'^2-\frac{300}{13}x'y'+\frac{100}{13}y'^2-\frac{216}{13}x'^2-\frac{180}{13}x'y'+\frac{216}{13}y'^2$$
$$+\frac{160}{13}x'^2+\frac{480}{13}x'y'+\frac{360}{13}y'^2-52x'=0$$

$$13x'^2 + 52y'^2 - 52x' = 0$$
$$x'^2 - 4x' + 4y'^2 = 0$$
$$(x' - 2)^2 + 4y'^2 = 4$$
$$\frac{(x'-2)^2}{4} + \frac{y'^2}{1} = 1$$

Ellipse; center at (2, 0), major axis is the x'-axis, vertices (4, 0) and (0, 0).



40. $34x^2 - 24xy + 41y^2 - 25 = 0$; $\theta \approx 36.9°$

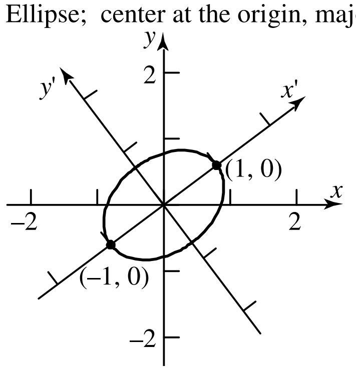
$$34\left(\frac{1}{5}(4x' - 3y')\right)^2 - 24\left(\frac{1}{5}(4x' - 3y')\right)\left(\frac{1}{5}(3x' + 4y')\right) + 41\left(\frac{1}{5}(3x' + 4y')\right)^2 - 25 = 0$$

$$\frac{34}{25}\left(16x'^2 - 24x'y' + 9y'^2\right) - \frac{24}{25}\left(12x'^2 + 7x'y' - 12y'^2\right) + \frac{41}{25}\left(9x'^2 + 24x'y' + 16y'^2\right) = 25$$

$$\frac{544}{25}x'^2 - \frac{816}{25}x'y' + \frac{306}{25}y'^2 - \frac{288}{25}x'^2 - \frac{168}{25}x'y' + \frac{288}{25}y'^2 + \frac{369}{25}x'^2 + \frac{984}{25}x'y' + \frac{656}{25}y'^2 = 25$$

$$25x'^2 + 50y'^2 = 25$$
$$x'^2 + 2y'^2 = 1$$
$$\frac{x'^2}{1} + \frac{y'^2}{\frac{1}{2}} = 1$$

Ellipse; center at the origin, major axis is the x'-axis, vertices (±1, 0).



41. $16x^2 + 24xy + 9y^2 - 130x + 90y = 0$

$A = 16,\ B = 24, \text{ and } C = 9;\ \cot(2\theta) = \dfrac{A-C}{B} = \dfrac{16-9}{24} = \dfrac{7}{24} \Rightarrow \cos(2\theta) = \dfrac{7}{25}$

$$\sin\theta = \sqrt{\frac{1-\frac{7}{25}}{2}} = \sqrt{\frac{9}{25}} = \frac{3}{5};\quad \cos\theta = \sqrt{\frac{1+\frac{7}{25}}{2}} = \sqrt{\frac{16}{25}} = \frac{4}{5} \Rightarrow \theta \approx 36.9^\circ$$

$$x = x'\cos\theta - y'\sin\theta = \frac{4}{5}x' - \frac{3}{5}y' = \frac{1}{5}(4x' - 3y')$$

$$y = x'\sin\theta + y'\cos\theta = \frac{3}{5}x' + \frac{4}{5}y' = \frac{1}{5}(3x' + 4y')$$

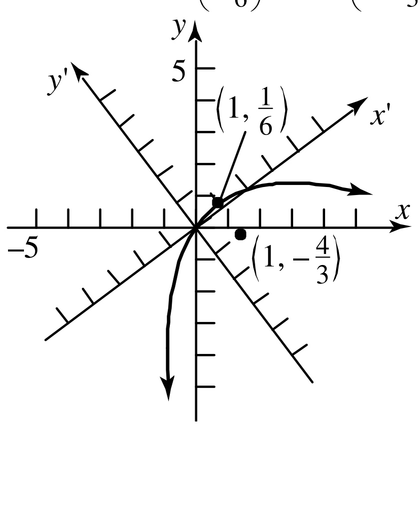
$$16\left(\frac{1}{5}(4x'-3y')\right)^2 + 24\left(\frac{1}{5}(4x'-3y')\right)\left(\frac{1}{5}(3x'+4y')\right) + 9\left(\frac{1}{5}(3x'+4y')\right)^2$$
$$-130\left(\frac{1}{5}(4x'-3y')\right) + 90\left(\frac{1}{5}(3x'+4y')\right) = 0$$

$$\frac{16}{25}\left(16x'^2 - 24x'y' + 9y'^2\right) + \frac{24}{25}\left(12x'^2 + 7x'y' - 12y'^2\right)$$
$$+\frac{9}{25}\left(9x'^2 + 24x'y' + 16y'^2\right) - 104x' + 78y' + 54x' + 72y' = 0$$

$$\frac{256}{25}x'^2 - \frac{384}{25}x'y' + \frac{144}{25}y'^2 + \frac{288}{25}x'^2 + \frac{168}{25}x'y' - \frac{288}{25}y'^2$$
$$+\frac{81}{25}x'^2 + \frac{216}{25}x'y' + \frac{144}{25}y'^2 - 50x' + 150y' = 0$$

$$25x'^2 - 50x' + 150y' = 0$$
$$x'^2 - 2x' = -6y'$$
$$(x'-1)^2 = -6y' + 1$$
$$(x'-1)^2 = -6\left(y' - \frac{1}{6}\right)$$

Parabola; vertex $\left(1, \frac{1}{6}\right)$, focus $\left(1, -\frac{4}{3}\right)$; axis of symmetry parallel to the y' axis.



42. $16x^2+24xy+9y^2-60x+80y=0$

$A=16$, $B=24$, and $C=9$; $\cot(2\theta)=\dfrac{A-C}{B}=\dfrac{16-9}{24}=\dfrac{7}{24}\Rightarrow\cos(2\theta)=\dfrac{7}{25}$

$$\sin\theta=\sqrt{\frac{1-\frac{7}{25}}{2}}=\sqrt{\frac{9}{25}}=\frac{3}{5};\quad \cos\theta=\sqrt{\frac{1+\frac{7}{25}}{2}}=\sqrt{\frac{16}{25}}=\frac{4}{5}\Rightarrow\theta\approx 36.9^\circ$$

$$x=x'\cos\theta-y'\sin\theta=\frac{4}{5}x'-\frac{3}{5}y'=\frac{1}{5}(4x'-3y')$$

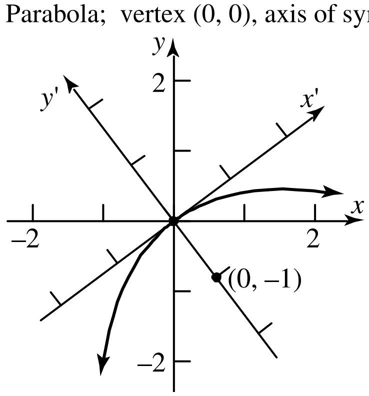
$$y=x'\sin\theta+y'\cos\theta=\frac{3}{5}x'+\frac{4}{5}y'=\frac{1}{5}(3x'+4y')$$

$$16\left(\frac{1}{5}(4x'-3y')\right)^2+24\left(\frac{1}{5}(4x'-3y')\right)\left(\frac{1}{5}(3x'+4y')\right)+9\left(\frac{1}{5}(3x'+4y')\right)^2$$
$$-60\left(\frac{1}{5}(4x'-3y')\right)+80\left(\frac{1}{5}(3x'+4y')\right)=0$$

$$\frac{16}{25}\left(16x'^2-24x'y'+9y'^2\right)+\frac{24}{25}\left(12x'^2+7x'y'-12y'^2\right)$$
$$+\frac{9}{25}\left(9x'^2+24x'y'+16y'^2\right)-48x'+36y'+48x'+64y'=0$$

$$\frac{256}{25}x'^2-\frac{384}{25}x'y'+\frac{144}{25}y'^2+\frac{288}{25}x'^2+\frac{168}{25}x'y'-\frac{288}{25}y'^2$$
$$+\frac{81}{25}x'^2+\frac{216}{25}x'y'+\frac{144}{25}y'^2+100y'=0$$
$$25x'^2+100y'=0$$
$$x'^2=-4y'$$

Parabola; vertex (0, 0), axis of symmetry is the y' axis, focus (0, –1).



43. $A=1,\ B=3,\ C=-2\qquad B^2-4AC=3^2-4(1)(-2)=17>0$; hyperbola

44. $A=2,\ B=-3,\ C=4\qquad B^2-4AC=(-3)^2-4(2)(4)=-23<0$; ellipse

45. $A=1,\ B=-7,\ C=3\qquad B^2-4AC=(-7)^2-4(1)(3)=37>0$; hyperbola

46. $A=2,\ B=-3,\ C=2\qquad B^2-4AC=(-3)^2-4(2)(2)=-7<0$; ellipse

47. $A=9,\ B=12,\ C=4\qquad B^2-4AC=12^2-4(9)(4)=0$; parabola

48. $A=10,\ B=12,\ C=4 \quad B^2-4AC=12^2-4(10)(4)=-16<0$; ellipse

49. $A=10,\ B=-12,\ C=4 \quad B^2-4AC=(-12)^2-4(10)(4)=-16<0$; ellipse

50. $A=4,\ B=12,\ C=9 \quad B^2-4AC=12^2-4(4)(9)=0$; parabola

51. $A=3,\ B=-2,\ C=1 \quad B^2-4AC=(-2)^2-4(3)(1)=-8<0$; ellipse

52. $A=3,\ B=2,\ C=1 \quad B^2-4AC=2^2-4(3)(1)=-8<0$; ellipse

53.
$$A'=A\cos^2\theta+B\sin\theta\cos\theta+C\sin^2\theta$$
$$B'=B(\cos^2\theta-\sin^2\theta)+2(C-A)(\sin\theta\cos\theta)$$
$$C'=A\sin^2\theta-B\sin\theta\cos\theta+C\cos^2\theta$$
$$D'=D\cos\theta+E\sin\theta$$
$$E'=-D\sin\theta+E\cos\theta$$
$$F'=F$$

54.
$$\begin{aligned}A'+C'&=\left(A\cos^2\theta+B\sin\theta\cos\theta+C\sin^2\theta\right)+\left(A\sin^2\theta-B\sin\theta\cos\theta+C\cos^2\theta\right)\\&=A\left(\cos^2\theta+\sin^2\theta\right)+C\left(\sin^2\theta+\cos^2\theta\right)=A(1)+C(1)=A+C\end{aligned}$$

55.
$$\begin{aligned}B'^2&=[B(\cos^2\theta-\sin^2\theta)+2(C-A)\sin\theta\cos\theta]^2\\&=[B\cos2\theta-(A-C)\sin2\theta]^2\\&=B^2\cos^2 2\theta-2B(A-C)\sin2\theta\cos2\theta+(A-C)^2\sin^2 2\theta\end{aligned}$$
$$\begin{aligned}4A'C'&=4[A\cos^2\theta+B\sin\theta\cos\theta+C\sin^2\theta][A\sin^2\theta-B\sin\theta\cos\theta+C\cos^2\theta]\\&=4\left[A\left(\tfrac{1+\cos2\theta}{2}\right)+B\left(\tfrac{\sin2\theta}{2}\right)+C\left(\tfrac{1-\cos2\theta}{2}\right)\right]\left[A\left(\tfrac{1-\cos2\theta}{2}\right)-B\left(\tfrac{\sin2\theta}{2}\right)+C\left(\tfrac{1+\cos2\theta}{2}\right)\right]\\&=[A(1+\cos2\theta)+B(\sin2\theta)+C(1-\cos2\theta)][A(1-\cos2\theta)-B(\sin2\theta)+C(1+\cos2\theta)]\\&=[(A+C)+B\sin2\theta+(A-C)\cos2\theta][(A+C)-(B\sin2\theta+(A-C)\cos2\theta)]\\&=(A+C)^2-[B\sin2\theta+(A-C)\cos2\theta]^2\\&=(A+C)^2-[B^2\sin^2 2\theta+2B(A-C)\sin2\theta\cos2\theta+(A-C)^2\cos^2 2\theta]\end{aligned}$$
$$\begin{aligned}B'^2-4A'C'&=B^2\cos^2 2\theta-2B(A-C)\sin2\theta\cos2\theta+(A-C)^2\sin^2 2\theta\\&\quad-(A+C)^2+B^2\sin^2 2\theta+2B(A-C)\sin2\theta\cos2\theta+(A-C)^2\cos^2 2\theta\\&=B^2(\cos^2 2\theta+\sin^2 2\theta)+(A-C)^2(\cos^2 2\theta+\sin^2 2\theta)-(A+C)^2\\&=B^2+(A-C)^2-(A+C)^2=B^2+(A^2-2AC+C^2)-(A^2+2AC+C^2)\\&=B^2-4AC\end{aligned}$$

56. Since $B^2 - 4AC = B'^2 - 4A'C'$ for any rotation θ (Problem 55), choose θ so that $B' = 0$. Then $B^2 - 4AC = -4A'C'$.

a. If $B^2 - 4AC = -4A'C' = 0$ then $A'C' = 0$. Using the theorem for identifying conics without completing the square, the equation is a parabola.

b. If $B^2 - 4AC = -4A'C' < 0$ then $A'C' > 0$. Thus, the equation is an ellipse (or circle).

c. If $B^2 - 4AC = -4A'C' > 0$ then $A'C' < 0$. Thus, the equation is a hyperbola.

57.
$$\begin{aligned}
d^2 &= (y_2 - y_1)^2 + (x_2 - x_1)^2 \\
&= \left(x_2' \sin\theta + y_2' \cos\theta - x_1' \sin\theta - y_1' \cos\theta\right)^2 + \left(x_2' \cos\theta - y_2' \sin\theta - x_1' \cos\theta + y_1' \sin\theta\right)^2 \\
&= \left(\left(x_2' - x_1'\right)\sin\theta + \left(y_2' - y_1'\right)\cos\theta\right)^2 + \left(\left(x_2' - x_1'\right)\cos\theta - \left(y_2' - y_1'\right)\sin\theta\right)^2 \\
&= \left(x_2' - x_1'\right)^2 \sin^2\theta + 2\left(x_2' - x_1'\right)\left(y_2' - y_1'\right)\sin\theta\cos\theta + \left(y_2' - y_1'\right)^2 \cos^2\theta \\
&\qquad + \left(x_2' - x_1'\right)^2 \cos^2\theta - 2\left(x_2' - x_1'\right)\left(y_2' - y_1'\right)\sin\theta\cos\theta + \left(y_2' - y_1'\right)^2 \sin^2\theta \\
&= \left(x_2' - x_1'\right)^2 \left(\sin^2\theta + \cos^2\theta\right) + \left(y_2' - y_1'\right)^2 \left(\cos^2\theta + \sin^2\theta\right) \\
&= \left[\left(x_2' - x_1'\right)^2 + \left(y_2' - y_1'\right)^2\right]\left(\sin^2\theta + \cos^2\theta\right) \\
&= \left(x_2' - x_1'\right)^2 + \left(y_2' - y_1'\right)^2
\end{aligned}$$

58.
$$\begin{aligned}
x^{1/2} + y^{1/2} &= a^{1/2} \\
y^{1/2} &= a^{1/2} - x^{1/2} \\
y &= \left(a^{1/2} - x^{1/2}\right)^2 \\
y &= a - 2a^{1/2}x^{1/2} + x \\
2a^{1/2}x^{1/2} &= (a + x) - y \\
4ax &= (a + x)^2 - 2y(a + x) + y^2 \\
4ax &= a^2 + 2ax + x^2 - 2ay - 2xy + y^2 \\
0 &= x^2 - 2xy + y^2 - 2ax - 2ay + a^2
\end{aligned}$$

$B^2 - 4AC = (-2)^2 - 4(1)(1) = 4 - 4 = 0$

The graph of the equation is part of a parabola.

59 – 60. Answers will vary.

Section 10.6

1. $r\cos\theta$; $r\sin\theta$

2. $r = 6\cos\theta$
Begin by multiplying both sides of the equation by r to get $r^2 = 6r\cos\theta$. Since $r\cos\theta = x$ and $r^2 = x^2 + y^2$, we obtain $x^2 + y^2 = 6x$.
Move all the variable terms to the left side of the equation and complete the square in x.

$$x^2 - 6x + y^2 = 0$$
$$\left(x^2 - 6x + 9\right) + y^2 = 9$$
$$\left(x-3\right)^2 + y^2 = 9$$

3. conic; focus; directrix

4. 1; < 1; > 1

5. True

6. True

7. $e = 1$; $p = 1$; parabola; directrix is perpendicular to the polar axis and 1 unit to the right of the pole.

8. $e = 1$; $p = 3$; parabola; directrix is parallel to the polar axis and 3 units below the pole.

9. $r = \dfrac{4}{2 - 3\sin\theta} = \dfrac{4}{2\left(1 - \frac{3}{2}\sin\theta\right)}$

$$= \frac{2}{1 - \frac{3}{2}\sin\theta}$$

$ep = 2$, $e = \dfrac{3}{2}$; $p = \dfrac{4}{3}$
Hyperbola; directrix is parallel to the polar axis and $\dfrac{4}{3}$ units below the pole.

10. $r = \dfrac{2}{1 + 2\cos\theta}$; $ep = 2$, $e = 2$; $p = 1$
Hyperbola; directrix is perpendicular to the polar axis and 1 unit to the right of the pole.

11. $r = \dfrac{3}{4 - 2\cos\theta} = \dfrac{3}{4\left(1 - \frac{1}{2}\cos\theta\right)}$

$$= \frac{\frac{3}{4}}{1 - \frac{1}{2}\cos\theta};$$

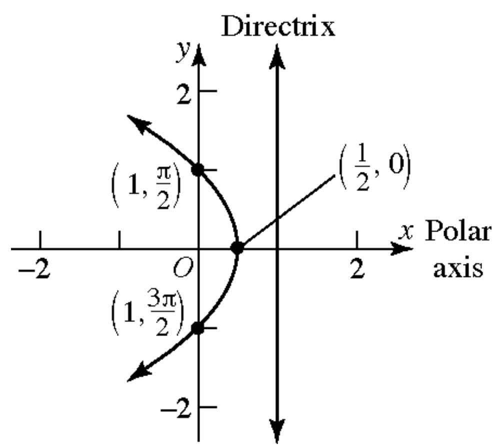
$ep = \dfrac{3}{4}$, $e = \dfrac{1}{2}$; $p = \dfrac{3}{2}$
Ellipse; directrix is perpendicular to the polar axis and $\dfrac{3}{2}$ units to the left of the pole.

12. $r = \dfrac{6}{8 + 2\sin\theta} = \dfrac{6}{8\left(1 + \frac{1}{4}\sin\theta\right)}$

$$= \frac{\frac{3}{4}}{1 + \frac{1}{4}\sin\theta}$$

$ep = \dfrac{3}{4}$, $e = \dfrac{1}{4}$; $p = 3$
Ellipse; directrix is parallel to the polar axis and 3 units above the pole.

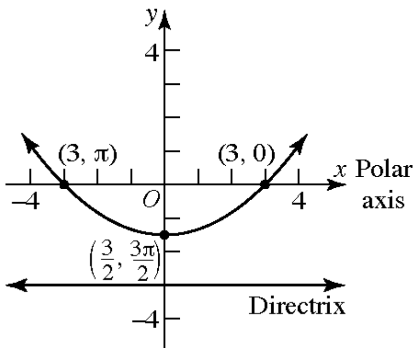
13. $r = \dfrac{1}{1 + \cos\theta}$
$ep = 1$, $e = 1$, $p = 1$
Parabola; directrix is perpendicular to the polar axis 1 unit to the right of the pole; vertex is $\left(\dfrac{1}{2}, 0\right)$.



14. $r = \dfrac{3}{1-\sin\theta}$

$ep = 3,\ \ e = 1,\ \ p = 3$

Parabola; directrix is parallel to the polar axis 3 units below the pole; vertex is $\left(\dfrac{3}{2}, \dfrac{3\pi}{2}\right)$.



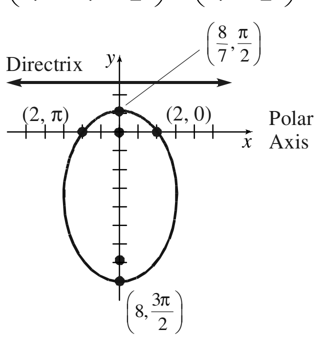
15. $r = \dfrac{8}{4+3\sin\theta}$

$r = \dfrac{8}{4\left(1+\dfrac{3}{4}\sin\theta\right)} = \dfrac{2}{1+\dfrac{3}{4}\sin\theta}$

$ep = 2,\ \ e = \dfrac{3}{4},\ \ p = \dfrac{8}{3}$

Ellipse; directrix is parallel to the polar axis $\dfrac{8}{3}$ units above the pole; vertices are $\left(\dfrac{8}{7}, \dfrac{\pi}{2}\right)$ and $\left(8, \dfrac{3\pi}{2}\right)$.

Also: $a = \dfrac{1}{2}\left(8+\dfrac{8}{7}\right) = \dfrac{32}{7}$ so the center is at $\left(8-\dfrac{32}{7}, \dfrac{3\pi}{2}\right) = \left(\dfrac{24}{7}, \dfrac{3\pi}{2}\right)$, and $c = \dfrac{24}{7} - 0 = \dfrac{24}{7}$ so that the second focus is at $\left(\dfrac{24}{7}+\dfrac{24}{7}, \dfrac{3\pi}{2}\right) = \left(\dfrac{48}{7}, \dfrac{3\pi}{2}\right)$



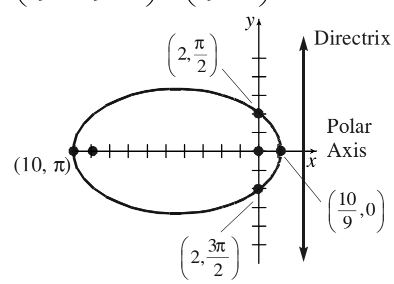
16. $r = \dfrac{10}{5+4\cos\theta}$

$r = \dfrac{10}{5\left(1+\dfrac{4}{5}\cos\theta\right)} = \dfrac{2}{1+\dfrac{4}{5}\cos\theta}$

$ep = 2,\ \ e = \dfrac{4}{5},\ \ p = \dfrac{5}{2}$

Ellipse; directrix is perpendicular to the polar axis $\dfrac{5}{2}$ units to the right of the pole; vertices are $\left(\dfrac{10}{9}, 0\right)$ and $(10, \pi)$.

Also: $a = \dfrac{1}{2}\left(10+\dfrac{10}{9}\right) = \dfrac{50}{9}$ so the center is at $\left(10-\dfrac{50}{9}, \pi\right) = \left(\dfrac{40}{9}, \pi\right)$, and $c = \dfrac{40}{9} - 0 = \dfrac{40}{9}$ so that the second focus is at $\left(\dfrac{40}{9}+\dfrac{40}{9}, \pi\right) = \left(\dfrac{80}{9}, \pi\right)$.



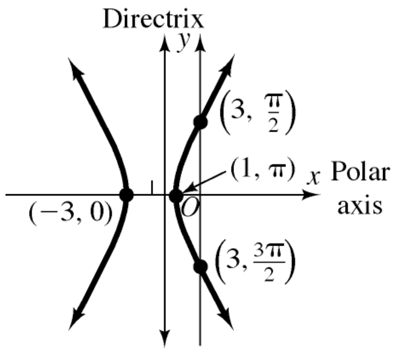
17. $r = \dfrac{9}{3-6\cos\theta}$

$r = \dfrac{9}{3(1-2\cos\theta)} = \dfrac{3}{1-2\cos\theta}$

$ep = 3,\ \ e = 2,\ \ p = \dfrac{3}{2}$

Hyperbola; directrix is perpendicular to the polar axis $\dfrac{3}{2}$ units to the left of the pole; vertices are $(-3, 0)$ and $(1, \pi)$.

Also: $a = \dfrac{1}{2}(3-1) = 1$ so the center is at $(1+1, \pi) = (2, \pi)$ [or $(-2, 0)$], and $c = 2 - 0 = 2$ so that the second focus is at $(2+2, \pi) = (4, \pi)$ [or $(-4, 0)$].



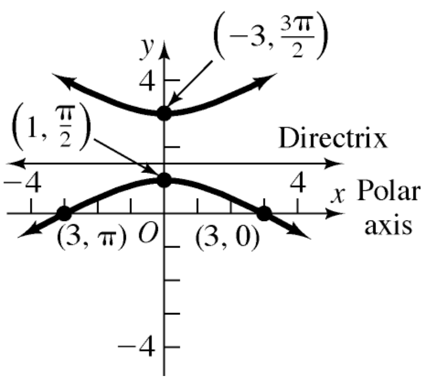
18. $r=\dfrac{12}{4+8\sin\theta}$

$r=\dfrac{12}{4(1+2\sin\theta)}=\dfrac{3}{1+2\sin\theta}$

$ep=3,\ e=2,\ p=\dfrac{3}{2}$

Hyperbola; directrix is parallel to the polar axis $\dfrac{3}{2}$ units above the pole; vertices are $\left(1,\dfrac{\pi}{2}\right)$ and $\left(-3,\dfrac{3\pi}{2}\right)$.

Also: $a=\dfrac{1}{2}(3-1)=1$ so the center is at $\left(1+1,\dfrac{\pi}{2}\right)=\left(2,\dfrac{\pi}{2}\right)$ [or $\left(-2,\dfrac{3\pi}{2}\right)$], and $c=2-0=2$ so that the second focus is at $\left(2+2,\dfrac{\pi}{2}\right)=\left(4,\dfrac{\pi}{2}\right)$ [or $\left(-4,\dfrac{3\pi}{2}\right)$].



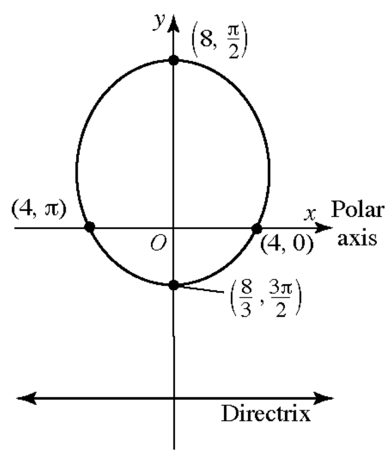
19. $r=\dfrac{8}{2-\sin\theta}$

$r=\dfrac{8}{2\left(1-\frac{1}{2}\sin\theta\right)}=\dfrac{4}{1-\frac{1}{2}\sin\theta}$

$ep=4,\ e=\dfrac{1}{2},\ p=8$

Ellipse; directrix is parallel to the polar axis 8 units below the pole; vertices are $\left(8,\dfrac{\pi}{2}\right)$ and $\left(\dfrac{8}{3},\dfrac{3\pi}{2}\right)$.

Also: $a=\dfrac{1}{2}\left(8+\dfrac{8}{3}\right)=\dfrac{16}{3}$ so the center is at $\left(8-\dfrac{16}{3},\dfrac{\pi}{2}\right)=\left(\dfrac{8}{3},\dfrac{\pi}{2}\right)$, and $c=\dfrac{8}{3}-0=\dfrac{8}{3}$ so that the second focus is at $\left(\dfrac{8}{3}+\dfrac{8}{3},\dfrac{\pi}{2}\right)=\left(\dfrac{16}{3},\dfrac{\pi}{2}\right)$.



20. $r=\dfrac{8}{2+4\cos\theta}$

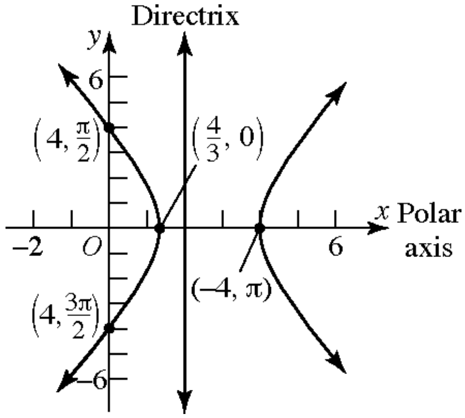
$r=\dfrac{8}{2(1+2\cos\theta)}=\dfrac{4}{1+2\cos\theta}$

$ep=4,\ e=2,\ p=2$

Hyperbola; directrix is perpendicular to the polar axis 2 units to the right of the pole; vertices are $\left(\dfrac{4}{3},0\right)$ and $(-4,\pi)$.

Also: $a=\dfrac{1}{2}\left(4-\dfrac{4}{3}\right)=\dfrac{4}{3}$ so the center is at $\left(\dfrac{4}{3}+\dfrac{4}{3},0\right)=\left(\dfrac{8}{3},0\right)$ [or $\left(-\dfrac{8}{3},\pi\right)$], and $c=\dfrac{8}{3}-0=\dfrac{8}{3}$ so that the second focus is at

$\left(\frac{8}{3}+\frac{8}{3},0\right)=\left(\frac{16}{3},0\right)$ [or $\left(-\frac{16}{3},\pi\right)$].



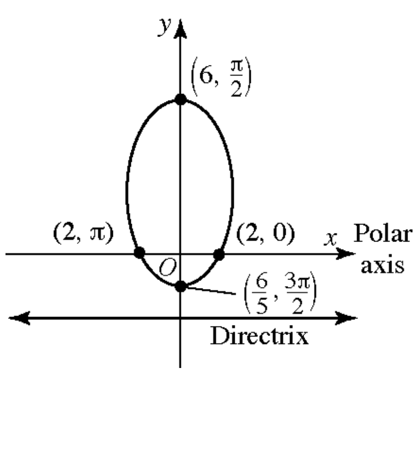
21. $r(3-2\sin\theta)=6 \Rightarrow r=\dfrac{6}{3-2\sin\theta}$

$r=\dfrac{6}{3\left(1-\frac{2}{3}\sin\theta\right)}=\dfrac{2}{1-\frac{2}{3}\sin\theta}$

$ep=2,\ e=\frac{2}{3},\ p=3$

Ellipse; directrix is parallel to the polar axis 3 units below the pole; vertices are $\left(6,\frac{\pi}{2}\right)$ and $\left(\frac{6}{5},\frac{3\pi}{2}\right)$.

Also: $a=\frac{1}{2}\left(6+\frac{6}{5}\right)=\frac{18}{5}$ so the center is at $\left(6-\frac{18}{5},\frac{\pi}{2}\right)=\left(\frac{12}{5},\frac{\pi}{2}\right)$, and $c=\frac{12}{5}-0=\frac{12}{5}$ so that the second focus is at $\left(\frac{12}{5}+\frac{12}{5},\frac{\pi}{2}\right)=\left(\frac{24}{5},\frac{\pi}{2}\right)$.



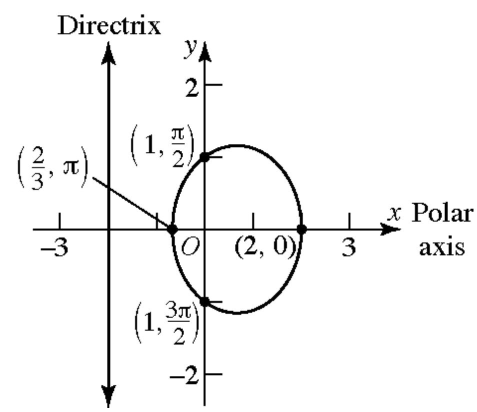
22. $r(2-\cos\theta)=2 \Rightarrow r=\dfrac{2}{2-\cos\theta}$

$r=\dfrac{2}{2\left(1-\frac{1}{2}\cos\theta\right)}=\dfrac{1}{1-\frac{1}{2}\cos\theta}$

$ep=1,\ e=\frac{1}{2},\ p=2$

Ellipse; directrix is perpendicular to the polar axis 2 units to the left of the pole; vertices are $(2,0)$ and $\left(\frac{2}{3},\pi\right)$.

Also: $a=\frac{1}{2}\left(2+\frac{2}{3}\right)=\frac{4}{3}$ so the center is at $\left(2-\frac{4}{3},0\right)=\left(\frac{2}{3},0\right)$, and $c=\frac{2}{3}-0=\frac{2}{3}$ so that the second focus is at $\left(\frac{2}{3}+\frac{2}{3},0\right)=\left(\frac{4}{3},0\right)$.



23. $r=\dfrac{6\sec\theta}{2\sec\theta-1}=\dfrac{6\left(\frac{1}{\cos\theta}\right)}{2\left(\frac{1}{\cos\theta}\right)-1}=\dfrac{\frac{6}{\cos\theta}}{\frac{2-\cos\theta}{\cos\theta}}$

$=\left(\dfrac{6}{\cos\theta}\right)\left(\dfrac{\cos\theta}{2-\cos\theta}\right)=\dfrac{6}{2-\cos\theta}$

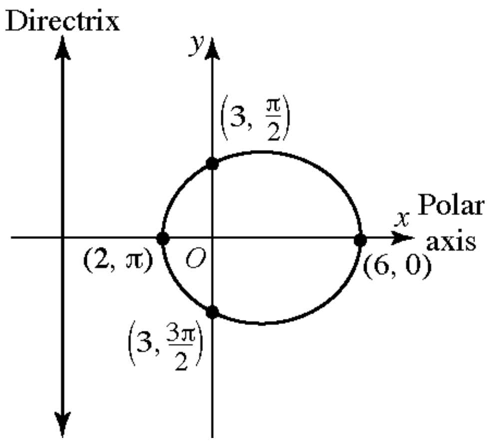
$r=\dfrac{6}{2\left(1-\frac{1}{2}\cos\theta\right)}=\dfrac{3}{1-\frac{1}{2}\cos\theta}$

$ep=3,\ e=\frac{1}{2},\ p=6$

Ellipse; directrix is perpendicular to the polar axis 6 units to the left of the pole; vertices are $(6,0)$ and $(2,\pi)$.

Also: $a=\frac{1}{2}(6+2)=4$ so the center is at

$(6-4,0)=(2,0)$, and $c=2-0=2$ so that the second focus is at $(2+2,0)=(4,0)$.

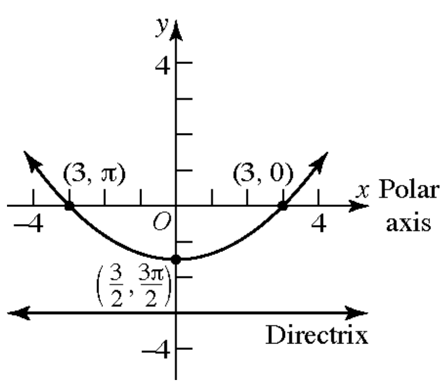


24. $r=\dfrac{3\csc\theta}{\csc\theta-1}=\dfrac{3\left(\dfrac{1}{\sin\theta}\right)}{\dfrac{1}{\sin\theta}-1}=\dfrac{\dfrac{3}{\sin\theta}}{\dfrac{1-\sin\theta}{\sin\theta}}$

$=\left(\dfrac{3}{\sin\theta}\right)\left(\dfrac{\sin\theta}{1-\sin\theta}\right)=\dfrac{3}{1-\sin\theta}$

$ep=3,\ e=1,\ p=3$

Parabola; directrix is parallel to the polar axis 3 units below the pole; vertex is $\left(\dfrac{3}{2},\dfrac{3\pi}{2}\right)$.



25. $r=\dfrac{1}{1+\cos\theta}$

$r+r\cos\theta=1$

$r=1-r\cos\theta$

$r^2=(1-r\cos\theta)^2$

$x^2+y^2=(1-x)^2$

$x^2+y^2=1-2x+x^2$

$y^2+2x-1=0$

26. $r=\dfrac{3}{1-\sin\theta}$

$r-r\sin\theta=3$

$r=3+r\sin\theta$

$r^2=(3+r\sin\theta)^2$

$x^2+y^2=(3+y)^2$

$x^2+y^2=9+6y+y^2$

$x^2-6y-9=0$

27. $r=\dfrac{8}{4+3\sin\theta}$

$4r+3r\sin\theta=8$

$4r=8-3r\sin\theta$

$16r^2=(8-3r\sin\theta)^2$

$16(x^2+y^2)=(8-3y)^2$

$16x^2+16y^2=64-48y+9y^2$

$16x^2+7y^2+48y-64=0$

28. $r=\dfrac{10}{5+4\cos\theta}$

$5r+4r\cos\theta=10$

$5r=10-4r\cos\theta$

$25r^2=(10-4r\cos\theta)^2$

$25(x^2+y^2)=(10-4x)^2$

$25x^2+25y^2=100-80x+16x^2$

$9x^2+25y^2+80x-100=0$

29. $r=\dfrac{9}{3-6\cos\theta}$

$3r-6r\cos\theta=9$

$3r=9+6r\cos\theta$

$r=3+2r\cos\theta$

$r^2=(3+2r\cos\theta)^2$

$x^2+y^2=(3+2x)^2$

$x^2+y^2=9+12x+4x^2$

$3x^2-y^2+12x+9=0$

30. $r = \dfrac{12}{4 + 8\sin\theta}$

$$4r + 8r\sin\theta = 12$$
$$4r = 12 - 8r\sin\theta$$
$$r = 3 - 2r\sin\theta$$
$$r^2 = (3 - 2r\sin\theta)^2$$
$$x^2 + y^2 = (3 - 2y)^2$$
$$x^2 + y^2 = 9 - 12y + 4y^2$$
$$x^2 - 3y^2 + 12y - 9 = 0$$

31. $r = \dfrac{8}{2 - \sin\theta}$

$$2r - r\sin\theta = 8$$
$$2r = 8 + r\sin\theta$$
$$4r^2 = (8 + r\sin\theta)^2$$
$$4(x^2 + y^2) = (8 + y)^2$$
$$4x^2 + 4y^2 = 64 + 16y + y^2$$
$$4x^2 + 3y^2 - 16y - 64 = 0$$

32. $r = \dfrac{8}{2 + 4\cos\theta}$

$$2r + 4r\cos\theta = 8$$
$$2r = 8 - 4r\cos\theta$$
$$r = 4 - 2r\cos\theta$$
$$r^2 = (4 - 2r\cos\theta)^2$$
$$x^2 + y^2 = (4 - 2x)^2$$
$$x^2 + y^2 = 16 - 16x + 4x^2$$
$$3x^2 - y^2 - 16x + 16 = 0$$

33. $r(3 - 2\sin\theta) = 6$

$$3r - 2r\sin\theta = 6$$
$$3r = 6 + 2r\sin\theta$$
$$9r^2 = (6 + 2r\sin\theta)^2$$
$$9(x^2 + y^2) = (6 + 2y)^2$$
$$9x^2 + 9y^2 = 36 + 24y + 4y^2$$
$$9x^2 + 5y^2 - 24y - 36 = 0$$

34. $r(2 - \cos\theta) = 2$

$$2r - r\cos\theta = 2$$
$$2r = 2 + r\cos\theta$$
$$4r^2 = (2 + r\cos\theta)^2$$
$$4(x^2 + y^2) = (2 + x)^2$$
$$4x^2 + 4y^2 = 4 + 4x + x^2$$
$$3x^2 + 4y^2 - 4x - 4 = 0$$

35. $r = \dfrac{6\sec\theta}{2\sec\theta - 1}$

$$r = \frac{6}{2 - \cos\theta}$$
$$2r - r\cos\theta = 6$$
$$2r = 6 + r\cos\theta$$
$$4r^2 = (6 + r\cos\theta)^2$$
$$4(x^2 + y^2) = (6 + x)^2$$
$$4x^2 + 4y^2 = 36 + 12x + x^2$$
$$3x^2 + 4y^2 - 12x - 36 = 0$$

36. $r = \dfrac{3\csc\theta}{\csc\theta - 1}$

$$r = \frac{3}{1 - \sin\theta}$$
$$r - r\sin\theta = 3$$
$$r = 3 + r\sin\theta$$
$$r^2 = (3 + r\sin\theta)^2$$
$$x^2 + y^2 = (3 + y)^2$$
$$x^2 + y^2 = 9 + 6y + y^2$$
$$x^2 - 6y - 9 = 0$$

37. $r = \dfrac{ep}{1 + e\sin\theta}$

$e = 1; \quad p = 1$

$$r = \frac{1}{1 + \sin\theta}$$

38. $r = \dfrac{ep}{1 - e\sin\theta}$

$e = 1; \quad p = 2$

$$r = \frac{2}{1 - \sin\theta}$$

39. $r = \dfrac{ep}{1 - e\cos\theta}$

$e = \dfrac{4}{5}; \quad p = 3$

$r = \dfrac{\frac{12}{5}}{1 - \frac{4}{5}\cos\theta} = \dfrac{12}{5 - 4\cos\theta}$

40. $r = \dfrac{ep}{1 + e\sin\theta}$

$e = \dfrac{2}{3}; \quad p = 3$

$r = \dfrac{2}{1 + \frac{2}{3}\sin\theta} = \dfrac{6}{3 + 2\sin\theta}$

41. $r = \dfrac{ep}{1 - e\sin\theta}$

$e = 6; \quad p = 2$

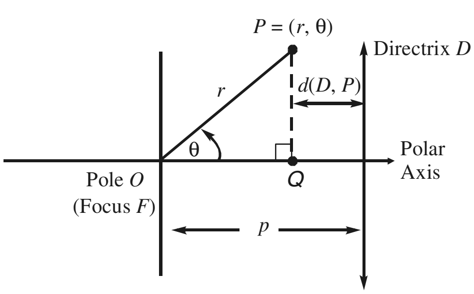
$r = \dfrac{12}{1 - 6\sin\theta}$

42. $r = \dfrac{ep}{1 + e\cos\theta}$

$e = 5; \quad p = 5$

$r = \dfrac{25}{1 + 5\cos\theta}$

43. Consider the following diagram:



$d(F, P) = e \cdot d(D, P)$

$d(D, P) = p - r\cos\theta$

$r = e(p - r\cos\theta)$

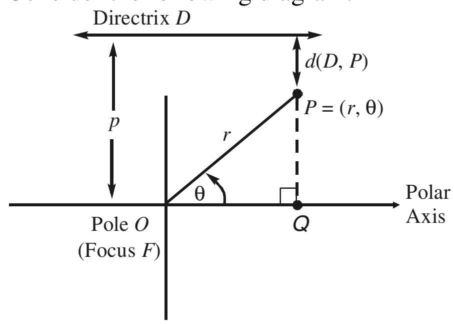
$r = ep - er\cos\theta$

$r + er\cos\theta = ep$

$r(1 + e\cos\theta) = ep$

$r = \dfrac{ep}{1 + e\cos\theta}$

44. Consider the following diagram:



$d(F, P) = e \cdot d(D, P)$

$d(D, P) = p - r\sin\theta$

$r = e(p - r\sin\theta)$

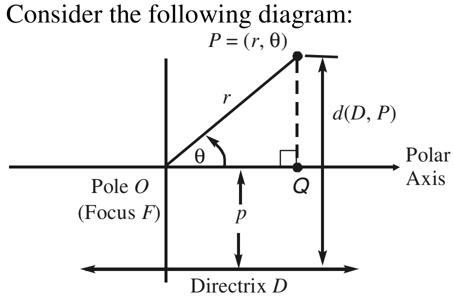
$r = ep - er\sin\theta$

$r + er\sin\theta = ep$

$r(1 + e\sin\theta) = ep$

$r = \dfrac{ep}{1 + e\sin\theta}$

45. Consider the following diagram:



$d(F, P) = e \cdot d(D, P)$

$d(D, P) = p + r\sin\theta$

$r = e(p + r\sin\theta)$

$r = ep + er\sin\theta$

$r - er\sin\theta = ep$

$r(1 - e\sin\theta) = ep$

$r = \dfrac{ep}{1 - e\sin\theta}$

46. $r=\dfrac{(3.442)10^7}{1-0.206\cos\theta}$

At aphelion, the greatest distance from the sun, $\cos\theta=1$.

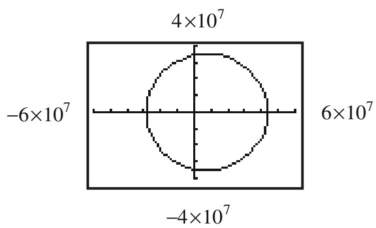
$r=\dfrac{(3.442)10^7}{1-0.206(1)}=\dfrac{(3.442)10^7}{0.794}$

$\approx 4.335\times10^7$ miles

At perihelion, the shortest distance from the sun, $\cos\theta=-1$.

$r=\dfrac{(3.442)10^7}{1-0.206(-1)}=\dfrac{(3.442)10^7}{1.206}$

$\approx 2.854\times10^7$ miles



Section 10.7

1. $|3|=3;\ \dfrac{2\pi}{4}=\dfrac{\pi}{2}$

2. plane curve; parameter

3. ellipse

4. cycloid

5. False; for example: $x=2\cos t$, $y=3\sin t$ define the same curve as in problem 3.

6. True

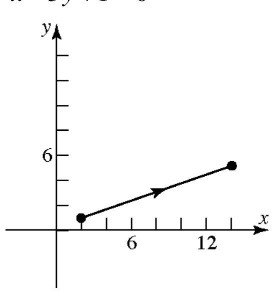
7. $x=3t+2,\ \ y=t+1,\ \ 0\le t\le 4$

$x=3(y-1)+2$

$x=3y-3+2$

$x=3y-1$

$x-3y+1=0$



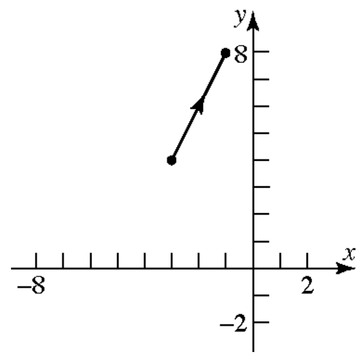
8. $x=t-3,\ \ y=2t+4,\ \ 0\le t\le 2$

$y=2(x+3)+4$

$y=2x+6+4$

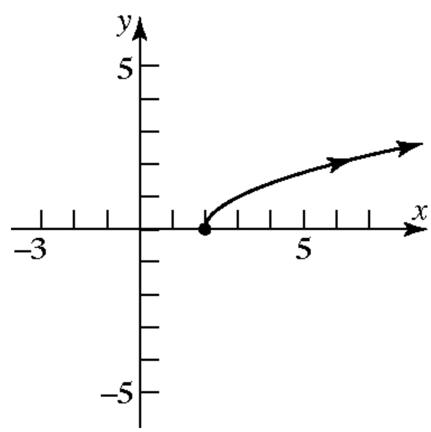
$y=2x+10$

$2x-y+10=0$



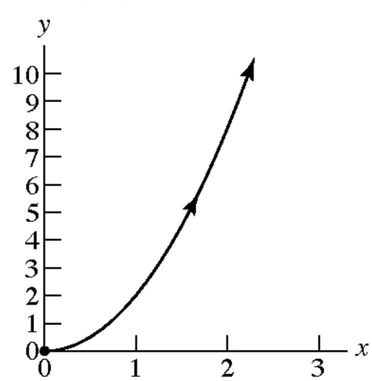
9. $x=t+2,\ \ y=\sqrt{t},\ \ t\ge 0$

$y=\sqrt{x-2}$



10. $x=\sqrt{2t},\ \ y=4t,\ \ t\ge 0$

$y=4\left(\dfrac{x^2}{2}\right)=2x^2,\ \ x\ge 0$

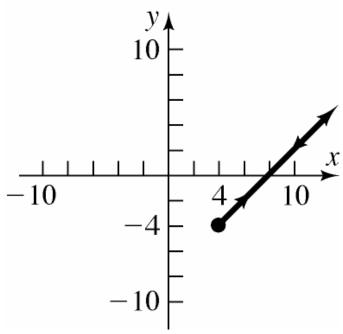


11. $x = t^2 + 4,\ \ y = t^2 - 4,\ \ -\infty < t < \infty$

$y = (x-4) - 4$

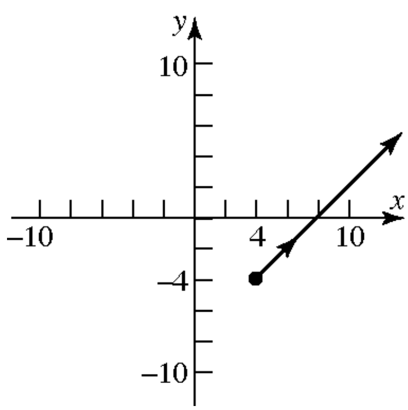
$y = x - 8$

For $-\infty < t < 0$ the movement is to the left. For $0 < t < \infty$ the movement is to the right.



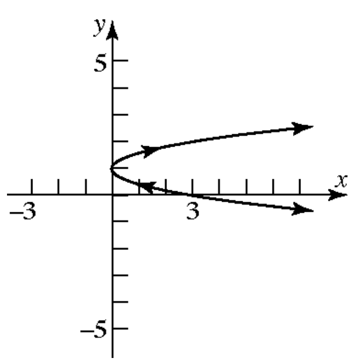
12. $x = \sqrt{t} + 4,\ \ y = \sqrt{t} - 4,\ \ t \ge 0$

$y = x - 4 - 4 = x - 8$



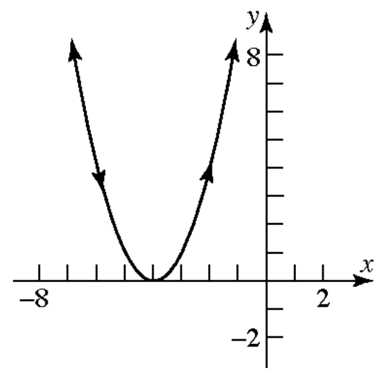
13. $x = 3t^2,\ \ y = t + 1,\ \ -\infty < t < \infty$

$x = 3(y-1)^2$



14. $x = 2t - 4,\ \ y = 4t^2,\ \ -\infty < t < \infty$

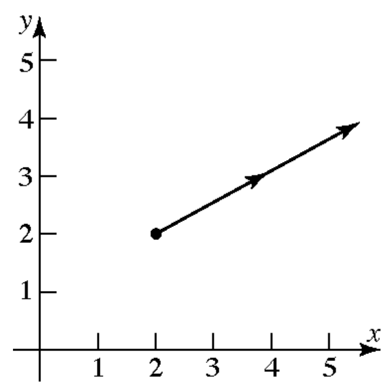
$$y = 4\left(\frac{x+4}{2}\right)^2 = (x+4)^2$$



15. $x = 2e^t,\ \ y = 1 + e^t,\ \ t \ge 0$

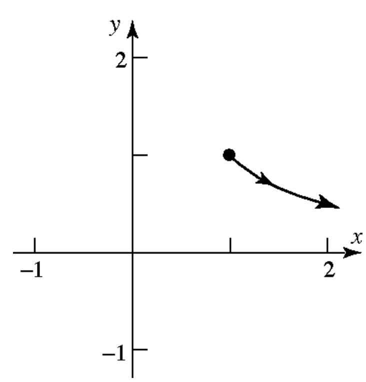
$$y = 1 + \frac{x}{2}$$

$2y = 2 + x$



16. $x = e^t,\ \ y = e^{-t},\ \ t \ge 0$

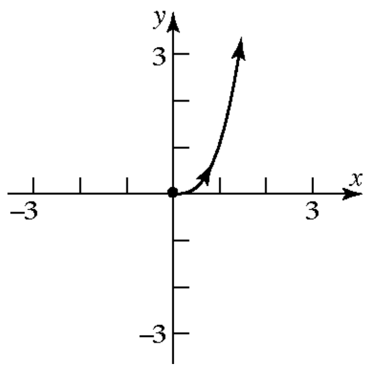
$$y = x^{-1} = \frac{1}{x}$$



17. $x=\sqrt{t},\ \ y=t^{3/2}\ ,\ \ t\geq 0$

$y=\left(x^{2}\right)^{3/2}$

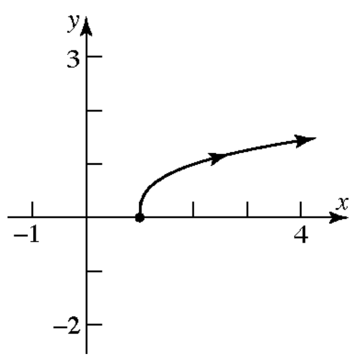
$y=x^{3}$



18. $x=t^{3/2}+1,\ \ y=\sqrt{t}\ ,\ \ t\geq 0$

$x=\left(y^{2}\right)^{3/2}+1$

$x=y^{3}+1$

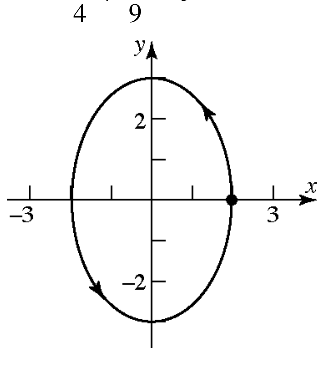


19. $x=2\cos t,\ \ y=3\sin t\ ,\ \ 0\leq t\leq 2\pi$

$$\frac{x}{2}=\cos t \qquad \frac{y}{3}=\sin t$$

$$\left(\frac{x}{2}\right)^{2}+\left(\frac{y}{3}\right)^{2}=\cos^{2} t+\sin^{2} t=1$$

$$\frac{x^{2}}{4}+\frac{y^{2}}{9}=1$$

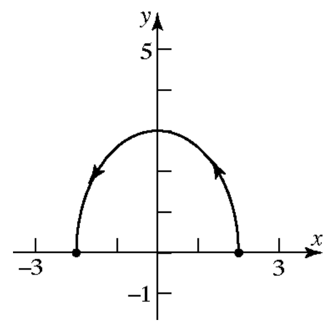


20. $x=2\cos t,\ \ y=3\sin t\ ,\ \ 0\leq t\leq \pi$

$$\frac{x}{2}=\cos t \qquad \frac{y}{3}=\sin t$$

$$\left(\frac{x}{2}\right)^{2}+\left(\frac{y}{3}\right)^{2}=\cos^{2} t+\sin^{2} t=1$$

$$\frac{x^{2}}{4}+\frac{y^{2}}{9}=1 \qquad y\geq 0$$

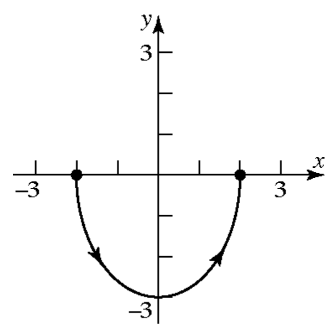


21. $x=2\cos t,\ \ y=3\sin t\ ,\ \ -\pi\leq t\leq 0$

$$\frac{x}{2}=\cos t \qquad \frac{y}{3}=\sin t$$

$$\left(\frac{x}{2}\right)^{2}+\left(\frac{y}{3}\right)^{2}=\cos^{2} t+\sin^{2} t=1$$

$$\frac{x^{2}}{4}+\frac{y^{2}}{9}=1 \qquad y\leq 0$$

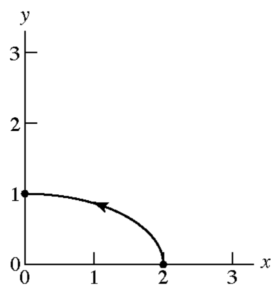


22. $x = 2\cos t,\ \ y = \sin t\ ,\ \ 0 \le t \le \dfrac{\pi}{2}$

$$\frac{x}{2} = \cos t \qquad y = \sin t$$

$$\left(\frac{x}{2}\right)^2 + (y)^2 = \cos^2 t + \sin^2 t = 1$$

$$\frac{x^2}{4} + y^2 = 1 \qquad x \ge 0,\ y \ge 0$$

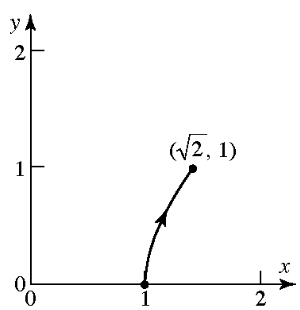


23. $x = \sec t,\ \ y = \tan t\ ,\ \ 0 \le t \le \dfrac{\pi}{4}$

$$\sec^2 t = 1 + \tan^2 t$$

$$x^2 = 1 + y^2$$

$$x^2 - y^2 = 1 \qquad 1 \le x \le \sqrt{2},\ \ 0 \le y \le 1$$

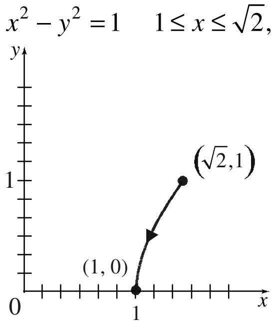


24. $x = \csc t,\ \ y = \cot t\ ,\ \ \dfrac{\pi}{4} \le t \le \dfrac{\pi}{2}$

$$\csc^2 t = 1 + \cot^2 t$$

$$x^2 = 1 + y^2$$

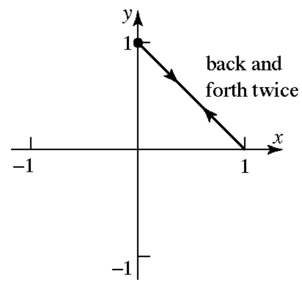
$$x^2 - y^2 = 1 \qquad 1 \le x \le \sqrt{2},\ \ 0 \le y \le 1$$



25. $x = \sin^2 t,\ \ y = \cos^2 t\ ,\ \ 0 \le t \le 2\pi$

$$\sin^2 t + \cos^2 t = 1$$

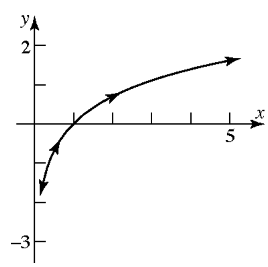
$$x + y = 1$$



26. $x = t^2,\ \ y = \ln t\ ,\ \ t > 0$

$$y = \ln\sqrt{x}$$

$$y = \frac{1}{2}\ln x$$



27. $x = t,\ \ y = 4t - 1 \qquad x = t + 1,\ \ y = 4t + 3$

28. $x = t,\ \ y = -8t + 3 \qquad x = t + 1,\ \ y = -8t - 5$

29. $x = t,\ \ y = t^2 + 1 \qquad x = t - 1,\ \ y = t^2 - 2t + 2$

30. $x = t,\ \ y = -2t^2 + 1$

$x = t - 1,\ \ y = -2t^2 + 4t - 1$

31. $x = t,\ \ y = t^3 \qquad x = \sqrt[3]{t},\ \ y = t$

32. $x = t,\ \ y = t^4 + 1 \qquad x = \sqrt[4]{t - 1},\ \ y = t$

33. $x = t^{3/2},\ \ y = t \qquad x = \sqrt{t},\ \ y = \sqrt[3]{t}$

34. $x = \sqrt{t},\ \ y = t \qquad x = t^2,\ \ y = t^4$

35. $x = t + 2,\ \ y = t;\ \ 0 \le t \le 5$

36. $x = t,\ \ y = -t + 1;\ \ -1 \le t \le 3$

37. $x = 3\cos t,\ \ y = 2\sin t;\ \ 0 \le t \le 2\pi$

38. $x = \cos\left(\frac{\pi}{2}(t-1)\right),\ y = 4\sin\left(\frac{\pi}{2}(t-1)\right);\ 0 \le t \le 2$

39. Since the motion begins at (2, 0), we want $x = 2$ and $y = 0$ when $t = 0$. For the motion to be clockwise, we must have x positive and y negative initially.

$x = 2\cos(\omega t),\quad y = -3\sin(\omega t)$

$\frac{2\pi}{\omega} = 2 \Rightarrow \omega = \pi$

$x = 2\cos(\pi t),\quad y = -3\sin(\pi t),\quad 0 \le t \le 2$

40. Since the motion begins at (0, 3), we want $x = 0$ and $y = 3$ when $t = 0$. For the motion to be counter-clockwise, we need x negative and y positive initially.

$x = -2\sin(\omega t),\quad y = 3\cos(\omega t)$

$\frac{2\pi}{\omega} = 1 \Rightarrow \omega = 2\pi$

$x = -2\sin(2\pi t),\quad y = 3\cos(2\pi t),\quad 0 \le t \le 1$

41. Since the motion begins at (0, 3), we want $x = 0$ and $y = 3$ when $t = 0$. For the motion to be clockwise, we need x positive and y positive initially.

$x = 2\sin(\omega t),\quad y = 3\cos(\omega t)$

$\frac{2\pi}{\omega} = 1 \Rightarrow \omega = 2\pi$

$x = 2\sin(2\pi t),\quad y = 3\cos(2\pi t),\quad 0 \le t \le 1$

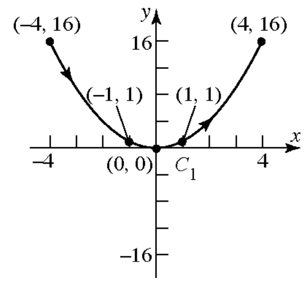
42. Since the motion begins at (2, 0), we want $x = 2$ and $y = 0$ when $t = 0$. For the motion to be counter-clockwise, we need x positive and y positive initially.

$x = 2\cos(\omega t),\quad y = 3\sin(\omega t)$

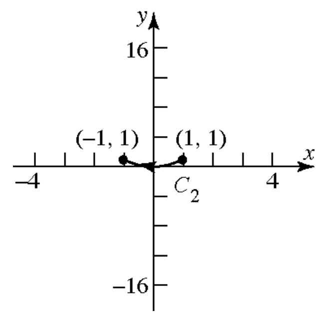
$\frac{2\pi}{\omega} = 3 \Rightarrow \omega = \frac{2\pi}{3}$

$x = 2\cos\left(\frac{2\pi}{3}t\right),\quad y = 3\sin\left(\frac{2\pi}{3}t\right),\quad 0 \le t \le 3$

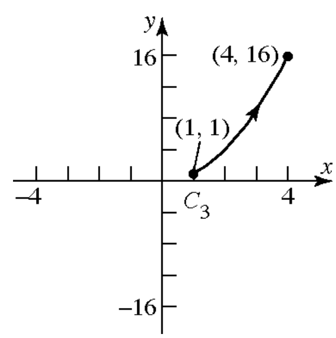
43. C_1



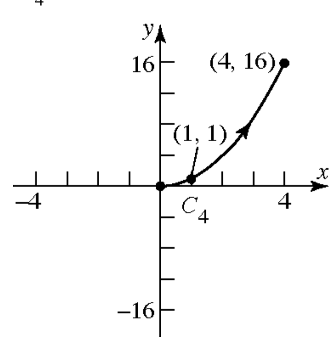
C_2



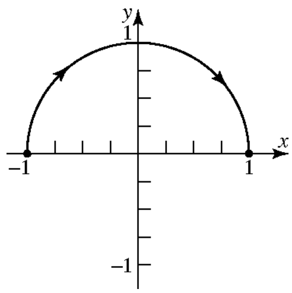
C_3



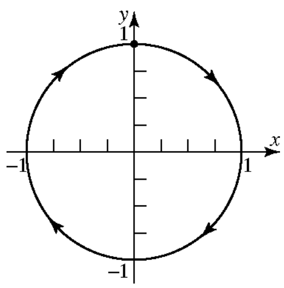
C_4



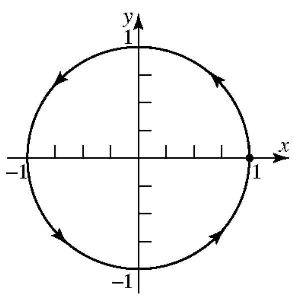
44. C_1



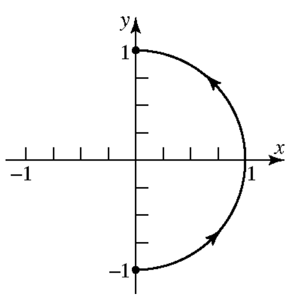
C_2



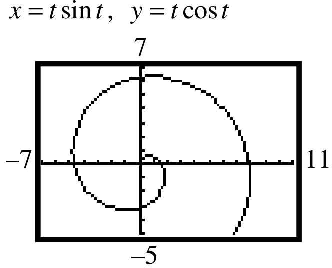
C_3



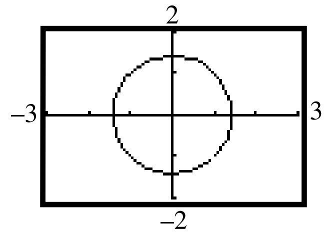
C_4



45. $x = t\sin t, \quad y = t\cos t$

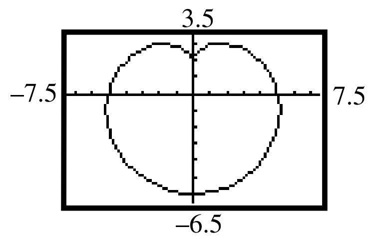


46. $x = \sin t + \cos t, \quad y = \sin t - \cos t$



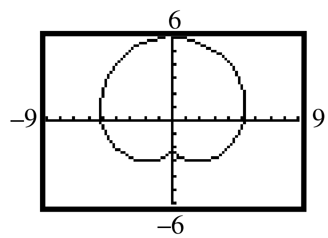
47. $x = 4\sin t - 2\sin(2t)$

$y = 4\cos t - 2\cos(2t)$



48. $x = 4\sin t + 2\sin(2t)$

$y = 4\cos t + 2\cos(2t)$



49. a. Use equations (1):

$$x = (50\cos 90°)t = 0$$

$$y = -\frac{1}{2}(32)t^2 + (50\sin 90°)t + 6$$
$$= -16t^2 + 50t + 6$$

b. The ball is in the air until $y = 0$. Solve:

$$-16t^2 + 50t + 6 = 0$$

$$t = \frac{-50 \pm \sqrt{50^2 - 4(-16)(6)}}{2(-16)}$$
$$= \frac{-50 \pm \sqrt{2884}}{-32}$$
$$\approx -0.12 \text{ or } 3.24$$

The ball is in the air for about 3.24 seconds. (The negative solution is extraneous.)

c. The maximum height occurs at the vertex of the quadratic function.

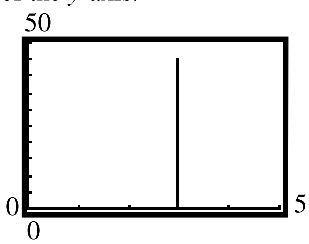
$$t = -\frac{b}{2a} = -\frac{50}{2(-16)} = 1.5625 \text{ seconds}$$

Evaluate the function to find the maximum height:

$$-16(1.5625)^2 + 50(1.5625) + 6 = 45.0625$$

The maximum height is 45.0625 feet.

d. We use $x = 3$ so that the line is not on top of the y-axis.



50. a. Use equations (1):

$$x = \left(40\cos 90^{\circ}\right)t = 0$$

$$y = -\frac{1}{2}(32)t^2 + \left(40\sin 90^{\circ}\right)t + 5$$
$$= -16t^2 + 40t + 5$$

b. The ball is in the air until $y = 0$. Solve:

$$-16t^2 + 40t + 5 = 0$$

$$t = \frac{-40 \pm \sqrt{40^2 - 4(-16)(5)}}{2(-16)}$$

$$= \frac{-40 \pm \sqrt{1920}}{-32} \approx -0.12 \text{ or } 2.62$$

The ball is in the air for about 2.62 seconds. (The negative solution is extraneous.)

c. The maximum height occurs at the vertex of the quadratic function.

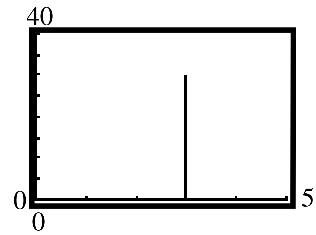
$$t = -\frac{b}{2a} = -\frac{40}{2(-16)} = 1.25 \text{ seconds}$$

Evaluate the function to find the maximum height:

$$-16(1.25)^2 + 40(1.25) + 5 = 30$$

The maximum height is 30 feet.

d. We use $x = 3$ so that the line is not on top of the y-axis.



51. Let $y_1 = 1$ be the train's path and $y_2 = 5$ be Bill's path.

a. Train: Using the hint,

$$x_1 = \frac{1}{2}(2)t^2 = t^2$$
$$y_1 = 1$$

Bill:

$$x_2 = 5(t - 5)$$
$$y_2 = 5$$

b. Bill will catch the train if $x_1 = x_2$.

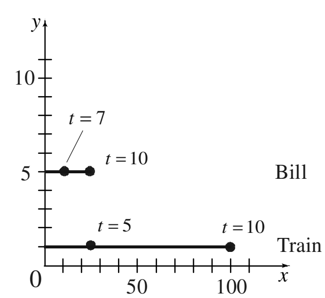
$$t^2 = 5(t - 5)$$
$$t^2 = 5t - 25$$
$$t^2 - 5t + 25 = 0$$

Since

$$b^2 - 4ac = (-5)^2 - 4(1)(25) \quad ,$$
$$= 25 - 100 = -75 < 0$$

the equation has no real solution. Thus, Bill will not catch the train.

c.



52. Let $y_1 = 1$ be the bus's path and $y_2 = 5$ be Jodi's path.

a. Bus: Using the hint,

$$x_1 = \frac{1}{2}(3)t^2 = 1.5t^2$$
$$y_1 = 1$$

Jodi:

$$x_2 = 5(t-2)$$
$$y_2 = 5$$

b. Jodi will catch the bus if $x_1 = x_2$.

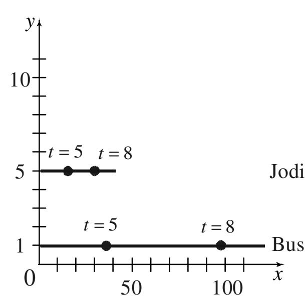
$$1.5t^2 = 5(t-2)$$
$$1.5t^2 = 5t - 10$$
$$1.5t^2 - 5t + 10 = 0$$

Since

$$b^2 - 4ac = (-5)^2 - 4(1.5)(10),$$
$$= 25 - 60 = -35 < 0$$

the equation has no real solution. Thus, Jodi will not catch the bus.

c.



53. a. Use equations (1):

$$x = (145\cos 20^\circ)t$$
$$y = -\frac{1}{2}(32)t^2 + (145\sin 20^\circ)t + 5$$

b. The ball is in the air until $y = 0$. Solve:

$$-16t^2 + (145\sin 20^\circ)t + 5 = 0$$
$$t = \frac{-145\sin 20^\circ \pm \sqrt{(145\sin 20^\circ)^2 - 4(-16)(5)}}{2(-16)}$$
$$\approx -0.10 \text{ or } 3.20$$

The ball is in the air for about 3.20 seconds. (The negative solution is extraneous.)

c. Find the horizontal displacement:

$$x = (145\cos 20^\circ)(3.20) \approx 436 \text{ feet}$$

d. The maximum height occurs at the vertex of the quadratic function.

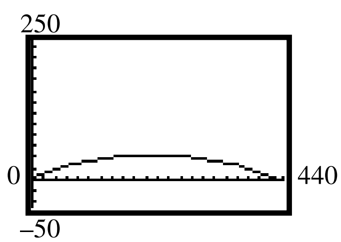
$$t = -\frac{b}{2a} = -\frac{145\sin 20^\circ}{2(-16)} \approx 1.55 \text{ seconds}$$

Evaluate the function to find the maximum height:

$$-16(1.55)^2 + (145\sin 20^\circ)(1.55) + 5 \approx 43.43$$

The maximum height is about 43.43 feet.

e.



54. a. Use equations (1):

$$x = (125\cos 40^\circ)t$$
$$y = -16t^2 + (125\sin 40^\circ)t + 3$$

The ball is in the air until $y = 0$. Solve:

$$-16t^2 + (125\sin 40^\circ)t + 3 = 0$$
$$t = \frac{-125\sin 40^\circ \pm \sqrt{(125\sin 40^\circ)^2 - 4(-16)(3)}}{2(-16)}$$
$$\approx -0.037 \quad \text{or} \quad 5.059$$

b. The ball is in the air for about 5.059 sec. (The negative solution is extraneous.)

c. Find the horizontal displacement:

$$x = (125\cos 40^\circ)(5.059) \approx 484.41 \text{ feet}$$

d. The maximum height occurs at the vertex of the quadratic function.

$$t = -\frac{b}{2a} = -\frac{125\sin 40^\circ}{2(-16)} \approx 2.51 \text{ seconds}$$

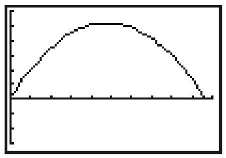
Evaluate the function to find the maximum height:

$$-16(2.51)^2 + (125\sin 40^\circ)(2.51) + 3 \approx 103.87$$

The maximum height is about 103.87 feet.

e. $x = (125\cos 40°)t$

$y = -16t^2 + (125\sin 40°)t + 3$



55. a. Use equations (1):

$x = (40\cos 45°)t = 20\sqrt{2}t$

$y = -\frac{1}{2}(9.8)t^2 + (40\sin 45°)t + 300$

$= -4.9t^2 + 20\sqrt{2}t + 300$

b. The ball is in the air until $y = 0$. Solve:

$-4.9t^2 + 20\sqrt{2}t + 300 = 0$

$t = \frac{-20\sqrt{2} \pm \sqrt{(20\sqrt{2})^2 - 4(-4.9)(300)}}{2(-4.9)}$

≈ -5.45 or 11.23

The ball is in the air for about 11.23 seconds. (The negative solution is extraneous.)

c. Find the horizontal displacement:

$x = (20\sqrt{2})(11.23) \approx 317.6$ meters

d. The maximum height occurs at the vertex of the quadratic function.

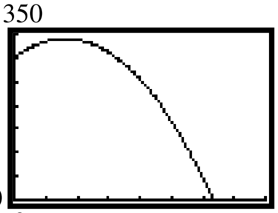
$t = -\frac{b}{2a} = -\frac{20\sqrt{2}}{2(-4.9)} \approx 2.89$ seconds

Evaluate the function to find the maximum height:

$-4.9(2.89)^2 + 20\sqrt{2}(2.89) + 300$

$= 340.8$ meters

e.



56. a. Use equations (1):

$x = (40\cos 45°)t = 20\sqrt{2}t$

$y = -\frac{1}{2}\cdot\frac{1}{6}(9.8)t^2 + (40\sin 45°)t + 300$

$= -\frac{4.9}{6}t^2 + 20\sqrt{2}\,t + 300$

b. The ball is in the air until $y = 0$. Solve:

$-\frac{4.9}{6}t^2 + 20\sqrt{2}t + 300 = 0$

$t = \frac{-20\sqrt{2} \pm \sqrt{(20\sqrt{2})^2 - 4\left(-\frac{4.9}{6}\right)(300)}}{2\left(\frac{-4.9}{6}\right)}$

≈ -8.51 or 43.15

The ball is in the air for about 43.15 seconds. (The negative solution is extraneous.)

c. Find the horizontal displacement:

$x = (20\sqrt{2})(43.15) \approx 1220.5$ meters

d. The maximum height occurs at the vertex of the quadratic function.

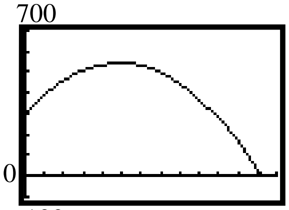
$t = -\frac{b}{2a} = -\frac{20\sqrt{2}}{2\left(\frac{-4.9}{6}\right)} \approx 17.32$ seconds

Evaluate the function to find the maximum height:

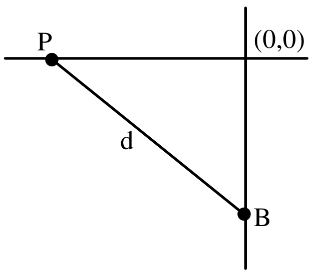
$-\frac{4.9}{6}(17.32)^2 + 20\sqrt{2}(17.32) + 300$

≈ 544.9 meters

e.

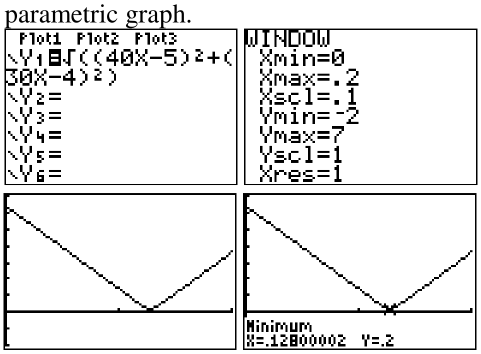


57. **a.** At $t=0$, the Camry is 5 miles from the intersection (at (0, 0)) traveling east (along the x-axis) at 40 mph. Thus, $x=40t-5$, $y=0$, describes the position of the Camry as a function of time. The Impala, at $t=0$, is 4 miles from the intersection traveling north (along the y-axis) at 30 mph. Thus, $x=0$, $y=30t-4$, describes the position of the Impala as a function of time.



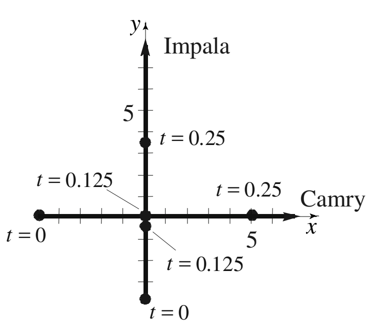
b. Let d represent the distance between the cars. Use the Pythagorean Theorem to find the distance: $d=\sqrt{(40t-5)^2+(30t-4)^2}$.

c. Note this is a function graph not a parametric graph.

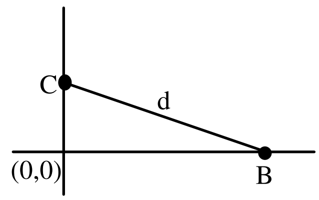


d. The minimum distance between the cars is 0.2 miles and occurs at 0.128 hours (7.68 min).

e.

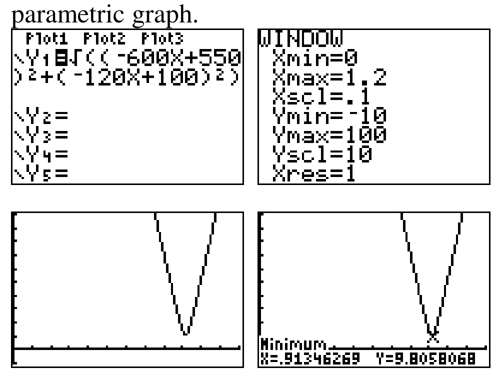


58. **a.** At $t=0$, the Boeing 747 is 550 miles from the intersection (at (0, 0)) traveling west (along the x-axis) at 600 mph. Thus, $x=-600t+550$, $y=0$, describes the position of the Boeing 747 as a function of time. The Cessna, at $t=0$, is 100 miles from the intersection traveling south (along the y-axis) at 120 mph. Thus, $x=0$, $y=-120t+100$ describes the position of the Cessna as a function of time.



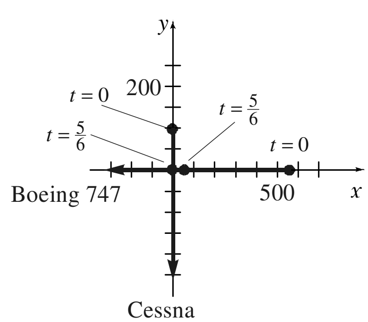
b. Let d represent the distance between the planes. Use the Pythagorean Theorem to find the distance:
$d=\sqrt{(-600t+550)^2+(-120t+100)^2}$.

c. Note this is a function graph not a parametric graph.



d. The minimum distance between the planes is 9.8 miles and occurs at 0.913 hours (54.8 min).

e.



59. a. We start with the parametric equations $x = (v_0 \cos\theta)t$ and $y = -\frac{1}{2}gt^2 + (v_0 \sin\theta)t + h$.

We are given that $\theta = 45°$, $g = 32$ ft/sec^2, and $h = 3$ feet. This gives us

$$x = (v_0 \cdot \cos 45°)t = \frac{\sqrt{2}}{2}v_0 t\ ; \quad y = -\frac{1}{2}(32)t^2 + (v_0 \sin 45°)t + 3 = -16t^2 + \frac{\sqrt{2}}{2}v_0 t + 3$$

where v_0 is the velocity at which the ball leaves the bat.

b. Letting $v_0 = 90$ miles/hr $= 132$ ft/sec, we have

$$y = -16t^2 + \frac{\sqrt{2}}{2}(132)t + 3 = -16t^2 + 66\sqrt{2}\ t + 3$$

The height is maximized when $t = -\frac{66\sqrt{2}}{2(-16)} = \frac{33\sqrt{2}}{16} \approx 2.9168$ sec

$$y = -16(2.9168)^2 + 66\sqrt{2}(2.9168) + 3 \approx 139.1$$

The maximum height of the ball is about 139.1 feet.

c. From part (b), the maximum height is reached after approximately 2.9168 seconds.

$$x = (v_0 \cos\theta)t = \left(132 \cdot \frac{\sqrt{2}}{2}\right)(2.9168) \approx 272.25$$

The ball will reach its maximum height when it is approximately 272.25 feet from home plate.

d. The ball will reach the left field fence when $x = 310$ feet.

$$x = (v_0 \cos\theta)t = 66\sqrt{2}\ t$$

$$310 = 66\sqrt{2}\ t$$

$$t = \frac{310}{66\sqrt{2}} \approx 3.3213 \text{ sec}$$

Thus, it will take about 3.3213 seconds to reach the left field fence.

$$y = -16(3.3213)^2 + 66\sqrt{2}(3.3213) + 3 \approx 136.5$$

Since the left field fence is 37 feet high, the ball will clear the Green Monster by $136.5 - 37 = 99.5$ feet.

60. a. $x = (v_0 \cos\theta)t, \quad y = (v_0 \sin\theta)t - 16t^2$

$$t = \frac{x}{v_0 \cos\theta}$$

$$y = v_0 \sin\theta\left(\frac{x}{v_0 \cos\theta}\right) - 16\left(\frac{x}{v_0 \cos\theta}\right)^2$$

$$y = (\tan\theta)x - \frac{16}{{v_0}^2 \cos^2\theta}x^2$$

y is a quadratic function of *x*; its graph is a parabola with $a = \frac{-16}{{v_0}^2 \cos^2\theta}$, $b = \tan\theta$, and $c = 0$.

b.

$$y = 0$$

$$(v_0 \sin\theta)t - 16t^2 = 0$$

$$t(v_0 \sin\theta - 16t) = 0$$

$$t = 0 \text{ or } v_0 \sin\theta - 16t = 0$$

$$t = \frac{v_0 \sin\theta}{16}$$

c. $x = (v_0 \cos\theta)t = (v_0 \cos\theta)\left(\frac{v_0 \sin\theta}{16}\right) = \frac{v_0^2 \sin 2\theta}{32}$ feet

d. $x = y$

$(v_0 \cos\theta)t = (v_0 \sin\theta)t - 16t^2$

$16t^2 + (v_0 \cos\theta)t - (v_0 \sin\theta)t = 0$

$t\left(16t + (v_0 \cos\theta) - (v_0 \sin\theta)\right) = 0$

$t = 0$ or $16t + (v_0 \cos\theta) - (v_0 \sin\theta) = 0$

$t = \frac{v_0 \sin\theta - v_0 \cos\theta}{16} = \frac{v_0}{16}(\sin\theta - \cos\theta)$

At $t = \frac{v_0}{16}(\sin\theta - \cos\theta)$:

$x = v_0 \cos\theta\left(\frac{v_0}{16}(\sin\theta - \cos\theta)\right) = \frac{v_0^2}{16}\cos\theta(\sin\theta - \cos\theta)$

$y = \frac{v_0^2}{16}\left(-\cos^2\theta + \sin\theta\cos\theta\right)$ (recall we want $x = y$)

$\sqrt{x^2 + y^2} = \sqrt{2\left(\frac{v_0^2}{16}\cos\theta(\sin\theta - \cos\theta)\right)^2} = \frac{v_0^2}{16}\sqrt{2}\cos\theta(\sin\theta - \cos\theta)$

Note: since θ must be greater than $45°$ ($\sin\theta - \cos\theta > 0$), thus absolute value is not needed.

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61. $x = (x_2 - x_1)t + x_1$,

$y = (y_2 - y_1)t + y_1$, $-\infty < t < \infty$

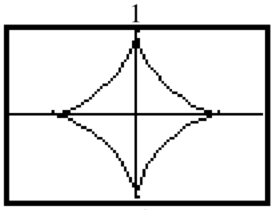
$\frac{x - x_1}{x_2 - x_1} = t$

$y = (y_2 - y_1)\left(\frac{x - x_1}{x_2 - x_1}\right) + y_1$

$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$

This is the two-point form for the equation of a line. Its orientation is from (x_1, y_1) to (x_2, y_2).

62. a. $x(t) = \cos^3 t$, $y(t) = \sin^3 t$, $0 \le t \le 2\pi$



b. $\cos^2 t + \sin^2 t = \left(x^{1/3}\right)^2 + \left(y^{1/3}\right)^2$

$x^{2/3} + y^{2/3} = 1$

63 – 64. Answers will vary.

Chapter 10 Review Exercises

1. $y^2 = -16x$

This is a parabola.

$a = 4$

Vertex: (0, 0)

Focus: (–4, 0)

Directrix: $x = 4$

2. $\frac{x^2}{25} - y^2 = 1$

This is a hyperbola.

$a = 5$, $b = 1$.

Find the value of c:

$c^2 = a^2 + b^2 = 25 + 1 = 26$

$c = \sqrt{26}$

Center: (0, 0)

Vertices: (5, 0), (–5, 0)

Foci: $(\sqrt{26}, 0)$, $(-\sqrt{26}, 0)$

Asymptotes: $y = \frac{1}{5}x$; $y = -\frac{1}{5}x$

3. $\dfrac{y^2}{25}+\dfrac{x^2}{16}=1$

This is an ellipse.

$a=5,\ b=4$.

Find the value of c:

$c^2=a^2-b^2=25-16=9$

$c=3$

Center: (0, 0)

Vertices: (0, 5), (0, –5)

Foci: (0, 3), (0, –3)

4. $x^2+4y=4$

This is a parabola.

Write in standard form:

$x^2=-4y+4$

$x^2=-4(y-1)$

$a=1$

Vertex: (0, 1)

Focus: (0, 0) Directrix: $y=2$

5. $4x^2-y^2=8$

This is a hyperbola.

Write in standard form:

$\dfrac{x^2}{2}-\dfrac{y^2}{8}=1$

$a=\sqrt{2},\ b=\sqrt{8}=2\sqrt{2}$.

Find the value of c:

$c^2=a^2+b^2=2+8=10$

$c=\sqrt{10}$

Center: (0, 0)

Vertices: $\left(-\sqrt{2},0\right),\left(\sqrt{2},0\right)$

Foci: $\left(-\sqrt{10},0\right),\ \left(\sqrt{10},0\right)$

Asymptotes: $y=2x;\quad y=-2x$

6. $x^2-4x=2y$

This is a parabola.

Write in standard form:

$x^2-4x+4=2y+4$

$(x-2)^2=2(y+2)$

$a=\dfrac{1}{2}$

Vertex: (2, –2)

Focus: $\left(2,-\dfrac{3}{2}\right)$

Directrix: $y=-\dfrac{5}{2}$

7. $y^2-4y-4x^2+8x=4$

This is a hyperbola.

Write in standard form:

$(y^2-4y+4)-4(x^2-2x+1)=4+4-4$

$(y-2)^2-4(x-1)^2=4$

$\dfrac{(y-2)^2}{4}-\dfrac{(x-1)^2}{1}=1$

$a=2,\ b=1$.

Find the value of c:

$c^2=a^2+b^2=4+1=5$

$c=\sqrt{5}$

Center: (1, 2)

Vertices: (1, 0), (1, 4)

Foci: $\left(1,2-\sqrt{5}\right),\ \left(1,2+\sqrt{5}\right)$

Asymptotes: $y-2=2(x-1);\quad y-2=-2(x-1)$

8. $4x^2+9y^2-16x-18y=11$

This is an ellipse.

Write in standard form:

$4x^2+9y^2-16x-18y=11$

$4(x^2-4x+4)+9(y^2-2y+1)=11+16+9$

$4(x-2)^2+9(y-1)^2=36$

$\dfrac{(x-2)^2}{9}+\dfrac{(y-1)^2}{4}=1$

$a=3,\ b=2$. Find the value of c:

$c^2=a^2-b^2=9-4=5\ \rightarrow\ c=\sqrt{5}$

Center: (2, 1); Vertices: (–1, 1), (5, 1)

Foci: $\left(2-\sqrt{5},1\right),\left(2+\sqrt{5},1\right)$

9. $4x^2-16x+16y+32=0$

This is a parabola.

Write in standard form:

$4(x^2-4x+4)=-16y-32+16$

$4(x-2)^2=-16(y+1)$

$(x-2)^2=-4(y+1)$

$a=1$

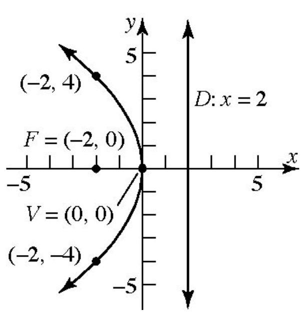
Vertex: (2, –1); Focus: $\left(2,-2\right)$;
Directrix: $y=0$

10. $9x^2+4y^2-18x+8y=23$
This is an ellipse.
Write in standard form:

$$9(x^2-2x+1)+4(y^2+2y+1)=23+9+4$$
$$9(x-1)^2+4(y+1)^2=36$$
$$\frac{(x-1)^2}{4}+\frac{(y+1)^2}{9}=1$$

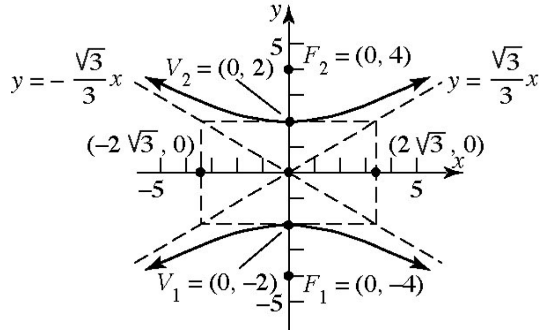
$a=3,\ b=2$.
Find the value of c:
$c^2=a^2-b^2=9-4=5$
$c=\sqrt{5}$
Center: (1, –1)
Vertices: (1, –4), (1, 2)
Foci: $\left(1,-1-\sqrt{5}\right),\left(1,-1+\sqrt{5}\right)$

11. Parabola: The focus is (–2, 0) and the directrix is $x=2$. The vertex is (0, 0). $a=2$ and since (–2, 0) is to the left of (0, 0), the parabola opens to the left. The equation of the parabola is:
$y^2=-4ax$
$y^2=-4\cdot 2\cdot x$
$y^2=-8x$



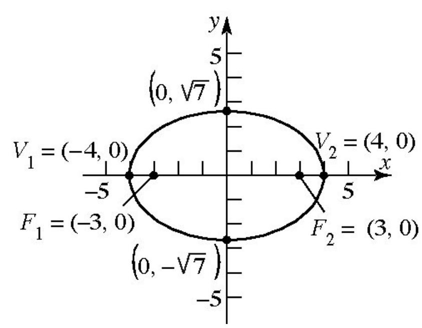
12. Hyperbola: Center: (0, 0);
Focus: (0, 4); Vertex: (0, –2);
Transverse axis is the y-axis; $a=2;\ c=4$.
Find b:
$b^2=c^2-a^2=16-4=12$
$b=\sqrt{12}=2\sqrt{3}$

Write the equation: $\frac{y^2}{4}-\frac{x^2}{12}=1$

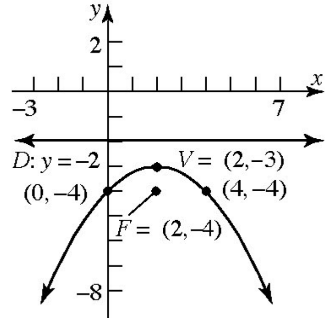


13. Ellipse: Foci: (–3, 0), (3, 0); Vertex: (4, 0);
Center: (0, 0); Major axis is the x-axis;
$a=4;\ c=3$. Find b:
$b^2=a^2-c^2=16-9=7$
$b=\sqrt{7}$

Write the equation: $\frac{x^2}{16}+\frac{y^2}{7}=1$



14. Parabola: The focus is (2, –4) and the vertex is (2, –3). Both lie on the vertical line $x=2$. $a=1$ and since (2, –4) is below (2, –3), the parabola opens down. The equation of the parabola is:
$(x-h)^2=-4a(y-k)$
$(x-2)^2=-4\cdot 1\cdot(y-(-3))$
$(x-2)^2=-4(y+3)$

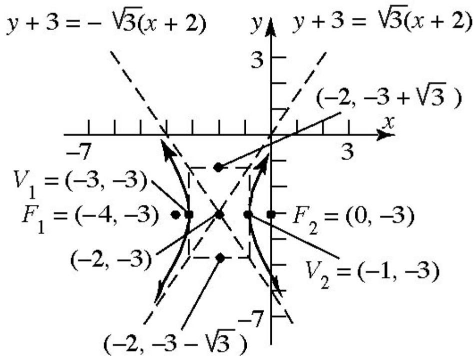


15. Hyperbola: Center: (–2, –3); Focus: (–4, –3); Vertex: (–3, –3); Transverse axis is parallel to the x-axis; $a = 1$; $c = 2$. Find b:

$b^2 = c^2 - a^2 = 4 - 1 = 3$

$b = \sqrt{3}$

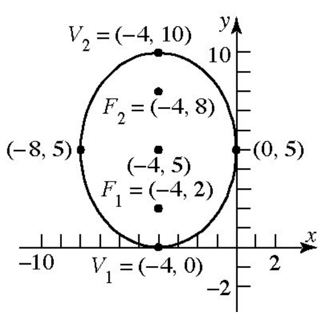
Write the equation: $\dfrac{(x+2)^2}{1} - \dfrac{(y+3)^2}{3} = 1$



16. Ellipse: Foci: (–4, 2), (–4, 8); Vertex: (–4, 10); Center: (–4, 5); Major axis is parallel to the y-axis; $a = 5$; $c = 3$. Find b:

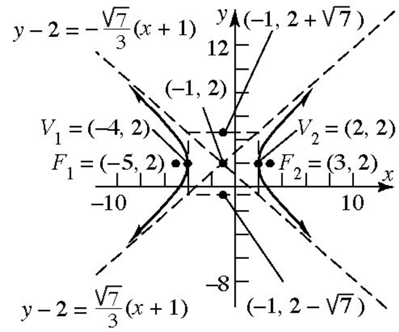
$b^2 = a^2 - c^2 = 25 - 9 = 16 \quad \rightarrow \quad b = 4$

Write the equation: $\dfrac{(x+4)^2}{16} + \dfrac{(y-5)^2}{25} = 1$



17. Hyperbola: Center: (–1, 2); $a = 3$; $c = 4$;
Transverse axis parallel to the x-axis;
Find b: $b^2 = c^2 - a^2 = 16 - 9 = 7 \quad \rightarrow \quad b = \sqrt{7}$

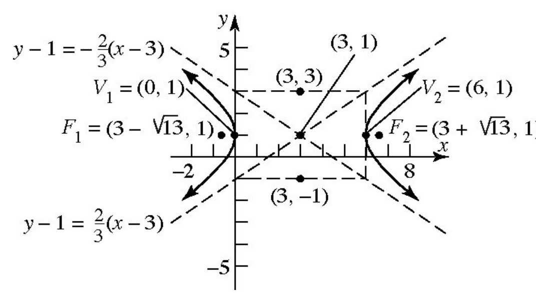
Write the equation: $\dfrac{(x+1)^2}{9} - \dfrac{(y-2)^2}{7} = 1$



18. Hyperbola: Vertices: (0, 1), (6, 1); Asymptote: $3y + 2x - 9 = 0$; Center: (3, 1); Transverse axis is parallel to the x-axis; $a = 3$; The slope of the asymptote is $-\dfrac{2}{3}$; Find b:

$\dfrac{-b}{a} = \dfrac{-b}{3} = \dfrac{-2}{3} \rightarrow -3b = -6 \rightarrow b = 2$

Write the equation: $\dfrac{(x-3)^2}{9} - \dfrac{(y-1)^2}{4} = 1$



19. $y^2 + 4x + 3y - 8 = 0$
$A = 0$ and $C = 1$; $AC = (0)(1) = 0$. Since $AC = 0$, the equation defines a parabola.

20. $x^2 + 2y^2 + 4x - 8y + 2 = 0$
$A = 1$ and $C = 2$; $AC = (1)(2) = 2$. Since $AC > 0$ and $A \neq C$, the equation defines an ellipse.

21. $9x^2 - 12xy + 4y^2 + 8x + 12y = 0$
$A = 9,\ B = -12,\ C = 4$
$B^2 - 4AC = (-12)^2 - 4(9)(4) = 0$
Parabola

22. $4x^2+10xy+4y^2-9=0$
$A=4,\ B=10,\ C=4$
$B^2-4AC=10^2-4(4)(4)=36>0$
Hyperbola

23. $x^2-2xy+3y^2+2x+4y-1=0$
$A=1,\ B=-2,\ C=3$
$B^2-4AC=(-2)^2-4(1)(3)=-8<0$
Ellipse

---

24. $2x^2+5xy+2y^2-\frac{9}{2}=0$

$A=2$, $B=5$, and $C=2$; $\cot(2\theta)=\frac{A-C}{B}=\frac{2-2}{5}=0 \ \rightarrow\ 2\theta=\frac{\pi}{2} \ \rightarrow\ \theta=\frac{\pi}{4}$

$x=x'\cos\theta-y'\sin\theta=\frac{\sqrt{2}}{2}x'-\frac{\sqrt{2}}{2}y'=\frac{\sqrt{2}}{2}(x'-y')$

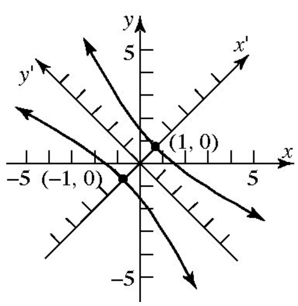
$y=x'\sin\theta+y'\cos\theta=\frac{\sqrt{2}}{2}x'+\frac{\sqrt{2}}{2}y'=\frac{\sqrt{2}}{2}(x'+y')$

$2\left(\frac{\sqrt{2}}{2}(x'-y')\right)^2+5\left(\frac{\sqrt{2}}{2}(x'-y')\right)\left(\frac{\sqrt{2}}{2}(x'+y')\right)+2\left(\frac{\sqrt{2}}{2}(x'+y')\right)^2-\frac{9}{2}=0$

$\left(x'^2-2x'y'+y'^2\right)+\frac{5}{2}\left(x'^2-y'^2\right)+\left(x'^2+2x'y'+y'^2\right)-\frac{9}{2}=0$

$\frac{9}{2}x'^2-\frac{1}{2}y'^2=\frac{9}{2}\rightarrow 9x'^2-y'^2=9\rightarrow\frac{x'^2}{1}-\frac{y'^2}{9}=1$

Hyperbola; center at (0, 0), transverse axis is the x'-axis, vertices at $(x',y')=(\pm1,0)$; foci at $(x',y')=(\pm\sqrt{10},0)$; asymptotes: $y'=\pm3x'$.



25. $6x^2+4xy+9y^2-20=0$

$A=6$, $B=4$, and $C=9$; $\cot(2\theta)=\frac{A-C}{B}=\frac{6-9}{4}=-\frac{3}{4} \ \rightarrow\ \cos(2\theta)=-\frac{3}{5}$

$\sin\theta=\sqrt{\frac{\left(1-\left(-\frac{3}{5}\right)\right)}{2}}=\sqrt{\frac{4}{5}}=\frac{2\sqrt{5}}{5}$; $\cos\theta=\sqrt{\frac{\left(1+\left(-\frac{3}{5}\right)\right)}{2}}=\sqrt{\frac{1}{5}}=\frac{\sqrt{5}}{5} \ \rightarrow\ \theta\approx 63.4°$

$x=x'\cos\theta-y'\sin\theta=\frac{\sqrt{5}}{5}x'-\frac{2\sqrt{5}}{5}y'=\frac{\sqrt{5}}{5}(x'-2y')$

$y=x'\sin\theta+y'\cos\theta=\frac{2\sqrt{5}}{5}x'+\frac{\sqrt{5}}{5}y'=\frac{\sqrt{5}}{5}(2x'+y')$

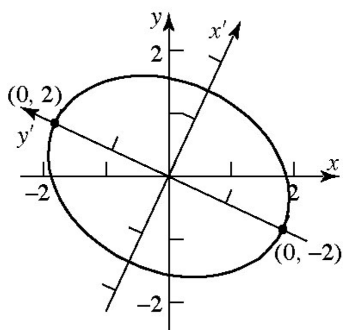
$$6\left(\frac{\sqrt{5}}{5}(x'-2y')\right)^2+4\left(\frac{\sqrt{5}}{5}(x'-2y')\right)\left(\frac{\sqrt{5}}{5}(2x'+y')\right)+9\left(\frac{\sqrt{5}}{5}(2x'+y')\right)^2-20=0$$

$$\frac{6}{5}\left(x'^2-4x'y'+4y'^2\right)+\frac{4}{5}\left(2x'^2-3x'y'-2y'^2\right)+\frac{9}{5}\left(4x'^2+4x'y'+y'^2\right)-20=0$$

$$\frac{6}{5}x'^2-\frac{24}{5}x'y'+\frac{24}{5}y'^2+\frac{8}{5}x'^2-\frac{12}{5}x'y'-\frac{8}{5}y'^2+\frac{36}{5}x'^2+\frac{36}{5}x'y'+\frac{9}{5}y'^2=20$$

$$10x'^2+5y'^2=20\rightarrow\frac{x'^2}{2}+\frac{y'^2}{4}=1$$

Ellipse; center at the origin, major axis is the y'-axis, vertices at $(x',y')=(0,\pm2)$; foci at $(x',y')=\left(0,\pm\sqrt{2}\right)$.



26. $4x^2-12xy+9y^2+12x+8y=0$

$A=4$, $B=-12$, and $C=9$; $\cot(2\theta)=\dfrac{A-C}{B}=\dfrac{4-9}{-12}=\dfrac{5}{12}\quad\rightarrow\ \cos(2\theta)=\dfrac{5}{13}$

$$\sin\theta=\sqrt{\frac{\left(1-\frac{5}{13}\right)}{2}}=\sqrt{\frac{4}{13}}=\frac{2\sqrt{13}}{13};\quad\cos\theta=\sqrt{\frac{\left(1+\frac{5}{13}\right)}{2}}=\sqrt{\frac{9}{13}}=\frac{3\sqrt{13}}{13}\quad\rightarrow\quad\theta\approx33.7^\circ$$

$$x=x'\cos\theta-y'\sin\theta=\frac{3\sqrt{13}}{13}x'-\frac{2\sqrt{13}}{13}y'=\frac{\sqrt{13}}{13}(3x'-2y')$$

$$y=x'\sin\theta+y'\cos\theta=\frac{2\sqrt{13}}{13}x'+\frac{3\sqrt{13}}{13}y'=\frac{\sqrt{13}}{13}(2x'+3y')$$

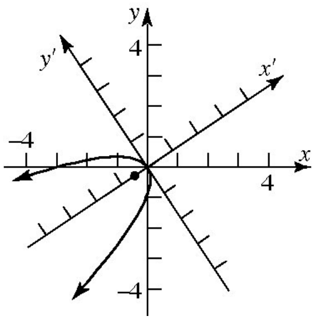
$$4\left(\frac{\sqrt{13}}{13}(3x'-2y')\right)^2-12\left(\frac{\sqrt{13}}{13}(3x'-2y')\right)\left(\frac{\sqrt{13}}{13}(2x'+3y')\right)+9\left(\frac{\sqrt{13}}{13}(2x'+3y')\right)^2$$
$$+12\left(\frac{\sqrt{13}}{13}(3x'-2y')\right)+8\left(\frac{\sqrt{13}}{13}(2x'+3y')\right)=0$$

$$\frac{4}{13}\left(9x'^2-12x'y'+4y'^2\right)-\frac{12}{13}\left(6x'^2+5x'y'-6y'^2\right)+\frac{9}{13}\left(4x'^2+12x'y'+9y'^2\right)$$
$$+\frac{36\sqrt{13}}{13}x'-\frac{24\sqrt{13}}{13}y'+\frac{16\sqrt{13}}{13}x'+\frac{24\sqrt{13}}{13}y'=0$$

$$\frac{36}{13}x'^2-\frac{48}{13}x'y'+\frac{16}{13}y'^2-\frac{72}{13}x'^2-\frac{60}{13}x'y'+\frac{72}{13}y'^2+\frac{36}{13}x'^2+\frac{108}{13}x'y'+\frac{81}{13}y'^2+4\sqrt{13}x'=0$$

$$13y'^2+4\sqrt{13}x'=0\quad\Rightarrow\quad y'^2=-\frac{4\sqrt{13}}{13}x'$$

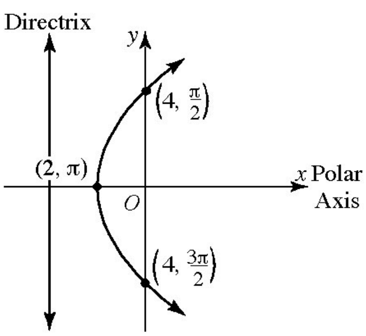
Parabola; vertex at the origin, focus at $(x',y')=\left(-\dfrac{\sqrt{13}}{13},0\right)$.



27. $r = \dfrac{4}{1-\cos\theta}$

$ep = 4,\ e = 1,\ p = 4$

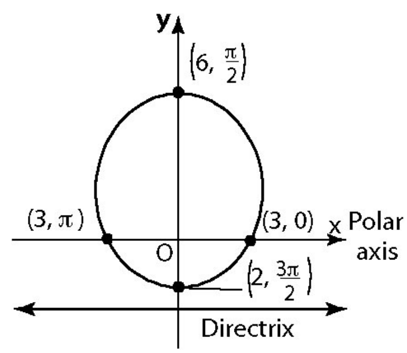
Parabola; directrix is perpendicular to the polar axis 4 units to the left of the pole; vertex is $(2, \pi)$.



28. $r = \dfrac{6}{2-\sin\theta} = \dfrac{3}{\left(1-\dfrac{1}{2}\sin\theta\right)}$

$ep = 3,\ e = \dfrac{1}{2},\ p = 6$

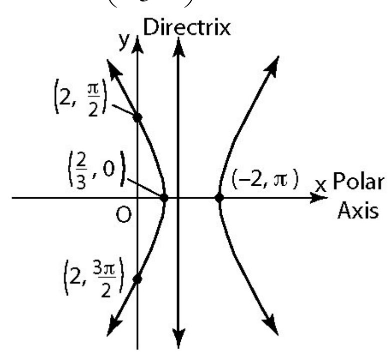
Ellipse; directrix is parallel to the polar axis 6 units below the pole; vertices are $\left(6, \dfrac{\pi}{2}\right)$ and $\left(2, \dfrac{3\pi}{2}\right)$. Center at $\left(2, \dfrac{\pi}{2}\right)$; other focus at $\left(4, \dfrac{\pi}{2}\right)$.



29. $r = \dfrac{8}{4+8\cos\theta} = \dfrac{2}{1+2\cos\theta}$

$ep = 2,\ e = 2,\ p = 1$

Hyperbola; directrix is perpendicular to the polar axis 1 unit to the right of the pole; vertices are $\left(\dfrac{2}{3}, 0\right)$ and $(-2, \pi)$. Center at $\left(\dfrac{4}{3}, 0\right)$; other focus at $\left(-\dfrac{8}{3}, \pi\right)$.



30. $r = \dfrac{4}{1-\cos\theta}$

$r - r\cos\theta = 4$

$r = 4 + r\cos\theta$

$r^2 = (4 + r\cos\theta)^2$

$x^2 + y^2 = (4+x)^2$

$x^2 + y^2 = 16 + 8x + x^2$

$y^2 - 8x - 16 = 0$

31. $r = \dfrac{8}{4 + 8\cos\theta}$

$4r + 8r\cos\theta = 8$

$4r = 8 - 8r\cos\theta$

$r = 2 - 2r\cos\theta$

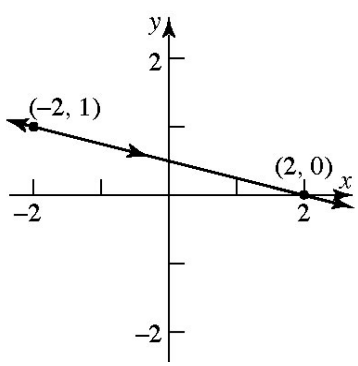
$r^2 = (2 - 2r\cos\theta)^2$

$x^2 + y^2 = (2-2x)^2$

$x^2 + y^2 = 4 - 8x + 4x^2$

$3x^2 - y^2 - 8x + 4 = 0$

32. $x = 4t - 2,\ \ y = 1 - t,\ \ -\infty < t < \infty$



$x = 4(1-y) - 2$

$x = 4 - 4y - 2$

$x + 4y = 2$

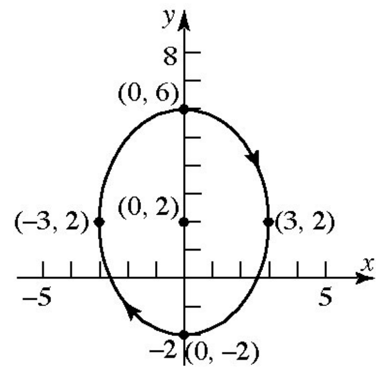
33. $x = 3\sin t,\ \ y = 4\cos t + 2,\ \ 0 \le t \le 2\pi$

$\dfrac{x}{3} = \sin t,\ \ \dfrac{y-2}{4} = \cos t$

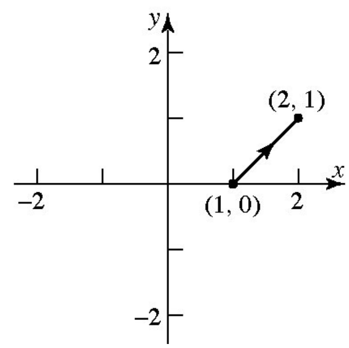
$\sin^2 t + \cos^2 t = 1$

$\left(\dfrac{x}{3}\right)^2 + \left(\dfrac{y-2}{4}\right)^2 = 1$

$\dfrac{x^2}{9} + \dfrac{(y-2)^2}{16} = 1$



34. $x = \sec^2 t,\ \ y = \tan^2 t,\ \ 0 \le t \le \dfrac{\pi}{4}$



$\tan^2 t + 1 = \sec^2 t \rightarrow y + 1 = x$

35. Answers will vary. One example:

$y = -2x + 4$

$x = t,\ \ y = -2t + 4$

$x = \dfrac{4-t}{2},\ \ y = t$

36. $\dfrac{x}{4} = \cos(\omega t);\ \ \dfrac{y}{3} = \sin(\omega t)$

$\dfrac{2\pi}{\omega} = 4 \Rightarrow 4\omega = 2\pi \Rightarrow \omega = \dfrac{2\pi}{4} = \dfrac{\pi}{2}$

$\therefore\ \ \dfrac{x}{4} = \cos\left(\dfrac{\pi}{2}t\right); \dfrac{y}{3} = \sin\left(\dfrac{\pi}{2}t\right)$

$\Rightarrow x = 4\cos\left(\dfrac{\pi}{2}t\right);\ \ y = 3\sin\left(\dfrac{\pi}{2}t\right),\ \ 0 \le t \le 4$

37. Write the equation in standard form:

$4x^2 + 9y^2 = 36 \ \rightarrow\ \dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$

The center of the ellipse is (0, 0). The major axis is the x-axis.

$a=3;\ b=2;$

$c^2=a^2-b^2=9-4=5\ \rightarrow\ c=\sqrt{5}.$

For the ellipse:
Vertices: (–3, 0), (3, 0);
Foci: $\left(-\sqrt{5},0\right),\left(\sqrt{5},0\right)$

For the hyperbola:
Foci: (–3, 0), (3, 0);
Vertices: $\left(-\sqrt{5},0\right),\left(\sqrt{5},0\right)$;
Center: (0, 0)

$a=\sqrt{5};\ c=3;$

$b^2=c^2-a^2=9-5=4\ \rightarrow\ b=2$

The equation of the hyperbola is: $\frac{x^2}{5}-\frac{y^2}{4}=1$

38. Let $(x,\ y)$ be any point in the collection of points.
The distance from
$(x,\ y)$ to $(3, 0)=\sqrt{(x-3)^2+y^2}$.
The distance from
$(x,\ y)$ to the line $x=\frac{16}{3}$ is $\left|x-\frac{16}{3}\right|$.
Relating the distances, we have:

$$\sqrt{(x-3)^2+y^2}=\frac{3}{4}\left|x-\frac{16}{3}\right|$$

$$(x-3)^2+y^2=\frac{9}{16}\left(x-\frac{16}{3}\right)^2$$

$$x^2-6x+9+y^2=\frac{9}{16}\left(x^2-\frac{32}{3}x+\frac{256}{9}\right)$$

$$16x^2-96x+144+16y^2=9x^2-96x+256$$

$$7x^2+16y^2=112$$

$$\frac{7x^2}{112}+\frac{16y^2}{112}=1$$

$$\frac{x^2}{16}+\frac{y^2}{7}=1$$

The set of points is an ellipse.

39. Locate the parabola so that the vertex is at (0, 0) and opens up. It then has the equation: $x^2=4ay$. Since the light source is located at the focus and is 1 foot from the base, $a=1$. Thus, $x^2=4y$. The width of the opening is 2, so the point (1, y) is located on the parabola. Solve for y:

$1^2=4y\rightarrow 1=4y\rightarrow y=0.25$ feet
The mirror is 0.25 feet, or 3 inches, deep.

40. Place the semi-elliptical arch so that the x-axis coincides with the water and the y-axis passes through the center of the arch. Since the bridge has a span of 60 feet, the length of the major axis is 60, or $2a=60$ or $a=30$. The maximum height of the bridge is 20 feet, so $b=20$. The equation is: $\frac{x^2}{900}+\frac{y^2}{400}=1$.

The height 5 feet from the center:

$$\frac{5^2}{900}+\frac{y^2}{400}=1$$

$$\frac{y^2}{400}=1-\frac{25}{900}$$

$$y^2=400\cdot\frac{875}{900}\rightarrow y\approx 19.72\text{ feet}$$

The height 10 feet from the center:

$$\frac{10^2}{900}+\frac{y^2}{400}=1$$

$$\frac{y^2}{400}=1-\frac{100}{900}$$

$$y^2=400\cdot\frac{800}{900}\rightarrow y\approx 18.86\text{ feet}$$

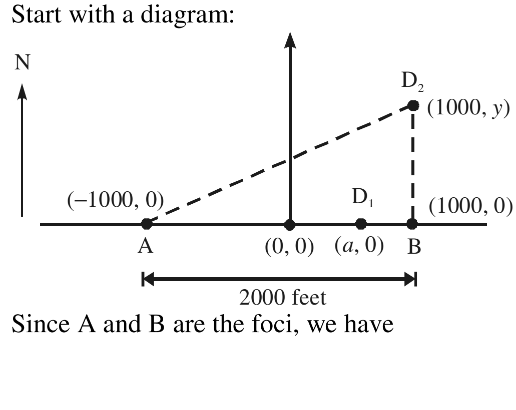
The height 20 feet from the center:

$$\frac{20^2}{900}+\frac{y^2}{400}=1$$

$$\frac{y^2}{400}=1-\frac{400}{900}$$

$$y^2=400\cdot\frac{500}{900}\rightarrow y\approx 14.91\text{ feet}$$

41. First note that all points where an explosion could take place, such that the time difference would be the same as that for the first detonation, would form a hyperbola with A and B as the foci.
Start with a diagram:



Since A and B are the foci, we have

$2c = 2000$

$c = 1000$

Since D_1 is on the transverse axis and is on the hyperbola, then it must be a vertex of the hyperbola. Since it is 200 feet from B, we have $a = 800$. Finally,

$b^2 = c^2 - a^2 = 1000^2 - 800^2 = 360{,}000$

Thus, the equation of the hyperbola is

$$\frac{x^2}{640{,}000} - \frac{y^2}{360{,}000} = 1$$

The point $(1000, y)$ needs to lie on the graph of the hyperbola. Thus, we have

$$\frac{(1000)^2}{640{,}000} - \frac{y^2}{360{,}000} = 1$$

$$-\frac{y^2}{360{,}000} = -\frac{9}{16}$$

$$y^2 = 202{,}500$$

$$y = 450$$

The second explosion should be set off 450 feet due north of point B.

42. **a.** Train:

$x_1 = \frac{3}{2}t^2;$

Let $y_1 = 1$ for plotting convenience.

Mary:

$x_2 = 6(t-2);$

Let $y_2 = 3$ for plotting convenience.

b. Mary will catch the train if $x_1 = x_2$.

$$\frac{3}{2}t^2 = 6(t-2)$$

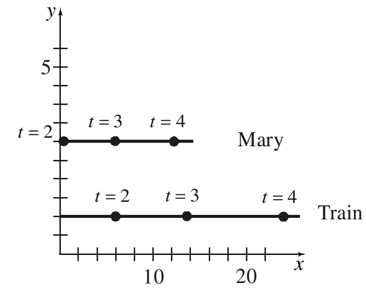
$$\frac{3}{2}t^2 = 6t - 12$$

$$t^2 - 4t + 8 = 0$$

Since $b^2 - 4ac = (-4)^2 - 4(1)(8) = 16 - 32 = -16 < 0$

the equation has no real solution. Thus, Mary will not catch the train.

c.



43. **a.** Use equations (1) in section 9.7.

$x = (80\cos(35°))t$

$y = -\frac{1}{2}(32)t^2 + (80\sin(35°))t + 6$

b. The ball is in the air until $y = 0$. Solve:

$-16t^2 + (80\sin(35°))t + 6 = 0$

$$t = \frac{-80\sin(35°) \pm \sqrt{(80\sin(35°))^2 - 4(-16)(6)}}{2(-16)}$$

$$\approx \frac{-45.89 \pm \sqrt{2489.54}}{-32}$$

$$\approx -0.13 \text{ or } 2.99$$

The ball is in the air for about 2.99 seconds. (The negative solution is extraneous.)

c. The maximum height occurs at the vertex of the quadratic function.

$$t = \frac{-b}{2a} = \frac{-80\sin(35°)}{2(-16)} \approx 1.43 \text{ seconds}$$

Evaluate the function to find the maximum height:

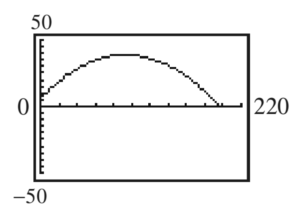
$-16(1.43)^2 + (80\sin(35°))(1.43) + 6 \approx 38.9$ ft

d. Find the horizontal displacement:

$x = (80\cos(35°))(2.99) \approx 196$ feet

(or roughly 65.3 yards)

e.



44. Answers will vary.

Chapter 10 Test

1. $\frac{(x+1)^2}{4}-\frac{y^2}{9}=1$

Rewriting the equation as $\frac{(x-(-1))^2}{2^2}-\frac{(y-0)^2}{3^2}=1$, we see that this is the equation of a hyperbola in the form $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$. Therefore, we have $h=-1$, $k=0$, $a=2$, and $b=3$. Since $a^2=4$ and $b^2=9$, we get $c^2=a^2+b^2=4+9=13$, or $c=\sqrt{13}$. The center is at $(-1,0)$ and the transverse axis is the x-axis. The vertices are at $(h\pm a,k)=(-1\pm 2,0)$, or $(-3,0)$ and $(1,0)$. The foci are at $(h\pm c,k)=\left(-1\pm\sqrt{13},0\right)$, or $\left(-1-\sqrt{13},0\right)$ and $\left(-1+\sqrt{13},0\right)$. The asymptotes are $y-0=\pm\frac{3}{2}(x-(-1))$, or $y=-\frac{3}{2}(x+1)$ and $y=\frac{3}{2}(x+1)$.

2. $8y=(x-1)^2-4$

Rewriting gives

$(x-1)^2=8y+4$

$(x-1)^2=8\left(y-\left(-\frac{1}{2}\right)\right)$

$(x-1)^2=4(2)\left(y-\left(-\frac{1}{2}\right)\right)$

This is the equation of a parabola in the form $(x-h)^2=4a(y-k)$. Therefore, the axis of symmetry is parallel to the y-axis and we have $(h,k)=\left(1,-\frac{1}{2}\right)$ and $a=2$. The vertex is at $(h,k)=\left(1,-\frac{1}{2}\right)$, the axis of symmetry is $x=1$, the focus is at $(h,k+a)=\left(1,-\frac{1}{2}+2\right)=\left(1,\frac{3}{2}\right)$, and the directrix is given by the line $y=k-a$, or $y=-\frac{5}{2}$.

3. $2x^2+3y^2+4x-6y=13$

Rewrite the equation by completing the square in *x* and *y*.

$$2x^2+3y^2+4x-6y=13$$
$$2x^2+4x+3y^2-6y=13$$
$$2\left(x^2+2x\right)+3\left(y^2-2y\right)=13$$
$$2\left(x^2+2x+1\right)+3\left(y^2-2y+1\right)=13+2+3$$
$$2(x+1)^2+3(y-1)^2=18$$
$$\frac{(x-(-1))^2}{9}+\frac{(y-1)^2}{6}=1$$

This is the equation of an ellipse with center at $(-1,1)$ and major axis parallel to the x-axis.

Since $a^2=9$ and $b^2=6$, we have $a=3$, $b=\sqrt{6}$, and $c^2=a^2-b^2=9-6=3$, or $c=\sqrt{3}$. The foci are $(h\pm c,k)=\left(-1\pm\sqrt{3},1\right)$ or $\left(-1-\sqrt{3},1\right)$ and $\left(-1+\sqrt{3},1\right)$. The vertices are at $(h\pm a,k)=(-1\pm 3,1)$, or $(-4,1)$ and $(2,1)$.

4. The vertex $(-1,3)$ and the focus $(-1,4.5)$ both lie on the vertical line $x=-1$ (the axis of symmetry). The distance *a* from the vertex to the focus is $a=1.5$. Because the focus lies above the vertex, we know the parabola opens upward. As a result, the form of the equation is

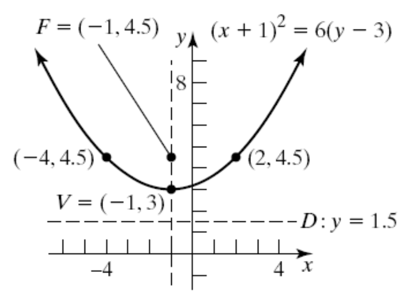
$(x-h)^2=4a(y-k)$

where $(h,k)=(-1,3)$ and $a=1.5$. Therefore, the equation is

$(x+1)^2=4(1.5)(y-3)$

$(x+1)^2=6(y-3)$

The points $(h\pm 2a,k)$, that is $(-4,4.5)$ and $(2,4.5)$, define the lattice rectum; the line $y=1.5$ is the directrix.



5. The center is $(h,k)=(0,0)$ so $h=0$ and $k=0$. Since the center, focus, and vertex all lie on the line $x=0$, the major axis is the y-axis. The distance from the center $(0,0)$ to a focus $(0,3)$ is $c=3$. The distance from the center $(0,0)$ to a vertex $(0,-4)$ is $a=4$. Then,

$$b^2=a^2-c^2=4^2-3^2=16-9=7$$

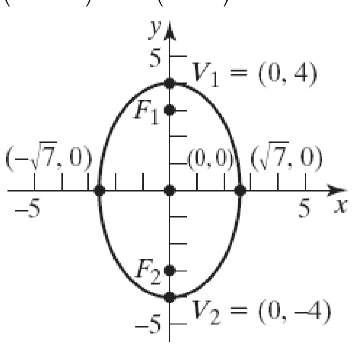
The form of the equation is

$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$

where $h=0$, $k=0$, $a=4$, and $b=\sqrt{7}$. Thus, we get

$$\frac{x^2}{7}+\frac{y^2}{16}=1$$

To graph the equation, we use the center $(h,k)=(0,0)$ to locate the vertices. The major axis is the y-axis, so the vertices are $a=4$ units above and below the center. Therefore, the vertices are $V_1=(0,4)$ and $V_2=(0,-4)$. Since $c=3$ and the major axis is the y-axis, the foci are 3 units above and below the center. Therefore, the foci are $F_1=(0,3)$ and $F_2=(0,-3)$. Finally, we use the value $b=\sqrt{7}$ to find the two points left and right of the center: $(-\sqrt{7},0)$ and $(\sqrt{7},0)$.



6. The center $(h,k)=(2,2)$ and vertex $(2,4)$ both lie on the line $x=2$, the transverse axis is parallel to the y-axis. The distance from the center $(2,2)$ to the vertex $(2,4)$ is $a=2$, so the other vertex must be $(2,0)$. The form of the equation is

$$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$$

where $h=2$, $k=2$, and $a=2$. This gives us

$$\frac{(y-2)^2}{4}-\frac{(x-2)^2}{b^2}=1$$

Since the graph contains the point $(x,y)=(2+\sqrt{10},5)$, we can use this point to determine the value for *b*.

$$\frac{(5-2)^2}{4}-\frac{\left(2+\sqrt{10}-2\right)^2}{b^2}=1$$

$$\frac{9}{4}-\frac{10}{b^2}=1$$

$$\frac{5}{4}=\frac{10}{b^2}$$

$$b^2=8$$

$$b=2\sqrt{2}$$

Therefore, the equation becomes

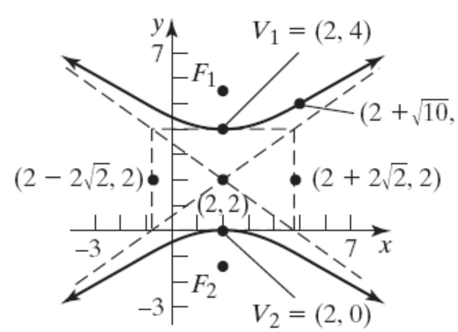
$$\frac{(y-2)^2}{4}-\frac{(x-2)^2}{8}=1$$

Since $c^2=a^2+b^2=4+8=12$, the distance from the center to either focus is $c=2\sqrt{3}$. Therefore, the foci are $c=2\sqrt{3}$ units above and below the center. The foci are $F_1=\left(2,2+2\sqrt{3}\right)$ and $F_2=\left(2,2-2\sqrt{3}\right)$. The asymptotes are given by the lines

$y-k=\pm\frac{a}{b}(x-h)$. Therefore, the asymptotes are

$$y-2=\pm\frac{2}{2\sqrt{2}}(x-2)$$

$$y=\pm\frac{\sqrt{2}}{2}(x-2)+2$$



7. $2x^2+5xy+3y^2+3x-7=0$ is in the form $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$ where $A=2$, $B=5$, and $C=3$.

$$B^2-4AC=5^2-4(2)(3)$$
$$=25-24$$
$$=1$$

Since $B^2-4AC>0$, the equation defines a hyperbola.

8. $3x^2-xy+2y^2+3y+1=0$ is in the form $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$ where $A=3$, $B=-1$, and $C=2$.

$$B^2-4AC=(-1)^2-4(3)(2)$$
$$=1-24$$
$$=-23$$

Since $B^2-4AC<0$, the equation defines an ellipse.

9. $x^2-6xy+9y^2+2x-3y-2=0$ is in the form $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$ where $A=1$, $B=-6$, and $C=9$.

$$B^2-4AC=(-6)^2-4(1)(9)$$
$$=36-36$$
$$=0$$

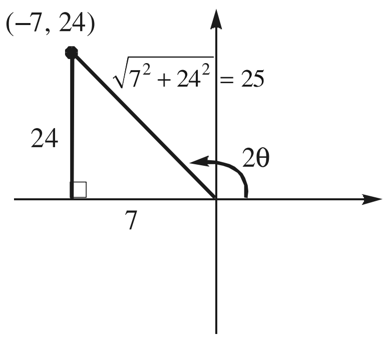
Since $B^2-4AC=0$, the equation defines a parabola.

10. $41x^2-24xy+34y^2-25=0$

Substituting $x=x'\cos\theta-y'\sin\theta$ and $y=x'\sin\theta+y'\cos\theta$ transforms the equation into one that represents a rotation through an angle θ. To eliminate the $x'y'$ term in the new equation, we need $\cot(2\theta)=\frac{A-C}{B}$. That is, we need to solve

$$\cot(2\theta)=\frac{41-34}{-24}$$
$$\cot(2\theta)=-\frac{7}{24}$$

Since $\cot(2\theta)<0$, we may choose $90°<2\theta<180°$, or $45°<\theta<90°$.



We have $\cot(2\theta)=-\frac{7}{24}$ so it follows that $\cos(2\theta)=-\frac{7}{25}$.

$$\sin\theta=\sqrt{\frac{1-\cos(2\theta)}{2}}=\sqrt{\frac{1-\left(-\frac{7}{25}\right)}{2}}=\sqrt{\frac{16}{25}}=\frac{4}{5}$$

$$\cos\theta=\sqrt{\frac{1+\cos(2\theta)}{2}}=\sqrt{\frac{1+\left(-\frac{7}{25}\right)}{2}}=\sqrt{\frac{9}{25}}=\frac{3}{5}$$

$$\theta=\cos^{-1}\left(\frac{3}{5}\right)\approx 53.13°$$

With these values, the rotation formulas are

$$x=\frac{3}{5}x'-\frac{4}{5}y' \quad \text{and} \quad y=\frac{4}{5}x'+\frac{3}{5}y'$$

Substituting these values into the original equation and simplifying, we obtain

$$41x^2-24xy+34y^2-25=0$$

$$41\left(\frac{3}{5}x'-\frac{4}{5}y'\right)^2-24\left(\frac{3}{5}x'-\frac{4}{5}y'\right)\left(\frac{4}{5}x'+\frac{3}{5}y'\right)$$

$$+34\left(\frac{4}{5}x'+\frac{3}{5}y'\right)^2=25$$

Multiply both sides by 25 and expand to obtain

$$41\left(9x'^2-24x'y'+16y'^2\right)-24\left(12x'^2-7x'y'-12y'^2\right)$$

$$+34\left(16x'^2+24x'y'+9y'^2\right)=625$$

$$625x'^2+1250y'^2=625$$

$$x'^2+2y'^2=1$$

$$\frac{x'^2}{1}+\frac{y'^2}{\frac{1}{2}}=1$$

Thus: $a=\sqrt{1}=1$ and $b=\sqrt{\frac{1}{2}}=\frac{\sqrt{2}}{2}$

This is the equation of an ellipse with center at $(0,0)$ in the $x'y'$–plane. The vertices are at

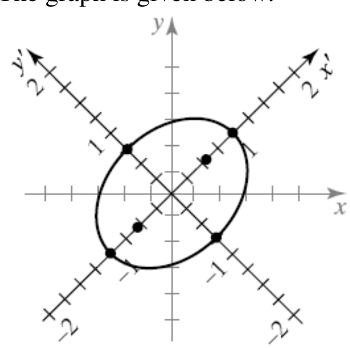
$(-1,0)$ and $(1,0)$ in the $x'y'-$plane.

$c^2 = a^2 - b^2 = 1 - \frac{1}{2} = \frac{1}{2} \quad \rightarrow \quad c = \frac{\sqrt{2}}{2}$

The foci are located at $\left(\pm\frac{\sqrt{2}}{2}, 0\right)$ in the $x'y'-$plane. In summary:

	$x'y'-plane$	$xy-plane$
center	$(0,0)$	$(0,0)$
vertices	$(\pm 1, 0)$	$\left(\frac{3}{5},\frac{4}{5}\right), \left(-\frac{3}{5},-\frac{4}{5}\right)$
minor axis intercepts	$\left(0, \pm\frac{\sqrt{2}}{2}\right)$	$\left(\frac{2\sqrt{2}}{5},\frac{3\sqrt{2}}{10}\right)$, $\left(-\frac{2\sqrt{2}}{5},-\frac{3\sqrt{2}}{10}\right)$
foci	$\left(\pm\frac{\sqrt{2}}{2}, 0\right)$	$\left(\frac{3\sqrt{2}}{10},\frac{2\sqrt{2}}{5}\right)$, $\left(-\frac{3\sqrt{2}}{10},-\frac{2\sqrt{2}}{5}\right)$

The graph is given below.



11. $r = \frac{ep}{1 - e\cos\theta} = \frac{3}{1 - 2\cos\theta}$

Therefore, $e = 2$ and $p = \frac{3}{2}$. Since $e > 1$, this is the equation of a hyperbola.

$$r = \frac{3}{1 - 2\cos\theta}$$

$r(1 - 2\cos\theta) = 3$

$r - 2r\cos\theta = 3$

Since $x = r\cos\theta$ and $x^2 + y^2 = r^2$, we get

$r - 2r\cos\theta = 3$

$r = 2r\cos\theta + 3$

$r^2 = (2r\cos\theta + 3)^2$

$x^2 + y^2 = (2x + 3)^2$

$x^2 + y^2 = 4x^2 + 12x + 9$

$3x^2 + 12x - y^2 = -9$

$3(x^2 + 4x) - y^2 = -9$

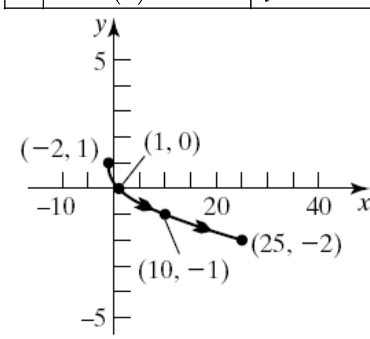
$3(x^2 + 4x + 4) - y^2 = -9 + 12$

$3(x + 2)^2 - y^2 = 3$

$\frac{(x+2)^2}{1} - \frac{y^2}{3} = 1$

12. $\begin{cases} x = 3t - 2 & (1) \\ y = 1 - \sqrt{t} & (2) \end{cases}$

t	x	y	(x, y)
0	$x = 3(0) - 2 = -2$	$y = 1 - \sqrt{0} = 1$	$(-2,1)$
1	$x = 3(1) - 2 = 1$	$y = 1 - \sqrt{1} = 0$	$(1,0)$
4	$x = 3(4) - 2 = 10$	$y = 1 - \sqrt{4} = -1$	$(10,-1)$
9	$x = 3(9) - 2 = 25$	$y = 1 - \sqrt{9} = -2$	$(25,-2)$



To find the rectangular equation for the curve, we need to eliminate the variable *t* from the equations.

We can start by solving equation (1) for *t*.

$x = 3t - 2$

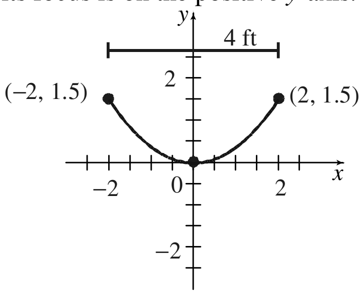
$3t = x + 2$

$t = \frac{x+2}{3}$

Substituting this result for *t* into equation (2) gives

$y = 1 - \sqrt{\frac{x+2}{3}}, \quad -2 \le x \le 25$

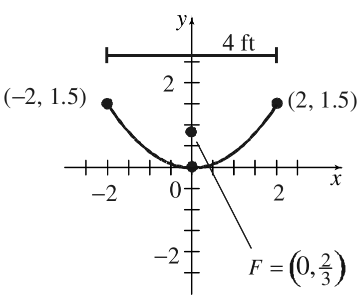
13. We can draw the parabola used to form the reflector on a rectangular coordinate system so that the vertex of the parabola is at the origin and its focus is on the positive y-axis.



The form of the equation of the parabola is $x^2 = 4ay$ and its focus is at $(0, a)$. Since the point $(2, 1.5)$ is on the graph, we have

$$2^2 = 4a(1.5)$$
$$4 = 6a$$
$$a = \frac{2}{3}$$

The microphone should be located $\frac{2}{3}$ feet (or 8 inches) from the base of the reflector, along its axis of symmetry.



Chapter 10 Cumulative Review

1. $\dfrac{f(x+h) - f(x)}{h}$

$$= \frac{-3(x+h)^2 + 5(x+h) - 2 - \left(-3x^2 + 5x - 2\right)}{h}$$

$$= \frac{-3\left(x^2 + 2xh + h^2\right) + 5x + 5h - 2 + 3x^2 - 5x + 2}{h}$$

$$= \frac{-3x^2 - 6xh - 3h^2 + 5x + 5h - 2 + 3x^2 - 5x + 2}{h}$$

$$= \frac{-6xh - 3h^2 + 5h}{h} = -6x - 3h + 5$$

2. $9x^4 + 33x^3 - 71x^2 - 57x - 10 = 0$

There are at most 4 real zeros.

Possible rational zeros:

$p = \pm 1, \pm 2, \pm 5, \pm 10;\quad q = \pm 1, \pm 3, \pm 9;$

$$\frac{p}{q} = \pm 1, \pm\frac{1}{3}, \pm\frac{1}{9}, \pm 2, \pm\frac{2}{3}, \pm\frac{2}{9}, \pm 5,$$
$$\pm\frac{5}{3}, \pm\frac{5}{9}, \pm 10, \pm\frac{10}{3}, \pm\frac{10}{9}$$

Graphing $y_1 = 9x^4 + 33x^3 - 71x^2 - 57x - 10$ indicates that there appear to be zeros at $x = -5$ and at $x = 2$.

Using synthetic division with $x = -5$:

–5)	9	33	–71	–57	–10
		–45	60	55	10
	9	–12	–11	–2	0

Since the remainder is 0, –5 is a zero for f. So $x - (-5) = x + 5$ is a factor.

The other factor is the quotient: $9x^3 - 12x^2 - 11x - 2$.

Thus, $f(x) = (x+5)\left(9x^3 - 12x^2 - 11x - 2\right)$.

Using synthetic division on the quotient and $x = 2$:

2)	9	–12	–11	–2
		18	12	2
	9	6	1	0

Since the remainder is 0, 2 is a zero for f. So $x - 2$ is a factor; thus,

$$f(x) = (x+5)(x-2)\left(9x^2 + 6x + 1\right)$$
$$= (x+5)(x-2)(3x+1)(3x+1)$$

Therefore, $x=-\frac{1}{3}$ is also a zero for *f* (with multiplicity 2). Solution set: $\left\{-5,-\frac{1}{3},2\right\}$.

3. $6-x\ge x^2$

$0\ge x^2+x-6$

$x^2+x-6\le 0$

$(x+3)(x-2)\le 0$

$f(x)=x^2+x-6$

$x=-3,\ x=2$ are the zeros of f.

Interval	$(-\infty,-3)$	$(-3,2)$	$(2,\infty)$
Test Value	–4	0	3
Value of *f*	6	–6	6
Conclusion	Positive	Negative	Positive

The solution set is $\{x\,|-3\le x\le 2\}$, or $[-3,2]$.

4. $f(x)=3^x+2$

a. Domain: $(-\infty,\infty)$; Range: $(2,\infty)$.

b. $f(x)=3^x+2$

$y=3^x+2$

$x=3^y+2$ Inverse

$x-2=3^y$

$\log_3(x-2)=y\Rightarrow f^{-1}(x)=\log_3(x-2)$

Domain of f^{-1} = range of f $=(2,\infty)$.

Range of f^{-1} = domain of f $=(-\infty,\infty)$.

5. $f(x)=\log_4(x-2)$

a. $f(x)=\log_4(x-2)=2$

$x-2=4^2$

$x-2=16$

$x=18$

The solution set is $\{18\}$.

b. $f(x)=\log_4(x-2)\le 2$

$x-2\le 4^2$ and $x-2>0$

$x-2\le 16$ and $x>2$

$x\le 18$ and $x>2$

$2<x\le 18$

$(2,18]$

6. a. This graph is a line containing points $(0,-2)$ and $(1,0)$.

$\text{slope}=\frac{\Delta y}{\Delta x}=\frac{0-(-2)}{1-0}=\frac{2}{1}=2$

using $y-y_1=m(x-x_1)$

$y-0=2(x-1)$

$y=2x-2$ or $2x-y-2=0$

b. This graph is a circle with center point (2, 0) and radius 2.

$(x-h)^2+(y-k)^2=r^2$

$(x-2)^2+(y-0)^2=2^2$

$(x-2)^2+y^2=4$

c. This graph is an ellipse with center point (0, 0); vertices $(\pm 3,0)$ and *y*-intercepts $(0,\pm 2)$.

$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$

$\frac{(x-0)^2}{3^2}+\frac{(y-0)^2}{2^2}=1 \Rightarrow \frac{x^2}{9}+\frac{y^2}{4}=1$

d. This graph is a parabola with vertex $(1,0)$ and *y*-intercept $(0,2)$.

$(x-h)^2=4a(y-k)$

$(x-1)^2=4ay$

$(0-1)^2=4a(2)$

$1=8a$

$a=\frac{1}{8}$

$(x-1)^2=\frac{1}{2}y$ or $y=2(x-1)^2$

e. This graph is a hyperbola with center $(0,0)$ and vertices $(0,\pm 1)$, containing the point $(3,2)$.

$$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$$
$$\frac{y^2}{1}-\frac{x^2}{b^2}=1$$
$$\frac{(2)^2}{1}-\frac{(3)^2}{b^2}=1$$
$$\frac{4}{1}-\frac{9}{b^2}=1$$
$$4-\frac{9}{b^2}=1$$
$$3=\frac{9}{b^2}$$
$$3b^2=9$$
$$b^2=3$$

The equation of the hyperbola is:

$$\frac{y^2}{1}-\frac{x^2}{3}=1$$

f. This is the graph of an exponential function with y-intercept $(0,1)$, containing the point $(1,4)$.

$y=A\cdot b^x$

y-intercept $(0,1)\Rightarrow 1=A\cdot b^0=A\cdot 1\Rightarrow A=1$

point $(1,4)\Rightarrow 4=b^1=b$

Therefore, $y=4^x$.

7. $\sin(2\theta)=0.5$

$$2\theta=\frac{\pi}{6}+2k\pi \quad\Rightarrow\quad \theta=\frac{\pi}{12}+k\pi$$
$$\text{or}\quad 2\theta=\frac{5\pi}{6}+2k\pi \quad\Rightarrow\quad \theta=\frac{5\pi}{12}+k\pi,$$

where k is any integer.

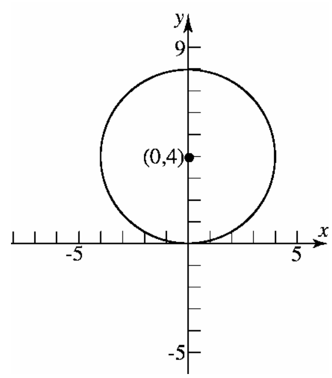
8. The line containing point (0,0), making an angle of 30° with the positive x-axis has polar equation: $\theta=\dfrac{\pi}{6}$.

9. Using rectangular coordinates, the circle with center point (0, 4) and radius 4 has the equation:

$$(x-h)^2+(y-k)^2=r^2$$
$$(x-0)^2+(y-4)^2=4^2$$
$$x^2+y^2-8y+16=16$$
$$x^2+y^2-8y=0$$

Converting to polar coordinates:

$$r^2-8r\sin\theta=0$$
$$r^2=8r\sin\theta$$
$$r=8\sin\theta$$



10. $f(x)=\dfrac{3}{\sin x+\cos x}$

f will be defined provided $\sin x+\cos x\neq 0$.

$$\sin x+\cos x=0$$
$$\sin x=-\cos x$$
$$\frac{\sin x}{\cos x}=-1$$
$$\tan x=-1$$
$$x=\frac{3\pi}{4}+k\pi,\ k \text{ is any integer}$$

The domain is

$$\left\{x \,\middle|\, x\neq\frac{3\pi}{4}+k\pi,\ \text{where } k \text{ is any integer}\right\}.$$

11. $\cot(2\theta)=1$, where $0^\circ<\theta<90^\circ$

$2\theta=\dfrac{\pi}{4}+k\pi\Rightarrow\theta=\dfrac{\pi}{8}+\dfrac{k\pi}{2}$, where k is any integer. On the interval $0^\circ<\theta<90^\circ$, the solution is $\theta=\frac{\pi}{8}=22.5^\circ$.

12. $x=5\tan t \ \rightarrow\ \frac{x}{5}=\tan t$

$$\sec^2 t=1+\tan^2 t$$
$$y=5\sec^2 t=5\left(1+\tan^2 t\right)$$
$$=5\left(1+\left(\frac{x}{5}\right)^2\right)=5+\frac{x^2}{5}$$

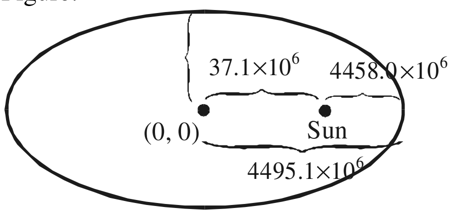
The rectangular equation is $y=\dfrac{x^2}{5}+5$.

Chapter 10 Projects

Project I – Internet Based Project

Project II

1. Figure:



$c = 37.2\times10^6$

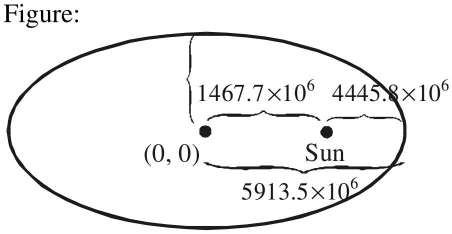
$b^2 = a^2 - c^2$

$b^2 = \left(4495.1\times10^6\right)^2 - \left(37.1\times10^6\right)^2$

$b = 4494.9\times10^6$

$$\frac{x^2}{(4495.1\,x10^6)^2} + \frac{y^2}{(4494.9\,x10^6)^2} = 1$$

2. Figure:



$77381.2\times10^6 + 4445.8\times10^6 = 11827\times10^6$

$a = 0.5\left(11827\times10^6\right) = 5913.5\times10^6$

$c = 1467.7\times10^6$

$b^2 = \left(5913.5\times10^6\right)^2 - \left(1467.7\times10^6\right)^2$

$b = 5728.5\times10^6$

$$\frac{x^2}{(5913.5\,x10^6)^2} + \frac{y^2}{(5728.5\,x10^6)^2} = 1$$

3. The two graphs are being graphed with the same center. Actually, the sun should remain in the same place for each graph. This means that the graph of Pluto needs to be adjusted.

4. Shift = Pluto's distance – Neptune's distance

$= 1467.7\times10^6 - 37.1\times10^6$

$= 1430.6\times10^6$

$$\frac{(x+1430.6\,x10^6)^2}{(5913.5\,x10^6)^2} + \frac{y^2}{(5728.5\,x10^6)^2} = 1$$

5. Yes. One must adjust the scale accordingly to see it.

6. $\left(4431.6\times10^6, 752.6\times10^6\right)$,

$\left(4431.6\times10^6, 752.6\times10^6\right)$

7. No, The timing is different. They do not both pass through those points at the same time.

Project III

1. As an example, T_1 will be used. (Note that any of the targets will yield the same result.)

$z = 4ax^2 + 4ay^2$

$0.5 = 4a(0)^2 + 4a(-2)^2$

$0.5 = 16a$

$a = \frac{1}{32}$

The focal length is 0.03125 m.

$z = \frac{1}{8}x^2 + \frac{1}{8}y^2$

2.

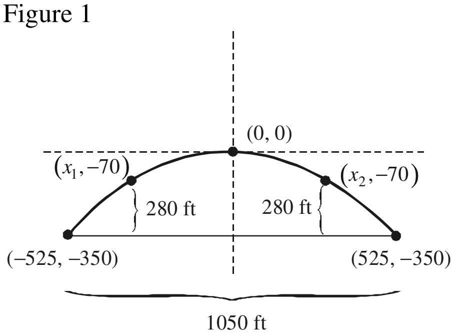
Target	x	R	θ	Z	y
T_1	0	9.551	–11.78	0.5	–1.950
T_2	0	9.948	–5.65	0.125	–0.979
T_3	0	9.928	5.90	0.125	1.021
T_4	0	9.708	11.89	0.5	2.000
T_5	9.510	9.722	–11.99	11.31	0
T_6	9.865	9.917	–5.85	12.165	0
T_7	9.875	9.925	5.78	12.189	0
T_8	9.350	9.551	11.78	10.928	0

$z = \frac{1}{8}x^2 + \frac{1}{8}y^2$, y = Rsinθ, x = Rcosθ

4. T_1 through T_4 do not need to be adjusted. T_5 must move 11.510 m toward the y-axis and the z coordinate must move down 10.81 m. T_6 must move 10.865 m toward the y-axis and the z coordinate must move down 12.04 m. T_7 must move 8.875 toward the y-axis and z must move down 12.064m. T_8 must move 7.35 m toward the y-axis and z must move down 10.425 m.

Project IV

Figure 1



1. $x^2 = -4ay$

$(525)^2 = -4a(-350)$

$272625 = 1400a$

$a = 196.875$

$x^2 = -787.5y$

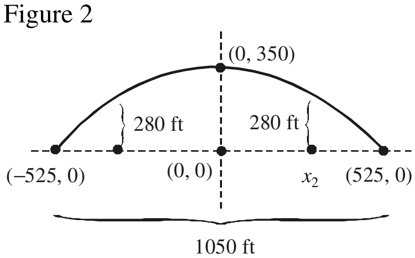
2. Let $y = -70$. (The arch needs to be 280 ft high. Remember the vertex is at (0, 0), so we must measure down to the arch from the x-axis at the point where the arch's height is 280 ft.)

$x^2 = -787.5(-70)$

$x^2 = 55125 \quad \to \quad x = \pm 234.8$

The channel will be 469.6 ft wide.

3. Figure 2



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{275625} + \frac{y^2}{122500} = 1$$

4. $\dfrac{x^2}{275625} + \dfrac{(280)^2}{122500} = 1$

$x^2 = \left(1 - \dfrac{(280)^2}{122500}\right)275625$

$x^2 = 99225$

$x = \pm 315$

The channel will be 630 feet wide.

5. If the river rises 10 feet, then we need to look for how wide the channel is when the height is 290 ft.

For the parabolic shape:

$x^2 = -787.5(-60)$

$x^2 = 47250$

$x = \pm 217.4$

There is still a 435 ft wide channel for the ship.

For the semi-ellipse:

$\dfrac{x^2}{275625} + \dfrac{(290)^2}{122500} = 1$

$x^2 = \left(1 - \dfrac{(290)^2}{122500}\right)275625$

$x^2 = 86400$

$x = \pm 293.9$

The ship has a 588-ft channel. A semi-ellipse would be more practical since the channel doesn't shrink in width in a flood as fast as a parabola.

Project V

1. $4t - 2 = \sec^2 t, \qquad 0 \le t \le \dfrac{\pi}{4}$

$1 - t = \tan^2 t, \qquad 0 \le t \le \dfrac{\pi}{4}$

For the x-values, t = 1.99, which is not in the domain [0, π/4]. Therefore there is no t-value that allows the two x values to be equal.

2. On the graphing utility, solve these in parametric form, using a t-step of π/32.
It appears that the two graphs intersect at about (1.14, 0.214). However, for the first pair, t = 0.785 at that point. That t-value gives the point (2, 1) on the second pair. There is no intersection point.

3. Since there were no solutions found for each method, the "solutions" matched.

4. $x_1 = 4t - 2$ $\quad y_1 = 1 - t$ D=**R**; R = **R**

$1 - y_1 = t$

$x = 4(1 - y) - 2$

$x + 4y = 2$

$x_2 = \sec^2 t \quad y_2 = \tan^2 t$

$1 + \tan^2 t = \sec^2 t$ $\quad$ D=[1,2], R=[0,1]

$1 + y = x$

$x - y = 1$

$\begin{cases} x + 4y = 1 \\ x - y = 1 \end{cases}$

$5y = 0$

$y = 0$

$x = 1$

The t-values that go with those x, y values are not the same for both pairs. Thus, again, there is no solution.

5. $x: \quad t^{3/2} = \ln t$

$y: \quad t^3 = 2t + 4$

Graphing each of these and finding the intersection: There is no intersection for the x-values, so there is no intersection for the system.

Graphing the two parametric pairs: The parametric equations show an intersection point. However, the t-value that gives that point for each parametric pair is not the same. Thus there is no solution for the system.

Putting each parametric pair into rectangular coordinates:

$x_1 = \ln t \to t = e^x \quad$ D = **R**, $R = (0, \infty)$

$y_1 = t^3 = e^{3x}$

$x_2 = t^{3/2} \to t = x^{2/3} \quad D = [0, \infty), \quad R = [4, \infty)$

$y_2 = 2t + 4 = 2x^{2/3} + 4$

Then solving that system:

$\begin{cases} y = e^{3x} \\ y = 2x^{2/3} + 4 \end{cases}$

This system has an intersection point at (0.56, 5.39).

However, ln t = 0.56, gives t = 1.75 and $t = (0.56)^{2/3} \approx 0.68$. Since the t-values are not the same, the point of intersection is false for the system.

6. x: 3 sin t = 2 cos t ➔ tan t = 2/3 ➔ t = 0.588
y: 4 cos t +2 = 4 sin t ➔ by graphing, the solution is t = 1.15 or t = 3.57.
Neither of these are the same as for the x-values, thus the system has no solution.
Graphing parametrically: If the graphs are done simultaneously on the graphing utility, the two graphs do not intersect at the same t-value. Tracing the graphs shows the same thing. This backs up the conclusion reached the first way.

$x_1 = 3\sin t \quad y_1 = 4\cos t + 2$

$\sin t = \dfrac{x}{3} \quad \cos t = \dfrac{y-2}{4}$

$\sin^2 t + \cos^2 t = 1$

$\dfrac{x^2}{9} + \dfrac{(y-2)^2}{16} = 1$

D=[-3, 3], R=[-2, 6]

$x_2 = 2\cos t \quad y_2 = 4\sin t$

$\cos t = \dfrac{x}{2} \quad \sin t = \dfrac{y}{4}$

$\cos^2 t + \sin^2 t = 1$

$\dfrac{x^2}{4} + \dfrac{y^2}{16} = 1$

D=[-2, 2], R=[-4,4]

Solving the system graphically: $x = 1.3$, $y = -3.05$. However, the t-values associated with these values are not the same. Thus there is no solution. (Similarly with the symmetric pair in the third quadrant.)

7. Efficiency depends upon the equations. Graphing the parametric pairs allows one to see immediately whether the t-values will be the same for each pair at any point of intersection. Sometimes, solving for t, as was done in the first method is easy and can be quicker. It leads straight to the t-values, so that allow the method to be efficient.
If the two graphs intersect, one must be careful to check that the t-values are the same for each curve at that point of intersection.