Chapter 8

Applications of Trigonometric Functions

Section 8.1

1. $a=\sqrt{5^2-3^2}=\sqrt{25-9}=\sqrt{16}=4$

2. $\tan\theta=\frac{1}{2},\quad 0<\theta<90°$

$\theta=\tan^{-1}\left(\frac{1}{2}\right)\approx 26.6°$

3. $\sin\theta=\frac{1}{2},\quad 0<\theta<90°$

$\theta=\sin^{-1}\frac{1}{2}=30°$

4. False; $\sin 52°=\cos 38°$

5. True

6. angle of elevation

7. True

8. False

9. opposite = 5; adjacent = 12; hypotenuse = ?
$(\text{hypotenuse})^2=5^2+12^2=169$

hypotenuse $=\sqrt{169}=13$

$\sin\theta=\frac{\text{opp}}{\text{hyp}}=\frac{5}{13}\qquad \csc\theta=\frac{\text{hyp}}{\text{opp}}=\frac{13}{5}$

$\cos\theta=\frac{\text{adj}}{\text{hyp}}=\frac{12}{13}\qquad \sec\theta=\frac{\text{hyp}}{\text{adj}}=\frac{13}{12}$

$\tan\theta=\frac{\text{opp}}{\text{adj}}=\frac{5}{12}\qquad \cot\theta=\frac{\text{adj}}{\text{opp}}=\frac{12}{5}$

10. opposite = 3; adjacent = 4, hypotenuse = ?
$(\text{hypotenuse})^2=3^2+4^2=25$

hypotenuse $=\sqrt{25}=5$

$\sin\theta=\frac{\text{opp}}{\text{hyp}}=\frac{3}{5}\qquad \csc\theta=\frac{\text{hyp}}{\text{opp}}=\frac{5}{3}$

$\cos\theta=\frac{\text{adj}}{\text{hyp}}=\frac{4}{5}\qquad \sec\theta=\frac{\text{hyp}}{\text{adj}}=\frac{5}{4}$

$\tan\theta=\frac{\text{opp}}{\text{adj}}=\frac{3}{4}\qquad \cot\theta=\frac{\text{adj}}{\text{opp}}=\frac{4}{3}$

11. opposite = 2; adjacent = 3; hypotenuse = ?
$(\text{hypotenuse})^2=2^2+3^2=13$

hypotenuse $=\sqrt{13}$

$\sin\theta=\frac{\text{opp}}{\text{hyp}}=\frac{2}{\sqrt{13}}=\frac{2}{\sqrt{13}}\cdot\frac{\sqrt{13}}{\sqrt{13}}=\frac{2\sqrt{13}}{13}$

$\cos\theta=\frac{\text{adj}}{\text{hyp}}=\frac{3}{\sqrt{13}}=\frac{3}{\sqrt{13}}\cdot\frac{\sqrt{13}}{\sqrt{13}}=\frac{3\sqrt{13}}{13}$

$\tan\theta=\frac{\text{opp}}{\text{adj}}=\frac{2}{3}$

$\csc\theta=\frac{\text{hyp}}{\text{opp}}=\frac{\sqrt{13}}{2}$

$\sec\theta=\frac{\text{hyp}}{\text{adj}}=\frac{\sqrt{13}}{3}$

$\cot\theta=\frac{\text{adj}}{\text{opp}}=\frac{3}{2}$

12. opposite = 3; adjacent = 3; hypotenuse = ?
$(\text{hypotenuse})^2=3^2+3^2=18$

hypotenuse $=\sqrt{18}=3\sqrt{2}$

$\sin\theta=\frac{\text{opp}}{\text{hyp}}=\frac{3}{3\sqrt{2}}=\frac{3}{3\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{2}}{2}$

$\cos\theta=\frac{\text{adj}}{\text{hyp}}=\frac{3}{3\sqrt{2}}=\frac{3}{3\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{2}}{2}$

$\tan\theta=\frac{\text{opp}}{\text{adj}}=\frac{3}{3}=1$

$\csc\theta=\frac{\text{hyp}}{\text{opp}}=\frac{3\sqrt{2}}{3}=\sqrt{2}$

$\sec\theta=\frac{\text{hyp}}{\text{adj}}=\frac{3\sqrt{2}}{3}=\sqrt{2}$

$\cot\theta=\frac{\text{adj}}{\text{opp}}=\frac{3}{3}=1$

13. adjacent = 2; hypotenuse = 4; opposite = ?

$(\text{opposite})^2 + 2^2 = 4^2$

$(\text{opposite})^2 = 16 - 4 = 12$

$\text{opposite} = \sqrt{12} = 2\sqrt{3}$

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{2}{4} = \frac{1}{2}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{2\sqrt{3}}{2} = \sqrt{3}$

$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{4}{2\sqrt{3}} = \frac{4}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{4}{2} = 2$

$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{2}{2\sqrt{3}} = \frac{2}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

14. opposite = 3; hypotenuse = 4; adjacent = ?

$3^2 + (\text{adjacent})^2 = 4^2$

$(\text{adjacent})^2 = 16 - 9 = 7$

$\text{adjacent} = \sqrt{7}$

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{3}{4}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{7}}{4}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{3}{\sqrt{7}} = \frac{3}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{3\sqrt{7}}{7}$

$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{4}{3}$

$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{4}{\sqrt{7}} = \frac{4}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{4\sqrt{7}}{7}$

$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{\sqrt{7}}{3}$

15. opposite = $\sqrt{2}$; adjacent = 1; hypotenuse = ?

$(\text{hypotenuse})^2 = \left(\sqrt{2}\right)^2 + 1^2 = 3$

$\text{hypotenuse} = \sqrt{3}$

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{2}}{1} = \sqrt{2}$

$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}}{2}$

$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{3}}{1} = \sqrt{3}$

$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

16. opposite = 2; adjacent = $\sqrt{3}$; hypotenuse = ?

$(\text{hypotenuse})^2 = 2^2 + \left(\sqrt{3}\right)^2 = 7$

$\text{hypotenuse} = \sqrt{7}$

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{2}{\sqrt{7}} = \frac{2}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{2\sqrt{7}}{7}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{3}}{\sqrt{7}} = \frac{\sqrt{3}}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{21}}{7}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{\sqrt{7}}{2}$

$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{7}}{\sqrt{3}} = \frac{\sqrt{7}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{21}}{3}$

$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{\sqrt{3}}{2}$

17. opposite = 1; hypotenuse = $\sqrt{5}$; adjacent = ?

$1^2 + (\text{adjacent})^2 = \left(\sqrt{5}\right)^2$

$(\text{adjacent})^2 = 5 - 1 = 4$

$\text{adjacent} = \sqrt{4} = 2$

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{1}{\sqrt{5}} = \frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{2}{\sqrt{5}} = \frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{1}{2}$

$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{\sqrt{5}}{1} = \sqrt{5}$

$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{5}}{2}$

$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{2}{1} = 2$

18. adjacent = 2; hypotenuse = $\sqrt{5}$; opposite = ?

$$(\text{opposite})^2 + 2^2 = \left(\sqrt{5}\right)^2$$
$$(\text{opposite})^2 = 5 - 4 = 1$$
$$\text{opposite} = \sqrt{1} = 1$$

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{1}{\sqrt{5}} = \frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{2}{\sqrt{5}} = \frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{1}{2}$$
$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{\sqrt{5}}{1} = \sqrt{5}$$
$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{5}}{2}$$
$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{2}{1} = 2$$

19. $\sin 38^\circ - \cos 52^\circ = \sin 38^\circ - \sin(90^\circ - 52^\circ)$
$= \sin 38^\circ - \sin 38^\circ$
$= 0$

20. $\tan 12^\circ - \cot 78^\circ = \tan 12^\circ - \tan(90^\circ - 78^\circ)$
$= \tan 12^\circ - \tan 12^\circ$
$= 0$

21. $\dfrac{\cos 10^\circ}{\sin 80^\circ} = \dfrac{\sin(90^\circ - 10^\circ)}{\sin 80^\circ} = \dfrac{\sin 80^\circ}{\sin 80^\circ} = 1$

22. $\dfrac{\cos 40^\circ}{\sin 50^\circ} = \dfrac{\sin(90^\circ - 40^\circ)}{\sin 50^\circ} = \dfrac{\sin 50^\circ}{\sin 50^\circ} = 1$

23. $1 - \cos^2 20^\circ - \cos^2 70^\circ = 1 - \cos^2 20^\circ - \sin^2(90^\circ - 70^\circ)$
$= 1 - \cos^2 20^\circ - \sin^2(20^\circ)$
$= 1 - \left(\cos^2 20^\circ + \sin^2(20^\circ)\right)$
$= 1 - 1$
$= 0$

24. $1 + \tan^2 5^\circ - \csc^2 85^\circ = \sec^2 5^\circ - \csc^2 85^\circ$
$= \sec^2 5^\circ - \sec^2(90^\circ - 85^\circ)$
$= \sec^2 5^\circ - \sec^2 5^\circ$
$= 0$

25. $\tan 20^\circ - \dfrac{\cos 70^\circ}{\cos 20^\circ} = \tan 20^\circ - \dfrac{\sin(90^\circ - 70^\circ)}{\cos 20^\circ}$
$= \tan 20^\circ - \dfrac{\sin 20^\circ}{\cos 20^\circ}$
$= \tan 20^\circ - \tan 20^\circ$
$= 0$

26. $\cot 40^\circ - \dfrac{\sin 50^\circ}{\sin 40^\circ} = \cot 40^\circ - \dfrac{\cos(90^\circ - 50^\circ)}{\sin 40^\circ}$
$= \cot 40^\circ - \dfrac{\cos 40^\circ}{\sin 40^\circ}$
$= \cot 40^\circ - \cot 40^\circ$
$= 0$

27. $\cos 35^\circ \cdot \sin 55^\circ + \cos 55^\circ \cdot \sin 35^\circ$
$= \cos 35^\circ \cdot \cos(90^\circ - 55^\circ) + \sin(90^\circ - 55^\circ) \cdot \sin 35^\circ$
$= \cos 35^\circ \cdot \cos 35^\circ + \sin 35^\circ \cdot \sin 35^\circ$
$= \cos^2 35^\circ + \sin^2 35^\circ$
$= 1$

28. $\sec 35^\circ \cdot \csc 55^\circ - \tan 35^\circ \cdot \cot 55^\circ$
$= \sec 35^\circ \cdot \sec(90^\circ - 55^\circ) - \tan 35^\circ \cdot \tan(90^\circ - 55^\circ)$
$= \sec 35^\circ \cdot \sec 35^\circ - \tan 35^\circ \cdot \tan 35^\circ$
$= \sec^2 35^\circ - \tan^2 35^\circ$
$= (1 + \tan^2 35^\circ) - \tan^2 35^\circ$
$= 1$

29. $b = 5,\ B = 20^\circ$

$$\tan B = \frac{b}{a}$$
$$\tan\left(20^\circ\right) = \frac{5}{a}$$
$$a = \frac{5}{\tan\left(20^\circ\right)} \approx \frac{5}{0.3640} \approx 13.74$$

$$\sin B = \frac{b}{c}$$
$$\sin\left(20^\circ\right) = \frac{5}{c}$$
$$c = \frac{5}{\sin\left(20^\circ\right)} \approx \frac{5}{0.3420} \approx 14.62$$

$$A = 90^\circ - B = 90^\circ - 20^\circ = 70^\circ$$

30. $b=4,\ B=10°$

$$\tan B=\frac{b}{a}$$
$$\tan(10°)=\frac{4}{a}$$
$$a=\frac{4}{\tan(10°)}\approx\frac{4}{0.1763}\approx 22.69$$
$$\sin B=\frac{b}{c}$$
$$\sin(10°)=\frac{4}{c}$$
$$c=\frac{4}{\sin(10°)}\approx\frac{4}{0.1736}\approx 23.04$$
$$A=90°-B=90°-10°=80°$$

31. $a=6,\ \ B=40°$

$$\tan B=\frac{b}{a}$$
$$\tan(40°)=\frac{b}{6}$$
$$b=6\tan(40°)\approx 6\cdot(0.8391)\approx 5.03$$
$$\cos B=\frac{a}{c}$$
$$\cos(40°)=\frac{6}{c}$$
$$c=\frac{6}{\cos(40°)}\approx\frac{6}{0.7660}\approx 7.83$$
$$A=90°-B=90°-40°=50°$$

32. $a=7,\ \ B=50°$

$$\tan B=\frac{b}{a}$$
$$\tan(50°)=\frac{b}{7}$$
$$b=7\tan(50°)\approx 7\cdot(1.1918)\approx 8.34$$
$$\cos B=\frac{a}{c}$$
$$\cos(50°)=\frac{7}{c}$$
$$c=\frac{7}{\cos(50°)}\approx\frac{7}{0.6428}\approx 10.89$$
$$A=90°-B=90°-50°=40°$$

33. $b=4,\ \ A=10°$

$$\tan A=\frac{a}{b}$$
$$\tan(10°)=\frac{a}{4}$$
$$a=4\tan(10°)\approx 4\cdot(0.1763)\approx 0.71$$
$$\cos A=\frac{b}{c}$$
$$\cos(10°)=\frac{4}{c}$$
$$c=\frac{4}{\cos(10°)}\approx\frac{4}{0.9848}\approx 4.06$$
$$B=90°-A=90°-10°=80°$$

34. $b=6,\ \ A=20°$

$$\tan A=\frac{a}{b}$$
$$\tan(20°)=\frac{a}{6}$$
$$a=6\tan(20°)\approx 6\cdot(0.3640)\approx 2.18$$
$$\cos A=\frac{b}{c}$$
$$\cos(20°)=\frac{6}{c}$$
$$c=\frac{6}{\cos(20°)}\approx\frac{6}{0.9397}\approx 6.39$$
$$B=90°-A=90°-20°=70°$$

35. $a=5,\ \ A=25°$

$$\cot A=\frac{b}{a}$$
$$\cot(25°)=\frac{b}{5}$$
$$b=5\cot(25°)\approx 5\cdot(2.1445)\approx 10.72$$
$$\csc A=\frac{c}{a}$$
$$\csc(25°)=\frac{c}{5}$$
$$c=5\csc(25°)\approx 5\cdot(2.3662)\approx 11.83$$
$$B=90°-A=B=90°-A=90°-25°=65°$$

36. $a = 6, \ \ A = 40º$

$$\cot A = \frac{a}{b}$$

$$\cot(40º) = \frac{b}{6}$$

$$b = 6\cot(40º) \approx 6\cdot(1.1918) \approx 7.15$$

$$\csc A = \frac{c}{a}$$

$$\csc(45°) = \frac{c}{6}$$

$$c = 6\csc(40º) \approx 6\cdot(1.5557) \approx 9.33$$

$$B = 90° - A = 90° - 40° = 50°$$

37. $c = 9, \ \ B = 20º$

$$\sin B = \frac{b}{c}$$

$$\sin(20º) = \frac{b}{9}$$

$$b = 9\sin(20º) \approx 9\cdot(0.3420) \approx 3.08$$

$$\cos B = \frac{a}{c}$$

$$\cos(20º) = \frac{a}{9}$$

$$a = 9\cos(20º) \approx 9\cdot(0.9397) \approx 8.46$$

$$A = 90° - A = 90° - 20° = 70°$$

38. $c = 10, \ \ A = 40º$

$$\sin A = \frac{a}{c}$$

$$\sin(40º) = \frac{a}{10}$$

$$a = 10\sin(40º) \approx 10\cdot(0.6428) \approx 6.43$$

$$\cos A = \frac{b}{c}$$

$$\cos(40º) = \frac{b}{10}$$

$$b = 10\cos(40º) \approx 10\cdot(0.7660) \approx 7.66$$

$$B = 90° - A = 90° - 40° = 50°$$

39. $a = 5, \ \ b = 3$

$$c^2 = a^2 + b^2 = 5^2 + 3^2 = 25 + 9 = 34$$

$$c = \sqrt{34} \approx 5.83$$

$$\tan A = \frac{a}{b} = \frac{5}{3}$$

$$A = \tan^{-1}\left(\frac{5}{3}\right) \approx 59.0°$$

$$B = 90° - A \approx 90° - 59.0° = 31.0°$$

40. $a = 2, \ \ b = 8$

$$c^2 = a^2 + b^2 = 2^2 + 8^2 = 4 + 64 = 68$$

$$c = \sqrt{68} \approx 8.25$$

$$\tan A = \frac{a}{b} = \frac{2}{8} = \frac{1}{4}$$

$$A = \tan^{-1}\left(\frac{1}{4}\right) \approx 14.0°$$

$$B = 90° - A \approx 90° - 14.0° = 76.0°$$

41. $a = 2, \ \ c = 5$

$$c^2 = a^2 + b^2$$

$$b^2 = c^2 - a^2 = 5^2 - 2^2 = 25 - 4 = 21$$

$$b = \sqrt{21} \approx 4.58$$

$$\sin A = \frac{a}{c} = \frac{2}{5}$$

$$A = \tan^{-1}\left(\frac{2}{5}\right) \approx 23.6º$$

$$B = 90° - A \approx 90° - 23.6° = 66.4°$$

42. $b = 4, \ \ c = 6$

$$c^2 = a^2 + b^2$$

$$a^2 = c^2 - b^2 = 6^2 - 4^2 = 36 - 16 = 20$$

$$a = \sqrt{20} \approx 4.47$$

$$\cos A = \frac{b}{c} = \frac{4}{6} = \frac{2}{3}$$

$$A = \tan^{-1}\left(\frac{2}{3}\right) \approx 48.2º$$

$$B = 90º - A \approx 90º - 48.2º = 41.8º$$

43. $c = 5, \ a = 2$

$\sin A = \dfrac{2}{5}$

$A = \sin^{-1}\left(\dfrac{2}{5}\right) \approx 23.6°$

$B = 90° - A \approx 90° - 23.6° = 66.4°$

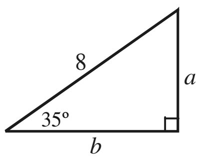
The two angles measure about $23.6°$ and $66.4°$.

44. $c = 3, \ a = 1$

$B = 90° - A \approx 90° - 19.5° = 70.5°$

The two angles measure about $19.5°$ and $70.5°$.

45. $c = 8, \ \theta = 35°$



$\sin(35°) = \dfrac{a}{8}$

$a = 8\sin(35°)$

$\approx 8(0.5736)$

≈ 4.59 in.

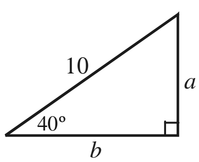
$\cos(35°) = \dfrac{b}{8}$

$b = 8\cos(35°)$

$\approx 8(0.8192)$

≈ 6.55 in.

46. $c = 10, \ \theta = 40°$



$\sin(40°) = \dfrac{a}{10}$

$a = 10\sin(40°)$

$\approx 10(0.6428)$

≈ 6.43 cm.

$\cos(40°) = \dfrac{b}{10}$

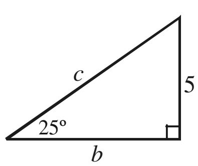
$b = 10\cos(40°)$

$\approx 10(0.7660)$

≈ 7.66 cm.

47. Case 1: $\theta = 25°, \ a = 5$

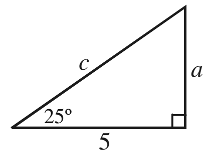
a.



$\sin(25°) = \dfrac{5}{c}$

$c = \dfrac{5}{\sin(25°)} \approx \dfrac{5}{0.4226} \approx 11.83$ in.

Case 2: $\theta = 25°, \ b = 5$



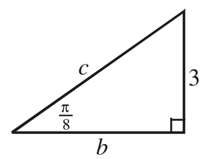
$\cos(25°) = \dfrac{5}{c}$

$c = \dfrac{5}{\cos(25°)} \approx \dfrac{5}{0.9063} \approx 5.52$ in.

b. There are two possible cases because the given side could be adjacent or opposite the given angle.

48. Case 1: $\theta = \dfrac{\pi}{8}, \ a = 3$

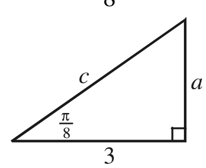
a.



$\sin\left(\dfrac{\pi}{8}\right) = \dfrac{3}{c}$

$c = \dfrac{3}{\sin\left(\dfrac{\pi}{8}\right)} \approx \dfrac{3}{0.3827} \approx 7.84$ m.

Case 2: $\theta = \dfrac{\pi}{8}, \ b = 3$



$\cos\left(\dfrac{\pi}{8}\right) = \dfrac{3}{c}$

$c = \dfrac{3}{\cos\left(\dfrac{\pi}{8}\right)} \approx \dfrac{3}{0.9239} \approx 3.25$ m.

b. There are two possible cases because the given side could be adjacent or opposite the given angle.

49. $\tan(35°) = \dfrac{|AC|}{100}$

$|AC| = 100\tan(35°) \approx 100(0.7002) \approx 70.02$ feet

50. $\tan(40^\circ) = \dfrac{|AC|}{100}$

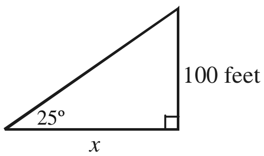
$|AC| = 100\tan(40^\circ) \approx 100(0.8391) \approx 83.91$ feet

51. Let x = the height of the Eiffel Tower.

$\tan(85.361^\circ) = \dfrac{x}{80}$

$x = 80\tan(85.361^\circ) \approx 80(12.3239) \approx 985.91$ feet

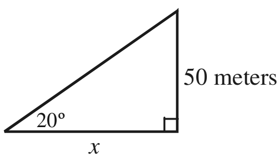
52. Let x = the distance to the shore.



$\tan(25^\circ) = \dfrac{100}{x}$

$x = \dfrac{100}{\tan(25^\circ)} \approx \dfrac{100}{0.4663} \approx 214.45$ feet

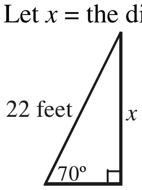
53. Let x = the distance to the base of the plateau.



$\tan(20^\circ) = \dfrac{50}{x}$

$x = \dfrac{50}{\tan(20^\circ)} \approx \dfrac{50}{0.3640} \approx 137.37$ meters

54. Let x = the distance up the building



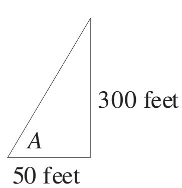
$\sin(70^\circ) = \dfrac{x}{22}$

$x = 22\sin(70^\circ) \approx 22(0.9397) \approx 20.67$ feet

55. $\tan A = \dfrac{300}{50} = 6$

$A = \tan^{-1} 6 \approx 80.5^\circ$

The angle of elevation of the sun is about 80.5°.



56. opposite side = 10 feet, adjacent side = 35 feet

$\tan\theta = \dfrac{10}{35}$

$\theta = \tan^{-1}\left(\dfrac{10}{35}\right) \approx 15.9^\circ$

57. **a.** Let x represent the distance the truck traveled in the 1 second time interval.

$\tan(15^\circ) = \dfrac{30}{x}$

$x = \dfrac{30}{\tan(15^\circ)} \approx \dfrac{30}{0.2679} \approx 111.96$ feet

The truck is traveling at 111.96 ft/sec, or

$\dfrac{111.96 \text{ ft}}{\text{sec}} \cdot \dfrac{1 \text{ mile}}{5280 \text{ ft}} \cdot \dfrac{3600 \text{ sec}}{\text{hr}} \approx 76.3$ mi/hr .

b. $\tan(20^\circ) = \dfrac{30}{x}$

$x = \dfrac{30}{\tan(20^\circ)} \approx \dfrac{30}{0.3640} \approx 82.42$ feet

The truck is traveling at 82.42 ft/sec, or

$\dfrac{82.42 \text{ ft}}{\text{sec}} \cdot \dfrac{1 \text{ mile}}{5280 \text{ ft}} \cdot \dfrac{3600 \text{ sec}}{\text{hr}} \approx 56.2$ mi/hr .

c. A ticket is issued for traveling at a speed of 60 mi/hr or more.

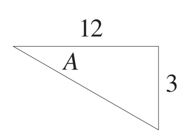
$\dfrac{60 \text{ mi}}{\text{hr}} \cdot \dfrac{5280 \text{ ft}}{\text{mi}} \cdot \dfrac{1\text{hr}}{3600 \text{ sec}} = 88$ ft/sec.

If $\tan\theta < \dfrac{30}{88}$, the trooper should issue a ticket. Now, $\tan^{-1}\left(\dfrac{30}{88}\right) \approx 18.8^\circ$, so a ticket is issued if $\theta < 18.8^\circ$.

58. If the camera is to be directed to a spot 6 feet above the floor 12 feet from the wall, then the "side opposite" the angle of depression is 3 feet. (see figure)

$\tan A = \dfrac{3}{12} = \dfrac{1}{4}$

$A = \tan^{-1}\dfrac{1}{4} \approx 14.0^\circ$



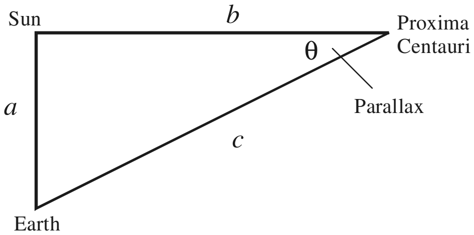
The angle of depression should be about 14.0°.

59. a. $4.22 \cdot (5.9 \times 10^{12}) = 24.898 \times 10^{12}$

$= 2.4898 \times 10^{13}$

Proxima Centauri is about 2.4898×10^{13} miles from Earth.

b. Construct a right triangle using the sun, Earth, and Proxima Centauri as shown. The hypotenuse is the distance between Earth and Proxima Centauri.



$\sin\theta = \frac{a}{c} = \frac{9.3 \times 10^7}{2.4898 \times 10^{13}}$

$\sin\theta \approx 0.000003735$

$\theta \approx \sin^{-1} 0.000003735$

$\theta \approx 0.000214°$

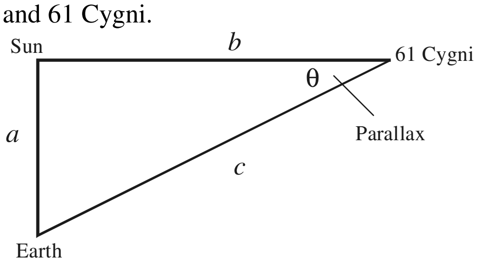
The parallax of Proxima Centauri is $0.000214°$.

60. a. $11.14 \cdot (5.9 \times 10^{12}) = 65.726 \times 10^{12}$

$= 6.5726 \times 10^{13}$

61 Cygni is about 6.5726×10^{13} miles from Earth.

b. Construct a right triangle using the sun, Earth, and 61 Cygni as shown. The hypotenuse is the distance between Earth and 61 Cygni.



$\sin\theta = \frac{a}{c} = \frac{9.3 \times 10^7}{6.5726 \times 10^{13}}$

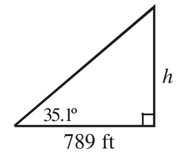
$\sin\theta \approx 0.000001415$

$\theta \approx \sin^{-1} 0.000001415$

$\theta \approx 0.00008°$

The parallax of 61 Cygni is $0.00008°$.

61. Let h = the height of the monument.



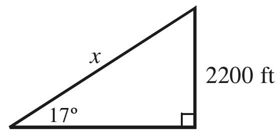
$\tan(35.1°) = \frac{h}{789}$

$h = 789 \tan(35.1°)$

$\approx 789(0.7028)$

≈ 554.52 ft

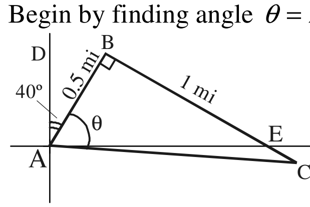
62. The elevation change is $11200 - 9000 = 2200$ ft. Let x = the length of the trail.



$\sin 17° = \frac{2200}{x}$

$x = \frac{2200}{\sin(17°)} \approx \frac{2200}{0.2924} \approx 7524.67$ ft.

63. Begin by finding angle $\theta = \angle BAC$: (see figure)



$\tan\theta = \frac{1}{0.5} = 2$

$\theta = 63.4°$

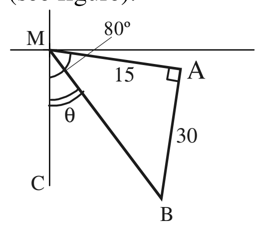
$\angle DAC = 40° + 63.4° = 103.4°$

$\angle EAC = 103.4° - 90° = 13.4°$

Now, $90° - 13.4° = 76.6°$

The control tower should use a bearing of S76.6˚E.

64. Find $\angle AMB$ and subtract from 80˚ to obtain θ (see figure).



$\angle CMA = 80°$

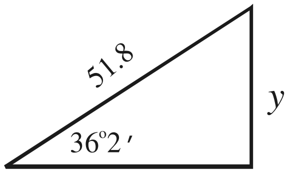
$\tan \angle AMB = \frac{30}{15} = 2$

$\angle AMB = \tan^{-1} 2 \approx 63.4°$

$\theta = 80° - 63.4° = 16.6°$

The bearing of the ship from port is S16.6ºE.

65. Let y = the height of the embankment.



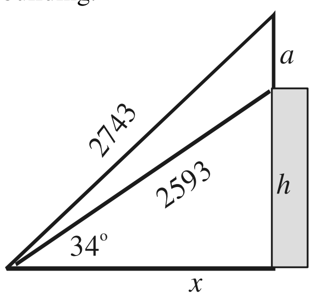
$\theta = 36°2' \approx 36.033°$

$\sin(36.033°) = \frac{y}{51.8}$

$y = 51.8\sin(36.033°) \approx 30.5$ meters

The embankment is about 30.5 meters high.

66. a. Let h = the height of the building, a = the height of the antenna, and x = the distance between the surveyor and the base of the building.



$\cos 34° = \frac{x}{2593}$

$x = 2593 \cdot \cos 34° \approx 2150$

The surveyor is located approximately 2150 feet from the building.

b. $\sin 34° = \frac{h}{2593}$

$h = 2593 \cdot \sin 34° \approx 1450$

The building is about 1450 feet high.

c. Let θ = the angle of inclination from the surveyor to the top of the antenna.

$\cos\theta = \frac{2150}{2743}$

$\theta = \cos^{-1}\left(\frac{2150}{2743}\right) \approx 38.4°$

d. $\sin\theta = \frac{h+a}{2743} = \frac{1450+a}{2743}$

$2743 \cdot \sin\theta = 1450 + a$

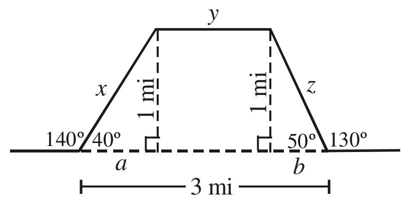
$a = 2743 \cdot \sin\theta - 1450$

$a = 2743\sin\left(\cos^{-1}\left(\frac{2150}{2743}\right)\right) - 1450$

$a \approx 253$

The antenna is about 253 feet tall.

67. Let x, y, and z = the three segments of the highway around the bay (see figure).



The length of the highway $= x + y + z$

$\sin(40°) = \frac{1}{x}$

$x = \frac{1}{\sin(40°)} \approx 1.5557$ mi

$\sin(50°) = \frac{1}{z}$

$z = \frac{1}{\sin(50°)} \approx 1.3054$ mi

$\tan(40°) = \frac{1}{a}$

$a = \frac{1}{\tan(40°)} \approx 1.1918$ mi

$\tan(50°) = \frac{1}{b}$

$b = \frac{1}{\tan(50°)} \approx 0.8391$ mi

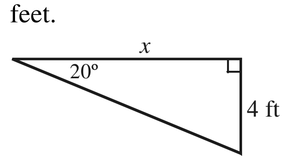
$a + y + b = 3$

$y = 3 - a - b$

$\approx 3 - 1.1918 - 0.8391 = 0.9691$ mi

The length of the highway is about:
$1.5557 + 0.9691 + 1.3054 \approx 3.83$ miles .

68. Let x = the distance from George at which the camera must be set in order to see his head and feet.

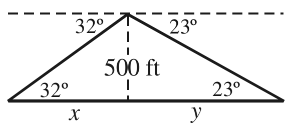


$\tan(20°) = \frac{4}{x}$

$x = \frac{4}{\tan(20°)} \approx 10.99$ feet

If the camera is set at a distance of 10 feet from George, his feet will not be seen by the lens. The camera would need to be moved back about 1 additional foot (11 feet total).

69. We construct the figure below:

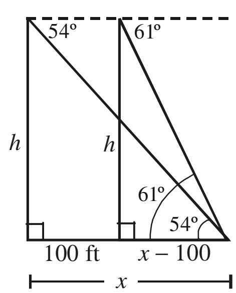


$$\tan(32°) = \frac{500}{x} \qquad \tan(23°) = \frac{500}{y}$$

$$x = \frac{500}{\tan(32°)} \qquad y = \frac{500}{\tan(23°)}$$

$$\text{Distance} = x + y = \frac{500}{\tan(32°)} + \frac{500}{\tan(23°)} \approx 1978.09 \text{ feet}$$

70. Let h = the height of the balloon.



$$\tan(54°) = \frac{h}{x}$$

$$x = \frac{h}{\tan(54°)}$$

$$\tan(61°) = \frac{h}{x-100}$$

$$h = (x-100)\tan(61°)$$

$$h = \left(\frac{h}{\tan(54°)} - 100\right)\tan(61°)$$

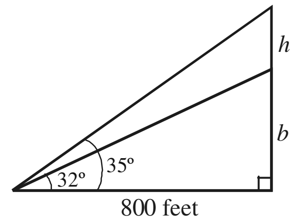
$$h - \frac{\tan(61°)}{\tan(54°)}h = -100\tan(61°)$$

$$h\left(1 - \frac{\tan(61°)}{\tan(54°)}\right) = -100\tan(61°)$$

$$h = \frac{-100\tan(61°)}{\left(1 - \frac{\tan(61°)}{\tan(54°)}\right)} \approx 580.61$$

Thus, the height of the balloon is approximately 580.61 feet.

71. Let h represent the height of Lincoln's face.



$$\tan(32°) = \frac{b}{800}$$

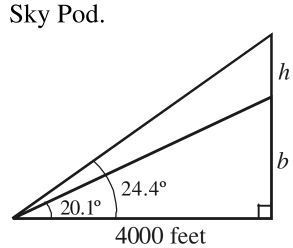
$$b = 800\tan(32°) \approx 499.90$$

$$\tan(35°) = \frac{b+h}{800}$$

$$b + h = 800\tan(35°) \approx 560.17$$

Thus, the height of Lincoln's face is:
$h = (b+h) - b = 560.17 - 499.90 \approx 60.27$ feet

72. Let h represent the height of tower above the Sky Pod.



$$\tan(20.1°) = \frac{b}{4000}$$

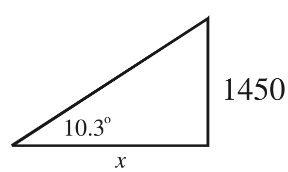
$$b = 4000\tan(20.1°) \approx 1463.79$$

$$\tan(24.4°) = \frac{b+h}{4000}$$

$$b + h = 4000\tan(24.4°) \approx 1814.48$$

Thus, the height of tower above the Sky Pod is:
$h = (b+h) - b = 1814.48 - 1463.79 \approx 350.69$ feet

73. Let x = the distance between the buildings.



$$\tan\theta = \frac{1450}{x}$$

$$x = \frac{1450}{\tan(10.3°)} \approx 7979$$

The two buildings are about 7979 feet apart.

74. $\tan(\alpha+\beta)=\frac{2070}{630}$

$$\alpha+\beta=\tan^{-1}\left(\frac{2070}{630}\right)$$
$$\alpha=\tan^{-1}\left(\frac{2070}{630}\right)-\beta$$
$$=\tan^{-1}\left(\frac{2070}{630}\right)-\cot^{-1}\left(\frac{67}{55}\right)$$
$$\approx 33.69°$$

Let x = the distance between the Arch and the boat on the Missouri side.

$$\tan\alpha=\frac{x}{630}$$
$$x=630\tan\alpha=630\tan(33.69°)$$
$$x=420$$

Therefore, the Mississippi River is approximately $2070-420=1650$ feet wide at the St. Louis riverfront.

75. The height of the beam above the wall is $46-20=26$ feet.

$$\tan\theta=\frac{26}{10}=2.6$$
$$\theta=\tan^{-1}2.6\approx 69.0°$$

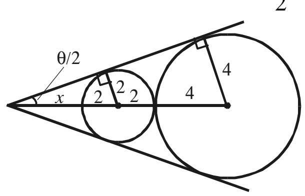
The pitch of the roof is about $69.0°$.

76. $\tan A=\frac{10-6}{15}=\frac{4}{15}$

$$A=\tan^{-1}\left(\frac{4}{15}\right)\approx 14.9°$$

The angle of elevation from the player's eyes to the center of the rim is about $14.9°$.

77. A line segment drawn from the vertex of the angle through the centers of the circles will bisect θ (see figure). Thus, the angle of the two right triangles formed is $\frac{\theta}{2}$.



Let x = the length of the segment from the vertex of the angle to the smaller circle. Then the hypotenuse of the smaller right triangle is $x+2$, and the hypotenuse of the larger right triangle is $x+2+2+4=x+8$. Since the two right triangles are similar, we have that:

$$\frac{x+2}{2}=\frac{x+8}{4}$$
$$4(x+2)=2(x+8)$$
$$4x+8=2x+16$$
$$2x=8$$
$$x=4$$

Thus, the hypotenuse of the smaller triangle is $4+2=6$. Now, $\sin\left(\frac{\theta}{2}\right)=\frac{2}{6}=\frac{1}{3}$, so we have that: $\frac{\theta}{2}=\sin^{-1}\left(\frac{1}{3}\right)$

$$\theta=2\cdot\sin^{-1}\left(\frac{1}{3}\right)\approx 38.9°$$

78. **a.** $\cos\left(\frac{\theta}{2}\right)=\frac{3960}{3960+h}$

b. $d=3960\theta$

c. $\cos\left(\frac{d}{7920}\right)=\frac{3960}{3960+h}$

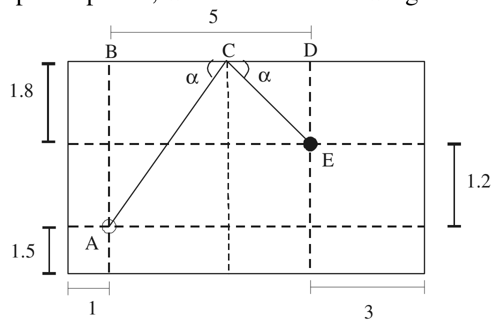
d. $\cos\left(\frac{2500}{7920}\right)=\frac{3960}{3960+h}$

$$0.9506\approx\frac{3960}{3960+h}$$
$$0.9506(3960+h)\approx 3960$$
$$3764+0.9506h\approx 3960$$
$$0.9506h\approx 196$$
$$h\approx 206 \text{ miles}$$

e. $\cos\left(\frac{d}{7920}\right)=\frac{3960}{3960+300}=\frac{3960}{4260}$

$$\frac{d}{7920}=\cos^{-1}\left(\frac{3960}{4260}\right)$$
$$d=7970\cos^{-1}\left(\frac{3960}{4260}\right)$$
$$\approx 2990 \text{ miles}$$

79. Adding some lines to the diagram and labeling special points, we obtain the following:

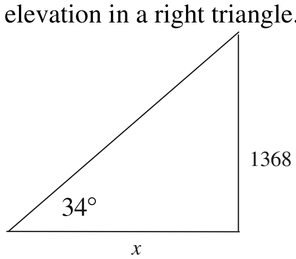


If we let x = length of side BC, we see that, in ΔABC, $\tan\alpha = \dfrac{3}{x}$. Also, in ΔEDC, $\tan\alpha = \dfrac{1.8}{5-x}$. Therefore, we have

$$\frac{3}{x} = \frac{1.8}{5-x}$$
$$15 - 3x = 1.8x$$
$$15 = 4.8x$$
$$x = \frac{15}{4.8} = 3.125 \text{ ft}$$
$$1 + 3.125 = 4.125 \text{ ft}$$

The player should hit the top cushion at a point that is 4.125 feet from upper left corner.

80. a. The distance between the buildings is the length of the side adjacent to the angle of elevation in a right triangle.

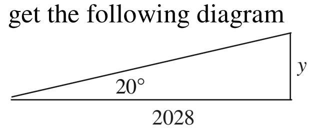


Since $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$ and we know the angle measure, we can use the tangent to find the distance. Let x = the distance between the buildings. This gives us

$$\tan 34° = \frac{1368}{x}$$
$$x = \frac{1368}{\tan 34°}$$
$$x \approx 2028$$

The office building is about 2028 feet from the base of the tower.

b. Let y = the difference in height between the Trade Center and the office building. Together with the result from part (a), we get the following diagram

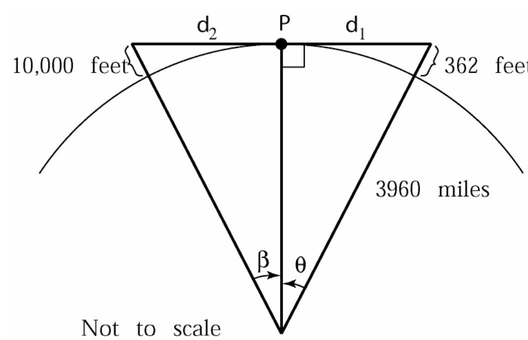


$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan 20° = \frac{y}{2028}$$
$$y \approx 738$$

The Trade Center is about 958 feet taller than the office building. Therefore, the office building is $1368 - 738 = 630$ feet tall.

81 – 82. Answers will vary.

83. Let θ = the central angle formed by the top of the lighthouse, the center of the Earth and the point P on the Earth's surface where the line of sight from the top of the lighthouse is tangent to the Earth. Note also that 362 feet = $\dfrac{362}{5280}$ miles.



$$\theta = \cos^{-1}\left(\frac{3960}{3960 + 362/5280}\right) \approx 0.33715°$$

<u>Verify the airplane information:</u>

Let β = the central angle formed by the plane, the center of the Earth and the point P.

$$\beta = \cos^{-1}\left(\frac{3960}{3960 + 10{,}000/5280}\right) \approx 1.77169°$$

Note that

$$\tan\theta = \frac{d_1}{3690} \qquad \text{and} \qquad \tan\beta = \frac{d_2}{3690}$$
$$d_1 = 3960\tan\theta \qquad\qquad d_2 = 3960\tan\beta$$

So,

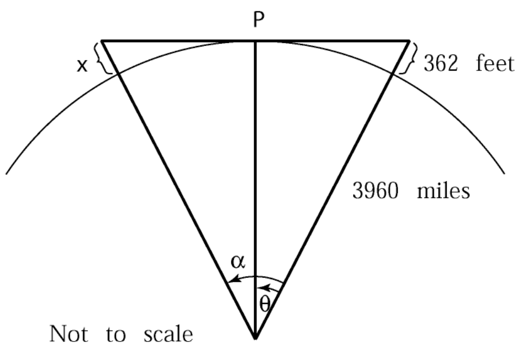
$$d_1 + d_2 = 3960\tan\theta + 3960\tan\beta$$
$$\approx 3960\tan(0.33715°) + 3960\tan(1.77169°)$$
$$\approx 146 \text{ miles}$$

To express this distance in nautical miles, we express the total angle $\theta+\beta$ in minutes. That is,

$\theta+\beta \approx \left(0.33715^{\circ}+1.77169^{\circ}\right)\cdot 60 \approx 126.5$

nautical miles. Therefore, a plane flying at an altitude of 10,000 feet can see the lighthouse 120 miles away.

Verify the ship information:



Let α = the central angle formed by 40 nautical miles then. $\alpha=\frac{40}{60}=\frac{2}{3}^{\circ}$

$$\cos(\alpha-\theta)=\frac{3960}{3960+x}$$
$$\cos\left((2/3)^{\circ}-0.33715^{\circ}\right)=\frac{3960}{3960+x}$$
$$\left(3960+x\right)\cos\left((2/3)^{\circ}-0.33715^{\circ}\right)=3960$$
$$3960+x=\frac{3960}{\cos\left((2/3)^{\circ}-0.33715^{\circ}\right)}$$
$$x=\frac{3960}{\cos\left((2/3)^{\circ}-0.33715^{\circ}\right)}-3960$$
$$\approx 0.06549 \text{ miles}$$
$$\approx \left(0.06549 \text{ mi}\right)\left(5280\frac{\text{ft}}{\text{mi}}\right)$$
$$\approx 346 \text{ feet}$$

Therefore, a ship that is 346 feet above sea level can see the lighthouse from a distance of 40 nautical miles.

Section 8.2

1. $\sin A\cos B-\cos A\sin B$

2. $\cos\theta=\frac{\sqrt{3}}{2}$

   $\theta=\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)=\frac{\pi}{6}=30^{\circ}$

   The solution set is $\left\{\frac{\pi}{6}=30^{\circ}\right\}$.

3. For similar triangles, corresponding pairs of sides occur in the same ratio. Therefore, we can write:

   $\frac{x}{3}=\frac{5}{2}$ or $x=\frac{15}{2}$

   The missing length is $\frac{15}{2}$.

4. oblique

5. $\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$

6. False

7. False: You must have at least one angle opposite one side.

8. ambiguous case

9. $c=5,\ B=45^{\circ},\ C=95^{\circ}$

   $A=180^{\circ}-B-\gamma=180^{\circ}-45^{\circ}-95^{\circ}=40^{\circ}$

   $$\frac{\sin A}{a}=\frac{\sin C}{c}$$
   $$\frac{\sin 40^{\circ}}{a}=\frac{\sin 95^{\circ}}{5}$$
   $$a=\frac{5\sin 40^{\circ}}{\sin 95^{\circ}}\approx 3.23$$
   $$\frac{\sin B}{b}=\frac{\sin C}{c}$$
   $$\frac{\sin 45^{\circ}}{b}=\frac{\sin 95^{\circ}}{5}$$
   $$b=\frac{5\sin 45^{\circ}}{\sin 95^{\circ}}\approx 3.55$$

10. $c = 4,\ A = 45°,\ B = 40°$
$C = 180° - A - B = 180° - 45° - 40° = 95°$

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin 45°}{a} = \frac{\sin 95°}{4}$$
$$a = \frac{4\sin 45°}{\sin 95°} \approx 2.84$$
$$\frac{\sin B}{b} = \frac{\sin C}{c}$$
$$\frac{\sin 40°}{b} = \frac{\sin 95°}{4}$$
$$b = \frac{4\sin 40°}{\sin 95°} \approx 2.58$$

11. $b = 3,\ A = 50°,\ C = 85°$
$B = 180° - A - C = 180° - 50° - 85° = 45°$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 50°}{a} = \frac{\sin 45°}{3}$$
$$a = \frac{3\sin 50°}{\sin 45°} \approx 3.25$$
$$\frac{\sin C}{c} = \frac{\sin B}{b}$$
$$\frac{\sin 85°}{c} = \frac{\sin 45°}{3}$$
$$c = \frac{3\sin 85°}{\sin 45°} \approx 4.23$$

12. $b = 10,\ B = 30°,\ C = 125°$
$A = 180° - B - C = 180° - 30° - 125° = 25°$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 25°}{a} = \frac{\sin 30°}{10}$$
$$a = \frac{10\sin 25°}{\sin 30°} \approx 8.45$$
$$\frac{\sin C}{c} = \frac{\sin B}{b}$$
$$\frac{\sin 125°}{c} = \frac{\sin 30°}{10}$$
$$c = \frac{10\sin 125°}{\sin 30°} \approx 16.38$$

13. $b = 7,\ A = 40°,\ B = 45°$
$C = 180° - A - B = 180° - 40° - 45° = 95°$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 40°}{a} = \frac{\sin 45°}{7}$$
$$a = \frac{7\sin 40°}{\sin 45°} \approx 6.36$$
$$\frac{\sin C}{c} = \frac{\sin B}{b}$$
$$\frac{\sin 95°}{c} = \frac{\sin 45°}{7}$$
$$c = \frac{7\sin 95°}{\sin 45°} \approx 9.86$$

14. $c = 5,\ A = 10°,\ B = 5°$
$C = 180° - A - B = 180° - 10° - 5° = 165°$

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin 10°}{a} = \frac{\sin 165°}{5}$$
$$a = \frac{5\sin 10°}{\sin 165°} \approx 3.35$$
$$\frac{\sin B}{b} = \frac{\sin C}{c}$$
$$\frac{\sin 5°}{b} = \frac{\sin 165°}{5}$$
$$b = \frac{5\sin 5°}{\sin 165°} \approx 1.68$$

15. $b = 2,\ B = 40°,\ C = 100°$
$A = 180° - B - C = 180° - 40° - 100° = 40°$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 40°}{a} = \frac{\sin 40°}{2}$$
$$a = \frac{2\sin 40°}{\sin 40°} = 2$$
$$\frac{\sin C}{c} = \frac{\sin B}{b}$$
$$\frac{\sin 100°}{c} = \frac{\sin 40°}{2}$$
$$c = \frac{2\sin 100°}{\sin 40°} \approx 3.06$$

16. $b = 6,\ A = 100°,\ B = 30°$
$C = 180° - A - B = 180° - 100° - 30° = 50°$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 100°}{a} = \frac{\sin 30°}{6}$$
$$a = \frac{6 \sin 100°}{\sin 30°} \approx 11.82$$
$$\frac{\sin C}{c} = \frac{\sin B}{b}$$
$$\frac{\sin 50°}{c} = \frac{\sin 30°}{6}$$
$$c = \frac{6 \sin 50°}{\sin 30°} \approx 9.19$$

17. $A = 40°,\ B = 20°,\ a = 2$
$C = 180° - A - B = 180° - 40° - 20° = 120°$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 40°}{2} = \frac{\sin 20°}{b}$$
$$b = \frac{2 \sin 20°}{\sin 40°} \approx 1.06$$
$$\frac{\sin C}{c} = \frac{\sin A}{a}$$
$$\frac{\sin 120°}{c} = \frac{\sin 40°}{2}$$
$$c = \frac{2 \sin 120°}{\sin 40°} \approx 2.69$$

18. $A = 50°,\ C = 20°,\ a = 3$
$B = 180° - A - C = 180° - 50° - 20° = 110°$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 50°}{3} = \frac{\sin 110°}{b}$$
$$b = \frac{3 \sin 110°}{\sin 50°} \approx 3.68$$
$$\frac{\sin C}{c} = \frac{\sin A}{a}$$
$$\frac{\sin 20°}{c} = \frac{\sin 50°}{3}$$
$$c = \frac{3 \sin 20°}{\sin 50°} \approx 1.34$$

19. $B = 70°,\ C = 10°,\ b = 5$
$A = 180° - B - C = 180° - 70° - 10° = 100°$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 100°}{a} = \frac{\sin 70°}{5}$$
$$a = \frac{5 \sin 100°}{\sin 70°} \approx 5.24$$
$$\frac{\sin C}{c} = \frac{\sin B}{b}$$
$$\frac{\sin 10°}{c} = \frac{\sin 70°}{5}$$
$$c = \frac{5 \sin 10°}{\sin 70°} \approx 0.92$$

20. $A = 70°,\ B = 60°,\ c = 4$
$C = 180° - A - B = 180° - 70° - 60° = 50°$

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin 70°}{a} = \frac{\sin 50°}{4}$$
$$a = \frac{4 \sin 70°}{\sin 50°} \approx 4.91$$
$$\frac{\sin B}{b} = \frac{\sin C}{c}$$
$$\frac{\sin 60°}{b} = \frac{\sin 50°}{4}$$
$$b = \frac{4 \sin 60°}{\sin 50°} \approx 4.52$$

21. $A = 110°,\ C = 30°,\ c = 3$
$B = 180° - A - C = 180° - 110° - 30° = 40°$

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin 110°}{a} = \frac{\sin 30°}{3}$$
$$a = \frac{3 \sin 110°}{\sin 30°} \approx 5.64$$
$$\frac{\sin C}{c} = \frac{\sin B}{b}$$
$$\frac{\sin 30°}{3} = \frac{\sin 40°}{b}$$
$$b = \frac{3 \sin 40°}{\sin 30°} \approx 3.86$$

22. $B = 10°,\ C = 100°,\ b = 2$

$A = 180° - B - C = 180° - 10° - 100° = 70°$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 70°}{a} = \frac{\sin 10°}{2}$$
$$a = \frac{2\sin 70°}{\sin 10°} \approx 10.82$$
$$\frac{\sin C}{c} = \frac{\sin B}{b}$$
$$\frac{\sin 100°}{c} = \frac{\sin 10°}{2}$$
$$c = \frac{2\sin 100°}{\sin 10°} \approx 11.34$$

23. $A = 40°,\ B = 40°,\ c = 2$

$C = 180° - A - B = 180° - 40° - 40° = 100°$

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin 40°}{a} = \frac{\sin 100°}{2}$$
$$a = \frac{2\sin 40°}{\sin 100°} \approx 1.31$$
$$\frac{\sin B}{b} = \frac{\sin C}{c}$$
$$\frac{\sin 40°}{b} = \frac{\sin 100°}{2}$$
$$b = \frac{2\sin 40°}{\sin 100°} \approx 1.31$$

24. $B = 20°,\ C = 70°,\ a = 1$

$A = 180° - B - C = 180° - 20° - 70° = 90°$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 90°}{1} = \frac{\sin 20°}{b}$$
$$b = \frac{1\sin 20°}{\sin 90°} \approx 0.34$$
$$\frac{\sin C}{c} = \frac{\sin A}{a}$$
$$\frac{\sin 70°}{c} = \frac{\sin 90°}{1}$$
$$c = \frac{1\sin 70°}{\sin 90°} \approx 0.94$$

25. $a = 3,\ b = 2,\ A = 50°$

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$
$$\frac{\sin B}{2} = \frac{\sin(50°)}{3}$$
$$\sin B = \frac{2\sin(50°)}{3} \approx 0.5107$$

$B = \sin^{-1}(0.5107)$

$B = 30.7°$ or $B = 149.3°$

The second value is discarded because $A + B > 180°$.

$C = 180° - A - B = 180° - 50° - 30.7° = 99.3°$

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$
$$\frac{\sin 99.3°}{c} = \frac{\sin 50°}{3}$$
$$c = \frac{3\sin 99.3°}{\sin 50°} \approx 3.86$$

One triangle: $B \approx 30.7°,\ C \approx 99.3°,\ c \approx 3.86$

26. $b = 4,\ c = 3,\ B = 40°$

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$
$$\frac{\sin 40°}{4} = \frac{\sin C}{3}$$
$$\sin C = \frac{3\sin 40°}{4} \approx 0.4821$$

$C = \sin^{-1}(0.4821)$

$C = 28.8°$ or $C = 151.2°$

The second value is discarded because $B + C > 180°$.

$A = 180° - B - C = 180° - 40° - 28.8° = 111.2°$

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$
$$\frac{\sin 40°}{4} = \frac{\sin 111.2°}{a}$$
$$a = \frac{4\sin 111.2°}{\sin 40°} \approx 5.80$$

One triangle: $A \approx 111.2°,\ C \approx 28.8°,\ a \approx 5.80$

27. $b = 5,\ c = 3,\ B = 100°$

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$
$$\frac{\sin 100°}{5} = \frac{\sin C}{3}$$
$$\sin C = \frac{3\sin 100°}{5} \approx 0.5909$$
$$C = \sin^{-1}(0.5909)$$
$$C = 36.2° \text{ or } C = 143.8°$$

The second value is discarded because $B + C > 180°$.

$$A = 180° - B - C = 180° - 100° - 36.2° = 43.8°$$
$$\frac{\sin B}{b} = \frac{\sin A}{a}$$
$$\frac{\sin 100°}{5} = \frac{\sin 43.8°}{a}$$
$$a = \frac{5\sin 43.8°}{\sin 100°} \approx 3.51$$

One triangle: $A \approx 43.8°,\ C \approx 36.2°,\ a \approx 3.51$

28. $a = 2,\ c = 1,\ A = 120°$

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$
$$\frac{\sin C}{1} = \frac{\sin 120°}{2}$$
$$\sin C = \frac{1\sin 120°}{2} \approx 0.4330$$
$$C = \sin^{-1}(0.4330)$$
$$C = 25.7° \text{ or } C = 154.3°$$

The second value is discarded because $A + C > 180°$.

$$B = 180° - A - C = 180° - 120° - 25.7° = 34.3°$$
$$\frac{\sin B}{b} = \frac{\sin A}{a}$$
$$\frac{\sin 34.3°}{b} = \frac{\sin 120°}{2}$$
$$b = \frac{2\sin 34.3°}{\sin 120°} \approx 1.30$$

One triangle: $B \approx 34.3°, C \approx 25.7°,\ b \approx 1.30$

29. $a = 4,\ b = 5,\ A = 60°$

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$
$$\frac{\sin B}{5} = \frac{\sin 60°}{4}$$
$$\sin B = \frac{5\sin 60°}{4} \approx 1.0825$$

There is no angle B for which $\sin B > 1$. Therefore, there is no triangle with the given measurements.

30. $b = 2,\ c = 3,\ B = 40°$

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$
$$\frac{\sin 40°}{2} = \frac{\sin C}{3}$$
$$\sin C = \frac{3\sin 40°}{2} \approx 0.9642$$
$$C = \sin^{-1}(0.9642)$$
$$C_1 = 74.6° \text{ or } C_2 = 105.4°$$

For both values, $B + C < 180°$. Therefore, there are two triangles.

$$A_1 = 180° - B - C_1 = 180° - 40° - 74.6° = 65.4°$$
$$\frac{\sin B}{b} = \frac{\sin A_1}{a_1}$$
$$\frac{\sin 40°}{2} = \frac{\sin 65.4°}{a_1}$$
$$a_1 = \frac{2\sin 65.4°}{\sin 40°} \approx 2.83$$
$$A_2 = 180° - B - C_2 = 180° - 40° - 105.4° = 34.6°$$
$$\frac{\sin B}{b} = \frac{\sin A_2}{a_2}$$
$$\frac{\sin 40°}{2} = \frac{\sin 34.6°}{a_2}$$
$$a_2 = \frac{2\sin 34.6°}{\sin 40°} \approx 1.77$$

Two triangles:
$A_1 \approx 65.4°,\ C_1 \approx 74.6°,\ a_1 \approx 2.83$ or
$A_2 \approx 34.6°, C_2 \approx 105.4°,\ a_2 \approx 1.77$

31. $b=4,\ c=6,\ B=20°$

$$\frac{\sin B}{b}=\frac{\sin C}{c}$$
$$\frac{\sin 20°}{4}=\frac{\sin C}{6}$$
$$\sin C=\frac{6\sin 20°}{4}\approx 0.5130$$
$$C=\sin^{-1}(0.5130)$$
$$C_1=30.9° \text{ or } C_2=149.1°$$

For both values, $B+C<180°$. Therefore, there are two triangles.

$$A_1=180°-B-C_1=180°-20°-30.9°=129.1°$$
$$\frac{\sin B}{b}=\frac{\sin A_1}{a_1}$$
$$\frac{\sin 20°}{4}=\frac{\sin 129.1°}{a_1}$$
$$a_1=\frac{4\sin 129.1°}{\sin 20°}\approx 9.08$$
$$A_2=180°-B-C_2=180°-20°-149.1°=10.9°$$
$$\frac{\sin B}{b}=\frac{\sin A_2}{a_2}$$
$$\frac{\sin 20°}{4}=\frac{\sin 10.9°}{a_2}$$
$$a_2=\frac{4\sin 10.9°}{\sin 20°}\approx 2.21$$

Two triangles:
$A_1\approx 129.1°,\ C_1\approx 30.9°,\ a_1\approx 9.08$ or
$A_2\approx 10.9°,\ C_2\approx 149.1°,\ a_2\approx 2.21$

32. $a=3,\ b=7,\ A=70°$

$$\frac{\sin B}{7}=\frac{\sin 70°}{3}$$
$$\sin B=\frac{7\sin 70°}{3}\approx 2.1926$$

There is no angle B for which $\sin B>1$. Thus, there is no triangle with the given measurements.

33. $a=2,\ c=1,\ C=100°$

$$\frac{\sin C}{c}=\frac{\sin A}{a}$$
$$\frac{\sin 100°}{1}=\frac{\sin A}{2}$$
$$\sin A=2\sin 100°\approx 1.9696$$

There is no angle A for which $\sin A>1$. Thus, there is no triangle with the given measurements.

34. $b=4,\ c=5,\ B=95°$

$$\frac{\sin C}{c}=\frac{\sin B}{b}$$
$$\frac{\sin C}{5}=\frac{\sin 95°}{4}$$
$$\sin C=\frac{5\sin 95°}{4}\approx 1.2452$$

There is no angle C for which $\sin C>1$. Thus, there is no triangle with the given measurements.

35. $a=2,\ c=1,\ C=25°$

$$\frac{\sin A}{a}=\frac{\sin C}{c}$$
$$\frac{\sin A}{2}=\frac{\sin 25°}{1}$$
$$\sin A=\frac{2\sin 25°}{1}\approx 0.8452$$
$$A=\sin^{-1}(0.8452)$$
$$A_1=57.7° \text{ or } A_2=122.3°$$

For both values, $A+C<180°$. Therefore, there are two triangles.

$$B_1=180°-A_1-C=180°-57.7°-25°=97.3°$$
$$\frac{\sin B_1}{b_1}=\frac{\sin C}{c}$$
$$\frac{\sin 97.3°}{b_1}=\frac{\sin 25°}{1}$$
$$b_1=\frac{1\sin 97.3°}{\sin 25°}\approx 2.35$$
$$B_2=180°-A_2-\gamma=180°-122.3°-25°=32.7°$$
$$\frac{\sin B_2}{b_2}=\frac{\sin C}{c}$$
$$\frac{\sin 32.7°}{b_2}=\frac{\sin 25°}{1}$$
$$b_2=\frac{1\sin 32.7°}{\sin 25°}\approx 1.28$$

Two triangles:
$A_1\approx 57.7°,\ B_1\approx 97.3°,\ b_1\approx 2.35$ or
$A_2\approx 122.3°,\ B_2\approx 32.7°,\ b_2\approx 1.28$

36. $b=4,\ c=5,\ B=40°$

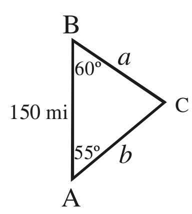
$$\frac{\sin B}{b}=\frac{\sin C}{c}$$
$$\frac{\sin 40°}{4}=\frac{\sin C}{5}$$
$$\sin C=\frac{5\sin 40°}{4}\approx 0.8035$$
$$C=\sin^{-1}(0.8035)$$
$$C_1=53.5° \text{ or } C_2=126.5°$$

For both values, $B+C<180°$. Therefore, there are two triangles.

$$A_1=180°-B-C_1=180°-40°-53.5°=86.5°$$
$$\frac{\sin B}{b}=\frac{\sin A_1}{a_1}$$
$$\frac{\sin 40°}{4}=\frac{\sin 86.5°}{a_1}$$
$$a_1=\frac{4\sin 86.5°}{\sin 40°}\approx 6.21$$
$$A_2=180°-B-C_2=180°-40°-126.5°=13.5°$$
$$\frac{\sin B}{b}=\frac{\sin A_2}{a_2}$$
$$\frac{\sin 40°}{4}=\frac{\sin 13.5°}{a_2}$$
$$a_2=\frac{4\sin 13.5°}{\sin 40°}\approx 1.45$$

Two triangles:
$A_1\approx 86.5°,\ C_1\approx 53.5°,\ a_1\approx 6.21$ or
$A_2\approx 13.5°,\ C_2\approx 126.5°,\ a_2\approx 1.45$

37. a. Find C; then use the Law of Sines (see figure):



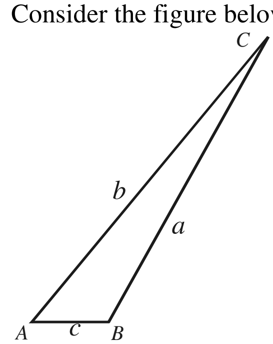
$$C=180°-60°-55°=65°$$
$$\frac{\sin 55°}{a}=\frac{\sin 65°}{150}$$
$$a=\frac{150\sin 55°}{\sin 65°}\approx 135.58 \text{ miles}$$
$$\frac{\sin 60°}{b}=\frac{\sin 65°}{150}$$
$$b=\frac{150\sin 60°}{\sin 65°}\approx 143.33 \text{ miles}$$

Station Able is about 143.33 miles from the ship, and Station Baker is about 135.58 miles from the ship.

b. $t=\dfrac{a}{r}=\dfrac{135.6}{200}\approx 0.68$ hours

$$0.68 \text{ hr}\cdot 60\ \frac{\text{min}}{\text{hr}}\approx 41 \text{ minutes}$$

38. Consider the figure below.



We are given that $A=49.8974°$,
$B=180°-49.9312°=130.0688°$, and
$c=300$ km.
Using angles A and B, we can find
$$C=180°-130.0688°-49.8974°$$
$$=0.0338°$$
Using the Law of Sines, we can determine b and a.

$$\frac{\sin B}{b}=\frac{\sin C}{c}$$
$$b=\frac{c\cdot\sin B}{\sin C}=\frac{300\cdot\sin 130.0688°}{\sin 0.0338°}\approx 389{,}173.319$$
$$\frac{\sin A}{a}=\frac{\sin C}{c}$$
$$a=\frac{c\cdot\sin A}{\sin C}=\frac{300\cdot\sin 49.8974°}{\sin 0.0338°}\approx 388{,}980.139$$

At the time of the measurements, the moon was about 389,000 km from Earth.

39. $\angle QPR = 180° - 25° = 155°$

$\angle PQR = 180° - 155° - 15° = 10°$

Let c represent the distance from P to Q.

$$\frac{\sin 15°}{c} = \frac{\sin 10°}{1000}$$

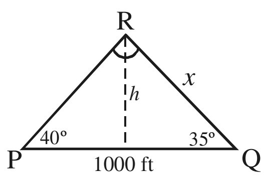
$$c = \frac{1000 \sin 15°}{\sin 10°} \approx 1490.48 \text{ feet}$$

40. From Problem 39, we have that the distance from P to Q is 1490.48 feet. Let h represent the distance from Q to D.

$$\sin 25° = \frac{h}{1490.48}$$

$$h = 1490.48 \sin 25° \approx 629.90 \text{ feet}$$

41. Let h = the height of the plane and x = the distance from Q to the plane (see figure).



$\angle PRQ = 180° - 40° - 35° = 105°$

$$\frac{\sin 40°}{x} = \frac{\sin 105°}{1000}$$

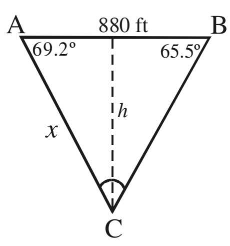
$$x = \frac{1000 \sin 40°}{\sin 105°} \approx 665.46 \text{ feet}$$

$$\sin 35° = \frac{h}{x} = \frac{h}{665.46}$$

$$h = (665.46) \sin 35° \approx 381.69 \text{ feet}$$

The plane is about 381.69 feet high.

42. Let h = the height of the bridge, x = the distance from C to point A (see figure).



$\angle ACB = 180° - 69.2° - 65.5° = 45.3°$

$$\frac{\sin 65.5°}{x} = \frac{\sin 45.3°}{880}$$

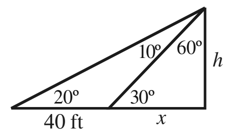
$$x = \frac{880 \sin 65.5°}{\sin 45.3°} \approx 1126.57 \text{ feet}$$

$$\sin 69.2° = \frac{h}{x} = \frac{h}{1126.57}$$

$$h = (1126.57) \sin 69.2° \approx 1053.15 \text{ feet}$$

The bridge is about 1053.15 feet high.

43. Let h = the height of the tree, and let x = the distance from the first position to the center of the tree (see figure).



Using the Law of Sines twice yields two equations relating x and h.

Equation 1: $\dfrac{\sin 30°}{h} = \dfrac{\sin 60°}{x}$

$$x = \frac{h \sin 60°}{\sin 30°}$$

Equation 2: $\dfrac{\sin 20°}{h} = \dfrac{\sin 70°}{x + 40}$

$$x = \frac{h \sin 70°}{\sin 20°} - 40$$

Set the two equations equal to each other and solve for h.

$$\frac{h \sin 60°}{\sin 30°} = \frac{h \sin 70°}{\sin 20°} - 40$$

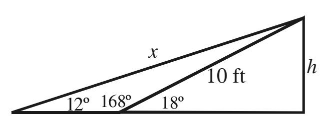
$$h\left(\frac{\sin 60°}{\sin 30°} - \frac{\sin 70°}{\sin 20°}\right) = -40$$

$$h = \frac{-40}{\dfrac{\sin 60°}{\sin 30°} - \dfrac{\sin 70°}{\sin 20°}}$$

$$\approx 39.4 \text{ feet}$$

The height of the tree is about 39.4 feet.

44. Let x = the length of the new ramp (see figure).



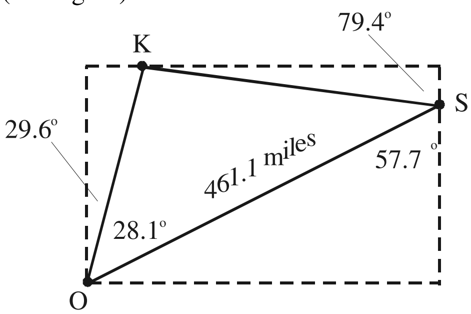
Using the Law of Sines:

$$\frac{\sin 162°}{x} = \frac{\sin 12°}{10}$$

$$x = \frac{10 \sin 162°}{\sin 12°} \approx 14.86 \text{ feet}$$

The new ramp is about 14.86 feet long.

45. Note that $\angle KOS = 57.7° - 29.6° = 28.1°$ (See figure)



From the diagram we find that
$\angle KSO = 180° - 79.4° - 57.7° = 42.9°$ and
$\angle OKS = 180° - 28.1° - 42.9° = 109.0°$.
We can use the Law of Sines to find the distance between Oklahoma City and Kansas City, as well as the distance between Kansas City and St. Louis.

$$\frac{\sin 42.9°}{OK} = \frac{\sin 109.0°}{461.1}$$

$$OK = \frac{461.1 \sin 42.9°}{\sin 109.0°} \approx 332.0$$

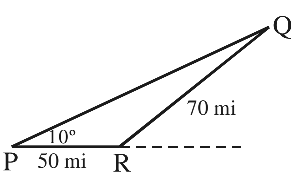
$$\frac{\sin 28.1°}{KS} = \frac{\sin 109.0°}{461.1}$$

$$KS = \frac{461.1 \sin 28.1°}{\sin 109.0°} \approx 229.7$$

Therefore, the total distance using the connecting flight is $332.0 + 229.7 = 561.7$ miles. Using the connecting flight, Adam would receive $561.7 - 461.1 = 100.6$ more frequent flyer miles.

46. The time of the actual trip was:

$$t = \frac{50+70}{250} = \frac{120}{250} = 0.48 \text{ hour}$$



$RQ = 70,\ PR = 50,\ P = 10°$
Solve the triangle:

$$\frac{\sin 10°}{70} = \frac{\sin Q}{50}$$

$$\sin Q = \frac{50 \sin 10°}{70} \approx 0.1240$$

$$Q = \sin^{-1}(0.1240) \approx 7.125°$$

$$R = 180° - 10° - 7.125° = 162.875°$$

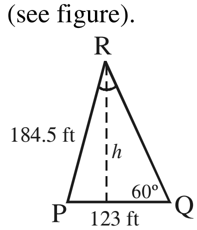
$$\frac{\sin 10°}{70} = \frac{\sin 162.875°}{PQ}$$

$$PQ = \frac{70 \sin 162.875°}{\sin 10°} \approx 118.7$$

$$t = \frac{118.67}{250} = 0.478 \text{ hour}$$

The trip should have taken 0.4748 hour but, because of the incorrect course, took 0.48 hour. Thus, the trip took 0.0052 hour, or about 18.7 seconds, longer.

47. Let h = the perpendicular distance from R to PQ, (see figure).



$$\frac{\sin R}{123} = \frac{\sin 60°}{184.5}$$

$$\sin R = \frac{123 \sin 60°}{184.5} \approx 0.5774$$

$$R = \sin^{-1}(0.5774) \approx 35.3°$$

$$\angle RPQ = 180° - 60° - 35.3° \approx 84.7°$$

$$\sin 84.7° = \frac{h}{184.5}$$

$$h = 184.5 \sin 84.7° \approx 183.71 \text{ feet}$$

48. Let $\theta = \angle AOP$

$$\frac{\sin\theta}{9} = \frac{\sin 15°}{3}$$

$$\sin\theta = \frac{9\sin 15°}{3} \approx 0.7765$$

$$\theta = \sin^{-1}(0.7765) \approx 50.94°$$

or $\theta \approx 180° - 50.94° = 129.06°$

$A = 114.06°$ or $A = 35.94°$

$$\frac{\sin 114.06°}{a} = \frac{\sin 15°}{3}$$

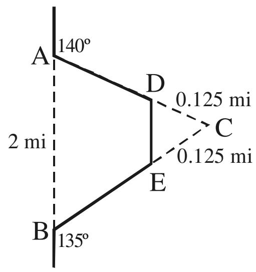
$$a = \frac{3\sin 114.06°}{\sin 15°} \approx 10.58 \text{ inches}$$

or $\dfrac{\sin 35.94°}{a} = \dfrac{\sin 15°}{3}$

$$a = \frac{3\sin 35.94°}{\sin 15°} \approx 6.80 \text{ inches}$$

The approximate distance from the piston to the center of the crankshaft is either 6.80 inches or 10.58 inches.

49. $A = 180° - 140° = 40°$; $B = 180° - 135° = 45°$;
$C = 180° - 40° - 45° = 95°$

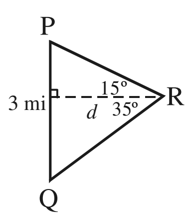


$$\frac{\sin 40°}{BC} = \frac{\sin 95°}{2}$$
$$BC = \frac{2\sin 40°}{\sin 95°} \approx 1.290 \text{ mi}$$
$$\frac{\sin 45°}{AC} = \frac{\sin 95°}{2}$$
$$AC = \frac{2\sin 45°}{\sin 95°} \approx 1.420 \text{ mi}$$
$$BE = 1.290 - 0.125 = 1.165 \text{ mi}$$
$$AD = 1.420 - 0.125 = 1.295 \text{ mi}$$

For the isosceles triangle,
$$\angle CDE = \angle CED = \frac{180° - 95°}{2} = 42.5°$$
$$\frac{\sin 95°}{DE} = \frac{\sin 42.5°}{0.125}$$
$$DE = \frac{0.125\sin 95°}{\sin 42.5°} \approx 0.184 \text{ miles}$$

The approximate length of the highway is
$AD + DE + BE = 1.295 + 0.184 + 1.165 \approx 2.64$ mi.

50. Let PR = the distance from lighthouse P to the ship, QR = the distance from lighthouse Q to the ship, and d = the distance from the ship to the shore. From the diagram, $\angle QPR = 75°$ and $\angle PQR = 55°$, where point R is the ship.

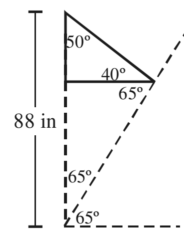


a. Use the Law of Sines:
$$\frac{\sin 50°}{3} = \frac{\sin 55°}{PR}$$
$$PR = \frac{3\sin 55°}{\sin 50°} \approx 3.21 \text{ miles}$$
The ship is about 3.21 miles from lighthouse P.

b. Use the Law of Sines:
$$\frac{\sin 50°}{3} = \frac{\sin 75°}{QR}$$
$$QR = \frac{3\sin 75°}{\sin 50°} \approx 3.78 \text{ miles}$$
The ship is about 3.78 miles from lighthouse Q.

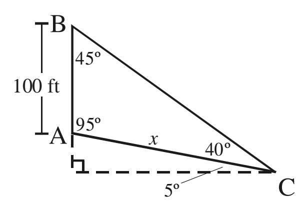
c. Use the Law of Sines:
$$\frac{\sin 90°}{3.2} = \frac{\sin 75°}{d}$$
$$d = \frac{3.2\sin 75°}{\sin 90°} \approx 3.10 \text{ miles}$$
The ship is about 3.10 miles from the shore.

51. Determine other angles in the figure:



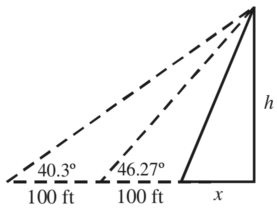
Using the Law of Sines:
$$\frac{\sin(65+40)°}{88} = \frac{\sin 25°}{L}$$
$$L = \frac{88\sin 25°}{\sin 105°} \approx 38.5 \text{ inches}$$
The awning is about 38.5 inches long.

52. The tower forms an angle of 95° with the ground. Let x be the distance from the ranger to the tower.



$$\angle ABC = 180° - 95° - 40° = 45°$$
$$\frac{\sin 40°}{100} = \frac{\sin 45°}{x}$$
$$x = \frac{100\sin 45°}{\sin 40°} \approx 110.01 \text{ feet}$$
The ranger is about 110.01 feet from the tower.

53. Let h = height of the pyramid, and let x = distance from the edge of the pyramid to the point beneath the tip of the pyramid (see figure).



Using the Law of Sines twice yields two equations relating x and y:

Equation 1: $$\frac{\sin 46.27°}{h} = \frac{\sin(90° - 46.27°)}{x+100}$$

$$(x+100)\sin 46.27° = h\sin 43.73°$$

$$x\sin 46.27° + 100\sin 46.27° = h\sin 43.73°$$

$$x = \frac{h\sin 43.73° - 100\sin 46.27°}{\sin 46.27°}$$

Equation 2: $$\frac{\sin 40.3°}{h} = \frac{\sin(90° - 40.3°)}{x+200}$$

$$(x+200)\sin 40.3° = h\sin 49.7°$$

$$x\sin 40.3° + 200\sin 40.3° = h\sin 49.7°$$

$$x = \frac{h\sin 49.7° - 200\sin 40.3°}{\sin 40.3°}$$

Set the two equations equal to each other and solve for h.

$$\frac{h\sin 43.73° - 100\sin 46.27°}{\sin 46.27°} = \frac{h\sin 49.7° - 200\sin 40.3°}{\sin 40.3°}$$

$$h\sin 43.73°\cdot\sin 40.3° - 100\sin 46.27°\cdot\sin 40.3° = h\sin 49.7°\cdot\sin 46.27° - 200\sin 40.3°\cdot\sin 46.27°$$

$$h\sin 43.73°\cdot\sin 40.3° - h\sin 49.7°\cdot\sin 46.27° = 100\sin 46.27°\cdot\sin 40.3° - 200\sin 40.3°\cdot\sin 46.27°$$

$$h = \frac{100\sin 46.27°\cdot\sin 40.3° - 200\sin 40.3°\cdot\sin 46.27°}{\sin 43.73°\cdot\sin 40.3° - \sin 49.7°\cdot\sin 46.27°}$$

$$\approx 449.36 \text{ feet}$$

The current height of the pyramid is about 449.36 feet.

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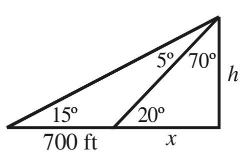
54. Let h = the height of the aircraft, and let x = the distance from the first sensor to a point on the ground beneath the airplane (see figure).



Using the Law of Sines twice yields two equations relating x and h.

Equation 1: $$\frac{\sin 20°}{h} = \frac{\sin 70°}{x}$$

$$x = \frac{h\sin 70°}{\sin 20°}$$

Equation 2: $$\frac{\sin 15°}{h} = \frac{\sin 75°}{x+700}$$

$$x = \frac{h\sin 75°}{\sin 15°} - 700$$

Set the equations equal to each other and solve for h.

$$\frac{h\sin 70°}{\sin 20°} = \frac{h\sin 75°}{\sin 15°} - 700$$

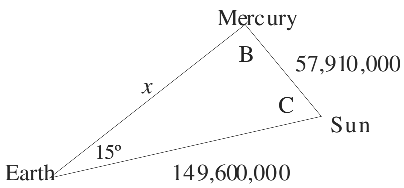
$$h\left(\frac{\sin 70°}{\sin 20°} - \frac{\sin 75°}{\sin 15°}\right) = -700$$

$$h = \frac{-700}{\frac{\sin 70°}{\sin 20°} - \frac{\sin 75°}{\sin 15°}}$$

$$\approx 710.97 \text{ feet}$$

The height of the aircraft is about 710.97 feet.

55. Using the Law of Sines:



$$\frac{\sin 15°}{57,910,000} = \frac{\sin B}{149,600,000}$$
$$\sin B = \frac{149,600,000 \cdot \sin 15°}{57,910,000} = \frac{14,960 \cdot \sin 15°}{5791}$$
$$B = \sin^{-1}\left(\frac{14,960 \cdot \sin 15°}{5791}\right) \approx 41.96°$$

or

$$B \approx 138.04°$$

$C \approx 180° - 41.96° - 15° = 123.04°$ or
$C \approx 180° - 138.04° - 15° = 26.96°$

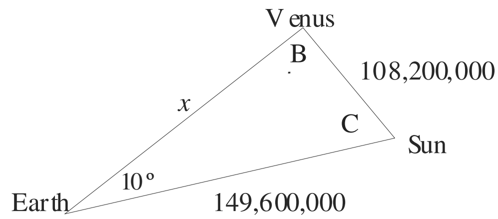
$$\frac{\sin 15°}{57,910,000} = \frac{\sin C}{x}$$
$$x = \frac{57,910,000 \cdot \sin C}{\sin 15°} = \frac{57,910,000 \cdot \sin 123.04°}{\sin 15°} \approx 187,564,951.5 \text{ km}$$

or

$$x = \frac{57,910,000 \cdot \sin 26.96°}{\sin 15°} \approx 101,439,834.5 \text{ km}$$

So the possible distances between Earth and Mercury are approximately 101,440,000 km and 187,600,000 km.

56. Using the Law of Sines:



$$\frac{\sin 10°}{108,200,000} = \frac{\sin B}{149,600,000}$$
$$\sin B = \frac{149,600,000 \cdot \sin 10°}{108,200,000} = \frac{1496 \cdot \sin 10°}{1082}$$
$$B = \sin^{-1}\left(\frac{1496 \cdot \sin 10°}{1082}\right) \approx 13.892°$$

or $B \approx 166.108°$

$C \approx 180° - 13.892° - 10° = 156.108°$ or
$C \approx 180° - 166.108° - 10° = 13.892°$

$$\frac{\sin 10°}{108,200,000} = \frac{\sin C}{x}$$
$$x = \frac{108,200,000 \cdot \sin C}{\sin 10°}$$
$$x = \frac{108,200,000 \cdot \sin 156.108°}{\sin 10°} \approx 252,363,760.4 \text{ km}$$

or

$$x = \frac{108,200,000 \cdot \sin 3.892°}{\sin 10°} \approx 42,293,457.3 \text{ km}$$

So the approximate possible distances between Earth and Venus are 42,300,000 km and 252,400,000 km.

57. Since there are 36 equally spaced cars, each car is separated by $\frac{360°}{36} = 10°$. The angle between the radius and a line segment connecting consecutive cars is $\frac{170°}{2} = 85°$ (see figure). If we let r = the radius of the wheel, we get

$$\frac{\sin 10°}{22} = \frac{\sin 85°}{r}$$
$$r = \frac{22 \sin 85°}{\sin 10°} \approx 126$$

The length of the diameter of the wheel is approximately $d = 2r = 2(126) = 252$ feet.

58. $\frac{a+b}{c}=\frac{a}{c}+\frac{b}{c}=\frac{\sin A}{\sin C}+\frac{\sin B}{\sin C}$

$$=\frac{\sin A+\sin B}{\sin C}$$

$$=\frac{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{2\sin\left(\frac{C}{2}\right)\cos\left(\frac{C}{2}\right)}$$

$$=\frac{\sin\left(\frac{\pi}{2}-\frac{C}{2}\right)\cos\left(\frac{A-B}{2}\right)}{\sin\left(C\right)\cos\left(\frac{C}{2}\right)}$$

$$=\frac{\cos\left(\frac{C}{2}\right)\cos\left(\frac{A-B}{2}\right)}{\sin\left(\frac{C}{2}\right)\cos\left(\frac{C}{2}\right)}$$

$$=\frac{\cos\left(\frac{A-B}{2}\right)}{\sin\left(\frac{C}{2}\right)}=\frac{\cos\left[\frac{1}{2}(A-B)\right]}{\sin\left(\frac{1}{2}C\right)}$$

59. $\frac{a-b}{c}=\frac{a}{c}-\frac{b}{c}=\frac{\sin A}{\sin C}-\frac{\sin B}{\sin C}=\frac{\sin A-\sin B}{\sin C}$

$$=\frac{2\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)}{\sin\left(2\cdot\frac{C}{2}\right)}$$

$$=\frac{2\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)}{2\sin\left(\frac{C}{2}\right)\cos\left(\frac{C}{2}\right)}$$

$$=\frac{\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{\pi}{2}-\frac{C}{2}\right)}{\sin\left(\frac{C}{2}\right)\cos\left(\frac{C}{2}\right)}$$

$$=\frac{\sin\left(\frac{A-B}{2}\right)\sin\left(\frac{C}{2}\right)}{\sin\left(\frac{C}{2}\right)\cos\left(\frac{C}{2}\right)}$$

$$=\frac{\sin\left(\frac{A-B}{2}\right)}{\cos\left(\frac{C}{2}\right)}=\frac{\sin\left[\frac{1}{2}(A-B)\right]}{\cos\left(\frac{1}{2}C\right)}$$

60. $a=\frac{b\sin A}{\sin B}=\frac{b\sin[180^\circ-(B+\gamma)]}{\sin B}$

$$=\frac{b}{\sin B}\sin(B+C)$$

$$=\frac{b}{\sin B}\left(\sin B\cos C+\cos B\sin C\right)$$

$$=b\cos C+\frac{b\sin C}{\sin B}\cos B$$

$$=b\cos C+c\cos B$$

61. $\frac{a-b}{a+b}=\frac{\frac{a-b}{c}}{\frac{a+b}{c}}$

$$=\frac{\frac{\sin\left[\frac{1}{2}(A-B)\right]}{\cos\left(\frac{1}{2}C\right)}}{\frac{\cos\left[\frac{1}{2}(A-B)\right]}{\sin\left(\frac{1}{2}C\right)}}$$

$$=\frac{\sin\left[\frac{1}{2}(A-B)\right]}{\cos\left(\frac{1}{2}C\right)}\cdot\frac{\sin\left(\frac{1}{2}C\right)}{\cos\left[\frac{1}{2}(A-B)\right]}$$

$$=\tan\left[\frac{1}{2}(A-B)\right]\tan\left(\frac{1}{2}C\right)$$

$$=\tan\left[\frac{1}{2}(A-B)\right]\tan\left[\frac{1}{2}\left(\pi-(A+B)\right)\right]$$

$$=\tan\left[\frac{1}{2}(A-B)\right]\tan\left[\frac{\pi}{2}-\left(\frac{A+B}{2}\right)\right]$$

$$=\tan\left[\frac{1}{2}(A-B)\right]\cot\left(\frac{A+B}{2}\right)$$

$$=\frac{\tan\left[\frac{1}{2}(A-B)\right]}{\tan\left[\frac{1}{2}(A+B)\right]}$$

62. $\sin B=\sin\left(\angle PQR\right)=\sin\left(\angle PP'R\right)=\frac{b}{2r}$

$$\frac{\sin B}{b}=\frac{1}{2r}$$

The result follows from the Law of Sines.

63 – 65. Answers will vary.

Section 8.3

1. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

2. $\cos\theta = \dfrac{\sqrt{2}}{2}$

$\theta = 45° \text{ or } \frac{\pi}{4}$

The solution set is $\{45°\}$ or $\{\frac{\pi}{4}\}$

3. Cosines

4. Sines

5. Cosines

6. False: Use the Law of Cosines

7. False

8. True

9. $a = 2,\ c = 4,\ A = 45°$

$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$b^2 = 2^2 + 4^2 - 2\cdot 2\cdot 4\cos 45°$$
$$= 20 - 16\cdot\frac{\sqrt{2}}{2}$$
$$= 20 - 8\sqrt{2}$$
$$b = \sqrt{20 - 8\sqrt{2}} \approx 2.95$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$2bc\cos A = b^2 + c^2 - a^2$$
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
$$\cos A = \frac{2.95^2 + 4^2 - 2^2}{2(2.95)(4)} = \frac{20.7025}{23.6}$$
$$A = \cos^{-1}\left(\frac{20.7025}{23.6}\right) \approx 28.7°$$
$$C = 180° - A - B \approx 180° - 28.7° - 45° \approx 106.3°$$

10. $b = 3,\ c = 4,\ A = 30°$

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$a^2 = 3^2 + 4^2 - 2\cdot 3\cdot 4\cos 30°$$
$$= 25 - 24\left(\frac{\sqrt{3}}{2}\right)$$
$$= 25 - 12\sqrt{3}$$
$$a = \sqrt{25 - 12\sqrt{3}} \approx 2.05$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$
$$\cos C = \frac{2.05^2 + 3^2 - 4^2}{2(2.05)(3)} = \frac{-2.7975}{12.3}$$
$$C = \cos^{-1}\left(\frac{-2.7975}{12.3}\right) \approx 103.1°$$
$$B = 180° - A - C \approx 180° - 30° - 103.1° = 46.9°$$

11. $a = 2,\ b = 3,\ C = 95°$

$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$c^2 = 2^2 + 3^2 - 2\cdot 2\cdot 3\cos 95° = 13 - 12\cos 95°$$
$$c = \sqrt{13 - 12\cos 95°} \approx 3.75$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
$$\cos A = \frac{3^2 + 3.75^2 - 2^2}{2(3)(3.75)} = \frac{19.0625}{22.5}$$
$$A = \cos^{-1}\left(\frac{19.0625}{22.5}\right) \approx 32.1°$$
$$B = 180° - A - C \approx 180° - 32.1° - 95° = 52.9°$$

12. $a = 2,\ c = 5,\ B = 20°$

$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$b^2 = 2^2 + 5^2 - 2\cdot 2\cdot 5\cos 20° = 29 - 20\cos 20°$$
$$b = \sqrt{29 - 20\cos 20°} \approx 3.19$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
$$\cos A = \frac{\left(\sqrt{29 - 20\cos 20°}\right)^2 + 5^2 - 2^2}{2(\sqrt{29 - 20\cos 20°})(5)} \approx 0.97681$$
$$A = \cos^{-1}(0.97681) \approx 12.4°$$
$$C = 180° - A - B \approx 180° - 12.4° - 20° = 147.6°$$

13. $a = 6,\ b = 5,\ c = 8$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{5^2 + 8^2 - 6^2}{2(5)(8)} = \frac{53}{80}$$

$$A = \cos^{-1}\left(\frac{53}{80}\right) \approx 48.5°$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos B = \frac{6^2 + 8^2 - 5^2}{2(6)(8)} = \frac{75}{96}$$

$$B = \cos^{-1}\left(\frac{75}{96}\right) \approx 38.6°$$

$$C = 180° - A - B \approx 180° - 48.5° - 38.6° = 92.9°$$

14. $a = 8,\ b = 5,\ c = 4$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{5^2 + 4^2 - 8^2}{2(5)(4)} = -\frac{23}{80}$$

$$A = \cos^{-1}\left(-\frac{23}{80}\right) \approx 125.1°$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{8^2 + 4^2 - 5^2}{2(8)(4)} = \frac{55}{64}$$

$$B = \cos^{-1}\left(\frac{55}{64}\right) \approx 30.8°$$

$$C = 180° - A - B \approx 180° - 125.1° - 30.8° = 24.1°$$

15. $a = 9,\ b = 6,\ c = 4$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{6^2 + 4^2 - 9^2}{2(6)(4)} = -\frac{29}{48}$$

$$A = \cos^{-1}\left(-\frac{29}{48}\right) \approx 127.2°$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{9^2 + 4^2 - 6^2}{2(9)(4)} = \frac{61}{72}$$

$$B = \cos^{-1}\left(\frac{61}{72}\right) \approx 32.1°$$

$$C = 180° - A - B \approx 180° - 127.2° - 32.1° = 20.7°$$

16. $a = 4,\ b = 3,\ c = 4$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{3^2 + 4^2 - 4^2}{2(3)(4)} = \frac{9}{24}$$

$$A = \cos^{-1}\left(\frac{9}{24}\right) \approx 68.0°$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{4^2 + 4^2 - 3^2}{2(4)(4)} = \frac{23}{32}$$

$$B = \cos^{-1}\left(\frac{23}{32}\right) \approx 44.0°$$

$$C = 180° - A - B \approx 180° - 68.0° - 44.0° = 68.0°$$

17. $a = 3,\ b = 4,\ C = 40°$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c^2 = 3^2 + 4^2 - 2\cdot 3\cdot 4\cos 40° = 25 - 24\cos 40°$$

$$c = \sqrt{25 - 24\cos 40°} \approx 2.57$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{4^2 + 2.57^2 - 3^2}{2(4)(2.57)} = \frac{13.6049}{20.56}$$

$$A = \cos^{-1}\left(\frac{13.6049}{20.56}\right) \approx 48.6°$$

$$B = 180° - A - C \approx 180° - 48.6° - 40° = 91.4°$$

18. $a = 2,\ c = 1,\ B = 10°$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$b^2 = 2^2 + 1^2 - 2\cdot 2\cdot 1\cos 10° = 5 - 4\cos 10°$$

$$b = \sqrt{5 - 4\cos 10°} \approx 1.03$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{1.03^2 + 1^2 - 2^2}{2(1.03)(1)} = -\frac{1.9391}{2.06}$$

$$A = \cos^{-1}\left(-\frac{1.9391}{2.06}\right) \approx 160.3°$$

$$C = 180° - A - B \approx 180° - 160.3° - 10° = 9.7°$$

19. $b = 1,\ c = 3,\ A = 80°$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$a^2 = 1^2 + 3^2 - 2\cdot 1\cdot 3\cos 80° = 10 - 6\cos 80°$$

$$a = \sqrt{10 - 6\cos 80°} \approx 2.99$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{2.99^2 + 3^2 - 1^2}{2(2.99)(3)} = \frac{16.9401}{17.94}$$
$$B = \cos^{-1}\left(\frac{16.9401}{17.94}\right) \approx 19.2°$$
$$C = 180° - A - B \approx 180° - 80° - 19.2° = 80.8°$$

20. $a = 6,\ b = 4,\ C = 60°$
$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$c^2 = 6^2 + 4^2 - 2\cdot 6\cdot 4\cos 60° = 28$$
$$c = \sqrt{28} \approx 5.29$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{4^2 + 5.29^2 - 6^2}{2(4)(5.29)} = \frac{7.9841}{42.32}$$
$$A = \cos^{-1}\left(\frac{7.9841}{42.32}\right) \approx 79.1°$$
$$B = 180° - A - C \approx 180° - 79.1° - 60° = 40.9°$$

21. $a = 3,\ c = 2,\ B = 110°$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$b^2 = 3^2 + 2^2 - 2\cdot 3\cdot 2\cos 110° = 13 - 12\cos 110°$$
$$b = \sqrt{13 - 12\cos 110°} \approx 4.14$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{3^2 + 4.14^2 - 2^2}{2(3)(4.14)} = \frac{22.1396}{24.84}$$
$$C = \cos^{-1}\left(\frac{22.1396}{24.84}\right) \approx 27.0°$$
$$A = 180° - B - C \approx 180° - 110° - 27.0° = 43.0°$$

22. $b = 4,\ c = 1,\ A = 120°$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$a^2 = 4^2 + 1^2 - 2\cdot 4\cdot 1\cos 120° = 21$$
$$a = \sqrt{21} \approx 4.58$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{4.58^2 + 4^2 - 1^2}{2(4.58)(4)} = \frac{35.9764}{36.64}$$
$$C = \cos^{-1}\left(\frac{35.9764}{36.64}\right) \approx 10.9°$$
$$B = 180° - A - C \approx 180° - 120° - 10.9° = 49.1°$$

23. $a = 2,\ b = 2,\ C = 50°$
$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$c^2 = 2^2 + 2^2 - 2\cdot 2\cdot 2\cos 50° = 8 - 8\cos 50°$$
$$c = \sqrt{8 - 8\cos 50°} \approx 1.69$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{2^2 + 1.69^2 - 2^2}{2(2)(1.69)} = \frac{2.8561}{6.76}$$
$$A = \cos^{-1}\left(\frac{2.8561}{6.76}\right) \approx 65.0°$$
$$B = 180° - A - C \approx 180° - 65.0° - 50° = 65.0°$$

24. $a = 3,\ c = 2,\ B = 90°$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$b^2 = 3^2 + 2^2 - 2\cdot 3\cdot 2\cos 90° = 13$$
$$b = \sqrt{13} \approx 3.61$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{(\sqrt{13})^2 + 2^2 - 3^2}{2(\sqrt{13})(2)} \approx 0.55470$$
$$A = \cos^{-1}(0.55470) \approx 56.3°$$
$$C = 180° - A - B \approx 10° - 56.3° - 90° = 33.7°$$

25. $a = 12,\ b = 13,\ c = 5$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{13^2 + 5^2 - 12^2}{2(13)(5)} = \frac{50}{130}$$
$$A = \cos^{-1}\left(\frac{50}{130}\right) \approx 67.4°$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{12^2 + 5^2 - 13^2}{2(12)(5)} = 0$$
$$B = \cos^{-1} 0 = 90°$$
$$C = 180° - A - B \approx 180° - 67.4° - 90° = 22.6°$$

26. $a = 4,\ b = 5,\ c = 3$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{5^2 + 3^2 - 4^2}{2(5)(3)} = 0.6$$

$$A = \cos^{-1} 0.6 \approx 53.1°$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{4^2 + 3^2 - 5^2}{2(4)(3)} = 0$$

$$B = \cos^{-1} 0 = 90°$$

$$C = 180° - A - B \approx 180° - 53.1° - 90° = 36.9°$$

27. $a = 2,\ b = 2,\ c = 2$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{2^2 + 2^2 - 2^2}{2(2)(2)} = 0.5$$

$$A = \cos^{-1} 0.5 = 60°$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{2^2 + 2^2 - 2^2}{2(2)(2)} = 0.5$$

$$B = \cos^{-1} 0.5 = 60°$$

$$C = 180° - A - B \approx 180° - 60° - 60° = 60°$$

28. $a = 3,\ b = 3,\ c = 2$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{3^2 + 2^2 - 3^2}{2(3)(2)} = \frac{1}{3}$$

$$A = \cos^{-1}\left(\frac{1}{3}\right) \approx 70.5°$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{3^2 + 2^2 - 3^2}{2(3)(2)} = \frac{1}{3}$$

$$B = \cos^{-1}\left(\frac{1}{3}\right) \approx 70.5°$$

$$C = 180° - A - B \approx 180° - 70.5° - 70.5° = 39.0°$$

29. $a = 5,\ b = 8,\ c = 9$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{8^2 + 9^2 - 5^2}{2(8)(9)} = \frac{120}{144}$$

$$A = \cos^{-1}\left(\frac{120}{144}\right) \approx 33.6°$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{5^2 + 9^2 - 8^2}{2(5)(9)} = \frac{42}{90}$$

$$B = \cos^{-1}\left(\frac{42}{90}\right) \approx 62.2°$$

$$C = 180° - A - B \approx 180° - 33.6° - 62.2° = 84.2°$$

30. $a = 4,\ b = 3,\ c = 6$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{3^2 + 6^2 - 4^2}{2(3)(6)} = \frac{29}{36}$$

$$A = \cos^{-1}\left(\frac{29}{36}\right) \approx 36.3°$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{4^2 + 6^2 - 3^2}{2(4)(6)} = \frac{43}{48}$$

$$B = \cos^{-1}\left(\frac{43}{48}\right) \approx 26.4°$$

$$C = 180° - A - B \approx 180° - 36.3° - 26.4° = 117.3°$$

31. $a = 10,\ b = 8,\ c = 5$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{8^2 + 5^2 - 10^2}{2(8)(5)} = -\frac{11}{80}$$

$$A = \cos^{-1}\left(-\frac{11}{80}\right) \approx 97.9°$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{10^2 + 5^2 - 8^2}{2(10)(5)} = \frac{61}{100}$$

$$B = \cos^{-1}\left(\frac{61}{100}\right) \approx 52.4°$$

$$C = 180° - A - B \approx 180° - 97.9° - 52.4° = 29.7°$$

32. $a=9,\ b=7,\ c=10$

$$a^2=b^2+c^2-2bc\cos A$$

$$\cos A=\frac{b^2+c^2-a^2}{2bc}=\frac{7^2+10^2-9^2}{2(7)(10)}=\frac{68}{140}$$

$$A=\cos^{-1}\left(\frac{68}{140}\right)\approx 60.9^\circ$$

$$b^2=a^2+c^2-2ac\cos B$$

$$\cos B=\frac{a^2+c^2-b^2}{2ac}=\frac{9^2+10^2-7^2}{2(9)(10)}=\frac{132}{180}$$

$$B=\cos^{-1}\left(\frac{132}{180}\right)\approx 42.8^\circ$$

$$C=180^\circ-A-B\approx 180^\circ-60.9^\circ-42.8^\circ=76.3^\circ$$

33. $B=20^\circ,\ C=75^\circ,\ b=5$

$$\frac{\sin B}{b}=\frac{\sin C}{c}$$

$$c=\frac{b\sin C}{\sin B}=\frac{5\sin 75^\circ}{\sin 20^\circ}\approx 14.12$$

$$A=180^\circ-20^\circ-75^\circ=85^\circ$$

$$\frac{\sin A}{a}=\frac{\sin B}{b}$$

$$a=\frac{b\sin A}{\sin B}=\frac{5\sin 85^\circ}{\sin 20^\circ}\approx 14.56$$

34. $A=50^\circ,\ B=55^\circ,\ c=9$

$$C=180^\circ-50^\circ-55^\circ=75^\circ$$

$$\frac{\sin C}{c}=\frac{\sin B}{b}$$

$$b=\frac{c\sin B}{\sin C}=\frac{9\sin 55^\circ}{\sin 75^\circ}\approx 7.63$$

$$\frac{\sin C}{c}=\frac{\sin A}{a}$$

$$a=\frac{c\sin A}{\sin C}=\frac{9\sin 50^\circ}{\sin 75^\circ}\approx 7.14$$

35. $a=6,\ b=8,\ c=9$

$$a^2=b^2+c^2-2bc\cos A$$

$$\cos A=\frac{b^2+c^2-a^2}{2bc}=\frac{8^2+9^2-6^2}{2(8)(9)}=\frac{109}{144}$$

$$A=\cos^{-1}\frac{109}{144}\approx 40.8^\circ$$

$$b^2=a^2+c^2-2ac\cos B$$

$$\cos B=\frac{a^2+c^2-b^2}{2ac}=\frac{6^2+9^2-8^2}{2(6)(9)}=\frac{53}{108}$$

$$B=\cos^{-1}\frac{53}{108}\approx 60.6^\circ$$

$$c^2=a^2+b^2-2ab\cos C$$

$$\cos C=\frac{a^2+b^2-c^2}{2ab}=\frac{6^2+8^2-9^2}{2(6)(8)}=\frac{19}{96}$$

$$C=\cos^{-1}\frac{19}{96}\approx 78.6^\circ$$

36. $a=14,\ b=7,\ A=85^\circ$

$$\frac{\sin 85^\circ}{14}=\frac{\sin B}{7}$$

$$\sin B=\frac{\sin 85^\circ}{2}\approx 0.49810$$

$$B=\sin^{-1}(0.49810)\approx 29.9^\circ \text{ or } 150.1^\circ$$

The second value is discarded since $A+B>180^\circ$. Therefore, $B\approx 29.9^\circ$.

$$C=180^\circ-29.9^\circ-85^\circ=65.1^\circ$$

$$\frac{\sin 85^\circ}{14}=\frac{\sin 65.1^\circ}{c}$$

$$c=\frac{14\cdot\sin 65.1^\circ}{\sin 85^\circ}\approx 12.75$$

37. $B=35^\circ,\ C=65^\circ,\ a=15$

$$A=180^\circ-35^\circ-65^\circ=80^\circ$$

$$\frac{\sin A}{a}=\frac{\sin C}{c}$$

$$c=\frac{a\sin C}{\sin A}=\frac{15\sin 65^\circ}{\sin 35^\circ}\approx 13.80$$

$$\frac{\sin A}{a}=\frac{\sin B}{b}$$

$$b=\frac{a\sin B}{\sin A}=\frac{15\sin 35^\circ}{\sin 80^\circ}\approx 8.74$$

38. $a=4$, $c=5$, $B=55°$

$b^2=a^2+c^2-2ac\cos B$

$b^2=4^2+5^2-2(4)(5)\cos 55°$

$b^2=41-40\cos 55°$

$b=\sqrt{41-40\cos 55°}\approx 4.25$

$\frac{\sin 55°}{\sqrt{41-40\cos 55°}}=\frac{\sin A}{4}$

$\sin A=\frac{4\sin 55°}{\sqrt{41-40\cos 55°}}\approx 0.77109$

$A=\sin^{-1}(0.77109)\approx 50.5°$ or $129.5°$

We discard the second value since $A+B>180°$. Therefore, $A\approx 50.5°$.

$C=180°-55°-50.5°=74.5°$

39. $a=3$, $b=10$, $A=10°$

$\frac{\sin 10°}{3}=\frac{\sin B}{10}$

$\sin B=\frac{10\sin 10°}{3}\approx 0.578827$

$B=\sin^{-1}(0.578827)\approx 35.4°$ or $144.6°$

Since both values yield $A+B<180°$, there are two triangles.

$B_1=35.4°$ and $B_2=144.6°$

$C_1=180°-10°-35.4°=134.6°$

$C_2=180°-10°-144.6°=25.4°$

Using the Law of Cosines we get

$c_1=\sqrt{3^2+10^2-2\cdot 3\cdot 10\cdot\cos 134.6°}$
≈ 12.29

$c_2=\sqrt{3^2+10^2-2\cdot 3\cdot 10\cdot\cos 25.4°}$
≈ 7.40

40. $A=65°$, $B=72°$, $b=7$

$\frac{\sin 72°}{7}=\frac{\sin 65°}{a}$

$a=\frac{7\sin 65°}{\sin 72°}\approx 6.67$

$C=180°-65°-72°=43°$

$\frac{\sin 43°}{c}=\frac{\sin 72°}{7}$

$c=\frac{7\sin 43°}{\sin 72°}\approx 5.02$

41. $b=5$, $c=12$, $A=60°$

$a^2=b^2+c^2-2bc\cos A$

$a^2=5^2+12^2-2\cdot 5\cdot 12\cdot\cos 60°$
$=109$

$a=\sqrt{109}\approx 10.44$

$\frac{\sin 60°}{\sqrt{109}}=\frac{\sin B}{5}$

$\sin B=\frac{5\sin 60°}{\sqrt{109}}\approx 0.414751$

$B=\sin^{-1}(0.414751)\approx 24.5°$ or $155.5°$

We discard the second value because it would give $A+B>180°$. Therefore, $B\approx 24.5°$.

$C=180°-60°-24.5°=95.5°$

42. $a=10$, $b=10$, $c=15$

$a^2=b^2+c^2-2bc\cos A$

$\cos A=\frac{b^2+c^2-a^2}{2bc}$
$=\frac{10^2+15^2-10^2}{2(10)(15)}=\frac{225}{300}=\frac{3}{4}$

$A=\cos^{-1}\frac{3}{4}\approx 41.4°$

$b^2=a^2+c^2-2ac\cos B$

$\cos B=\frac{a^2+c^2-b^2}{2ac}$
$=\frac{10^2+15^2-10^2}{2(10)(15)}=\frac{225}{300}=\frac{3}{4}$

$B=\cos^{-1}\frac{3}{4}\approx 41.4°$

$c^2=a^2+b^2-2ab\cos C$

$\cos C=\frac{a^2+b^2-c^2}{2ab}$
$=\frac{10^2+10^2-15^2}{2(10)(10)}=\frac{-25}{200}=-\frac{1}{8}$

$C=\cos^{-1}\left(-\frac{1}{8}\right)\approx 97.2°$

43. Find the third side of the triangle using the Law of Cosines: $a = 150,\ b = 35,\ C = 110°$

$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$= 150^2 + 35^2 - 2 \cdot 150 \cdot 35\cos 110°$$
$$= 23,725 - 10,500\cos 110°$$
$$c = \sqrt{23,725 - 10,500\cos 110°} \approx 165$$

The ball is approximately 165 yards from the center of the green.

44. a. The angle inside the triangle at Sarasota is $180° - 50° = 130°$. Use the Law of Cosines to find the third side:

$a = 150,\ b = 100,\ C = 130°$

$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$= 150^2 + 100^2 - 2 \cdot 150 \cdot 100\cos 130°$$
$$= 32,500 - 30,000\cos 130°$$
$$c = \sqrt{32,500 - 30,000\cos 130°} \approx 227.56 \text{ mi}$$

b. Use the Law of Sines to find the angle inside the triangle at Ft. Myers:

$$\frac{\sin A}{100} = \frac{\sin 130°}{227.56}$$
$$\sin A = \frac{100\sin 130°}{227.56}$$
$$A = \sin^{-1}\left(\frac{100\sin 130°}{227.56}\right) \approx 19.7°$$

Since the angle of the triangle is 19.7°, the pilot should fly at a bearing of N 19.7° E.

45. After 10 hours the ship will have traveled 150 nautical miles along its altered course. Use the Law of Cosines to find the distance from Barbados on the new course.

$a = 600,\ b = 150,\ C = 20°$

$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$= 600^2 + 150^2 - 2 \cdot 600 \cdot 150\cos 20°$$
$$= 382,500 - 180,000\cos 20°$$
$$c = \sqrt{382,500 - 180,000\cos 20°}$$
$$\approx 461.9 \text{ nautical miles}$$

a. Use the Law of Cosines to find the angle opposite the side of 600:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
$$\cos A = \frac{150^2 + 461.9^2 - 600^2}{2(150)(461.9)} = -\frac{124,148.39}{138,570}$$
$$A = \cos^{-1}\left(-\frac{124,148.39}{138,570}\right) \approx 153.6°$$

The captain needs to turn the ship through an angle of $180° - 153.6° = 26.4°$.

b. $t = \dfrac{461.9 \text{ nautical miles}}{15 \text{ knots}} \approx 30.8$ hours are required for the second leg of the trip. (The total time for the trip will be about 40.8 hours.)

46. a. After 15 minutes, the plane would have flown 220(0.25) = 55 miles. Find the third side of the triangle:

$a = 55,\ b = 330,\ \gamma = 10°$

$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$= 55^2 + 330^2 - 2 \cdot 55 \cdot 330\cos 10°$$
$$= 111,925 - 36,300\cos 10°$$
$$c = \sqrt{111,925 - 36,300\cos 10°} \approx 276$$

Find the measure of the angle opposite the 330-mile side:

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$
$$= \frac{55^2 + 276^2 - 330^2}{2(55)(276)} = -\frac{29,699}{30,360}$$
$$B = \cos^{-1}\left(-\frac{29,699}{30,360}\right) \approx 168.0°$$

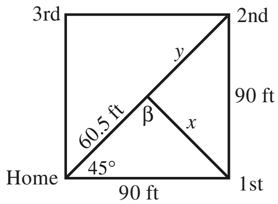
The pilot should turn through an angle of $180° - 168.0° = 12.0°$.

b. If the total trip is to be done in 90 minutes, and 15 minutes were used already, then there are 75 minutes or 1.25 hours to complete the trip. The plane must travel 276 miles in 1.25 hours:

$$r = \frac{276}{1.25} = 220.8 \text{ miles/hour}$$

The pilot must maintain a speed of 220.8 mi/hr to complete the trip in 90 minutes.

47. a. Find x in the figure:



$$x^2 = 60.5^2 + 90^2 - 2(60.5)(90)\cos 45°$$
$$= 11{,}760.25 - 10{,}980\left(\frac{\sqrt{2}}{2}\right)$$
$$= 11{,}760.25 - 5445\sqrt{2}$$
$$x = \sqrt{11{,}760.25 - 5445\sqrt{2}} \approx 63.7 \text{ feet}$$

It is about 63.7 feet from the pitching rubber to first base.

b. Use the Pythagorean Theorem to find y in the figure:

$$90^2 + 90^2 = (60.5 + y)^2$$
$$16{,}200 = (60.5 + y)^2$$
$$60.5 + y = \sqrt{16{,}200} \approx 127.3$$
$$y \approx 66.8 \text{ feet}$$

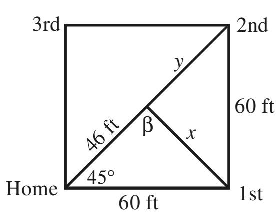
It is about 66.8 feet from the pitching rubber to second base.

c. Find B in the figure by using the Law of Cosines:

$$\cos B = \frac{60.5^2 + 63.7^2 - 90^2}{2(60.5)(63.7)} = -\frac{382.06}{7707.7}$$
$$B = \cos^{-1}\left(-\frac{382.06}{7707.7}\right) \approx 92.8°$$

The pitcher needs to turn through an angle of about 92.8° to face first base.

48. a. Find x in the figure:



$$x^2 = 46^2 + 60^2 - 2(46)(60)\cos 45°$$
$$= 5716 - 5520\left(\frac{\sqrt{2}}{2}\right) = 5716 - 2760\sqrt{2}$$
$$x = \sqrt{5716 - 2760\sqrt{2}} \approx 42.58 \text{ feet}$$

It is about 42.58 feet from the pitching rubber to first base.

b. Use the Pythagorean Theorem to find y in the figure:

$$60^2 + 60^2 = (46 + y)^2$$
$$7200 = (46 + y)^2$$
$$46 + y = \sqrt{7200} \approx 84.85$$
$$y \approx 38.85 \text{ feet}$$

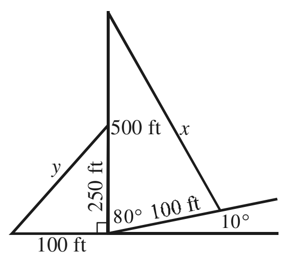
It is about 38.85 feet from the pitching rubber to second base.

c. Find B in the figure by using the Law of Cosines:

$$\cos B = \frac{46^2 + 42.58^2 - 60^2}{2(46)(42.58)} = \frac{329.0564}{3917.36}$$
$$B = \cos^{-1}\left(\frac{329.0564}{3917.36}\right) \approx 85.2°$$

The pitcher needs to turn through an angle of 85.2° to face first base.

49. a. Find x by using the Law of Cosines:



$$x^2 = 500^2 + 100^2 - 2(500)(100)\cos 80°$$
$$= 260{,}000 - 100{,}000\cos 80°$$
$$x = \sqrt{260{,}000 - 100{,}000\cos 80°} \approx 492.6 \text{ ft}$$

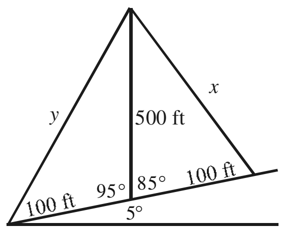
The guy wire needs to be about 492.6 feet long.

b. Use the Pythagorean Theorem to find the value of y:

$$y^2 = 100^2 + 250^2 = 72{,}500$$
$$y = 269.3 \text{ feet}$$

The guy wire needs to be about 269.3 feet long.

50. Find x by using the Law of Cosines:



$$x^2 = 500^2 + 100^2 - 2(500)(100)\cos 85°$$
$$= 260,000 - 100,000\cos 85°$$
$$x = \sqrt{260,000 - 100,000\cos 85°} \approx 501.28 \text{ feet}$$

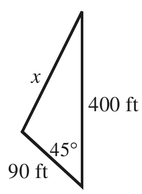
The guy wire needs to be about 501.28 feet long.

Find y by using the Law of Cosines:

$$y^2 = 500^2 + 100^2 - 2(500)100\cos 95°$$
$$= 260,000 - 100,000\cos 95°$$
$$y = \sqrt{260,000 - 100,000\cos 95°} \approx 518.38 \text{ feet}$$

The guy wire needs to be about 518.38 feet long.

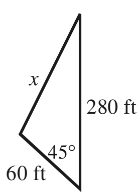
51. Find x by using the Law of Cosines:



$$x^2 = 400^2 + 90^2 - 2(400)(90)\cos(45°)$$
$$= 168,100 - 36,000\sqrt{2}$$
$$x = \sqrt{168,100 - 36,000\sqrt{2}} \approx 342.33 \text{ feet}$$

It is approximately 342.33 feet from dead center to third base.

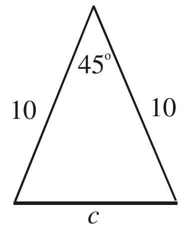
52. Find x by using the Law of Cosines:



$$x^2 = 280^2 + 60^2 - 2(280)(60)\cos(45°)$$
$$= 82,000 - 16,800\sqrt{2}$$
$$x = \sqrt{82,000 - 16,800\sqrt{2}} \approx 241.33 \text{ feet}$$

It is approximately 241.33 feet from dead center to third base.

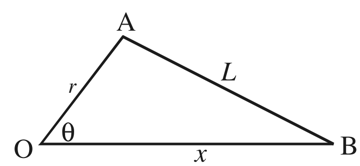
53. Use the Law of Cosines:



$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$= 10^2 + 10^2 - 2(10)(10)\cos(45°)$$
$$= 100 + 100 - 200\left(\frac{\sqrt{2}}{2}\right)$$
$$= 200 - 100\sqrt{2} = 100\left(2 - \sqrt{2}\right)$$
$$c = \sqrt{100\left(2 - \sqrt{2}\right)} = 10\sqrt{2 - \sqrt{2}} \approx 7.65$$

The footings should be approximately 7.65 feet apart.

54. Use the Law of Cosines:



$$L^2 = x^2 + r^2 - 2xr\cos\theta$$
$$x^2 - 2xr\cos\theta + r^2 - L^2 = 0$$

Using the quadratic formula:

$$x = \frac{2r\cos\theta + \sqrt{(2r\cos\theta)^2 - 4(1)(r^2 - L^2)}}{2(1)}$$
$$x = \frac{2r\cos\theta + \sqrt{4r^2\cos^2\theta - 4\left(r^2 - L^2\right)}}{2}$$
$$x = \frac{2r\cos\theta + \sqrt{4\left(r^2\cos^2\theta - r^2 + L^2\right)}}{2}$$
$$x = \frac{2r\cos\theta + 2\sqrt{r^2\cos^2\theta - r^2 + L^2}}{2}$$
$$x = r\cos\theta + \sqrt{r^2\cos^2\theta + L^2 - r^2}$$

55. Use the Law of Cosines to find the length of side d:

$$d^2 = r^2 + r^2 - 2 \cdot r \cdot r \cdot \cos\theta$$
$$= 2r^2 - 2r^2\cos\theta = 2r^2(1-\cos\theta)$$
$$= 4r^2\left(\frac{1-\cos\theta}{2}\right) = 2r\sqrt{\frac{1-\cos\theta}{2}}$$
$$= 2r\sin\left(\frac{\theta}{2}\right)$$

If $s = r\theta$ is the length of the arc subtended by θ, then $d < s$, and we have $2r\sin\left(\frac{\theta}{2}\right) < r\theta$ or $2\sin\left(\frac{\theta}{2}\right) < \theta$. Given $\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$ and $\cos\frac{\theta}{2} \le 1$, we have $\sin\theta \le 2\sin\frac{\theta}{2} < \theta$.
Therefore, $\sin\theta < \theta$ for any angle $\theta > 0$.

56. $$\cos\frac{C}{2} = \sqrt{\frac{1+\cos C}{2}}$$
$$= \sqrt{\frac{1+\frac{a^2+b^2-c^2}{2ab}}{2}}$$
$$= \sqrt{\frac{2ab+a^2+b^2-c^2}{4ab}}$$
$$= \sqrt{\frac{(a+b)^2-c^2}{4ab}}$$
$$= \sqrt{\frac{(a+b+c)(a+b-c)}{4ab}}$$
$$= \sqrt{\frac{2s(2s-c-c)}{4ab}}$$
$$= \sqrt{\frac{4s(s-c)}{4ab}} = \sqrt{\frac{s(s-c)}{ab}}$$

57. $$\sin\frac{C}{2} = \sqrt{\frac{1-\cos C}{2}}$$
$$= \sqrt{\frac{1-\frac{a^2+b^2-c^2}{2ab}}{2}}$$
$$= \sqrt{\frac{2ab-a^2-b^2+c^2}{4ab}}$$
$$= \sqrt{\frac{-(a^2-2ab+b^2-c^2)}{4ab}}$$
$$= \sqrt{\frac{-\left((a-b)^2-c^2\right)}{4ab}}$$
$$= \sqrt{\frac{-(a-b+c)(a-b-c)}{4ab}}$$
$$= \sqrt{\frac{(a-b+c)(b+c-a)}{4ab}}$$
$$= \sqrt{\frac{(2s-2b)(2s-2a)}{4ab}}$$
$$= \sqrt{\frac{4(s-b)(s-a)}{4ab}}$$
$$= \sqrt{\frac{(s-a)(s-b)}{ab}}$$

58. $$\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c}$$
$$= \frac{b^2+c^2-a^2}{2bca} + \frac{a^2+c^2-b^2}{2acb} + \frac{a^2+b^2-c^2}{2abc}$$
$$= \frac{b^2+c^2-a^2+a^2+c^2-b^2+a^2+b^2-c^2}{2abc}$$
$$= \frac{a^2+b^2+c^2}{2abc}$$

59 – 63. Answers will vary.

Section 8.4

1. $K = \frac{1}{2}bh$

2. $K = \frac{1}{2}ab\sin C$

3. $\sqrt{s(s-a)(s-b)(s-c)}$; $\frac{1}{2}(a+b+c)$

4. True

5. $a = 2,\ c = 4,\ B = 45°$
   $K = \frac{1}{2}ac\sin B = \frac{1}{2}(2)(4)\sin 45° \approx 2.83$

6. $b = 3,\ c = 4,\ A = 30°$
   $K = \frac{1}{2}bc\sin\alpha = \frac{1}{2}(3)(4)\sin 30° = 3$

7. $a = 2,\ b = 3,\ C = 95°$
   $K = \frac{1}{2}ab\sin C = \frac{1}{2}(2)(3)\sin 95° \approx 2.99$

8. $a=2,\ c=5,\ B=20°$

$$K=\frac{1}{2}ac\sin B=\frac{1}{2}(2)(5)\sin 20°\approx 1.71$$

9. $a=6,\ b=5,\ c=8$

$$s=\frac{1}{2}(a+b+c)=\frac{1}{2}(6+5+8)=\frac{19}{2}$$
$$K=\sqrt{s(s-a)(s-b)(s-c)}$$
$$=\sqrt{\left(\frac{19}{2}\right)\left(\frac{7}{2}\right)\left(\frac{9}{2}\right)\left(\frac{3}{2}\right)}=\sqrt{\frac{3591}{16}}\approx 14.98$$

10. $a=8,\ b=5,\ c=4$

$$s=\frac{1}{2}(a+b+c)=\frac{1}{2}(8+5+4)=\frac{17}{2}$$
$$K=\sqrt{s(s-a)(s-b)(s-c)}$$
$$=\sqrt{\left(\frac{17}{2}\right)\left(\frac{1}{2}\right)\left(\frac{7}{2}\right)\left(\frac{9}{2}\right)}=\sqrt{\frac{1071}{16}}\approx 8.18$$

11. $a=9,\ b=6,\ c=4$

$$s=\frac{1}{2}(a+b+c)=\frac{1}{2}(9+6+4)=\frac{19}{2}$$
$$K=\sqrt{s(s-a)(s-b)(s-c)}$$
$$=\sqrt{\left(\frac{19}{2}\right)\left(\frac{1}{2}\right)\left(\frac{7}{2}\right)\left(\frac{11}{2}\right)}=\sqrt{\frac{1463}{16}}\approx 9.56$$

12. $a=4,\ b=3,\ c=4$

$$s=\frac{1}{2}(a+b+c)=\frac{1}{2}(4+3+4)=\frac{11}{2}$$
$$K=\sqrt{s(s-a)(s-b)(s-c)}$$
$$=\sqrt{\left(\frac{11}{2}\right)\left(\frac{3}{2}\right)\left(\frac{5}{2}\right)\left(\frac{3}{2}\right)}=\sqrt{\frac{495}{16}}\approx 5.56$$

13. $a=3,\ b=4,\ C=40°$

$$K=\frac{1}{2}ab\sin C=\frac{1}{2}(3)(4)\sin 40°\approx 3.86$$

14. $a=2,\ c=1,\ B=10°$

$$K=\frac{1}{2}ac\sin B=\frac{1}{2}(2)(1)\sin 10°\approx 0.17$$

15. $b=1,\ c=3,\ A=80°$

$$K=\frac{1}{2}bc\sin A=\frac{1}{2}(1)(3)\sin 80°\approx 1.48$$

16. $a=6,\ b=4,\ C=60°$

$$K=\frac{1}{2}ab\sin C=\frac{1}{2}(6)(4)\sin 60°\approx 10.39$$

17. $a=3,\ c=2,\ B=110°$

$$K=\frac{1}{2}ac\sin B=\frac{1}{2}(3)(2)\sin 110°\approx 2.82$$

18. $b=4,\ c=1,\ A=120°$

$$K=\frac{1}{2}bc\sin A=\frac{1}{2}(4)(1)\sin 120°\approx 1.73$$

19. $a=12,\ b=13,\ c=5$

$$s=\frac{1}{2}(a+b+c)=\frac{1}{2}(12+13+5)=15$$
$$K=\sqrt{s(s-a)(s-b)(s-c)}$$
$$=\sqrt{(15)(3)(2)(10)}=\sqrt{900}=30$$

20. $a=4,\ b=5,\ c=3$

$$s=\frac{1}{2}(a+b+c)=\frac{1}{2}(4+5+3)=6$$
$$K=\sqrt{s(s-a)(s-b)(s-c)}$$
$$=\sqrt{(6)(2)(1)(3)}=\sqrt{36}=6$$

21. $a=2,\ b=2,\ c=2$

$$s=\frac{1}{2}(a+b+c)=\frac{1}{2}(2+2+2)=3$$
$$K=\sqrt{s(s-a)(s-b)(s-c)}$$
$$=\sqrt{(3)(1)(1)(1)}=\sqrt{3}\approx 1.73$$

22. $a=3,\ b=3,\ c=2$

$$s=\frac{1}{2}(a+b+c)=\frac{1}{2}(3+3+2)=4$$
$$K=\sqrt{s(s-a)(s-b)(s-c)}$$
$$=\sqrt{(4)(1)(1)(2)}=\sqrt{8}\approx 2.83$$

23. $a=5,\ b=8,\ c=9$

$$s=\frac{1}{2}(a+b+c)=\frac{1}{2}(5+8+9)=11$$
$$K=\sqrt{s(s-a)(s-b)(s-c)}$$
$$=\sqrt{(11)(6)(3)(2)}=\sqrt{396}\approx 19.90$$

24. $a = 4,\ b = 3,\ c = 6$

$$s = \frac{1}{2}(a+b+c) = \frac{1}{2}(4+3+6) = \frac{13}{2}$$
$$K = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{\left(\frac{13}{2}\right)\left(\frac{5}{2}\right)\left(\frac{7}{2}\right)\left(\frac{1}{2}\right)} = \sqrt{\frac{455}{16}} \approx 5.33$$

25. From the Law of Sines we know $\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$.

Solving for b, so we have that $b = \dfrac{a\sin B}{\sin A}$. Thus,

$$K = \frac{1}{2}ab\sin C = \frac{1}{2}a\left(\frac{a\sin B}{\sin A}\right)\sin C$$
$$= \frac{a^2 \sin B \sin C}{2\sin A}$$

26. From $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$, we have that

$c = \dfrac{b\sin C}{\sin B}$ and $a = \dfrac{c\sin A}{\sin C}$. Thus,

$$K = \frac{1}{2}bc\sin A = \frac{1}{2}b\left(\frac{b\sin C}{\sin B}\right)\sin A$$
$$= \frac{b^2 \sin A \sin C}{2\sin B}$$
$$K = \frac{1}{2}ac\sin B = \frac{1}{2}\left(\frac{c\sin A}{\sin C}\right)c\sin B$$
$$= \frac{c^2 \sin A \sin B}{2\sin C}$$

27. $A = 40°,\ B = 20°,\ a = 2$

$$C = 180° - A - B = 180° - 40° - 20° = 120°$$
$$K = \frac{a^2 \sin B \sin C}{2\sin A} = \frac{2^2 \sin 20° \cdot \sin 120°}{2\sin 40°} \approx 0.92$$

28. $A = 50°,\ C = 20°,\ a = 3$

$$B = 180° - A - C = 180° - 50° - 20° = 110°$$
$$K = \frac{a^2 \sin B \sin C}{2\sin A} = \frac{3^2 \sin 110° \cdot \sin 20°}{2\sin 50°} \approx 1.89$$

29. $B = 70°,\ C = 10°,\ b = 5$

$$A = 180° - B - C = 180° - 70° - 10° = 100°$$
$$K = \frac{b^2 \sin A \sin C}{2\sin B} = \frac{5^2 \sin 100° \cdot \sin 10°}{2\sin 70°} \approx 2.27$$

30. $A = 70°,\ B = 60°,\ c = 4$

$$C = 180° - A - B = 180° - 70° - 60° = 50°$$
$$K = \frac{c^2 \sin A \sin B}{2\sin C} = \frac{4^2 \sin 70° \cdot \sin 60°}{2\sin 50°} \approx 8.50$$

31. $A = 110°,\ C = 30°,\ c = 3$

$$B = 180° - A - C = 180° - 110° - 30° = 40°$$
$$K = \frac{c^2 \sin A \sin B}{2\sin C} = \frac{3^2 \sin 110° \cdot \sin 40°}{2\sin 30°} \approx 5.44$$

32. $B = 10°,\ C = 100°,\ b = 2$

$$A = 180° - B - C = 180° - 10° - 100° = 70°$$
$$K = \frac{b^2 \sin A \sin C}{2\sin B} = \frac{2^2 \sin 70° \cdot \sin 100°}{2\sin 10°} \approx 10.66$$

33. Area of a sector $= \dfrac{1}{2}r^2\theta$ where θ is in radians.

$$\theta = 70 \cdot \frac{\pi}{180} = \frac{7\pi}{18}$$
$$A_{\text{Sector}} = \frac{1}{2} \cdot 8^2 \cdot \frac{7\pi}{18} = \frac{112\pi}{9}\ \text{ft}^2$$
$$A_{\text{Triangle}} = \frac{1}{2} \cdot 8 \cdot 8\sin 70° = 32\sin 70°\ \text{ft}^2$$
$$A_{\text{Segment}} = \frac{112\pi}{9} - 32\sin 70° \approx 9.03\ \text{ft}^2$$

34. Area of a sector $= \dfrac{1}{2}r^2\theta$ where θ is in radians.

$$\theta = 40 \cdot \frac{\pi}{180} = \frac{2\pi}{9}$$
$$A_{\text{Sector}} = \frac{1}{2} \cdot 5^2 \cdot \frac{2\pi}{9} = \frac{25\pi}{9}\ \text{in}^2$$
$$A_{\text{Triangle}} = \frac{1}{2} \cdot 5 \cdot 5\sin 40° = \frac{25}{2}\sin 40°\ \text{in}^2$$
$$A_{\text{Segment}} = \frac{25\pi}{9} - \frac{25}{2}\sin 40° \approx 0.69\ \text{in}^2$$

35. Find the area of the lot using Heron's Formula:

$a = 100,\ b = 50,\ c = 75$

$$s = \frac{1}{2}(a+b+c) = \frac{1}{2}(100+50+75) = \frac{225}{2}$$

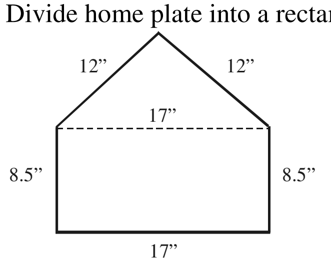
$$K = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{\left(\frac{225}{2}\right)\left(\frac{25}{2}\right)\left(\frac{125}{2}\right)\left(\frac{75}{2}\right)}$$
$$= \sqrt{\frac{52,734,375}{16}}$$
$$\approx 1815.46$$
$$\text{Cost} = (\$3)(1815.46) = \$5446.38$$

36. Diameter of canvas is 24 feet; radius of canvas is 12 feet; angle is 260°.

Area of a sector $= \frac{1}{2}r^2\theta$ where θ is in radians.

$$\theta = 260 \cdot \frac{\pi}{180} = \frac{13\pi}{9}$$
$$A_{\text{Sector}} = \frac{1}{2} \cdot 12^2 \cdot \frac{13\pi}{9} = \frac{936\pi}{9} = 104\pi \approx 326.73 \text{ ft}^2$$

37. Divide home plate into a rectangle and a triangle.



$$A_{\text{Rectangle}} = lw = (17)(8.5) = 144.5 \text{ in}^2$$
Using Heron's formula we get
$$A_{\text{Triangle}} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$s = \frac{1}{2}(a+b+c) = \frac{1}{2}(12+12+17) = 20.5$$
Thus,
$$A_{\text{Triangle}} = \sqrt{(20.5)(20.5-12)(20.5-12)(20.5-17)}$$
$$= \sqrt{(20.5)(8.5)(8.5)(3.5)}$$
$$= \sqrt{5183.9375}$$
$$\approx 72.0 \text{ sq. in.}$$
So, $A_{\text{Total}} = A_{\text{Rectangle}} + A_{\text{Triangle}}$
$$= 144.5 + 72.0$$
$$= 216.5 \text{ in}^2$$
The area of home plate is about 216.5 in^2.

38. Find the area of the shaded region by subtracting the area of the triangle from the area of the semicircle.
Area of the semicircle
$$A_{\text{Semicircle}} = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(5)^2 = \frac{25}{2}\pi \text{ in}^2$$

The triangle is a right triangle. Find the other leg:
$$8^2 + b^2 = 10^2$$
$$b^2 = 100 - 64 = 36$$
$$b = \sqrt{36} = 6$$
$$A_{\text{Triangle}} = \frac{1}{2} \cdot 8 \cdot 6 = 24 \text{ in}^2$$
$$A_{\text{Shaded region}} = 12.5\pi - 24 \approx 15.27 \text{ in}^2$$

39. The area is the sum of the area of a triangle and a sector.
$$A_{\text{Triangle}} = \frac{1}{2} r \cdot r \sin(\pi - \theta) = \frac{1}{2} r^2 \sin(\pi - \theta)$$
$$A_{\text{Sector}} = \frac{1}{2} r^2 \theta$$

$$K = \frac{1}{2}r^2 \sin(\pi - \theta) + \frac{1}{2}r^2\theta$$
$$= \frac{1}{2}r^2\left(\sin(\pi - \theta) + \theta\right)$$
$$= \frac{1}{2}r^2\left(\sin\pi\cos\theta - \cos\pi\sin\theta + \theta\right)$$
$$= \frac{1}{2}r^2\left(0 \cdot \cos\theta - (-1)\sin\theta + \theta\right)$$
$$= \frac{1}{2}r^2\left(\theta + \sin\theta\right)$$

40. Use the Law of Cosines to find the lengths of the diagonals of the polygon.
$$x^2 = 35^2 + 80^2 - 2 \cdot 35 \cdot 80 \cos 15^\circ$$
$$= 7625 - 5600 \cos 15^\circ$$
$$x = \sqrt{7625 - 5600\cos 15^\circ} \approx 47.072 \text{ feet}$$
The interior angle of the third triangle is: $180° - 100° = 80°$.
$$y^2 = 45^2 + 20^2 - 2 \cdot 45 \cdot 20 \cos 80^\circ$$
$$= 2425 - 1800 \cos 80^\circ$$
$$y = \sqrt{2425 - 1800\cos 80^\circ} \approx 45.961 \text{ feet}$$

Find the area of the three triangles:

$$s_1 \approx \frac{1}{2}(35 + 80 + 47.072) = 81.036$$
$$s_1 - a_1 = 81.036 - 35 = 46.036$$
$$s_1 - b_1 = 81.036 - 80 = 1.036$$
$$s_1 - c_1 = 81.036 - 47.072 = 33.964$$
$$K_1 \approx \sqrt{81.036(46.036)(1.036)(33.964)} \approx 362.31 \text{ ft}^2$$

$s_2 \approx \frac{1}{2}(40+45.961+47.072) = 66.5165$
$s_2 - a_2 = 66.5165 - 40 = 26.5165$
$s_2 - b_2 = 66.5165 - 45.961 = 20.5555$
$s_2 - c_2 = 66.5165 - 47.072 = 19.4445$
$K_2 \approx \sqrt{66.5165(26.5165)(20.5555)(19.4445)}$
$\approx 839.62 \text{ ft}^2$

$s_3 \approx \frac{1}{2}(45+20+45.961) = 55.4805$
$s_3 - a_3 = 55.4805 - 45 = 10.4805$
$s_3 - b_3 = 55.4805 - 20 = 35.4805$
$s_3 - c_3 = 55.4805 - 45.961 = 9.5195$
$K_3 \approx \sqrt{55.4805(10.4805)(35.4805)(9.5195)}$
$\approx 443.16 \text{ ft}^2$

The approximate area of the lake is
$362.31 + 839.62 + 443.16 = 1645.09 \text{ ft}^2$

41. Use Heron's formula:
$a = 87$, $b = 190$, $c = 173$

$$s = \frac{1}{2}(a+b+c) = \frac{1}{2}(87+190+173) = 225$$
$$K = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{225(225-87)(225-190)(225-173)}$$
$$= \sqrt{225(138)(35)(52)}$$
$$= \sqrt{56{,}511{,}000} \approx 7517.4$$

The building covers approximately 7517.4 square feet of ground area.

42. Use Heron's formula:
$a = 1028$, $b = 1046$, $c = 965$

$$s = \frac{1}{2}(a+b+c) = \frac{1}{2}(1028+1046+965) = 1519.5$$
$$K = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{1519.5(491.5)(473.5)(554.5)}$$
$$\approx 442{,}816$$

The area of the Bermuda Triangle is approximately 442,816 square miles.

43. a. Area $\Delta OAC = \frac{1}{2}|OC|\cdot|AC|$

$$= \frac{1}{2}\cdot\frac{|OC|}{1}\cdot\frac{|AC|}{1}$$
$$= \frac{1}{2}\cos\alpha\sin\alpha$$
$$= \frac{1}{2}\sin\alpha\cos\alpha$$

b. Area $\Delta OCB = \frac{1}{2}|OC|\cdot|BC|$

$$= \frac{1}{2}\cdot|OB|^2\cdot\frac{|OC|}{|OB|}\cdot\frac{|BC|}{|OB|}$$
$$= \frac{1}{2}|OB|^2\cos\beta\sin\beta$$
$$= \frac{1}{2}|OB|^2\sin\beta\cos\beta$$

c. Area $\Delta OAB = \frac{1}{2}|BD|\cdot|OA|$

$$= \frac{1}{2}|BD|\cdot 1$$
$$= \frac{1}{2}\cdot|OB|\cdot\frac{|BD|}{|OB|}$$
$$= \frac{1}{2}|OB|\sin(\alpha+\beta)$$

d. $$\frac{\cos A}{\cos B} = \frac{\frac{|OC|}{|OA|}}{\frac{|OC|}{|OB|}} = \frac{|OC|}{1}\cdot\frac{|OB|}{|OC|} = |OB|$$

e. Area ΔOAB = Area ΔOAC + Area ΔOCB

$$\frac{1}{2}|OB|\sin(\alpha+\beta) = \frac{1}{2}\sin\alpha\cos\alpha + \frac{1}{2}|OB|^2\sin\beta\cos\beta$$
$$\frac{\cos\alpha}{\cos\beta}\sin(\alpha+\beta) = \sin\alpha\cos\alpha + \frac{\cos^2\alpha}{\cos^2\beta}\sin\beta\cos\beta$$
$$\sin(\alpha+\beta) = \frac{\cos\beta}{\cos\alpha}\sin\alpha\cos\alpha + \frac{\cos\alpha}{\cos\beta}\sin\beta\cos\beta$$
$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

44. a. Area of $\Delta OBC = \frac{1}{2}\cdot 1\cdot 1\cdot\sin\theta = \frac{\sin\theta}{2}$

b. Area of $\Delta OBD = \frac{1}{2}\cdot 1\cdot\tan\theta = \frac{\tan\theta}{2} = \frac{\sin\theta}{2\cos\theta}$

c. Area $\Delta OBC <$ Area $\widehat{OBC} <$ Area ΔOBD

$$\frac{1}{2}\sin\theta < \frac{1}{2}\theta < \frac{\sin\theta}{2\cos\theta}$$

$$\sin\theta < \theta < \frac{\sin\theta}{\cos\theta}$$

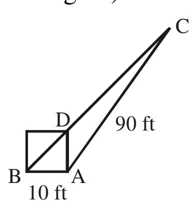
$$\frac{\sin\theta}{\sin\theta} < \frac{\theta}{\sin\theta} < \frac{\sin\theta}{\sin\theta\cos\theta}$$

$$1 < \frac{\theta}{\sin\theta} < \frac{1}{\cos\theta}$$

45. The grazing area must be considered in sections. Region A_1 represents three-fourth of a circle with radius 100 feet. Thus,

$$A_1 = \frac{3}{4}\pi(100)^2 = 7500\pi \approx 23{,}561.94 \text{ ft}^2$$

Angles are needed to find regions A_2 and A_3 : (see the figure)



In ΔABC , $\angle CBA = 45°$, $AB = 10$, $AC = 90$.
Find $\angle BCA$:

$$\frac{\sin\angle CBA}{90} = \frac{\sin\angle BCA}{10}$$

$$\frac{\sin 45°}{90} = \frac{\sin\angle BCA}{10}$$

$$\sin\angle BCA = \frac{10\sin 45°}{90} \approx 0.0786$$

$$\angle BCA = \sin^{-1}\left(\frac{10\sin 45°}{90}\right) \approx 4.51°$$

$$\angle BAC = 180° - 45° - 4.51° = 130.49°$$

$$\angle DAC = 130.49° - 90° = 40.49°$$

$$A_3 = \frac{1}{2}(10)(90)\sin 40.49° \approx 292.19 \text{ ft}^2$$

The angle for the sector A_2 is $90° - 40.49° = 49.51°$.

$$A_2 = \frac{1}{2}(90)^2\left(49.51\cdot\frac{\pi}{180}\right) \approx 3499.66 \text{ ft}^2$$

Since the cow can go in either direction around the barn, both A_2 and A_3 must be doubled.

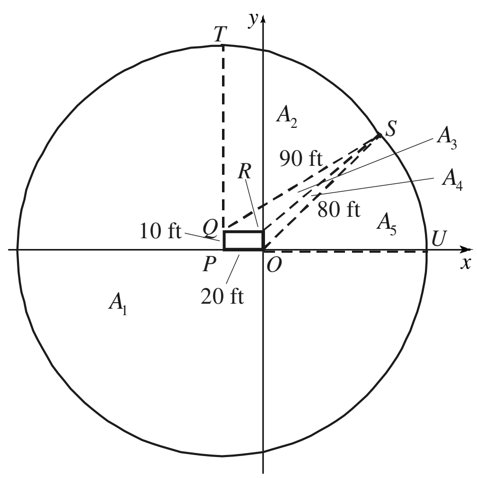
Thus, the total grazing area is:

$$23{,}561.94 + 2(3499.66) + 2(292.19)$$
$$\approx 31{,}146 \text{ ft}^2$$

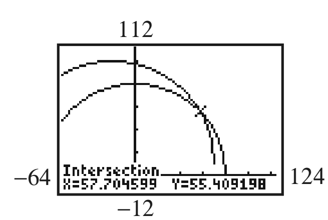
46. We begin by dividing the grazing area into five regions: three sectors and two triangles (see figure). Region A_1 is a sector representing three-fourths of a circle with radius 100 feet:

Thus, $A_1 = \frac{3}{4}\pi(100)^2 = 7500\pi \approx 23{,}561.9 \text{ ft}^2$.

To find the areas of regions A_2 , A_3 , A_4 , and A_5 , we first position the rectangular barn on a rectangular coordinate system so that the lower right corner is at the origin. The coordinates of the corners of the barn must then be $O(0,0)$, $P(-20,0)$, $Q(-20,10)$, and $R(0,10)$.



Now, region A_2 is a sector of a circle with center $Q(-20,10)$ and radius 90 feet. The equation of the circle then is $(x+20)^2 + (y-10)^2 = 90^2$. Likewise, region A_5 is a sector of a circle with center $O(0,0)$ and radius 80 feet. The equation of this circle then is $x^2 + y^2 = 80^2$. We use a graphing calculator to find the intersection point S of the two sectors. Let $Y_1 = \sqrt{80^2 - x^2}$ and $Y_2 = \sqrt{90^2 - (x+20)^2} + 10$.



The approximate coordinates are $S(57.7, 55.4)$. Now, consider ΔQRS (i.e. region A_3). The "base" of this triangle is 20 feet and the "height" is approximately $55.4 - 10 = 45.4$ feet (the *y*-coordinate of the intersection point minus the side of the barn). Thus, the area of region A_3 is $A_3 \approx \frac{1}{2}\cdot 20\cdot 45.4 = 454\text{ ft}^2$. Likewise, consider ΔORS (i.e. region A_4). The "base" of this triangle is 10 feet and the "height" is about 57.7 feet (the *x*-coordinate of the intersection point). Thus, $A_4 \approx \frac{1}{2}\cdot 10\cdot 57.7 = 288.5\text{ ft}^2$.

To find the area of sectors A_2 and A_5, we must determine their angles: $\angle TQS$ and $\angle SOU$, respectively. Now, we know $A_3 \approx 454\text{ ft}^2$. Also, we know $A_3 = \frac{1}{2}\cdot 90\cdot 20\sin\left(\angle SQR\right)$.

$$\text{Thus, } \frac{1}{2}\cdot 90\cdot 20\sin\left(\angle SQR\right) \approx 454$$
$$\sin\left(\angle SQR\right) \approx 0.5044$$
$$\angle SQR \approx 0.5287\text{ rad.}$$

Since $\angle TQR$ is a right angle, we have $\angle TQS \approx \frac{\pi}{2} - .5287 \approx 1.0421$ rad. So, $A_2 = \frac{1}{2}r^2\theta \approx \frac{1}{2}\cdot 90^2\cdot 1.0421 \approx 4220.5\text{ ft}^2$.

$$\text{Similarly, } \frac{1}{2}\cdot 80\cdot 10\sin\left(\angle SOR\right) \approx 288.5$$
$$\sin\left(\angle SOR\right) \approx 0.72125$$
$$\angle SOR \approx 0.8056\text{ rad.}$$

So, $\angle SOU \approx \frac{\pi}{2} - .8056 \approx 0.7652$ rad. and

$$A_5 \approx \frac{1}{2}\cdot 80^2\cdot 0.7652 \approx 2448.6\text{ ft}^2$$

Thus, the total grazing area is:
$$23{,}561.9 + 4220.5 + 454 + 288.5 + 2448.6$$
$$= 30{,}973\text{ ft}^2$$

47. **a.** Perimeter:
$$P = a+b+c = 9+10+17 = 36$$

Area:
$$s = \frac{1}{2}\left(a+b+c\right) = \frac{1}{2}\left(9+10+17\right) = 18$$
$$\begin{aligned} K &= \sqrt{s\left(s-a\right)\left(s-b\right)\left(s-c\right)} \\ &= \sqrt{18\left(18-9\right)\left(18-10\right)\left(18-17\right)} \\ &= \sqrt{18\left(9\right)\left(8\right)\left(1\right)} \\ &= \sqrt{1296} = 36 \end{aligned}$$

Since the perimeter and area are numerically equal, the given triangle is a perfect triangle.

b. Perimeter:
$$P = a+b+c = 6+25+29 = 60$$

Area:
$$s = \frac{1}{2}\left(a+b+c\right) = \frac{1}{2}\left(6+25+29\right) = 30$$
$$\begin{aligned} K &= \sqrt{s\left(s-a\right)\left(s-b\right)\left(s-c\right)} \\ &= \sqrt{30\left(30-6\right)\left(30-25\right)\left(30-29\right)} \\ &= \sqrt{30\left(24\right)\left(5\right)\left(1\right)} \\ &= \sqrt{3600} = 60 \end{aligned}$$

Since the perimeter and area are numerically equal, the given triangle is a perfect triangle.

48. $K = \frac{1}{2}h_1 a$, so $h_1 = \frac{2K}{a}$. Similarly, $h_2 = \frac{2K}{b}$ and $h_3 = \frac{2K}{c}$. Thus,

$$\begin{aligned} \frac{1}{h_1} + \frac{1}{h_2} + \frac{1}{h_3} &= \frac{a}{2K} + \frac{b}{2K} + \frac{c}{2K} \\ &= \frac{a+b+c}{2K} = \frac{2s}{2K} \\ &= \frac{s}{K} \end{aligned}$$

49. We know $K = \frac{1}{2}ah$ and $K = \frac{1}{2}ab\sin C$, which means $h = b\sin C$. From the Law of Sines, we know $\frac{\sin A}{a} = \frac{\sin B}{b}$, so $b = \frac{a\sin B}{\sin A}$. Therefore,

$$h = \left(\frac{a\sin B}{\sin A}\right)\sin C = \frac{a\sin B\sin C}{\sin A}.$$

50. $h = \dfrac{a\sin B \sin C}{\sin A}$ where h is the altitude to side a.

In ΔOPQ, c is opposite $\angle POQ$. The two adjacent angles are $\dfrac{A}{2}$ and $\dfrac{B}{2}$. Then

$r = \dfrac{c\sin\frac{A}{2}\sin\frac{B}{2}}{\sin(\angle POQ)}$. Now, $\angle POQ = \pi - \left(\dfrac{A}{2}+\dfrac{B}{2}\right)$,

so

$$\begin{aligned}\sin(\angle POQ) &= \sin\left[\pi - \left(\frac{A}{2}+\frac{B}{2}\right)\right]\\ &= \sin\left(\frac{A}{2}+\frac{B}{2}\right)\\ &= \sin\left(\frac{A+B}{2}\right)\\ &= \cos\left[\frac{\pi}{2}-\left(\frac{A+B}{2}\right)\right]\\ &= \cos\left[\frac{\pi-(A+B)}{2}\right]\\ &= \cos\frac{C}{2}\end{aligned}$$

Thus, $r = \dfrac{c\sin\frac{A}{2}\sin\frac{B}{2}}{\cos\frac{C}{2}}$.

51. $$\begin{aligned}\cot\frac{C}{2} = \frac{\cos\frac{C}{2}}{\sin\frac{C}{2}} &= \frac{\dfrac{c\cdot\sin\frac{A}{2}\cdot\sin\frac{B}{2}}{r}}{\sqrt{\dfrac{(s-a)(s-b)}{ab}}}\\ &= \frac{c\cdot\sqrt{\dfrac{(s-b)(s-c)}{bc}}\sqrt{\dfrac{(s-a)(s-c)}{ac}}}{r\sqrt{\dfrac{(s-a)(s-b)}{ab}}}\\ &= \frac{c}{r}\sqrt{\frac{(s-b)(s-c)}{bc}}\sqrt{\frac{(s-a)(s-c)}{ac}}\sqrt{\frac{ab}{(s-a)(s-b)}}\\ &= \frac{c}{r}\sqrt{\frac{ab(s-a)(s-b)(s-c)^2}{abc^2(s-a)(s-b)}}\\ &= \frac{c}{r}\sqrt{\frac{(s-c)^2}{c^2}} = \frac{c}{r}\cdot\frac{s-c}{c}\\ &= \frac{s-c}{r}\end{aligned}$$

52. Use the result of Problem 51:

$$\begin{aligned}\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2} &= \frac{s-a}{r}+\frac{s-b}{r}+\frac{s-c}{r}\\ &= \frac{s-a+s-b+s-c}{r}\\ &= \frac{3s-(a+b+c)}{r}\\ &= \frac{3s-2s}{r}\\ &= \frac{s}{r}\end{aligned}$$

53. $$\begin{aligned}K &= \text{Area } \Delta POQ + \text{ Area } \Delta POR + \text{ Area } \Delta QOR\\ &= \frac{1}{2}rc+\frac{1}{2}rb+\frac{1}{2}ra\\ &= \frac{1}{2}r(a+b+c)\\ &= rs\end{aligned}$$

Now, $K = \sqrt{s(s-a)(s-b)(s-c)}$, so

$$\begin{aligned}rs &= \sqrt{s(s-a)(s-b)(s-c)}\\ r &= \frac{\sqrt{s(s-a)(s-b)(s-c)}}{s}\\ &= \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}\end{aligned}$$

54 – 56. Answers will vary.

Section 8.5

1. $|5| = 5$; $\dfrac{2\pi}{4} = \dfrac{\pi}{2}$
2. simple harmonic; amplitude
3. simple harmonic; damped
4. True
5. $d = -5\cos(\pi t)$
6. $d = -10\cos\left(\dfrac{2\pi}{3}t\right)$
7. $d = -6\cos(2t)$
8. $d = -4\cos(4t)$

9. $d=-5\sin(\pi t)$

10. $d=-10\sin\left(\frac{2\pi}{3}t\right)$

11. $d=-6\sin(2t)$

12. $d=-4\sin(4t)$

13. $d=5\sin(3t)$

a. Simple harmonic

b. 5 meters

c. $\frac{2\pi}{3}$ seconds

d. $\frac{3}{2\pi}$ oscillation/second

14. $d=4\sin(2t)$

a. Simple harmonic

b. 4 meters

c. π seconds

d. $\frac{1}{\pi}$ oscillation/second

15. $d=6\cos(\pi t)$

a. Simple harmonic

b. 6 meters

c. 2 seconds

d. $\frac{1}{2}$ oscillation/second

16. $d=5\cos\left(\frac{\pi}{2}t\right)$

a. Simple harmonic

b. 5 meters

c. 4 seconds

d. $\frac{1}{4}$ oscillation/second

17. $d=-3\sin\left(\frac{1}{2}t\right)$

a. Simple harmonic

b. 3 meters

c. 4π seconds

d. $\frac{1}{4\pi}$ oscillation/second

18. $d=-2\cos(2t)$

a. Simple harmonic

b. 2 meters

c. π seconds

d. $\frac{1}{\pi}$ oscillation/second

19. $d=6+2\cos(2\pi t)$

a. Simple harmonic

b. 2 meters

c. 1 second

d. 1 oscillation/second

20. $d=4+3\sin(\pi t)$

a. Simple harmonic

b. 3 meters

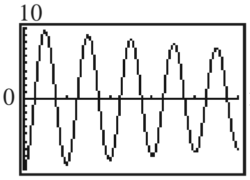
c. 2 seconds

d. $\frac{1}{2}$ oscillation/second

21. a. $d=-10e^{-0.7t/2(25)}\cos\left(\sqrt{\left(\frac{2\pi}{5}\right)^2-\frac{(0.7)^2}{4(25)^2}}\;t\right)$

$d=-10e^{-0.7t/50}\cos\left(\sqrt{\frac{4\pi^2}{25}-\frac{0.49}{2500}}\;t\right)$

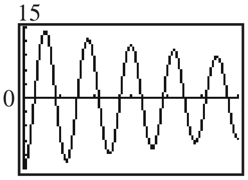
b.



22. a. $d=-15e^{-0.75t/2(20)}\cos\left(\sqrt{\left(\frac{2\pi}{6}\right)^2-\frac{(0.75)^2}{4(20)^2}}\;t\right)$

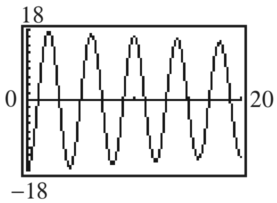
$d=-15e^{-0.75t/40}\cos\left(\sqrt{\frac{\pi^2}{9}-\frac{0.5625}{1600}}\;t\right)$

b.



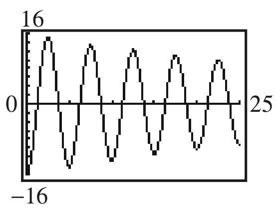
23. a. $d=-18e^{-0.6t/2(30)}\cos\left(\sqrt{\left(\frac{\pi}{2}\right)^2-\frac{(0.6)^2}{4(30)^2}}\,t\right)$

$d=-18e^{-0.6t/60}\cos\left(\sqrt{\frac{\pi^2}{4}-\frac{0.36}{3600}}\,t\right)$

b. 

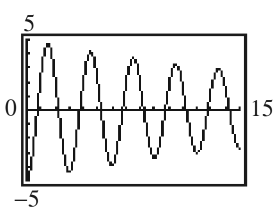
24. a. $d=-16e^{-0.65t/2(15)}\cos\left(\sqrt{\left(\frac{2\pi}{5}\right)^2-\frac{(0.65)^2}{4(15)^2}}\,t\right)$

$d=-16e^{-0.65t/30}\cos\left(\sqrt{\frac{4\pi^2}{25}-\frac{0.4225}{900}}\,t\right)$

b. 

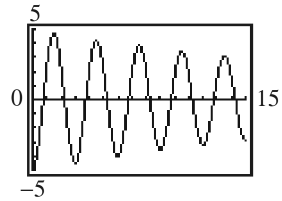
25. a. $d=-5e^{-0.8t/2(10)}\cos\left(\sqrt{\left(\frac{2\pi}{3}\right)^2-\frac{(0.8)^2}{4(10)^2}}\,t\right)$

$d=-5e^{-0.8t/20}\cos\left(\sqrt{\frac{4\pi^2}{9}-\frac{0.64}{400}}\,t\right)$

b. 

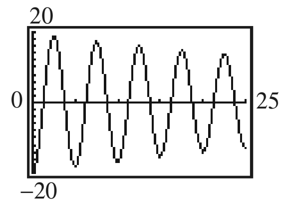
26. a. $d=-5e^{-0.7t/2(10)}\cos\left(\sqrt{\left(\frac{2\pi}{3}\right)^2-\frac{(0.7)^2}{4(10)^2}}\,t\right)$

$d=-5e^{-0.7t/20}\cos\left(\sqrt{\frac{4\pi^2}{9}-\frac{0.49}{400}}\,t\right)$

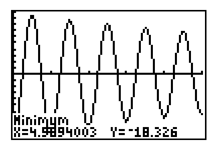
b. 

27. a. Damped motion with a bob of mass 20 kg and a damping factor of 0.7 kg/s.

b. 20 meters leftward

c. 

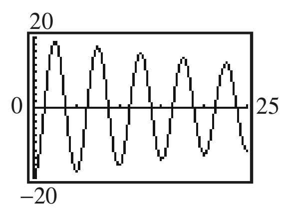
d. The displacement of the bob at the start of the second oscillation is about 18.33 meters leftward.



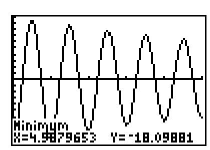
e. The displacement of the bob approaches zero, since $e^{-0.7t/40}\to 0$ as $t\to\infty$.

28. a. Damped motion with a bob of mass 20 kg and a damping factor of 0.8 kg/s.

b. 20 meters leftward

c. 

d. The displacement of the bob at the start of the second oscillation is about 18.10 meters leftward.

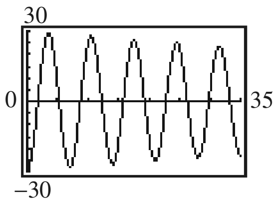


e. The displacement of the bob approaches zero, since $e^{-0.8t/40}\to 0$ as $t\to\infty$.

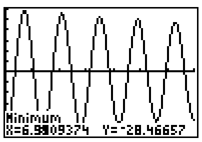
29. a. Damped motion with a bob of mass 40 kg and a damping factor of 0.6 kg/s.

b. 30 meters leftward

c.



d. The displacement of the bob at the start of the second oscillation is about 28.47 meters leftward.

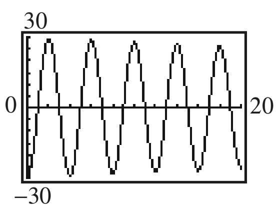


e. The displacement of the bob approaches zero, since $e^{-0.6t/80} \to 0$ as $t \to \infty$.

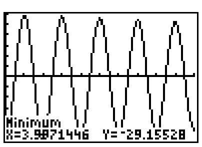
30. a. Damped motion with a bob of mass 35 kg and a damping factor of 0.5 kg/s.

b. 30 meters leftward

c.



d. The displacement of the bob at the start of the second oscillation is about 29.15 meters leftward.

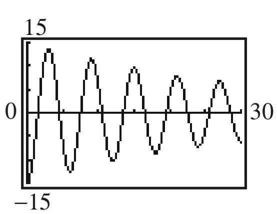


e. The displacement of the bob approaches zero, since $e^{-0.5t/70} \to 0$ as $t \to \infty$.

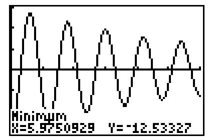
31. a. Damped motion with a bob of mass 15 kg and a damping factor of 0.9 kg/s.

b. 15 meters leftward

c.



d. The displacement of the bob at the start of the second oscillation is about 12.53 meters leftward.

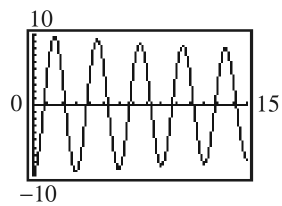


e. The displacement of the bob approaches zero, since $e^{-0.9t/30} \to 0$ as $t \to \infty$.

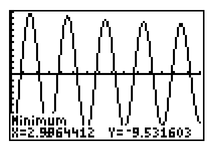
32. a. Damped motion with a bob of mass 25 kg and a damping factor of 0.8 kg/s.

b. 10 meters leftward

c.

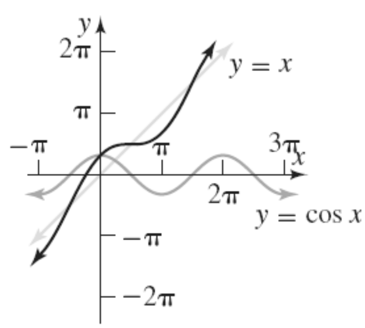


d. The displacement of the bob at the start of the second oscillation is 9.53 meters leftward.

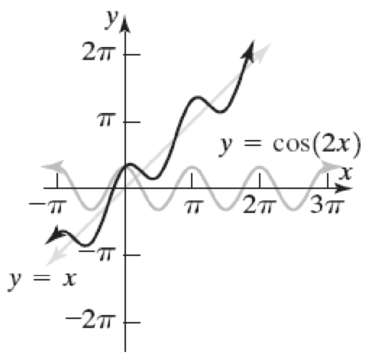


e. The displacement of the bob approaches zero, since $e^{-0.8t/50} \to 0$ as $t \to \infty$.

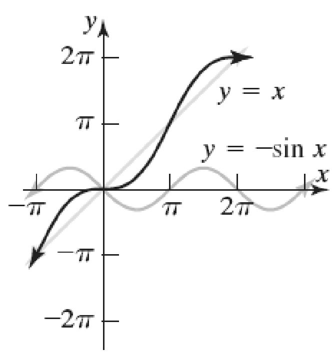
33. $f(x) = x + \cos x$



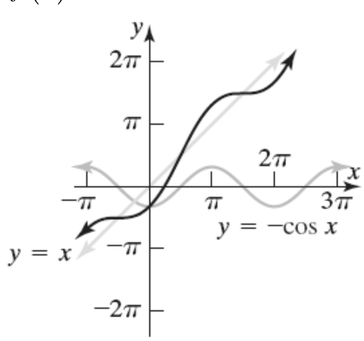
34. $f(x) = x + \cos(2x)$



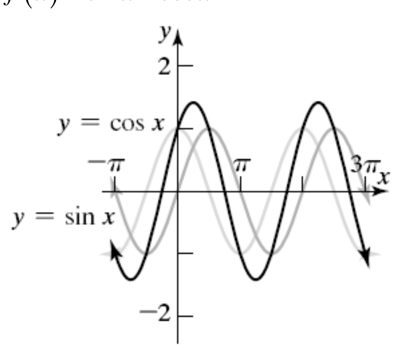
35. $f(x) = x - \sin x$



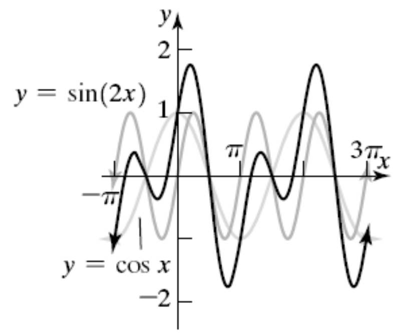
36. $f(x) = x - \cos x$



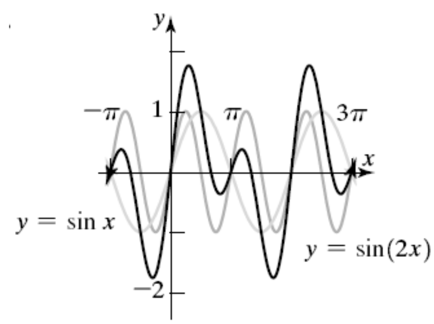
37. $f(x) = \sin x + \cos x$



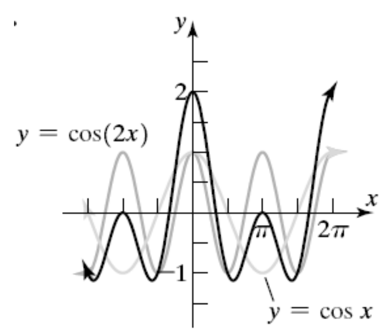
38. $f(x) = \sin(2x) + \cos x$



39. $f(x) = \sin x + \sin(2x)$



40. $f(x) = \cos(2x) + \cos x$

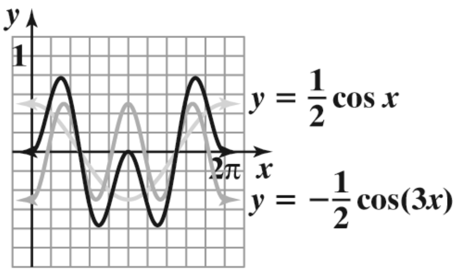


41. a. $f(x) = \sin(2x)\sin x$

$$= \frac{1}{2}\left[\cos(2x - x) - \cos(2x + x)\right]$$

$$= \frac{1}{2}\left[\cos(x) - \cos(3x)\right]$$

b.

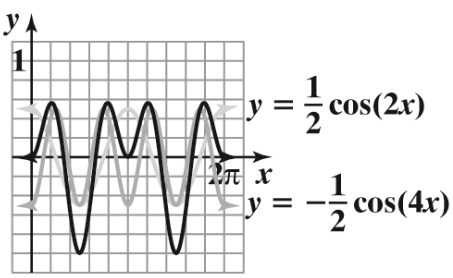


42. a. $F(x) = \sin(3x)\sin x$

$$= \frac{1}{2}[\cos(3x - x) - \cos(3x + x)]$$

$$= \frac{1}{2}[\cos(2x) - \cos(4x)]$$

b.

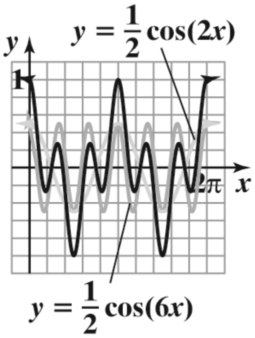


43. a. $G(x) = \cos(4x)\cos(2x)$

$$= \frac{1}{2}[\cos(4x - 2x) + \cos(4x + 2x)]$$

$$= \frac{1}{2}[\cos(2x) + \cos(6x)]$$

b.

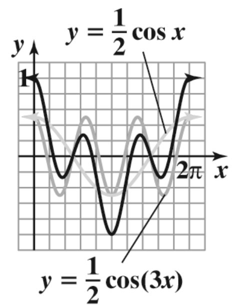


44. a. $h(x) = \cos(2x)\cos(x)$

$$= \frac{1}{2}[\cos(2x - x) + \cos(2x + x)]$$

$$= \frac{1}{2}[\cos(x) + \cos(3x)]$$

b.

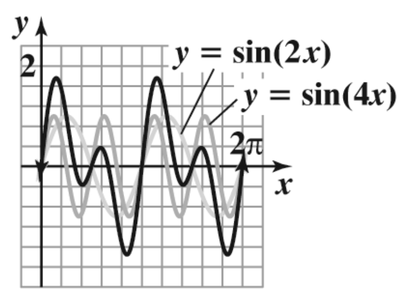


45. a. $H(x) = 2\sin(3x)\cos(x)$

$$= (2)\left(\frac{1}{2}\right)[\sin(3x + x) + \sin(3x - x)]$$

$$= \sin(4x) + \sin(2x)$$

b.

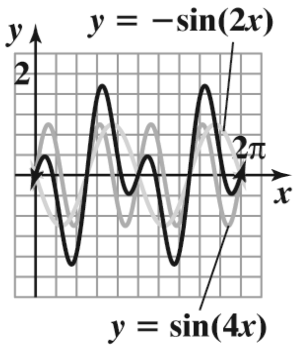


46. a. $g(x) = 2\sin(x)\cos(3x)$

$$= (2)\left(\frac{1}{2}\right)[\sin(x + 3x) + \sin(x - 3x)]$$

$$= \sin(4x) + \sin(-2x)$$

$$= \sin(4x) - \sin(2x)$$



47. The maximum displacement is the amplitude so we have $a = 0.80$. The frequency is given by $f = \frac{\omega}{2\pi} = 520$. Therefore, $\omega = 1040\pi$ and the motion of the diaphragm is described by the equation $d = 0.80\cos(1040\pi t)$.

48. If we consider a horizontal line through the center of the wheel as the equilibrium line, then the amplitude is $|a| = \left|\frac{165}{2}\right| = 82.5$. The wheel completes 1.6 so the period is $\frac{2\pi}{\omega} = \frac{1}{1.6}$ and $\omega = 3.2\pi$. We want the rider to be at the lowest position at time $t = 0$. Since $a\cos(\omega t)$ peaks at $t = 0$ if $a > 0$ and is at its lowest if $a < 0$, we select a cosine model and need $a = -82.5$. Using the model $d = a\cos(\omega t) + b$, we have $d = -82.5\cos(wt) + b$. When $t = 0$, the rider should be 15 feet above the ground. That is,

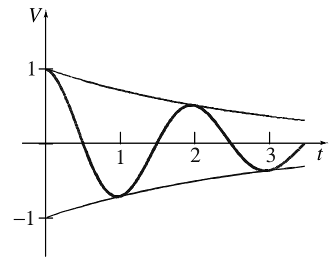
$$15 = -82.5\cos(0) + b$$
$$15 = -82.5 + b$$
$$97.5 = b$$

Therefore, the equation that describes the rider's motion is $d = 97.5 - 82.5\cos(3.2\pi t)$.

49. The maximum displacement is the amplitude so we have $a = 0.01$. The frequency is given by $f = \frac{\omega}{2\pi} = 440$. Therefore, $\omega = 880\pi$ and the movement of the tuning fork is described by the equation $d = 0.01\sin(880\pi t)$.

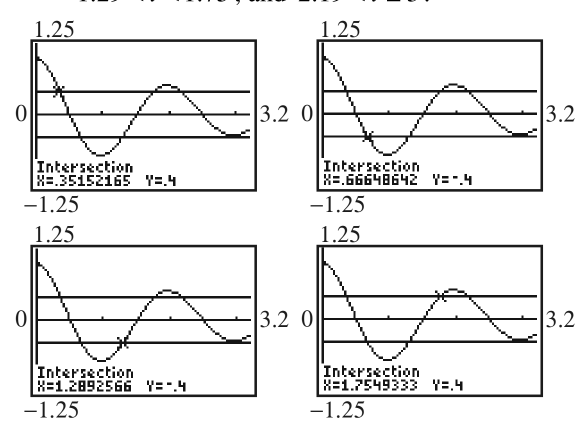
50. The maximum displacement is the amplitude so we have $a = 0.025$. The frequency is given by $f = \frac{\omega}{2\pi} = 329.63$. Therefore, $\omega = 659.26\pi$ and the movement of the tuning fork is described by the equation $d = 0.025\sin(659.26\pi t)$.

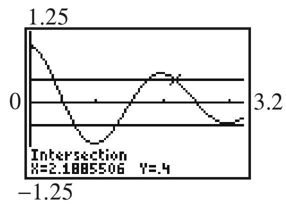
51. a.

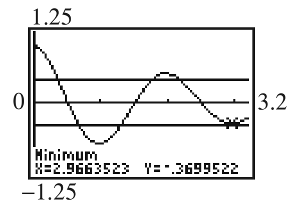


b. On the interval $0 \le t \le 3$, the graph of V touches the graph of $y = e^{-t/3}$ when $t = 0, 2$. The graph of V touches the graph of $y = -e^{-t/3}$ when $t = 1, 3$.

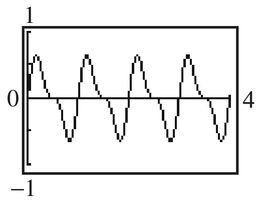
c. We need to solve the inequality $-0.4 < e^{-t/3}\cos(\pi t) < 0.4$ on the interval $0 \le t \le 3$. To do so, we consider the graphs of $y = -0.4$, $y = e^{-t/3}\cos(\pi t)$, and $y = 0.4$. On the interval $0 \le t \le 3$, we can use the INTERSECT feature on a calculator to determine that $y = e^{-t/3}\cos(\pi t)$ intersects $y = 0.4$ when $t \approx 0.35$, $t \approx 1.75$, and $t \approx 2.19$, $y = e^{-t/3}\cos(\pi t)$ intersects $y = -0.4$ when $t \approx 0.67$ and $t \approx 1.29$ and the graph shows that $-0.4 < e^{-t/3}\cos(\pi t) < 0.4$ when $t = 3$. Therefore, the voltage V is between –0.4 and 0.4 on the intervals $0.35 < t < 0.67$, $1.29 < t < 1.75$, and $2.19 < t \le 3$.



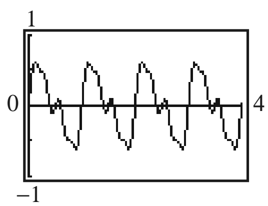




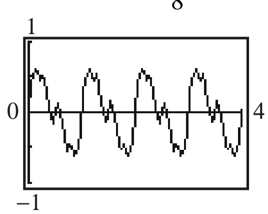
52. a. Let $Y_1 = \frac{1}{2}\sin(2\pi) + \frac{1}{4}\sin(4\pi x)$.



b. Let $Y_1 = \frac{1}{2}\sin(2\pi) + \frac{1}{4}\sin(4\pi x) + \frac{1}{8}\sin(8\pi x)$.

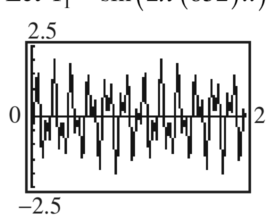


c. Let $Y_1 = \frac{1}{2}\sin(2\pi) + \frac{1}{4}\sin(4\pi x) + \frac{1}{8}\sin(8\pi x) + \frac{1}{16}\sin(16\pi x)$



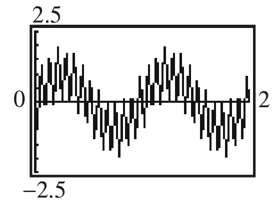
d. $f(x) = \frac{1}{2}\sin(2\pi x) + \frac{1}{4}\sin(4\pi x) + \frac{1}{8}\sin(8\pi x) + \frac{1}{16}\sin(16\pi x) + \frac{1}{32}\sin(32\pi x)$

53. Let $Y_1 = \sin(2\pi(852)x) + \sin(2\pi(1209)x)$.



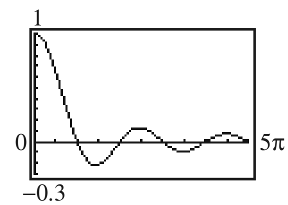
54. The sound emitted by touching * is $y = \sin(2\pi(941)t) + \sin(2\pi(1209)t)$.

Let $Y_1 = \sin(2\pi(941)x) + \sin(2\pi(1209)x)$.



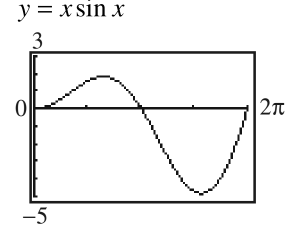
55 – 56. CBL Experiments

57. Let $Y_1 = \frac{\sin x}{x}$.

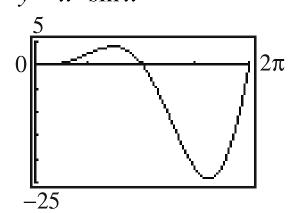


As x approaches 0, $\frac{\sin x}{x}$ approaches 1.

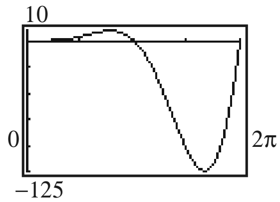
58. $y = x\sin x$



$y = x^2 \sin x$

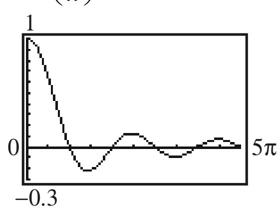


$y = x^3 \sin x$

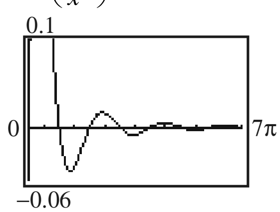


Possible observations: The graph lies between the bounding curves $y = \pm x$, $y = \pm x^2$, $y = \pm x^3$, respectively, touching them at odd multiples of $\frac{\pi}{2}$. The x-intercepts of each graph are the multiples of π.

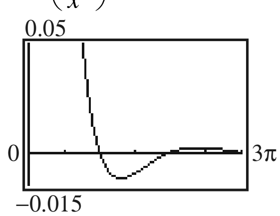
59. $y = \left(\frac{1}{x}\right)\sin x$



$y = \left(\frac{1}{x^2}\right)\sin x$



$y = \left(\frac{1}{x^3}\right)\sin x$



Possible observation: As x gets larger, the graph of $y = \left(\frac{1}{x^n}\right)\sin x$ gets closer to $y = 0$.

60. Answers will vary.

Chapter 8 Review Exercises

1. opposite = 4; adjacent = 3; hypotenuse = ?

$(\text{hypotenuse})^2 = 4^2 + 3^2 = 25$

$\text{hypotenuse} = \sqrt{25} = 5$

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{4}{5}$ $\qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{5}{4}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{3}{5}$ $\qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{5}{3}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{4}{3}$ $\qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{3}{4}$

2. adjacent = 2; hypotenuse = 4; opposite = ?

$(\text{opposite})^2 + 2^2 = 4^2$

$(\text{opposite})^2 = 16 - 4 = 12$

$\text{opposite} = \sqrt{12} = 2\sqrt{3}$

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{2}{4} = \frac{1}{2}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{2\sqrt{3}}{2} = \sqrt{3}$

$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{4}{2\sqrt{3}} = \frac{4}{2\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{4}{2} = 2$

$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{2}{2\sqrt{3}} = \frac{2}{2\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

3. $\cos 62^\circ - \sin 28^\circ = \cos 62^\circ - \cos(90^\circ - 28^\circ)$

$= \cos 62^\circ - \cos 62^\circ$

$= 0$

4. $\frac{\sec 55^\circ}{\csc 35^\circ} = \frac{\csc(90^\circ - 55^\circ)}{\csc 35^\circ} = \frac{\csc 35^\circ}{\csc 35^\circ} = 1$

5. $\cos^2 40^\circ + \cos^2 50^\circ = \sin^2(90^\circ - 40^\circ) + \cos^2 50^\circ$

$= \sin^2 50^\circ + \cos^2 50^\circ$

$= 1$

6. $c = 10, \ B = 20°$

$$\sin B = \frac{b}{c}$$

$$\sin 20° = \frac{b}{10}$$

$$b = 10\sin 20° \approx 3.42$$

$$\cos B = \frac{a}{c}$$

$$\cos 20° = \frac{a}{10}$$

$$a = 10\cos 20° \approx 9.40$$

$$A = 90° - B = 90° - 20° = 70°$$

7. $b = 2, \ c = 5$

$$c^2 = a^2 + b^2$$

$$a^2 = c^2 - b^2 = 5^2 - 2^2 = 25 - 4 = 21$$

$$a = \sqrt{21} \approx 4.58$$

$$\sin B = \frac{b}{c} = \frac{2}{5}$$

$$B = \sin^{-1}\left(\frac{2}{5}\right) \approx 23.6°$$

$$A = 90° - B \approx 90° - 23.6° = 66.4°$$

8. $A = 50°, \ B = 30°, \ a = 1$

$$C = 180° - A - B = 180° - 50° - 30° = 100°$$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 50°}{1} = \frac{\sin 30°}{b}$$

$$b = \frac{1\sin 30°}{\sin 50°} \approx 0.65$$

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\frac{\sin 100°}{c} = \frac{\sin 50°}{1}$$

$$c = \frac{1\sin 100°}{\sin 50°} \approx 1.29$$

9. $A = 100°, \ c = 2, \ a = 5$

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\frac{\sin C}{2} = \frac{\sin 100°}{5}$$

$$\sin C = \frac{2\sin 100°}{5} \approx 0.3939$$

$$C = \sin^{-1}\left(\frac{2\sin 100°}{5}\right)$$

$C \approx 23.2°$ or $C \approx 156.8°$

The value $156.8°$ is discarded because $A + C > 180°$. Thus, $\alpha \approx 23.2°$.

$$B = 180° - A - C \approx 180° - 100° - 23.2° = 56.8°$$

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin 56.8°}{b} = \frac{\sin 100°}{5}$$

$$b = \frac{5\sin 56.8°}{\sin 100°} \approx 4.25$$

10. $a = 3, \ c = 1, \ C = 110°$

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\frac{\sin 110°}{1} = \frac{\sin A}{3}$$

$$\sin A = \frac{3\sin 110°}{1} \approx 2.8191$$

No angle A exists for which $\sin A > 1$. Thus, there is no triangle with the given measurements.

11. $a = 3, \ c = 1, \ B = 100°$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$= 3^2 + 1^2 - 2\cdot 3\cdot 1\cos 100°$$

$$= 10 - 6\cos 100°$$

$$b = \sqrt{10 - 6\cos 100°} \approx 3.32$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{3.32^2 + 1^2 - 3^2}{2(3.32)(1)} = \frac{3.0224}{6.64}$$

$$A = \cos^{-1}\left(\frac{3.0224}{6.64}\right) \approx 62.9°$$

$$C = 180° - A - B \approx 180° - 62.9° - 100° = 17.1°$$

12. $a=3,\ b=5,\ B=80^\circ$

$$\frac{\sin A}{a}=\frac{\sin B}{b}$$
$$\frac{\sin A}{3}=\frac{\sin 80^\circ}{5}$$
$$\sin A=\frac{3\sin 80^\circ}{5}\approx 0.5909$$
$$A=\sin^{-1}\left(\frac{3\sin 80^\circ}{5}\right)$$
$$A\approx 36.2^\circ \text{ or } A\approx 143.8^\circ$$

The value 143.8° is discarded because $A+B>180^\circ$. Thus, $A\approx 36.2^\circ$.

$$C=180^\circ-A-B\approx 180^\circ-36.2^\circ-80^\circ=63.8^\circ$$
$$\frac{\sin C}{c}=\frac{\sin B}{b}$$
$$\frac{\sin 63.8^\circ}{c}=\frac{\sin 80^\circ}{5}$$
$$c=\frac{5\sin 63.8^\circ}{\sin 80^\circ}\approx 4.56$$

13. $a=2,\ b=3,\ c=1$

$$a^2=b^2+c^2-2bc\cos A$$
$$\cos A=\frac{b^2+c^2-a^2}{2bc}=\frac{3^2+1^2-2^2}{2(3)(1)}=1$$
$$A=\cos^{-1}1=0^\circ$$

No triangle exists with an angle of 0°.

14. $a=10,\ b=7,\ c=8$

$$a^2=b^2+c^2-2bc\cos A$$
$$\cos A=\frac{b^2+c^2-a^2}{2bc}=\frac{7^2+8^2-10^2}{2(7)(8)}=\frac{13}{112}$$
$$A=\cos^{-1}\left(\frac{13}{112}\right)\approx 83.3^\circ$$
$$b^2=a^2+c^2-2ac\cos B$$
$$\cos B=\frac{a^2+c^2-b^2}{2ac}=\frac{10^2+8^2-7^2}{2(10)(8)}=\frac{115}{160}$$
$$B=\cos^{-1}\left(\frac{115}{160}\right)\approx 44.0^\circ$$
$$C=180^\circ-A-B\approx 180^\circ-83.3^\circ-44.0^\circ\approx 52.7^\circ$$

15. $a=1,\ b=3,\ C=40^\circ$

$$c^2=a^2+b^2-2ab\cos C$$
$$=1^2+3^2-2\cdot 1\cdot 3\cos 40^\circ$$
$$=10-6\cos 40^\circ$$
$$c=\sqrt{10-6\cos 40^\circ}\approx 2.32$$
$$a^2=b^2+c^2-2bc\cos A$$
$$\cos A=\frac{b^2+c^2-a^2}{2bc}=\frac{3^2+2.32^2-1^2}{2(3)(2.32)}=\frac{13.3824}{13.92}$$
$$A=\cos^{-1}\left(\frac{13.3824}{13.92}\right)\approx 16.0^\circ$$
$$B=180^\circ-A-C\approx 180^\circ-16.0^\circ-40^\circ=124.0^\circ$$

16. $a=5,\ b=3,\ A=80^\circ$

$$\frac{\sin B}{b}=\frac{\sin A}{a}$$
$$\frac{\sin B}{3}=\frac{\sin 80^\circ}{5}$$
$$\sin B=\frac{3\sin 80^\circ}{5}\approx 0.5909$$
$$B=\sin^{-1}\left(\frac{3\sin 80^\circ}{5}\right)$$
$$B\approx 36.2^\circ \text{ or } B\approx 143.8^\circ$$

The value 143.8° is discarded because $A+B>180^\circ$. Thus, $B\approx 36.2^\circ$.

$$C=180^\circ-A-B\approx 180^\circ-80^\circ-36.2^\circ=63.8^\circ$$
$$\frac{\sin C}{c}=\frac{\sin A}{a}$$
$$\frac{\sin 63.8^\circ}{c}=\frac{\sin 80^\circ}{5}$$
$$c=\frac{5\sin 63.8^\circ}{\sin 80^\circ}\approx 4.56$$

17. $a=1,\ b=\frac{1}{2},\ c=\frac{4}{3}$

$$a^2=b^2+c^2-2bc\cos A$$
$$\cos A=\frac{b^2+c^2-a^2}{2bc}=\frac{\left(\frac{1}{2}\right)^2+\left(\frac{4}{3}\right)^2-1^2}{2\left(\frac{1}{2}\right)\left(\frac{4}{3}\right)}=\frac{27}{37}$$
$$A=\cos^{-1}\left(\frac{27}{37}\right)\approx 39.6^\circ$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$\cos B = \frac{a^2+c^2-b^2}{2ac} = \frac{1^2+\left(\frac{4}{3}\right)^2-\left(\frac{1}{2}\right)^2}{2(1)\left(\frac{4}{3}\right)} = \frac{91}{96}$$

$$B = \cos^{-1}\left(\frac{91}{96}\right) \approx 18.6^\circ$$

$$C = 180^\circ - A - B \approx 180^\circ - 39.6^\circ - 18.6^\circ \approx 121.8^\circ$$

18. $a = 3,\ A = 10^\circ,\ b = 4$

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin B}{4} = \frac{\sin 10^\circ}{3}$$

$$\sin B = \frac{4\sin 10^\circ}{3} \approx 0.2315$$

$$B = \sin^{-1}\left(\frac{4\sin 10^\circ}{3}\right)$$

$B_1 \approx 13.4^\circ$ or $B_2 \approx 166.6^\circ$

For both values, $A + B < 180^\circ$. Therefore, there are two triangles.

$$C_1 = 180^\circ - A - B_1 \approx 180^\circ - 10^\circ - 13.4^\circ \approx 156.6^\circ$$

$$\frac{\sin A}{a} = \frac{\sin C_1}{c_1}$$

$$\frac{\sin 10^\circ}{3} = \frac{\sin 156.6^\circ}{c_1}$$

$$c_1 = \frac{3\sin 156.6^\circ}{\sin 10^\circ} \approx 6.86$$

$$C_2 = 180^\circ - A - B_2 = 180^\circ - 10^\circ - 166.6^\circ \approx 3.4^\circ$$

$$\frac{\sin A}{a} = \frac{\sin C_2}{c_2}$$

$$\frac{\sin 10^\circ}{3} = \frac{\sin 3.4^\circ}{c_2}$$

$$c_2 = \frac{3\sin 3.4^\circ}{\sin 10^\circ} \approx 1.02$$

Two triangles: $B_1 \approx 13.4^\circ, C_1 \approx 156.6^\circ, c_1 \approx 6.86$

or $B_2 \approx 166.6^\circ, C_2 \approx 3.4^\circ, c_2 \approx 1.02$

19. $a = 4,\ A = 20^\circ,\ B = 100^\circ$

$$C = 180^\circ - A - B = 180^\circ - 20^\circ - 100^\circ = 60^\circ$$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 20^\circ}{4} = \frac{\sin 100^\circ}{b}$$

$$b = \frac{4\sin 100^\circ}{\sin 20^\circ} \approx 11.52$$

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\frac{\sin 60^\circ}{c} = \frac{\sin 20^\circ}{4}$$

$$c = \frac{4\sin 60^\circ}{\sin 20^\circ} \approx 10.13$$

20. $c = 5,\ b = 4,\ A = 70^\circ$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$a^2 = 4^2 + 5^2 - 2\cdot 4\cdot 5\cos 70^\circ = 41 - 40\cos 70^\circ$$

$$a = \sqrt{41 - 40\cos 70^\circ} \approx 5.23$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

$$= \frac{5.23^2 + 4^2 - 5^2}{2(5.23)(4)} = \frac{18.3529}{41.48}$$

$$C = \cos^{-1}\left(\frac{18.3529}{41.48}\right) \approx 64.0^\circ$$

$$B = 180^\circ - A - C \approx 180^\circ - 70^\circ - 64.0^\circ \approx 46.0^\circ$$

21. $a = 2,\ b = 3,\ C = 40^\circ$

$$K = \frac{1}{2}ab\sin C = \frac{1}{2}(2)(3)\sin 40^\circ \approx 1.93$$

22. $b = 4,\ c = 10,\ A = 70^\circ$

$$K = \frac{1}{2}bc\sin A = \frac{1}{2}(4)(10)\sin 70^\circ \approx 18.79$$

23. $a = 4,\ b = 3,\ c = 5$

$$s = \frac{1}{2}(a+b+c) = \frac{1}{2}(4+3+5) = 6$$

$$K = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{(6)(2)(3)(1)} = \sqrt{36} = 6$$

24. $a = 4,\ b = 2,\ c = 5$

$$s = \frac{1}{2}(a+b+c) = \frac{1}{2}(4+2+5) = 5.5$$
$$K = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{(5.5)(1.5)(3.5)(0.5)} = \sqrt{14.4375} \approx 3.80$$

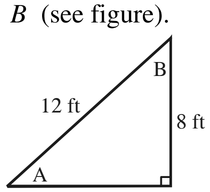
25. $A = 50°,\ B = 30°,\ a = 1$

$$C = 180° - A - B = 180° - 50° - 30° = 100°$$
$$K = \frac{a^2 \sin B \sin C}{2\sin A} = \frac{1^2 \sin 30° \cdot \sin 100°}{2\sin 50°} \approx 0.32$$

26. To find the area of the segment, we subtract the area of the triangle from the area of the sector.

$$A_{\text{Sector}} = \frac{1}{2}r^2\theta = \frac{1}{2}\cdot 6^2\left(50\cdot\frac{\pi}{180}\right) \approx 15.708 \text{ in}^2$$
$$A_{\text{Triangle}} = \frac{1}{2}ab\sin\theta = \frac{1}{2}\cdot 6\cdot 6\sin 50° \approx 13.789 \text{ in}^2$$
$$A_{\text{Segment}} = 15.708 - 13.789 \approx 1.92 \text{ in}^2$$

27. $c = 12$ feet, $a = 8$ feet. We need to find A and B (see figure).



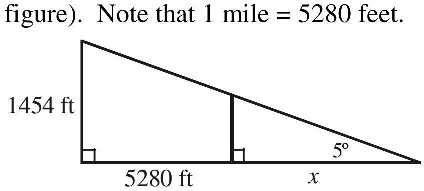
$$\sin A = \frac{8}{12} \Rightarrow A = \sin^{-1}\left(\frac{8}{12}\right) \approx 41.8°$$
$$B = 180° - 90° - A = 180° - 90° - 41.8° = 48.2°$$

28. Let x = the distance across the river.

$$\tan(25°) = \frac{x}{50}$$
$$x = 50\tan(25°) \approx 23.32$$

Thus, the distance across the river is 23.32 feet.

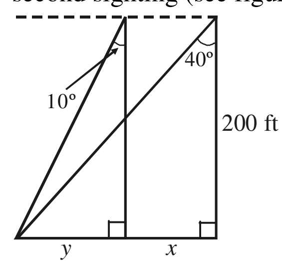
29. Let x = the distance the boat is from shore (see figure). Note that 1 mile = 5280 feet.



$$\tan(5°) = \frac{1454}{x+5280}$$
$$x + 5280 = \frac{1454}{\tan(5°)}$$
$$x = \frac{1454}{\tan(5°)} - 5280 \approx 16{,}619.30 - 5280 = 11{,}339.30$$

Thus, the boat is approximately 11,339.30 feet, or $\frac{11{,}339.30}{5280} \approx 2.15$ miles, from shore.

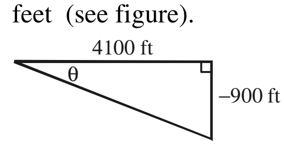
30. Let x = the distance traveled by the glider between the two sightings, and let y = the distance from the stationary object to a point on the ground beneath the glider at the time of the second sighting (see figure).



$$\tan(10°) = \frac{y}{200}$$
$$y = 200\tan(10°)$$
$$\tan(40°) = \frac{x+y}{200}$$
$$x + y = 200\tan(40°)$$
$$x = 200\tan(40°) - y = 200\tan(40°) - 200\tan(10°) \approx 167.82 - 35.27 = 132.55$$

The glider traveled 132.55 feet in 1 minute, so the speed of the glider is 132.55 ft/min.

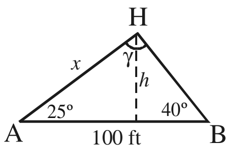
31. Let θ = the inclination (grade) of the trail. The "rise" of the trail is $4100 - 5000 = -900$ feet (see figure).



$$\sin\theta = \frac{900}{4100}$$
$$\theta = \sin^{-1}\left(\frac{900}{4100}\right) \approx 12.7°$$

The trail is inclined about 12.7° from the lake to the hotel.

32. Let h = the height of the helicopter, x = the distance from observer A to the helicopter, and $\gamma = \angle AHB$ (see figure).



$\gamma = 180° - 40° - 25° = 115°$

$$\frac{\sin 40°}{x} = \frac{\sin 115°}{100}$$

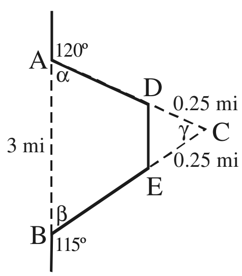
$$x = \frac{100 \sin 40°}{\sin 115°} \approx 70.92 \text{ feet}$$

$$\sin 25° = \frac{h}{x} \approx \frac{h}{70.92}$$

$h \approx 70.92 \sin 25° \approx 29.97$ feet

The helicopter is about 29.97 feet high.

33. $\alpha = 180° - 120° = 60°$; $\beta = 180° - 115° = 65°$;
$\gamma = 180° - 60° - 65° = 55°$



$$\frac{\sin 60°}{BC} = \frac{\sin 55°}{3}$$

$$BC = \frac{3 \sin 60°}{\sin 55°} \approx 3.17 \text{ mi}$$

$$\frac{\sin 65°}{AC} = \frac{\sin 55°}{3}$$

$$AC = \frac{3 \sin 65°}{\sin 55°} \approx 3.32 \text{ mi}$$

$BE = 3.17 - 0.25 = 2.92$ mi

$AD = 3.32 - 0.25 = 3.07$ mi

For the isosceles triangle,

$$\angle CDE = \angle CED = \frac{180° - 55°}{2} = 62.5°$$

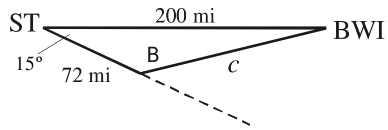
$$\frac{\sin 55°}{DE} = \frac{\sin 62.5°}{0.25}$$

$$DE = \frac{0.25 \sin 55°}{\sin 62.5°} \approx 0.23 \text{ miles}$$

The length of the highway is
$2.92 + 3.07 + 0.23 = 6.22$ miles.

34. a. After 4 hours, the sailboat would have sailed $18(4) = 72$ miles.

Find the third side of the triangle to determine the distance from the island:



$a = 72,\ b = 200,\ C = 15°$

$c^2 = a^2 + b^2 - 2ab \cos C$

$c^2 = 72^2 + 200^2 - 2 \cdot 72 \cdot 200 \cos 15°$

$\quad = 45{,}184 - 28{,}800 \cos 15°$

$c \approx 131.8$ miles

The sailboat is about 131.8 miles from the island.

b. Find the measure of the angle opposite the 200 side:

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos B = \frac{72^2 + 131.8^2 - 200^2}{2(72)(131.8)} = -\frac{17{,}444.76}{18{,}979.2}$$

$$B = \cos^{-1}\left(-\frac{17{,}444.76}{18{,}979.2}\right) \approx 156.8°$$

The sailboat should turn through an angle of $180° - 156.8° = 23.2°$ to correct its course.

c. The original trip would have taken:

$t = \frac{200}{18} \approx 11.11$ hours . The actual trip takes:

$t = 4 + \frac{131.8}{18} \approx 11.32$ hours . The trip takes about 0.21 hour, or about 12.6 minutes longer.

35. Find the lengths of the two unknown sides of the middle triangle:

$x^2 = 100^2 + 125^2 - 2(100)(125)\cos 50°$

$\quad = 25{,}625 - 25{,}000 \cos 50°$

$x = \sqrt{25{,}625 - 25{,}000 \cos 50°} \approx 97.75$ feet

$y^2 = 70^2 + 50^2 - 2(70)(50)\cos 100°$

$\quad = 7400 - 7000 \cos 100°$

$y = \sqrt{7400 - 7000 \cos 100°} \approx 92.82$ feet

Find the areas of the three triangles:

$$K_1 = \frac{1}{2}(100)(125)\sin 50° \approx 4787.78 \text{ ft}^2$$

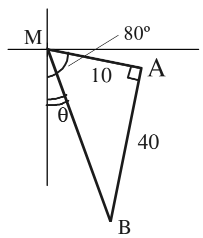
$$K_2 = \frac{1}{2}(50)(70)\sin 100° \approx 1723.41 \text{ ft}^2$$

$$s = \frac{1}{2}(50 + 97.75 + 92.82) = 120.285$$

$$K_3 = \sqrt{(120.285)(70.285)(22.535)(27.465)} \approx 2287.47 \text{ ft}^2$$

The approximate area of the lake is
$4787.78 + 1723.41 + 2287.47 = 8798.67 \text{ ft}^2$.

36. Find angle AMB and subtract from 80° to obtain θ.



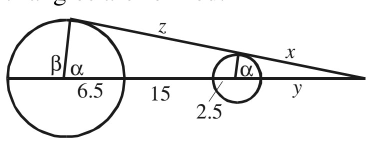
$$\tan \angle AMB = \frac{40}{10} = 4$$

$$\angle AMB = \tan^{-1} 4 \approx 76.0°$$

$$\theta \approx 80° - 76.0° \approx 4.0°$$

The bearing is S4.0°E .

37. Extend the tangent line until it meets a line extended through the centers of the pulleys. Label these extensions x and y. The distance between the points of tangency is z. Two similar triangles are formed.



Thus, $\frac{24+y}{y} = \frac{6.5}{2.5}$ where $24 + y$ is the hypotenuse of the larger triangle and y is the hypotenuse of the smaller triangle. Solve for y:

$$6.5y = 2.5(24 + y)$$

$$6.5y = 60 + 2.5y$$

$$4y = 60$$

$$y = 15$$

Use the Pythagorean Theorem to find x:

$$x^2 + 2.5^2 = 15^2$$

$$x^2 = 225 - 6.25 = 218.75$$

$$x = \sqrt{218.75} \approx 14.79$$

Use the Pythagorean Theorem to find z:

$$(z + 14.79)^2 + 6.5^2 = (24 + 15)^2$$

$$(z + 14.79)^2 = 1521 - 42.25 = 1478.75$$

$$z + 14.79 = \sqrt{1478.75} \approx 38.45$$

$$z \approx 23.66$$

Find α: $\cos\alpha = \frac{2.5}{15} \approx 0.1667$

$$\alpha = \cos^{-1}\left(\frac{2.5}{15}\right) \approx 1.4033 \text{ radians}$$

$$\beta \approx \pi - 1.4033 \approx 1.7383 \text{ radians}$$

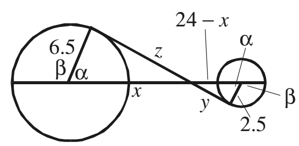
The arc length on the top of the larger pulley is about: 6.5(1.7383) = 11.30 inches.

The arc length on the top of the smaller pulley is about: 2.5(1.4033) = 3.51 inches.

The distance between the points of tangency is about 23.66 inches.

The length of the belt is about:
2(11.30 + 3.51 + 23.66) = 76.94 inches.

38. Let x = the hypotenuse of the larger right triangle in the figure below. Then $24 - x$ is the hypotenuse of the smaller triangle. The two triangles are similar.



$$\frac{6.5}{2.5} = \frac{x}{24 - x}$$

$$2.5x = 6.5(24 - x)$$

$$2.5x = 156 - 6.5x$$

$$9x = 156$$

$$x = \frac{52}{3}$$

$$24 - x = \frac{20}{3}$$

$$\cos\alpha = \frac{6.5}{\frac{52}{3}} = \frac{3}{8}$$

$$\alpha = \cos^{-1}\left(\frac{3}{8}\right) \approx 67.98°$$

$$\beta = 180° - 67.98° = 112.02°$$

$$z = 6.5\tan 67.98° \approx 16.07 \text{ in}$$

$$y = 2.5\tan 67.98° \approx 6.18 \text{ in}$$

The arc length on the top of the larger pulley is:
$6.5\left(112.02\cdot\frac{\pi}{180}\right)\approx 12.71$ inches.

The arc length on the top of the smaller pulley is
$2.5\left(112.02\cdot\frac{\pi}{180}\right)\approx 4.89$ inches.

The distance between the points of tangency is
$z+y\approx 16.07+6.18=22.25$ inches.

The length of the belt is about:
2(12.71 + 4.89 + 22.25) = 79.7 inches.

39. $d=-3\cos\left(\frac{\pi}{2}t\right)$

40. $d=6\sin(2t)$

a. Simple harmonic

b. 6 feet

c. π seconds

d. $\frac{1}{\pi}$ oscillation/second

41. $d=-2\cos(\pi t)$

a. Simple harmonic

b. 2 feet

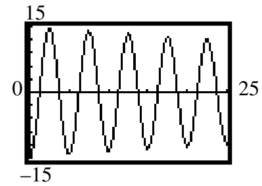
c. 2 seconds

d. $\frac{1}{2}$ oscillation/second

42. a. $d=-15e^{-0.75t/2(40)}\cos\left(\sqrt{\left(\frac{2\pi}{5}\right)^2-\frac{(0.75)^2}{4(40)^2}}\,t\right)$

$d=-15e^{-0.75t/80}\cos\left(\sqrt{\frac{4\pi^2}{25}-\frac{0.5625}{6400}}\,t\right)$

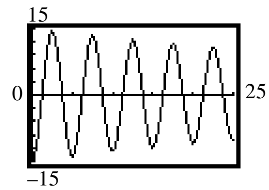
b.



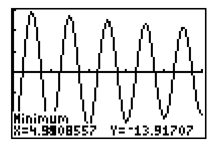
43. a. Damped motion with a bob of mass 20 kg and a damping factor of 0.6 kg/sec.

b. 15 meters leftward

c.

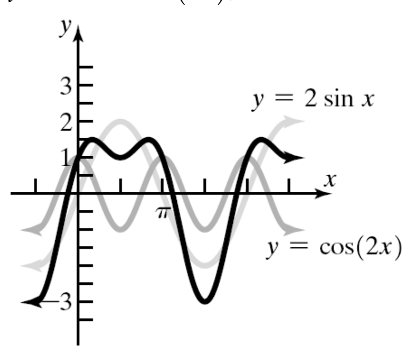


d. The displacement of the bob at the start of the second oscillation is about 13.92 meters.



e. It approaches zero, since $e^{-0.6t/40}\to 0$ as $t\to\infty$.

44. $y=2\sin x+\cos(2x),\quad 0\le x\le 2\pi$



Chapter 8 Test

1. opposite = 3; adjacent = 6; hypotenuse = ?

$(\text{hypotenuse})^2 = 3^2 + 6^2 = 45$

$\text{hypotenuse} = \sqrt{45} = 3\sqrt{5}$

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{3}{3\sqrt{5}} = \frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{6}{3\sqrt{5}} = \frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{3}{6} = \frac{1}{2}$

$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{3\sqrt{5}}{3} = \sqrt{5}$

$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{3\sqrt{5}}{6} = \frac{\sqrt{5}}{2}$

$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{6}{3} = 2$

2. $\sin 40° - \cos 50° = \sin 40° - \sin(90° - 50°)$

$= \sin 40° - \sin 40°$

$= 0$

3. Use the law of cosines to find a:

$a^2 = b^2 + c^2 - 2bc\cos A$

$= (17)^2 + (19)^2 - 2(17)(19)\cos 52°$

$\approx 289 + 361 - 646(0.616)$

$= 252.064$

$a = \sqrt{252.064} \approx 15.88$

Use the law of sines to find B.

$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{15.88}{\sin 52°} = \frac{17}{\sin B}$

$\sin B = \frac{17}{15.88}(\sin 52°) \approx 0.8436$

Since b is not the longest side of the triangle, we know that $B < 90^\circ$. Therefore,

$B = \sin^{-1}(0.8436) \approx 57.5°$

$C = 180° - A - B = 180° - 52° - 57.5° = 70.5°$

4. Use the Law of Sines to find b:

$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{12}{\sin 41°} = \frac{b}{\sin 22°}$

$b = \frac{12 \cdot \sin 22°}{\sin 41°} \approx 6.85$

$C = 180° - A - B = 180° - 41° - 22° = 117°$

Use the Law of Sines to find c:

$\frac{a}{\sin A} = \frac{c}{\sin C}$

$\frac{12}{\sin 41°} = \frac{c}{\sin 117°}$

$c = \frac{12 \cdot \sin 117°}{\sin 41°} \approx 16.30$

5. Use the law of cosines to find A.

$a^2 = b^2 + c^2 - 2bc\cos A$

$8^2 = (5)^2 + (10)^2 - 2(5)(10)\cos A$

$64 = 25 + 100 - 100\cos A$

$100\cos\alpha = 61$

$\cos A = 0.61$

$A = \cos^{-1}(0.61) \approx 52.41°$

Use the law of sines to find B.

$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{8}{\sin 52.41°} = \frac{5}{\sin B}$

$\sin B = \frac{5}{8}(\sin 52.41°)$

$\sin B \approx 0.495$

Since b is not the longest side of the triangle, we have that $B < 90°$. Therefore

$B = \sin^{-1}(0.495) \approx 29.67°$

$C = 180° - A - B$

$= 180° - 52.41° - 29.67°$

$= 97.92°$

6. $A = 55°,\ C = 20°,\ a = 4$

$B = 180° - A - C = 180° - 55° - 20° = 105°$

Use the law of sines to find b.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 55^\circ}{4} = \frac{\sin 105^\circ}{b}$$
$$b = \frac{4\sin 105^\circ}{\sin 55^\circ} \approx 4.72$$

Use the law of sines to find c.

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$
$$\frac{\sin 20^\circ}{c} = \frac{\sin 55^\circ}{4}$$
$$c = \frac{4\sin 20^\circ}{\sin 55^\circ} \approx 1.67$$

7. $a = 3,\ b = 7,\ A = 40^\circ$

Use the law of sines to find B

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$
$$\frac{\sin B}{7} = \frac{\sin 40^\circ}{3}$$
$$\sin B = \frac{7\sin 40^\circ}{3} \approx 1.4998$$

There is no angle B for which $\sin B > 1$. Therefore, there is no triangle with the given measurements.

8. $a = 8,\ b = 4,\ C = 70^\circ$

$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$c^2 = 8^2 + 4^2 - 2\cdot 8\cdot 4\cos 70^\circ$$
$$= 80 - 64\cos 70^\circ$$
$$c = \sqrt{80 - 64\cos 70^\circ} \approx 7.62$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{4^2 + 7.62^2 - 8^2}{2(4)(7.62)} = \frac{10.0644}{60.96}$$
$$A = \cos^{-1}\left(\frac{10.0644}{60.96}\right) \approx 80.5^\circ$$
$$B = 180^\circ - A - C \approx 180^\circ - 80.5^\circ - 70^\circ = 29.5^\circ$$

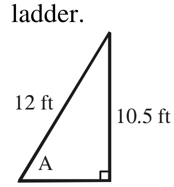
9. $a = 8,\ b = 4,\ C = 70^\circ$

$$K = \frac{1}{2}ab\sin C$$
$$= \frac{1}{2}(8)(4)\sin 70^\circ \approx 15.04 \text{ square units}$$

10. $a = 8,\ b = 5,\ c = 10$

$$s = \frac{1}{2}(a + b + c) = \frac{1}{2}(8 + 5 + 10) = 11.5$$
$$K = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{11.5(11.5-8)(11.5-5)(11.5-10)}$$
$$= \sqrt{11.5(3.5)(6.5)(1.5)}$$
$$= \sqrt{392.4375}$$
$$\approx 19.81 \text{ square units}$$

11. Let α = the angle formed by the ground and the ladder.

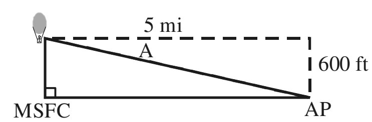


Then $\sin A = \dfrac{10.5}{12}$

$$A = \sin^{-1}\left(\frac{10.5}{12}\right) \approx 61.0^\circ$$

The angle formed by the ladder and ground is about 61.0°.

12. Let A = the angle of depression from the balloon to the airport.



Note that 5 miles = 26,400 feet.

$$\tan A = \frac{600}{26400} = \frac{1}{44}$$
$$A = \tan^{-1}\left(\frac{1}{44}\right) = 1.3^\circ$$

The angle of depression from the balloon to the airport is about 1.3°.

13. We can find the area of the shaded region by subtracting the area of the triangle from the area of the semicircle.
Since triangle ABC is a right triangle, we can use the Pythagorean theorem to find the length of the third side.

$$a^2 + b^2 = c^2$$
$$a^2 + 6^2 = 8^2$$
$$a^2 = 64 - 36 = 28$$
$$a = \sqrt{28} = 2\sqrt{7}$$

The area of the triangle is

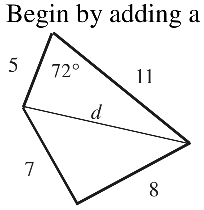
$$A=\frac{1}{2}bh=\frac{1}{2}\left(2\sqrt{7}\right)(6)=6\sqrt{7}\text{ square cm}.$$

The area of the semicircle is

$$A=\frac{1}{2}\pi r^2=\frac{1}{2}\pi(4)^2=8\pi\text{ square cm}.$$

Therefore, the area of the shaded region is $8\pi-6\sqrt{7}\approx 9.26$ square centimeters.

14. Begin by adding a diagonal to the diagram.



$$A_{\text{upper }\triangle}=\frac{1}{2}(5)(11)(\sin 72°)\approx 26.15\text{ sq. units}$$

By the law of cosines,

$$\begin{aligned}d^2&=(5)^2+(11)^2-2(5)(11)(\cos 72°)\\&=25+121-110(0.309)\\&=112.008\\d&=\sqrt{112.008}\approx 10.58\end{aligned}$$

Using Heron's formula for the lower triangle,

$$s=\frac{7+8+10.58}{2}=12.79$$

$$\begin{aligned}A_{\text{lower }\triangle}&=\sqrt{12.79(5.79)(4.79)(2.21)}\\&=\sqrt{783.9293}\\&\approx 28.00\text{ sq. units}\end{aligned}$$

Total Area $=26.15+28.00=54.15$ sq. units

15. Use the law of cosines to find c:

$$\begin{aligned}c^2&=a^2+b^2-2ab\cos C\\&=(4.2)^2+(3.5)^2-2(4.2)(3.5)\cos 32°\\&\approx 17.64+12.25-29.4(0.848)\\&=4.9588\\c&=\sqrt{4.9588}\approx 2.23\end{aligned}$$

Madison will have to swim about 2.23 miles.

16. Since ΔOAB is isosceles, we know that

$$\angle A=\angle B=\frac{180°-40°}{2}=70°$$

Then,

$$\frac{\sin A}{OB}=\frac{\sin 40°}{AB}$$

$$\frac{\sin 70°}{5}=\frac{\sin 40°}{AB}$$

$$AB=\frac{5\sin 40°}{\sin 70°}\approx 3.420$$

Now, AB is the diameter of the semicircle, so the radius is $\frac{3.420}{2}=1.710$.

$$A_{\text{Semicircle}}=\frac{1}{2}\pi r^2=\frac{1}{2}\pi(1.710)^2\approx 4.593\text{ sq. units}$$

$$\begin{aligned}A_{\text{Triangle}}&=\frac{1}{2}ab\sin(\measuredangle O)\\&=\frac{1}{2}(5)(5)(\sin 40°)\approx 8.035\text{ sq. units}\end{aligned}$$

$$\begin{aligned}A_{\text{Total}}&=A_{\text{Semicircle}}+A_{\text{Triangle}}\\&\approx 4.593+8.035\approx 12.63\text{ sq. units}\end{aligned}$$

17. Using Heron's formula:

$$s=\frac{5x+6x+7x}{2}=\frac{18x}{2}=9x$$

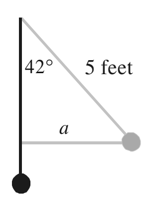
$$\begin{aligned}K&=\sqrt{9x(9x-5x)(9x-6x)(9x-7x)}\\&=\sqrt{9x\cdot 4x\cdot 3x\cdot 2x}\\&=\sqrt{216x^4}=\left(6\sqrt{6}\right)x^2\end{aligned}$$

Thus,

$$\begin{aligned}(6\sqrt{6})x^2&=54\sqrt{6}\\x^2&=9\\x&=3\end{aligned}$$

The sides are 15, 18, and 21.

18. Since we ignore all resistive forces, this is simple harmonic motion. Since the rest position $(t=0)$ is the vertical position $(d=0)$, the equation will have the form $d=a\sin(\omega t)$.



Now, the period is 6 seconds, so

$$6=\frac{2\pi}{\omega}$$

$$\omega=\frac{2\pi}{6}$$

$$\omega=\frac{\pi}{3}\text{ radians/sec}$$

From the diagram we see

$$\frac{a}{5}=\sin 42°$$

$$a=5(\sin 42°)$$

Thus, $d=5(\sin 42°)\left(\sin\frac{\pi t}{3}\right)$ or

$$d\approx 3.346\cdot\sin\left(\frac{\pi t}{3}\right).$$

Chapter 8 Cumulative Review

1. $3x^2+1=4x$

$$3x^2-4x+1=0$$

$$(3x-1)(x-1)=0$$

$$x=\frac{1}{3}\text{ or }x=1$$

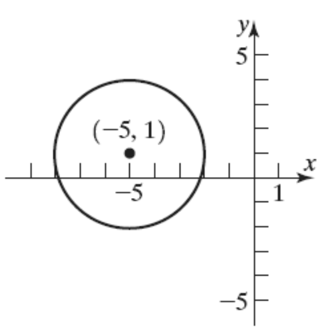
The solution set is $\left\{\frac{1}{3},1\right\}$.

2. Center $(-5,\ 1)$; Radius 3

$$(x-h)^2+(y-k)^2=r^2$$

$$(x-(-5))^2+(y-1)^2=3^2$$

$$(x+5)^2+(y-1)^2=9$$



3. $f(x)=\sqrt{x^2-3x-4}$

f will be defined provided

$g(x)=x^2-3x-4\ge 0$.

$$x^2-3x-4\ge 0$$

$$(x-4)(x+1)\ge 0$$

$x=4,\ x=-1$ are the zeros.

Interval	Test Number	$g(x)$	Pos./Neg.
$-\infty<x<-1$	–2	6	Positive
$-1<x<4$	0	–4	Negative
$4<x<\infty$	5	6	Positive

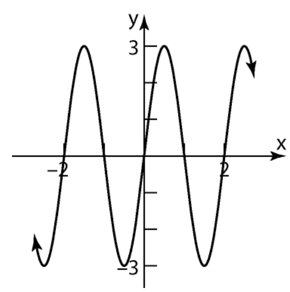
The domain of $f(x)=\sqrt{x^2-3x-4}$ is $\{x\mid x\le -1\text{ or }x\ge 4\}$.

4. $y=3\sin(\pi x)$

Amplitude: $|A|=|3|=3$

Period: $T=\frac{2\pi}{\pi}=2$

Phase Shift: $\frac{\phi}{\omega}=\frac{0}{\pi}=0$

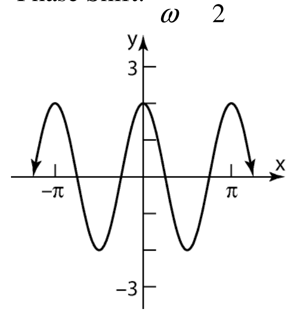


5. $y=-2\cos(2x-\pi)=-2\cos\left[2\left(x-\frac{\pi}{2}\right)\right]$

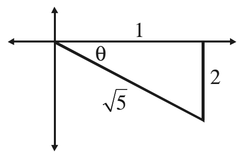
Amplitude: $|A|=|-2|=2$

Period: $T=\frac{2\pi}{2}=\pi$

Phase Shift: $\frac{\phi}{\omega}=\frac{\pi}{2}$



6. $\tan\theta = -2,\ \frac{3\pi}{2} < \theta < 2\pi$, so θ lies in quadrant IV.



a. $\frac{3\pi}{2} < \theta < 2\pi$, so $\sin\theta < 0$.

$$\sin\theta = -\frac{2}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$$

b. $\frac{3\pi}{2} < \theta < 2\pi$, so $\cos\theta > 0$

$$\cos\theta = \frac{1}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

c.
$$\sin(2\theta) = 2\sin\theta\cos\theta = 2\left(-\frac{2\sqrt{5}}{5}\right)\left(\frac{\sqrt{5}}{5}\right) = -\frac{20}{25} = -\frac{4}{5}$$

d.
$$\cos(2\theta) = \cos^2\theta - \sin^2\theta = \left(\frac{\sqrt{5}}{5}\right)^2 - \left(-\frac{2\sqrt{5}}{5}\right)^2 = \frac{5}{25} - \frac{20}{25} = -\frac{15}{25} = -\frac{3}{5}$$

e. $\frac{3\pi}{2} < \theta < 2\pi$

$\frac{3\pi}{4} < \frac{1}{2}\theta < \pi$

Since $\frac{1}{2}\theta$ lies in Quadrant II, $\sin\left(\frac{1}{2}\theta\right) > 0$.

$$\sin\left(\frac{1}{2}\theta\right) = \sqrt{\frac{1-\cos\theta}{2}} = \sqrt{\frac{1-\frac{\sqrt{5}}{5}}{2}} = \sqrt{\frac{\frac{5-\sqrt{5}}{5}}{2}} = \sqrt{\frac{5-\sqrt{5}}{10}}$$

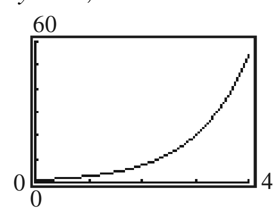
f. $\frac{3\pi}{2} < \theta < 2\pi$

$\frac{3\pi}{4} < \frac{1}{2}\theta < \pi$

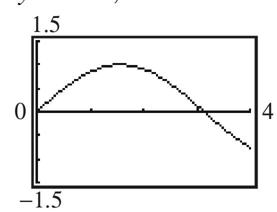
Since $\frac{1}{2}\theta$ lies in Quadrant II, $\cos\left(\frac{1}{2}\theta\right) < 0$.

$$\cos\left(\frac{1}{2}\theta\right) = -\sqrt{\frac{1+\cos\theta}{2}} = -\sqrt{\frac{1+\frac{\sqrt{5}}{5}}{2}} = -\sqrt{\frac{\frac{5+\sqrt{5}}{5}}{2}} = -\sqrt{\frac{5+\sqrt{5}}{10}}$$

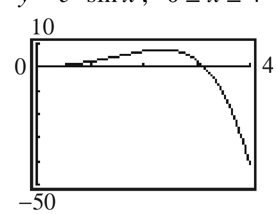
7. a. $y = e^x,\ 0 \le x \le 4$



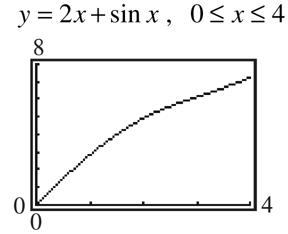
b. $y = \sin x,\ 0 \le x \le 4$



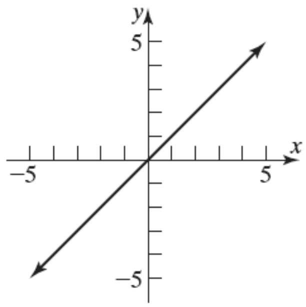
c. $y = e^x \sin x,\ 0 \le x \le 4$



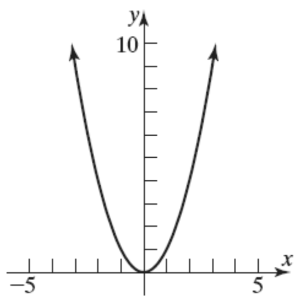
d. $y = 2x + \sin x,\ 0 \le x \le 4$



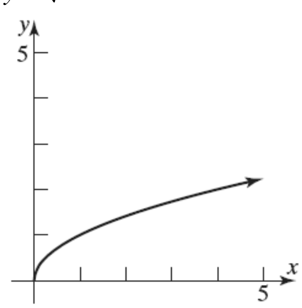
8. a. $y = x$



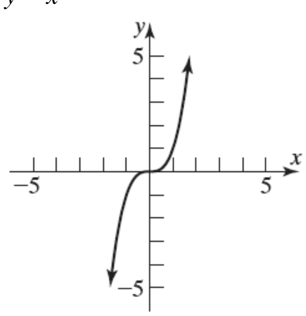
b. $y = x^2$



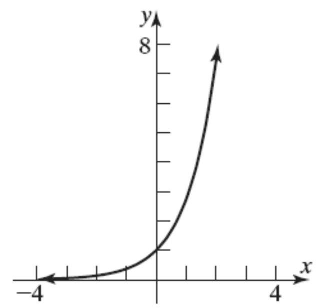
c. $y = \sqrt{x}$



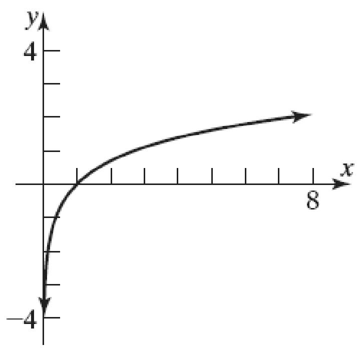
d. $y = x^3$



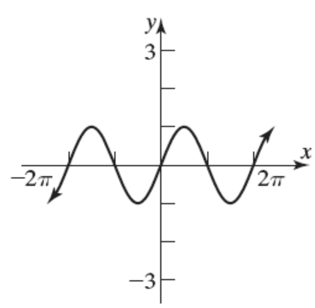
e. $y = e^x$



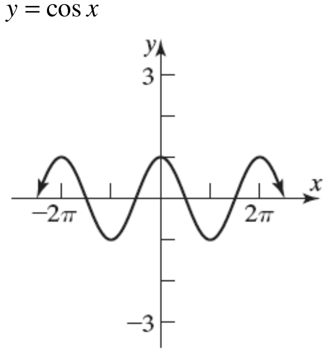
f. $y = \ln x$



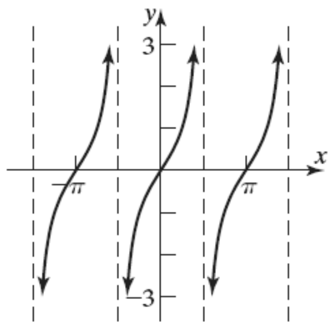
g. $y = \sin x$



h. $y = \cos x$



i. $y = \tan x$



9. $a = 20,\ c = 15,\ C = 40^\circ$

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\frac{\sin 40^o}{15} = \frac{\sin A}{20}$$

$$\sin A = \frac{20 \sin 40^o}{15}$$

$$A = \sin^{-1}\left(\frac{20\sin 40^o}{15}\right)$$

$A_1 \approx 59.0^\circ$ or $A_2 \approx 121.0^\circ$

For both values, $A + C < 180^\circ$. Therefore, there are two triangles.

$$B_1 = 180^\circ - A_1 - C \approx 180^\circ - 40^\circ - 59.0^\circ = 81.0^\circ$$

$$\frac{\sin C}{c} = \frac{\sin B_1}{b_1}$$

$$\frac{\sin 40^\circ}{15} = \frac{\sin 81.0^\circ}{b_1}$$

$$b_1 \approx \frac{15 \sin 81.0^\circ}{\sin 40^\circ} \approx 23.05$$

$$B_2 = 180^\circ - A_2 - C \approx 180^\circ - 40^\circ - 121.0^\circ = 19.0^\circ$$

$$\frac{\sin C}{c} = \frac{\sin B_2}{b_2}$$

$$\frac{\sin 40^\circ}{15} = \frac{\sin 19.0^\circ}{b_2}$$

$$b_2 \approx \frac{15 \sin 19.0^\circ}{\sin 40^\circ} \approx 7.60$$

Two triangles: $A_1 \approx 59.0^\circ,\ B_1 \approx 81.0^\circ,\ b_1 \approx 23.05$

or $A_2 \approx 121.0^\circ,\ B_2 \approx 19.0^\circ,\ b_2 \approx 7.60$.

10. $3x^5 - 10x^4 + 21x^3 - 42x^2 + 36x - 8 = 0$

Let $f(x) = 3x^5 - 10x^4 + 21x^3 - 42x^2 + 36x - 8 = 0$

$f(x)$ has at most 5 real zeros.

Possible rational zeros:

$p = \pm 1, \pm 2, \pm 4, \pm 8;\quad q = \pm 1, \pm 2, \pm 3;$

$$\frac{p}{q} = \pm 1, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm 2, \pm\frac{2}{3}, \pm 4, \pm\frac{4}{3}, \pm 8, \pm\frac{8}{3}$$

Using the Bounds on Zeros Theorem:

$$f(x) = 3\left(x^5 - \frac{10}{3}x^4 + 7x^3 - 14x^2 + 12x - \frac{8}{3}\right)$$

$$a_4 = -\frac{10}{3},\ a_3 = 7,\ a_2 = -14,\ a_1 = 12,\ a_0 = -\frac{8}{3}$$

$$\text{Max}\left\{1, \left|-\frac{8}{3}\right| + |\ 12| + |\ -14\ | + |7| + \left|-\frac{10}{3}\right|\right\}$$

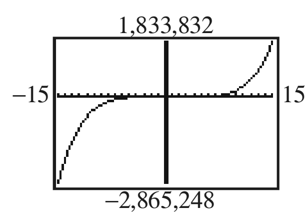
$$= \text{Max}\{1, 39\} = 39$$

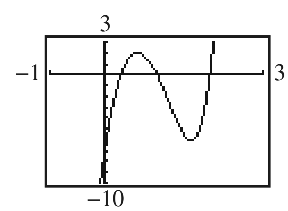
$$1 + \text{Max}\left\{\left|-\frac{8}{3}\right|, |\ 12|, |-14|, |7|, \left|-\frac{10}{3}\right|\right\}$$

$$= 1 + 14 = 15$$

The smaller of the two numbers is 15. Thus, every zero of f lies between −15 and 15.

Graphing using the bounds: (Second graph has a better window.)





From the graph it appears that there are x-intercepts at $\frac{1}{3}$, 1, and 2.

Using synthetic division with 1:

$$\begin{array}{r|rrrrrr} 1 & 3 & -10 & 21 & -42 & 36 & -8 \\ & & 3 & -7 & 14 & -28 & 8 \\ \hline & 3 & -7 & 14 & -28 & 8 & 0 \end{array}$$

Since the remainder is 0, $x - 1$ is a factor. The other factor is the quotient:

$3x^4 - 7x^3 + 14x^2 - 28x + 8$.

Using synthetic division with 2 on the quotient:

$$\begin{array}{r|rrrrr} 2 & 3 & -7 & 14 & -28 & 8 \\ & & 6 & -2 & 24 & -8 \\ \hline & 3 & -1 & 12 & -4 & 0 \end{array}$$

Since the remainder is 0, $x-2$ is a factor. The other factor is the quotient: $3x^3-x^2+12x-4$.

Using synthetic division with $\frac{1}{3}$ on the quotient:

$$\begin{array}{r|rrrr} \frac{1}{3} & 3 & -1 & 12 & 4 \\ & & 1 & 0 & 4 \\ \hline & 3 & 0 & 12 & 0 \end{array}$$

Since the remainder is 0, $x-\frac{1}{3}$ is a factor. The other factor is the quotient:
$3x^2+12=3\left(x^2+4\right)=3\left(x+2i\right)\left(x-2i\right)$.

Factoring,
$f(x)=3(x-1)\left(x-2\right)\left(x-\frac{1}{3}\right)\left(x+2i\right)\left(x-2i\right)$

The real zeros are 1,2, and $\frac{1}{3}$. The imaginary zeros are $-2i$ and $2i$.

Therefore, over the complex numbers, the equation $3x^5-10x^4+21x^3-42x^2+36x-8=0$ has solution set $\left\{-2i,\ 2i,\ \frac{1}{3},\ 1,\ 2\right\}$.

11. $R(x)=\dfrac{2x^2-7x-4}{x^2+2x-15}=\dfrac{(2x+1)(x-4)}{(x-3)(x+5)}$

$p(x)=2x^2-7x-4;\ q(x)=x^2+2x-15;$
$n=2;\ m=2$

Domain: $\{x \mid x\neq -5, x\neq 3\}$

R is in lowest terms.

The x-intercepts are the zeros of $p(x)$:

$$\begin{aligned} 2x^2-7x-4&=0 \\ \left(2x+1\right)\left(x-4\right)&=0 \\ 2x+1=0 \quad &\text{or} \quad x-4=0 \\ x=-\frac{1}{2} \quad &\qquad x=4 \end{aligned}$$

The y-intercept is
$R(0)=\dfrac{2\cdot 0^2-7\cdot 0-4}{0^2+2\cdot 0-15}=\dfrac{-4}{-15}=\dfrac{4}{15}$.

$R(-x)=\dfrac{2(-x)^2-7(-x)-4}{(-x)^2+2(-x)-15}=\dfrac{2x^2+7x-4}{x^2-2x-15}$ which is neither $R(x)$ nor $-R(x)$, so there is no symmetry.

The vertical asymptotes are the zeros of $q(x)$:

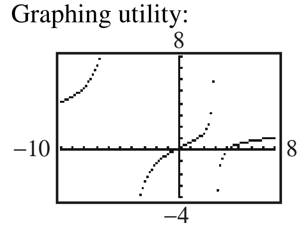
$$\begin{aligned} x^2+2x-15&=0 \\ \left(x+5\right)\left(x-3\right)&=0 \\ x+5=0 \quad &\text{or} \quad x-3=0 \\ x=-5 \quad &\qquad x=3 \end{aligned}$$

Since $n=m$, the line $y=2$ is the horizontal asymptote.

$R(x)$ intersects $y=2$ at $\left(\frac{26}{11},2\right)$, since:

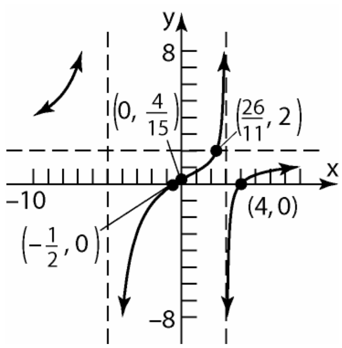
$$\begin{aligned} \frac{2x^2-7x-4}{x^2+2x-15}&=2 \\ 2x^2-7x-4&=2\left(x^2+2x-15\right) \\ 2x^2-7x-4&=2x^2+4x-30 \\ -11x&=-26 \\ x&=\frac{26}{11} \end{aligned}$$

Graphing utility:



Graph by hand:

Interval	Test number	Value of f	Location	Point
$(-\infty,-5)$	-6	≈ 12.22	Above x-axis	$(-6,12.22)$
$(-5,-0.5)$	-1	-0.3125	Below x-axis	$(-1,-0.3125)$
$(-0.5,3)$	0	≈ 0.27	Above x-axis	$(0,0.27)$
$(3,4)$	3.5	≈ -0.94	Below x-axis	$(3.5,-0.94)$
$(4,\infty)$	5	0.55	Above x-axis	$(5,0.55)$



12. $3^x = 12$

$\ln\left(3^x\right) = \ln(12)$

$x\ln(3) = \ln(12)$

$x = \dfrac{\ln(12)}{\ln(3)} \approx 2.26$

The solution set is $\{2.26\}$.

13. $\log_3(x+8) + \log_3 x = 2$

$\log_3\left[(x+8)(x)\right] = 2$

$(x+8)(x) = 3^2$

$x^2 + 8x = 9$

$x^2 + 8x - 9 = 0$

$(x+9)(x-1) = 0$

$x = -9$ or $x = 1$

$x = -9$ is extraneous because it makes the original logarithms undefined. The solution set is $\{1\}$.

14. $f(x) = 4x + 5$; $g(x) = x^2 + 5x - 24$

a. $f(x) = 0$

$4x + 5 = 0$

$4x = -5$

$x = -\dfrac{5}{4}$

The solution set is $\left\{-\dfrac{5}{4}\right\}$.

b. $f(x) = 13$

$4x + 5 = 13$

$4x = 8$

$x = 2$

The solution set is $\{2\}$.

c. $f(x) = g(x)$

$4x + 4 = x^2 + 5x - 24$

$0 = x^2 + x - 28$

$x = \dfrac{-1 \pm \sqrt{1^2 - 4(1)(-29)}}{2(1)}$

$= \dfrac{-1 \pm \sqrt{117}}{2} = \dfrac{-1 \pm 3\sqrt{13}}{2}$

The solution set is $\left\{\dfrac{-1-3\sqrt{13}}{2}, \dfrac{-1+3\sqrt{13}}{2}\right\}$.

d. $f(x) > 0$

$4x + 5 > 0$

$4x > -5$

$x > -\dfrac{5}{4}$

The solution set is $\left\{x \,\middle|\, x > -\dfrac{5}{4}\right\}$ or $\left(-\dfrac{5}{4}, \infty\right)$.

e. $g(x) \le 0$

$x^2 + 5x - 24 \le 0$

$(x+8)(x-3) \le 0$

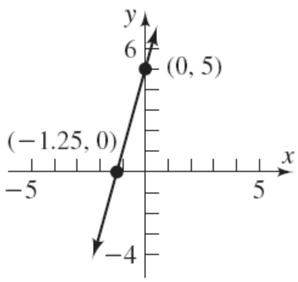
$x = -8, x = 3$ are the zeros.

Interval	Test number	$g(x)$	Pos./Neg.
$(-\infty,-8)$	-9	12	Positive
$(-8,3)$	0	-24	Negative
$(3,\infty)$	4	12	Positive

The solution set is $\{x \mid -8 \le x \le 3\}$ or $[-8,3]$.

f. $y = f(x) = 4x + 5$

The graph of *f* is a line with slope 4 and *y*-intercept 5.



g. $y = g(x) = x^2 + 5x - 24$

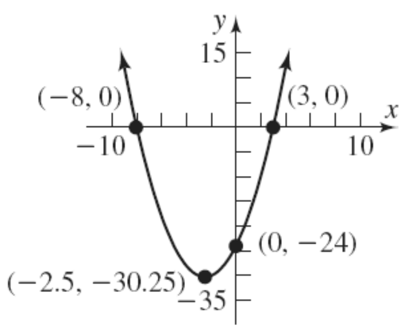
The graph of *g* is a parabola with *y*-intercept -24 and *x*-intercepts -8 and 3. The *x*-coordinate of the vertex is

$$x = -\frac{b}{2a} = -\frac{5}{2(1)} = -\frac{5}{2} = -2.5.$$

The *y*-coordinate of the vertex is

$$y = f\left(-\frac{b}{2a}\right) = f(-2.5)$$
$$= (-2.5)^2 + 5(-2.5) - 24 = -30.25$$

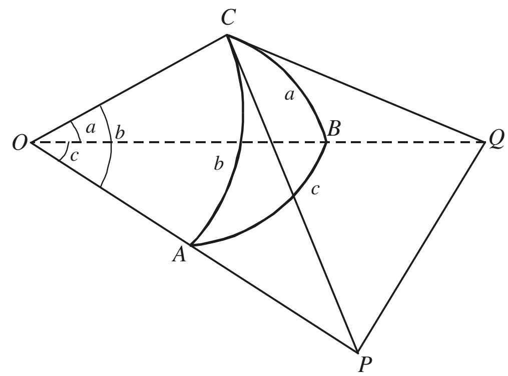
The vertex is $(-2.5, -30.25)$.



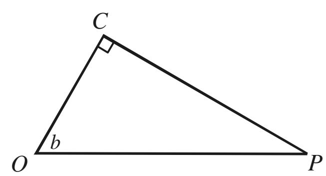
Chapter 8 Projects

Project I

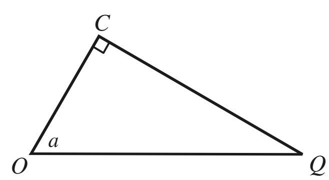
1.



Triangles ΔOCP and ΔOCQ are plane right triangles.



$$\sin b = \frac{CP}{OP}, \quad OC^2 + CP^2 = OP^2$$



$$\sin a = \frac{CQ}{OQ}, \quad OC^2 + CQ^2 = OQ^2$$

2. For ΔOPQ, we have that

$$(PQ)^2 = (OP)^2 + (OQ)^2 - 2(OQ)(OP)\cos c$$

For ΔCPQ, we have that

$$(PQ)^2 = (CQ)^2 + (CP)^2 - 2(CQ)(CP)\cos C$$

3.
$$0 = (OP)^2 + (OQ)^2 - 2(OQ)(OP)\cos c - \left[(CQ)^2 + (CP)^2 - 2(CQ)(CP)\cos C\right]$$
$$2(OQ)(OP)\cos c = (OQ)^2 - (CQ)^2 + (OP)^2 - (CQ)^2 + 2(CQ)(CP)\cos C$$

4. From part a: $(OC)^2 = (OQ)^2 - (CQ)^2$

$$(OC)^2 = (OP)^2 - (CP)^2$$

Thus,

$$2(OQ)(OP)\cos c = OQ^2 - CQ^2 + OP^2 - CQ^2 + 2(CQ)(CP)\cos C$$

$$2(OQ)(OP)\cos c = OC^2 + OC^2 + 2(CQ)(CP)\cos C$$

$$\cos c = \frac{2OC^2}{2(OQ)(OP)} + \frac{2(CQ)(CP)\cos C}{2(OQ)(OP)}$$

$$\cos c = \frac{OC^2}{(OQ)(OP)} + \frac{(CQ)(CP)\cos C}{(OQ)(OP)}$$

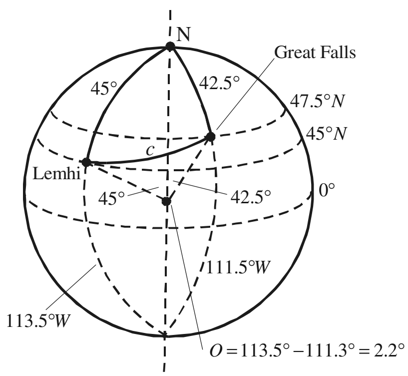
5. $$\cos c = \frac{OC^2}{(OQ)(OP)} + \frac{(CQ)(CP)\cos C}{(OQ)(OP)}$$

$$= \frac{OC}{OQ} \cdot \frac{OC}{OP} + \frac{CQ}{OQ} \cdot \frac{CP}{OP} \cdot \cos C$$

$$= \cos a \cos b + \sin a \sin b \cos C$$

Project II

1. Putting this on a circle of radius 1 in order to apply the Law of Cosines from A:



$$\cos c = \cos 45° \cos 42.5° + \sin 45° \sin 42.5° \cos 2.2°$$

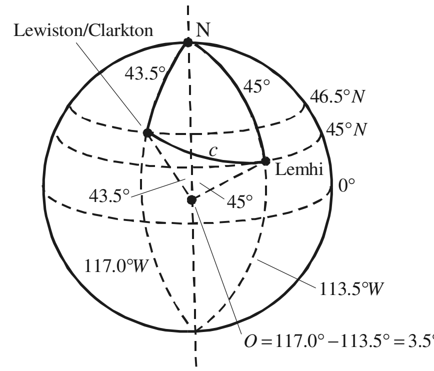
$$\cos c = 0.99870$$

$$c = 2.93°$$

$$s = rc = 3960\left(2.93° \cdot \frac{\pi}{180°}\right) \approx 202.5$$

It is 202.5 miles from Great Falls to Lemhi.

2.



$$\cos c = \cos 45° \cos 43.5° + \sin 45° \sin 43.5° \cos 3.5°$$

$$\cos c = 0.99875$$

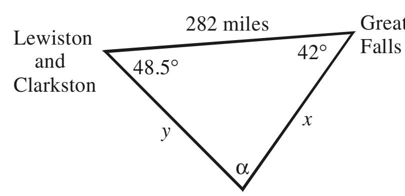
$$c = 2.87°$$

$$s = rc = 3960\left(2.87° \cdot \frac{\pi}{180°}\right) \approx 198.4$$

It is about 198.4 miles from Lemhi to Lewiston and Clarkston.

3. They traveled $202.5 + 198.4$ miles just to go from Great Falls to Lewiston and Clarkston.

4.



$$\alpha = 180° - 48.5° - 42° = 89.5°$$

$$\frac{x}{\sin 48.5°} = \frac{282}{\sin 89.5°}$$

$$x = \frac{282 \sin 48.5°}{\sin 89.5°} \approx 211.2$$

$$\frac{y}{\sin 42°} = \frac{282}{\sin 89.5°}$$

$$y = \frac{282 \sin 42°}{\sin 89.5°} \approx 188.8$$

Using a plane triangle, they traveled $211.2 + 188.7 = 399.9$ miles.

The mileage by using spherical triangles and that by using a plane triangle are relatively close. The total mileage is basically the same in each case. This is because compared to the surface of the Earth, these three towns are very close to each other and the surface can be approximated very closely by a plane.

Project III

1. $f_1 = \sin(\pi t)$

$f_3 = \frac{1}{3}\sin(3\pi t)$

$f_5 = \frac{1}{5}\sin(5\pi t)$

$f_7 = \frac{1}{7}\sin(7\pi t)$

$f_9 = \frac{1}{9}\sin(9\pi t)$

2. $f_1 = \sin(\pi t)$

$f_1 + f_3 = \sin(\pi t) + \frac{1}{3}\sin(3\pi t)$

$f_1 + f_3 + f_5 = \sin(\pi t) + \frac{1}{3}\sin(3\pi t) + \frac{1}{5}\sin(5\pi t)$

$f_1 + f_3 + f_5 + f_7$
$= \sin(\pi t) + \frac{1}{3}\sin(3\pi t) + \frac{1}{5}\sin(5\pi t) + \frac{1}{7}\sin(7\pi t)$

$f_1 + f_3 + f_5 + f_7 + f_9$
$= \sin(\pi t) + \frac{1}{3}\sin(3\pi t) + \frac{1}{5}\sin(5\pi t) + \frac{1}{7}\sin(7\pi t) + \frac{1}{9}\sin(9\pi t)$

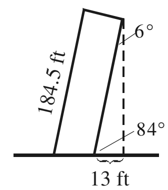
3. If one graphs each of these functions, one observes that with each iteration, the function becomes more square.

4. $f_1 + f_{13} + f_3 = \sin(\pi t) + \frac{1}{2}\cos(2\pi t) + \sin(3\pi t)$

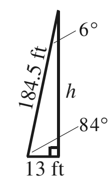
By adding in the cosine term, the curve does not become as flat. The waves at the "tops" and the "bottoms" become deeper.

Project IV

a.



b. Let h = the height of the tower with a lean of $6°$.

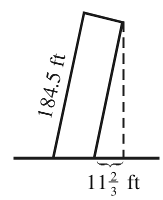


$\frac{h}{184.5} = \sin 84°$

$h = 184.5\sin 84°$

$= 183.5$ feet

c. $13 \text{ ft} - 16 \text{ in} = 11 \text{ ft } 8 \text{ in} = 11\frac{2}{3}$ ft



d. $\sin\alpha = \frac{11.67}{184.5}$

$\alpha = \sin^{-1}\left(\frac{11.67}{184.5}\right) \approx 4°$

e. $\frac{h}{184.5} = \sin 86°$

$h = 184.5\sin 86° \approx 184.1$ ft

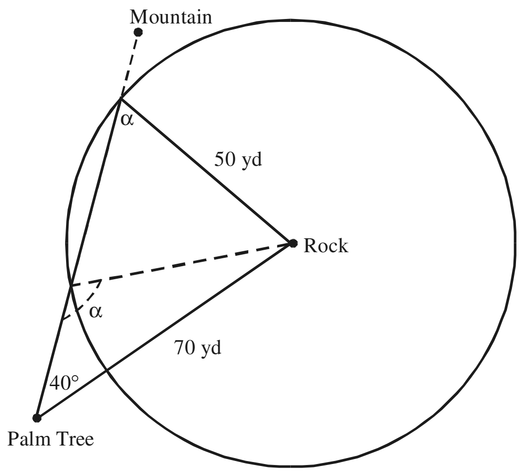
f. The angles are relatively small and for part (d), where $4°$ was acquired, it was arrived at by rounding.

g. Answers will vary.

Project V

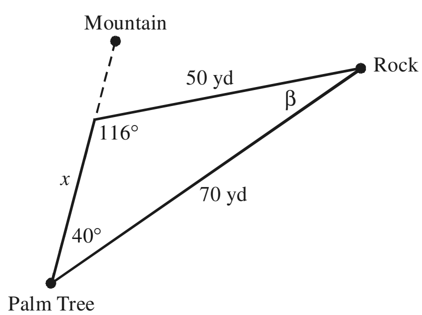
a. Answers will vary.

b.



$$\frac{\sin\alpha}{70}=\frac{\sin 40^\circ}{50}$$
$$\sin\alpha=\frac{70\sin 40^\circ}{50}\approx 0.8999$$
$$\alpha=\sin^{-1}\left(\frac{70\sin 40^\circ}{50}\right)\approx 64^\circ$$

c. $\alpha=180^\circ-64^\circ=116^\circ$



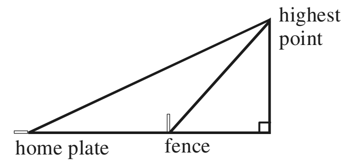
The third angle (i.e., at the rock) is 24°.

$$\frac{x}{\sin 24^\circ}=\frac{70}{\sin 116^\circ}$$
$$x=\frac{70\sin 24^\circ}{\sin 116^\circ}\approx 32$$

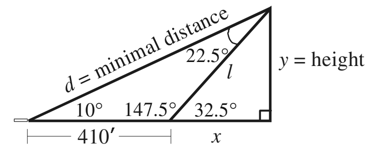
The treasure is about 32 yards away from the palm tree.

Project VI

a.



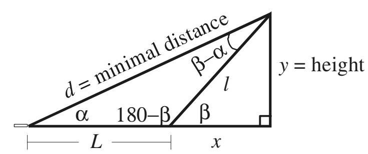
b.



$$\frac{d}{\sin 147.5^\circ}=\frac{410}{\sin 22.5^\circ}$$
$$d=\frac{410\sin 147.5^\circ}{\sin 22.5^\circ}$$
$$d=575.7\text{ feet}$$

$$\frac{l}{\sin 10^\circ}=\frac{410}{\sin 22.5^\circ}$$
$$l=\frac{410\sin 10^\circ}{\sin 22.5^\circ}$$
$$l=186.0\text{ feet}$$

c.



$$\frac{d}{\sin(180^\circ-\beta)}=\frac{L}{\sin(\beta-\alpha)}$$
$$d=\frac{L\sin(180^\circ-\beta)}{\sin(\beta-\alpha)}$$

$$\frac{l}{\sin\alpha}=\frac{L}{\sin(\beta-\alpha)}$$
$$l=\frac{L\sin\alpha}{\sin(\beta-\alpha)}$$

d. $$\frac{d}{\sin(180^\circ-\beta)}=\frac{L}{\sin(\beta-\alpha)}$$
$$d=\frac{L\sin(180^\circ-\beta)}{\sin(\beta-\alpha)}$$
$$d=\frac{L\sin(180^\circ)\cos(-\beta)+\cos 180^\circ\sin(-\beta)}{\sin(\beta-\alpha)}$$
$$d=\frac{L\left[(0)\cos(\beta)+(-1)\left(-\sin(\beta)\right)\right]}{\sin(\beta-\alpha)}$$
$$d=\frac{L\sin(\beta)}{\sin(\beta-\alpha)}$$

$$\sin\beta=\frac{y}{l}$$
$$y=l\sin\beta$$
$$y=\frac{L\sin\alpha\sin\beta}{\sin(\beta-\alpha)}$$