Chapter 6

Trigonometric Functions

Section 6.1

1. $C = 2\pi r$; $A = \pi r^2$

2. $d = r \cdot t$

3. standard position

4. central angle

5. radian

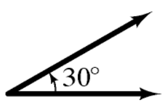
6. $r\theta$; $\frac{1}{2}r^2\theta$

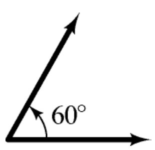
7. π

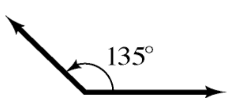
8. $\frac{s}{t}$; $\frac{\theta}{t}$

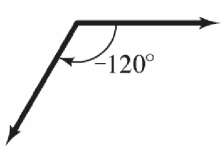
9. True

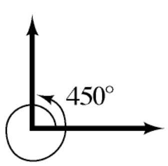
10. False; $v = r\omega$

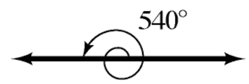
11.


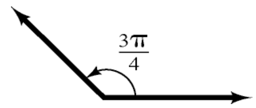
12.


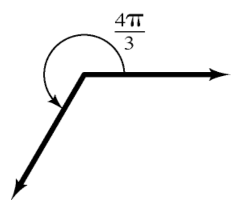
13.


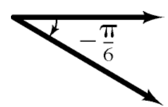
14.


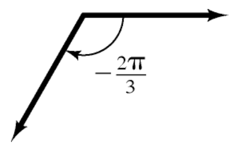
15.


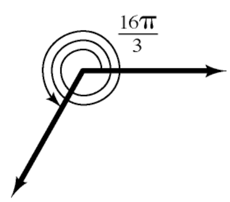
16.


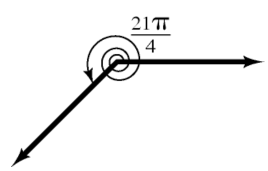
17.


18.


19.


20.


21.


22.


23. $$40°10'25'' = \left(40 + 10 \cdot \frac{1}{60} + 25 \cdot \frac{1}{60} \cdot \frac{1}{60}\right)^{\circ}$$
$$\approx (40 + 0.1667 + 0.00694)^{\circ}$$
$$\approx 40.17^{\circ}$$

24. $$61°42'21'' = \left(61 + 42 \cdot \frac{1}{60} + 21 \cdot \frac{1}{60} \cdot \frac{1}{60}\right)^{\circ}$$
$$\approx (61 + 0.7000 + 0.00583)^{\circ}$$
$$\approx 61.71^{\circ}$$

25. $1°2'3'' = \left(1 + 2 \cdot \frac{1}{60} + 3 \cdot \frac{1}{60} \cdot \frac{1}{60}\right)^{\circ}$
$\approx (1 + 0.0333 + 0.00083)°$
$\approx 1.03°$

26. $73°40'40'' = \left(73 + 40 \cdot \frac{1}{60} + 40 \cdot \frac{1}{60} \cdot \frac{1}{60}\right)^{\circ}$
$\approx (73 + 0.6667 + 0.0111)°$
$\approx 73.68°$

27. $9°9'9'' = \left(9 + 9 \cdot \frac{1}{60} + 9 \cdot \frac{1}{60} \cdot \frac{1}{60}\right)^{\circ}$
$= (9 + 0.15 + 0.0025)°$
$\approx 9.15°$

28. $98°22'45'' = \left(98 + 22 \cdot \frac{1}{60} + 45 \cdot \frac{1}{60} \cdot \frac{1}{60}\right)^{\circ}$
$\approx (98 + 0.3667 + 0.0125)°$
$\approx 98.38°$

29. $40.32° = 40° + 0.32°$
$= 40° + 0.32(60')$
$= 40° + 19.2'$
$= 40° + 19' + 0.2'$
$= 40° + 19' + 0.2(60'')$
$= 40° + 19' + 12''$
$= 40°19'12''$

30. $61.24° = 61° + 0.24°$
$= 61° + 0.24(60')$
$= 61° + 14.4'$
$= 61° + 14' + 0.4'$
$= 61° + 14' + 0.4(60'')$
$= 61° + 14' + 24''$
$= 61°14'24''$

31. $18.255° = 18° + 0.255°$
$= 18° + 0.255(60')$
$= 18° + 15.3'$
$= 18° + 15' + 0.3'$
$= 18° + 15' + 0.3(60'')$
$= 18° + 15' + 18''$
$= 18°15'18''$

32. $29.411° = 29° + 0.411°$
$= 29° + 0.411(60')$
$= 29° + 24.66'$
$= 29° + 24' + 0.66'$
$= 29° + 0.66(60'')$
$= 29° + 24' + 39.6''$
$\approx 29°24'40''$

33. $19.99° = 19° + 0.99°$
$= 19° + 0.99(60')$
$= 19° + 59.4'$
$= 19° + 59' + 0.4'$
$= 19° + 59' + 0.4(60'')$
$= 19° + 59' + 24''$
$= 19°59'24''$

34. $44.01° = 44° + 0.01°$
$= 44° + 0.01(60')$
$= 44° + 0.6'$
$= 44° + 0' + 0.6'$
$= 44° + 0' + 0.6(60'')$
$= 44° + 0' + 36''$
$= 44°0'36''$

35. $30° = 30 \cdot \frac{\pi}{180}$ radian $= \frac{\pi}{6}$ radian

36. $120° = 120 \cdot \frac{\pi}{180}$ radian $= \frac{2\pi}{3}$ radians

37. $240° = 240 \cdot \frac{\pi}{180}$ radian $= \frac{4\pi}{3}$ radians

38. $330° = 330 \cdot \frac{\pi}{180}$ radian $= \frac{11\pi}{6}$ radians

39. $-60° = -60 \cdot \frac{\pi}{180}$ radian $= -\frac{\pi}{3}$ radian

40. $-30° = -30 \cdot \frac{\pi}{180}$ radian $= -\frac{\pi}{6}$ radian

41. $180° = 180 \cdot \frac{\pi}{180}$ radian $= \pi$ radians

42. $270° = 270 \cdot \frac{\pi}{180}$ radian $= \frac{3\pi}{2}$ radians

43. $-135° = -135 \cdot \frac{\pi}{180}$ radian $= -\frac{3\pi}{4}$ radians

44. $-225° = -225 \cdot \frac{\pi}{180}$ radian $= -\frac{5\pi}{4}$ radians

45. $-90° = -90 \cdot \frac{\pi}{180}$ radian $= -\frac{\pi}{2}$ radians

46. $-180° = -180 \cdot \frac{\pi}{180}$ radian $= -\pi$ radians

47. $\frac{\pi}{3} = \frac{\pi}{3} \cdot \frac{180}{\pi}$ degrees $= 60°$

48. $\frac{5\pi}{6} = \frac{5\pi}{6} \cdot \frac{180}{\pi}$ degrees $= 150°$

49. $-\frac{5\pi}{4} = -\frac{5\pi}{4} \cdot \frac{180}{\pi}$ degrees $= -225°$

50. $-\frac{2\pi}{3} = -\frac{2\pi}{3} \cdot \frac{180}{\pi}$ degrees $= -120°$

51. $\frac{\pi}{2} = \frac{\pi}{2} \cdot \frac{180}{\pi}$ degrees $= 90°$

52. $4\pi = 4\pi \cdot \frac{180}{\pi}$ degrees $= 720°$

53. $\frac{\pi}{12} = \frac{\pi}{12} \cdot \frac{180}{\pi}$ degrees $= 15°$

54. $\frac{5\pi}{12} = \frac{5\pi}{12} \cdot \frac{180}{\pi}$ degrees $= 75°$

55. $-\frac{\pi}{2} = -\frac{\pi}{2} \cdot \frac{180}{\pi}$ degrees $= -90°$

56. $-\pi = -\pi \cdot \frac{180}{\pi}$ degrees $= -180°$

57. $-\frac{\pi}{6} = -\frac{\pi}{6} \cdot \frac{180}{\pi}$ degrees $= -30°$

58. $-\frac{3\pi}{4} = -\frac{3\pi}{4} \cdot \frac{180}{\pi}$ degrees $= -135°$

59. $17° = 17 \cdot \frac{\pi}{180}$ radian $= \frac{17\pi}{180}$ radian ≈ 0.30 radian

60. $73° = 73 \cdot \frac{\pi}{180}$ radian
$= \frac{73\pi}{180}$ radians
≈ 1.27 radians

61. $-40° = -40 \cdot \frac{\pi}{180}$ radian
$= -\frac{2\pi}{9}$ radian
≈ -0.70 radian

62. $-51° = -51 \cdot \frac{\pi}{180}$ radian
$= -\frac{17\pi}{60}$ radian
≈ -0.89 radian

63. $125° = 125 \cdot \frac{\pi}{180}$ radian
$= \frac{25\pi}{36}$ radians
≈ 2.18 radians

64. $350° = 350 \cdot \frac{\pi}{180}$ radian
$= \frac{35\pi}{18}$ radians
≈ 6.11 radians

65. 3.14 radians $= 3.14 \cdot \frac{180}{\pi}$ degrees $\approx 179.91°$

66. 0.75 radian $= 0.75 \cdot \frac{180}{\pi}$ degrees $\approx 42.97°$

67. 2 radians $= 2 \cdot \frac{180}{\pi}$ degrees $\approx 114.59°$

68. 3 radians $= 3 \cdot \frac{180}{\pi}$ degrees $\approx 171.89°$

69. 6.32 radians $= 6.32 \cdot \frac{180}{\pi}$ degrees $\approx 362.11°$

70. $\sqrt{2}$ radians $= \sqrt{2} \cdot \frac{180}{\pi}$ degrees $\approx 81.03°$

71. $r = 10$ meters; $\theta = \frac{1}{2}$ radian;

$s = r\theta = 10 \cdot \frac{1}{2} = 5$ meters

72. $r = 6$ feet; $\theta = 2$ radian; $s = r\theta = 6 \cdot 2 = 12$ feet

73. $\theta = \frac{1}{3}$ radian; $s = 2$ feet;

$s = r\theta$

$r = \frac{s}{\theta} = \frac{2}{(1/3)} = 6$ feet

74. $\theta = \frac{1}{4}$ radian; $s = 6$ cm;

$s = r\theta$

$r = \frac{s}{\theta} = \frac{6}{(1/4)} = 24$ cm

75. $r = 5$ miles; $s = 3$ miles;

$s = r\theta$

$\theta = \frac{s}{r} = \frac{3}{5} = 0.6$ radian

76. $r = 6$ meters; $s = 8$ meters;

$s = r\theta$

$\theta = \frac{s}{r} = \frac{8}{6} = \frac{4}{3} \approx 1.333$ radians

77. $r = 2$ inches; $\theta = 30° = 30 \cdot \frac{\pi}{180} = \frac{\pi}{6}$ radian;

$s = r\theta = 2 \cdot \frac{\pi}{6} = \frac{\pi}{3} \approx 1.047$ inches

78. $r = 3$ meters; $\theta = 120° = 120 \cdot \frac{\pi}{180} = \frac{2\pi}{3}$ radians

$s = r\theta = 3 \cdot \frac{2\pi}{3} = 2\pi \approx 6.283$ meters

79. $r = 10$ meters; $\theta = \frac{1}{2}$ radian

$A = \frac{1}{2}r^2\theta = \frac{1}{2}(10)^2\left(\frac{1}{2}\right) = \frac{100}{4} = 25 \text{ m}^2$

80. $r = 6$ feet; $\theta = 2$ radians

$A = \frac{1}{2}r^2\theta = \frac{1}{2}(6)^2(2) = 36 \text{ ft}^2$

81. $\theta = \frac{1}{3}$ radian; $A = 2 \text{ ft}^2$

$A = \frac{1}{2}r^2\theta$

$2 = \frac{1}{2}r^2\left(\frac{1}{3}\right)$

$2 = \frac{1}{6}r^2$

$12 = r^2$

$r = \sqrt{12} = 2\sqrt{3} \approx 3.464$ feet

82. $\theta = \frac{1}{4}$ radian; $A = 6 \text{ cm}^2$

$A = \frac{1}{2}r^2\theta$

$6 = \frac{1}{2}r^2\left(\frac{1}{4}\right)$

$6 = \frac{1}{8}r^2$

$48 = r^2$

$r = \sqrt{48} = 4\sqrt{3} \approx 6.928$ cm

83. $r = 5$ miles; $A = 3 \text{ mi}^2$

$A = \frac{1}{2}r^2\theta$

$3 = \frac{1}{2}(5)^2\theta$

$3 = \frac{25}{2}\theta$

$\theta = \frac{6}{25} = 0.24$ radian

84. $r = 6$ meters; $A = 8 \text{ m}^2$

$A = \frac{1}{2}r^2\theta$

$8 = \frac{1}{2}(6)^2\theta$

$8 = 18\theta$

$\theta = \frac{8}{18} = \frac{4}{9} \approx 0.444$ radian

85. $r = 2$ inches; $\theta = 30° = 30 \cdot \frac{\pi}{180} = \frac{\pi}{6}$ radian

$A = \frac{1}{2}r^2\theta = \frac{1}{2}(2)^2\left(\frac{\pi}{6}\right) = \frac{\pi}{3} \approx 1.047 \text{ in}^2$

86. $r = 3$ meters; $\theta = 120° = 120 \cdot \frac{\pi}{180} = \frac{2\pi}{3}$ radians

$A = \frac{1}{2} r^2 \theta = \frac{1}{2}(3)^2 \left(\frac{2\pi}{3}\right) = 3\pi \approx 9.425 \text{ m}^2$

87. $r = 2$ feet; $\theta = \frac{\pi}{3}$ radians

$s = r\theta = 2 \cdot \frac{\pi}{3} = \frac{2\pi}{3} \approx 2.094$ feet

$A = \frac{1}{2} r^2 \theta = \frac{1}{2}(2)^2 \left(\frac{\pi}{3}\right) = = \frac{2\pi}{3} \approx 2.094 \text{ ft}^2$

88. $r = 4$ meters; $\theta = \frac{\pi}{6}$ radian

$s = r\theta = 4 \cdot \frac{\pi}{6} = \frac{2\pi}{3} \approx 2.094$ meters

$A = \frac{1}{2} r^2 \theta = \frac{1}{2}(4)^2 \left(\frac{\pi}{6}\right) = \frac{4\pi}{3} \approx 4.189 \text{ m}^2$

89. $r = 12$ yards; $\theta = 70° = 70 \cdot \frac{\pi}{180} = \frac{7\pi}{18}$ radians

$s = r\theta = 12 \cdot \frac{7\pi}{18} \approx 14.661$ yards

$A = \frac{1}{2} r^2 \theta = \frac{1}{2}(12)^2 \left(\frac{7\pi}{18}\right) = 28\pi \approx 87.965 \text{ yd}^2$

90. $r = 9$ cm; $\theta = 50° = 50 \cdot \frac{\pi}{180} = \frac{5\pi}{18}$ radian

$s = r\theta = 9 \cdot \frac{5\pi}{18} \approx 7.854$ cm

$A = \frac{1}{2} r^2 \theta = \frac{1}{2}(9)^2 \left(\frac{5\pi}{18}\right) = \frac{45\pi}{4} \approx 35.343 \text{ cm}^2$

91. $r = 6$ inches

In 15 minutes,

$\theta = \frac{15}{60} \text{ rev} = \frac{1}{4} \cdot 360° = 90° = \frac{\pi}{2}$ radians

$s = r\theta = 6 \cdot \frac{\pi}{2} = 3\pi \approx 9.42$ inches

In 25 minutes,

$\theta = \frac{25}{60} \text{ rev} = \frac{5}{12} \cdot 360° = 150° = \frac{5\pi}{6}$ radians

$s = r\theta = 6 \cdot \frac{5\pi}{6} = 5\pi \approx 15.71$ inches

92. $r = 40$ inches; $\theta = 20° = \frac{\pi}{9}$ radian

$s = r\theta = 40 \cdot \frac{\pi}{9} = \frac{40\pi}{9} \approx 13.96$ inches

93. $r = 4$ m; $\theta = 45° = 45 \cdot \frac{\pi}{180} = \frac{\pi}{4}$ radian

$A = \frac{1}{2} r^2 \theta = \frac{1}{2}(4)^2 \left(\frac{\pi}{4}\right) = 2\pi \approx 6.28 \text{ m}^2$

94. $r = 3$ cm; $\theta = 60° = 60 \cdot \frac{\pi}{180} = \frac{\pi}{3}$ radians

$A = \frac{1}{2} r^2 \theta = \frac{1}{2}(3)^2 \left(\frac{\pi}{3}\right) = \frac{3\pi}{2} \approx 4.71 \text{ cm}^2$

95. $r = 30$ feet; $\theta = 135° = 135 \cdot \frac{\pi}{180} = \frac{3\pi}{4}$ radians

$A = \frac{1}{2} r^2 \theta = \frac{1}{2}(30)^2 \left(\frac{3\pi}{4}\right) = \frac{675\pi}{2} \approx 1060.29 \text{ ft}^2$

96. $r = 15$ yards; $A = 100 \text{ yd}^2$

$A = \frac{1}{2} r^2 \theta$

$100 = \frac{1}{2}(15)^2 \theta$

$100 = 112.5\theta$

$\theta = \frac{100}{112.5} = \frac{8}{9} \approx 0.89$ radian

or $\frac{8}{9} \cdot \frac{180}{\pi} = \left(\frac{160}{\pi}\right)^\circ \approx 50.93°$

97. $r = 5$ cm; $t = 20$ seconds; $\theta = \frac{1}{3}$ radian

$\omega = \frac{\theta}{t} = \frac{(1/3)}{20} = \frac{1}{3} \cdot \frac{1}{20} = \frac{1}{60}$ radian/sec

$v = \frac{s}{t} = \frac{r\theta}{t} = \frac{5 \cdot (1/3)}{20} = \frac{5}{3} \cdot \frac{1}{20} = \frac{1}{12}$ cm/sec

98. $r = 2$ meters; $t = 20$ seconds; $s = 5$ meters

$\omega = \frac{\theta}{t} = \frac{(s/r)}{t} = \frac{(5/2)}{20} = \frac{5}{2} \cdot \frac{1}{20} = \frac{1}{8}$ radian/sec

$v = \frac{s}{t} = \frac{5}{20} = \frac{1}{4}$ m/sec

99. $d = 26$ inches; $r = 13$ inches; $v = 35$ mi/hr

$$v = \frac{35 \text{ mi}}{\text{hr}} \cdot \frac{5280 \text{ ft}}{\text{mi}} \cdot \frac{12 \text{ in.}}{\text{ft}} \cdot \frac{1 \text{ hr}}{60 \text{ min}} = 36{,}960 \text{ in./min}$$

$$\omega = \frac{v}{r} = \frac{36{,}960 \text{ in./min}}{13 \text{ in.}} \approx 2843.08 \text{ radians/min} \approx \frac{2843.08 \text{ rad}}{\text{min}} \cdot \frac{1 \text{ rev}}{2\pi \text{ rad}} \approx 452.5 \text{ rev/min}$$

100. $r = 15$ inches; $\omega = 3 \text{ rev/sec} = 6\pi \text{ rad/sec}$

$$v = r\omega = 15 \cdot 6\pi \text{ in./sec} = 90\pi \approx 282.7 \text{ in/sec}$$

$$v = 90\pi \frac{\text{in.}}{\text{sec}} \cdot \frac{1\text{ft}}{12\text{in.}} \cdot \frac{1\text{mi}}{5280\text{ft}} \cdot \frac{3600\text{sec}}{1\text{hr}} \approx 16.1 \text{ mi/hr}$$

101. $r = 3960$ miles

$$\theta = 35°9' - 29°57' = 5°12' = 5.2° = 5.2 \cdot \frac{\pi}{180} \approx 0.09076 \text{ radian}$$

$$s = r\theta = 3960(0.09076) \approx 359 \text{ miles}$$

102. $r = 3960$ miles

$$\theta = 38°21' - 30°20' = 8°1' \approx 8.017° = 8.017 \cdot \frac{\pi}{180} \approx 0.1399 \text{ radian}$$

$$s = r\theta = 3960(0.1399) \approx 554 \text{ miles}$$

103. $r = 3429.5$ miles

$$\omega = 1 \text{ rev/day} = 2\pi \text{ radians/day} = \frac{\pi}{12} \text{ radians/hr}$$

$$v = r\omega = 3429.5 \cdot \frac{\pi}{12} \approx 898 \text{ miles/hr}$$

104. $r = 3033.5$ miles

$$\omega = 1 \text{ rev/day} = 2\pi \text{ radians/day} = \frac{\pi}{12} \text{ radians/hr}$$

$$v = r\omega = 3033.5 \cdot \frac{\pi}{12} \approx 794 \text{ miles/hr}$$

105. $r = 2.39 \times 10^5$ miles

$$\omega = 1 \text{ rev/27.3 days} = 2\pi \text{ radians/27.3 days} = \frac{\pi}{12 \cdot 27.3} \text{ radians/hr}$$

$$v = r\omega = \left(2.39 \times 10^5\right) \cdot \frac{\pi}{327.6} \approx 2292 \text{ miles/hr}$$

106. $r = 9.29 \times 10^7$ miles

$$\omega = 1 \text{ rev/365 days} = 2\pi \text{ radians/365 days} = \frac{\pi}{12 \cdot 365} \text{ radians/hr}$$

$$v = r\omega = \left(9.29 \times 10^7\right) \cdot \frac{\pi}{4380} \approx 66{,}633 \text{ miles/hr}$$

107. $r_1 = 2$ inches; $r_2 = 8$ inches;

$\omega_1 = 3 \text{ rev/min} = 6\pi$ radians/min

Find ω_2:

$$v_1 = v_2$$

$$r_1\omega_1 = r_2\omega_2$$

$$2(6\pi) = 8\omega_2$$

$$\omega_2 = \frac{12\pi}{8} = 1.5\pi \text{ radians/min} = \frac{1.5\pi}{2\pi} \text{ rev/min} = \frac{3}{4} \text{ rev/min}$$

108. $r = 30$ feet

$$\omega = \frac{1 \text{ rev}}{70 \text{ sec}} = \frac{2\pi}{70 \text{ sec}} = \frac{\pi}{35} \approx 0.09 \text{ radian/sec}$$

$$v = r\omega = 30 \text{ feet} \cdot \frac{\pi \text{ rad}}{35 \text{ sec}} = \frac{6\pi \text{ ft}}{7 \text{ sec}} \approx 2.69 \text{ feet/sec}$$

109. $r = 4$ feet; $\omega = 10 \text{ rev/min} = 20\pi$ radians/min

$$v = r\omega = 4 \cdot 20\pi = 80\pi \frac{\text{ft}}{\text{min}} = \frac{80\pi \text{ ft}}{\text{min}} \cdot \frac{1 \text{ mi}}{5280 \text{ ft}} \cdot \frac{60 \text{ min}}{\text{hr}} \approx 2.86 \text{ mi/hr}$$

110. $d = 26$ inches; $r = 13$ inches;

$\omega = 480$ rev/min $= 960\pi$ radians/min

$$v = r\omega$$
$$= 13 \cdot 960\pi$$
$$= 12480\pi \ \frac{\text{in}}{\text{min}}$$
$$= \frac{12480\pi \text{ in}}{\text{min}} \cdot \frac{1 \text{ ft}}{12 \text{ in}} \cdot \frac{1 \text{ mi}}{5280 \text{ ft}} \cdot \frac{60 \text{ min}}{\text{hr}}$$
$$\approx 37.13 \text{ mi/hr}$$
$$\omega = \frac{v}{r}$$
$$= \frac{80 \text{ mi/hr}}{13 \text{ in}} \cdot \frac{12 \text{ in}}{1 \text{ ft}} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} \cdot \frac{1 \text{ hr}}{60 \text{ min}} \cdot \frac{1 \text{ rev}}{2\pi \text{ rad}}$$
$$\approx 1034.26 \text{ rev/min}$$

111. $d = 8.5$ feet; $r = 4.25$ feet; $v = 9.55$ mi/hr

$$\omega = \frac{v}{r} = \frac{9.55 \text{ mi/hr}}{4.25 \text{ ft}}$$
$$= \frac{9.55 \text{ mi}}{\text{hr}} \cdot \frac{1}{4.25 \text{ ft}} \cdot \frac{5280 \text{ ft}}{\text{mi}} \cdot \frac{1 \text{ hr}}{60 \text{ min}} \cdot \frac{1 \text{ rev}}{2\pi}$$
$$\approx 31.47 \text{ rev/min}$$

112. Let t represent the time for the earth to rotate 90 miles.

$$\frac{t}{90} = \frac{24}{2\pi(3559)}$$
$$t = \frac{90(24)}{2\pi(3559)} \approx 0.0966 \text{ hours} \approx 5.8 \text{ minutes}$$

113. The earth makes one full rotation in 24 hours. The distance traveled in 24 hours is the circumference of the earth. At the equator the circumference is $2\pi(3960)$ miles. Therefore, the linear velocity a person must travel to keep up with the sun is:

$$v = \frac{s}{t} = \frac{2\pi(3960)}{24} \approx 1037 \text{ miles/hr}$$

114. Find s, when $r = 3960$ miles and $\theta = 1'$.

$$\theta = 1' \cdot \frac{1 \text{ degree}}{60 \text{ min}} \cdot \frac{\pi \text{ radians}}{180 \text{ degrees}} \approx 0.00029 \text{ radian}$$
$$s = r\theta = 3960(0.00029) \approx 1.15 \text{ miles}$$

Thus, 1 nautical mile is approximately 1.15 statute miles.

115. We know that the distance between Alexandria and Syene to be $s = 500$ miles. Since the measure of the Sun's rays in Alexandria is $7.2°$, the central angle formed at the center of Earth between Alexandria and Syene must also be $7.2°$. Converting to radians, we have

$7.2° = 7.2° \cdot \frac{\pi}{180°} = \frac{\pi}{25}$ radian. Therefore,

$$s = r\theta$$
$$500 = r \cdot \frac{\pi}{25}$$
$$r = \frac{25}{\pi} \cdot 500 = \frac{12{,}500}{\pi} \approx 3979 \text{ miles}$$
$$C = 2\pi r = 2\pi \cdot \frac{12{,}500}{\pi} = 25{,}000 \text{ miles.}$$

The radius of Earth is approximately 3979 miles, and the circumference is approximately 25,000 miles.

116. a. The length of the outfield fence is the arc length subtended by a central angle $\theta = 96°$ with $r = 200$ feet.

$$s = r \cdot \theta = 200 \cdot 96° \cdot \frac{\pi}{180°} \approx 335.10 \text{ feet}$$

The outfield fence is approximately 335.1 feet long.

b. The area of the warning track is the difference between the areas of two sectors with central angle $\theta = 96°$. One sector with $r = 200$ feet and the other with $r = 190$ feet.

$$A = \frac{1}{2}R^2\theta - \frac{1}{2}r^2\theta = \frac{\theta}{2}\left(R^2 - r^2\right)$$
$$= \frac{96°}{2} \cdot \frac{\pi}{180°}\left(200^2 - 190^2\right)$$
$$= \frac{4\pi}{15}(3900) \approx 3267.26$$

The area of the warning track is about 3267.26 square feet.

117. r_1 rotates at ω_1 rev/min , so $v_1 = r_1\omega_1$.
r_2 rotates at ω_2 rev/min , so $v_2 = r_2\omega_2$.
Since the linear speed of the belt connecting the pulleys is the same, we have:

$$v_1 = v_2$$
$$r_1\omega_1 = r_2\omega_2$$
$$\frac{r_1\omega_1}{r_2\omega_1} = \frac{r_2\omega_2}{r_2\omega_1}$$
$$\frac{r_1}{r_2} = \frac{\omega_2}{\omega_1}$$

118. Answers will vary.

119. If the radius of a circle is *r* and the length of the arc subtended by the central angle is also *r*, then the measure of the angle is 1 radian. Also,

$1 \text{ radian} = \dfrac{180}{\pi}$ degrees .

$1° = \dfrac{1}{360}$ revolution

120. Note that $1° = 1° \cdot \left(\dfrac{\pi \text{ radians}}{180°}\right) \approx 0.017$ radian

and $1 \text{ radian} \cdot \left(\dfrac{180°}{\pi \text{ radians}}\right) \approx 57.296°$.

Therefore, an angle whose measure is 1 radian is larger than an angle whose measure is 1 degree.

121. Linear speed measures the distance traveled per unit time, and angular speed measures the change in a central angle per unit time. In other words, linear speed describes distance traveled by a point located on the edge of a circle, and angular speed describes the turning rate of the circle itself.

122. This is a true statement. That is, since an angle measured in degrees can be converted to radian measure by using the formula
180 degrees $= \pi$ radians , the arc length formula can be rewritten as follows: $s = r\theta = \dfrac{\pi}{180} r\theta$.

123 – 125. Answers will vary.

Section 6.2

1. $c^2 = a^2 + b^2$

2. $f(5) = 3(5) - 7 = 15 - 7 = 8$

3. True

4. equal; proportional

5. $\left(-\dfrac{1}{2}, \dfrac{\sqrt{3}}{2}\right)$

6. $-\dfrac{1}{2}$

7. cosine

8. $(0,1)$

9. $\left(\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2}\right)$

10. $\left(\dfrac{1}{2}, \dfrac{\sqrt{3}}{2}\right)$

11. $\dfrac{y}{r}; \dfrac{x}{r}$

12. False

13. $P = \left(\dfrac{\sqrt{3}}{2}, \dfrac{1}{2}\right) \Rightarrow x = \dfrac{\sqrt{3}}{2},\ y = \dfrac{1}{2}$

$$\sin t = y = \frac{1}{2}$$
$$\cos t = x = \frac{\sqrt{3}}{2}$$
$$\tan t = \frac{y}{x} = \frac{\left(\frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$
$$\csc t = \frac{1}{y} = \frac{1}{\left(\frac{1}{2}\right)} = 1 \cdot \frac{2}{1} = 2$$

$$\sec t = \frac{1}{x} = \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} = 1 \cdot \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\cot t = \frac{x}{y} = \frac{\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{2}\right)} = \frac{\sqrt{3}}{2} \cdot \frac{2}{1} = \sqrt{3}$$

14. $P = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right) \Rightarrow x = \frac{1}{2},\ y = -\frac{\sqrt{3}}{2}$

$$\sin t = y = -\frac{\sqrt{3}}{2}$$

$$\cos t = x = \frac{1}{2}$$

$$\tan t = \frac{y}{x} = \frac{\left(-\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{2}\right)} = -\frac{\sqrt{3}}{2} \cdot \frac{2}{1} = -\sqrt{3}$$

$$\csc t = \frac{1}{y} = \frac{1}{\left(-\frac{\sqrt{3}}{2}\right)} = -\frac{2}{\sqrt{3}} = -\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$$

$$\sec t = \frac{1}{x} = \frac{1}{\left(\frac{1}{2}\right)} = 1 \cdot \frac{2}{1} = 2$$

$$\cot t = \frac{x}{y} = \frac{\left(\frac{1}{2}\right)}{\left(-\frac{\sqrt{3}}{2}\right)} = -\frac{1}{2} \cdot \frac{2}{\sqrt{3}} = -\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

15. $P = \left(-\frac{2}{5}, \frac{\sqrt{21}}{5}\right) \Rightarrow x = -\frac{2}{5},\ y = \frac{\sqrt{21}}{5}$

$$\sin t = y = \frac{\sqrt{21}}{5}$$

$$\cos t = x = -\frac{2}{5}$$

$$\tan t = \frac{y}{x} = \frac{\left(\frac{\sqrt{21}}{5}\right)}{\left(-\frac{2}{5}\right)} = \frac{\sqrt{21}}{5}\left(-\frac{5}{2}\right) = -\frac{\sqrt{21}}{2}$$

$$\csc t = \frac{1}{y} = \frac{1}{\left(\frac{\sqrt{21}}{5}\right)} = 1 \cdot \frac{5}{\sqrt{21}} = \frac{5}{\sqrt{21}} \cdot \frac{\sqrt{21}}{\sqrt{21}} = \frac{5\sqrt{21}}{21}$$

$$\sec t = \frac{1}{x} = \frac{1}{\left(-\frac{2}{5}\right)} = 1\left(-\frac{5}{2}\right) = -\frac{5}{2}$$

$$\cot t = \frac{x}{y} = \frac{\left(-\frac{2}{5}\right)}{\left(\frac{\sqrt{21}}{5}\right)} = -\frac{2}{5} \cdot \frac{5}{\sqrt{21}}$$

$$= -\frac{2}{\sqrt{21}} \cdot \frac{\sqrt{21}}{\sqrt{21}} = -\frac{2\sqrt{21}}{21}$$

16. $P = \left(-\frac{1}{5}, \frac{2\sqrt{6}}{5}\right) \Rightarrow x = -\frac{1}{5},\ y = \frac{2\sqrt{6}}{5}$

$$\sin t = y = \frac{2\sqrt{6}}{5}$$

$$\cos t = x = -\frac{1}{5}$$

$$\tan t = \frac{y}{x} = \frac{\left(\frac{2\sqrt{6}}{5}\right)}{\left(-\frac{1}{5}\right)} = \frac{2\sqrt{6}}{5}\left(-\frac{5}{1}\right) = -2\sqrt{6}$$

$$\csc t = \frac{1}{y} = \frac{1}{\left(\frac{2\sqrt{6}}{5}\right)} = 1 \cdot \frac{5}{2\sqrt{6}} = \frac{5}{2\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{5\sqrt{6}}{12}$$

$$\sec t = \frac{1}{x} = \frac{1}{\left(-\frac{1}{5}\right)} = 1\left(-\frac{5}{1}\right) = -5$$

$$\cot t = \frac{x}{y} = \frac{\left(-\frac{1}{5}\right)}{\left(\frac{2\sqrt{6}}{5}\right)} = -\frac{1}{5}\left(\frac{5}{2\sqrt{6}}\right)$$

$$= -\frac{1}{2\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = -\frac{\sqrt{6}}{12}$$

17. $P = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \Rightarrow x = -\frac{\sqrt{2}}{2},\ y = \frac{\sqrt{2}}{2}$

$$\sin t = \frac{\sqrt{2}}{2}$$

$$\cos t = x = -\frac{\sqrt{2}}{2}$$

$$\tan t = \frac{y}{x} = \frac{\left(\frac{\sqrt{2}}{2}\right)}{\left(-\frac{\sqrt{2}}{2}\right)} = -1$$

$$\csc t = \frac{1}{y} = \frac{1}{\left(\frac{\sqrt{2}}{2}\right)} = 1 \cdot \frac{2}{\sqrt{2}} = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2}$$

$$\sec t = \frac{1}{x} = \frac{1}{\left(-\frac{\sqrt{2}}{2}\right)} = 1\left(-\frac{2}{\sqrt{2}}\right) = -\frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\sqrt{2}$$

$$\cot t = \frac{x}{y} = \frac{\left(-\frac{\sqrt{2}}{2}\right)}{\left(\frac{\sqrt{2}}{2}\right)} = -1$$

18. $P = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \Rightarrow x = \frac{\sqrt{2}}{2},\ y = \frac{\sqrt{2}}{2}$

$$\sin t = y = \frac{\sqrt{2}}{2}$$

$$\cos t = x = \frac{\sqrt{2}}{2}$$

$$\tan t = \frac{y}{x} = \frac{\left(\frac{\sqrt{2}}{2}\right)}{\left(\frac{\sqrt{2}}{2}\right)} = 1$$

$$\csc t = \frac{1}{y} = \frac{1}{\left(\frac{\sqrt{2}}{2}\right)} = 1 \cdot \frac{2}{\sqrt{2}} = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2}$$

$$\sec t = \frac{1}{x} = \frac{1}{\left(\frac{\sqrt{2}}{2}\right)} = 1 \cdot \frac{2}{\sqrt{2}} = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2}$$

$$\cot t = \frac{x}{y} = \frac{\left(\frac{\sqrt{2}}{2}\right)}{\left(\frac{\sqrt{2}}{2}\right)} = 1$$

19. $P = \left(\frac{2\sqrt{2}}{3}, -\frac{1}{3}\right) \Rightarrow x = \frac{2\sqrt{2}}{3},\ y = -\frac{1}{3}$

$$\sin t = y = -\frac{1}{3}$$

$$\cos t = x = \frac{2\sqrt{2}}{3}$$

$$\tan t = \frac{y}{x} = \frac{\left(-\frac{1}{3}\right)}{\left(\frac{2\sqrt{2}}{3}\right)} = -\frac{1}{3} \cdot \frac{3}{2\sqrt{2}}$$

$$= -\frac{1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{4}$$

$$\csc t = \frac{1}{y} = \frac{1}{\left(-\frac{1}{3}\right)} = 1\left(-\frac{3}{1}\right) = -3$$

$$\sec t = \frac{1}{x} = \frac{1}{\left(\frac{2\sqrt{2}}{3}\right)} = 1\left(\frac{3}{2\sqrt{2}}\right) = \frac{3}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{4}$$

$$\cot t = \frac{x}{y} = \frac{\left(\frac{2\sqrt{2}}{3}\right)}{\left(-\frac{1}{3}\right)} = \frac{2\sqrt{2}}{3}\left(-\frac{3}{1}\right) = -2\sqrt{2}$$

20. $P = \left(-\frac{\sqrt{5}}{3}, -\frac{2}{3}\right) \Rightarrow x = -\frac{\sqrt{5}}{3},\ y = -\frac{2}{3}$

$$\sin t = y = -\frac{2}{3}$$

$$\cos t = x = -\frac{\sqrt{5}}{3}$$

$$\tan t = \frac{y}{x} = \frac{\left(-\frac{2}{3}\right)}{\left(-\frac{\sqrt{5}}{3}\right)} = -\frac{2}{3}\left(-\frac{3}{\sqrt{5}}\right)$$

$$= \frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\csc t = \frac{1}{y} = \frac{1}{\left(-\frac{2}{3}\right)} = 1\left(-\frac{3}{2}\right) = -\frac{3}{2}$$

$$\sec t = \frac{1}{x} = \frac{1}{\left(-\frac{\sqrt{5}}{3}\right)} = 1\left(-\frac{3}{\sqrt{5}}\right)$$
$$= -\frac{3}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = -\frac{3\sqrt{5}}{5}$$
$$\cot t = \frac{x}{y} = \frac{\left(-\frac{\sqrt{5}}{3}\right)}{\left(-\frac{2}{3}\right)} = -\frac{\sqrt{5}}{3}\left(-\frac{3}{2}\right) = \frac{\sqrt{5}}{2}$$

21. $\sin\left(\frac{11\pi}{2}\right) = \sin\left(\frac{3\pi}{2} + \frac{8\pi}{2}\right)$
$= \sin\left(\frac{3\pi}{2} + 4\pi\right)$
$= \sin\left(\frac{3\pi}{2} + 2 \cdot 2\pi\right)$
$= \sin\left(\frac{3\pi}{2}\right)$
$= -1$

22. $\cos(7\pi) = \cos(\pi + 6\pi)$
$= \cos(\pi + 3 \cdot 2\pi) = \cos(\pi) = -1$

23. $\tan(6\pi) = \tan(0 + 6\pi) = \tan(0) = 0$

24. $\cot\left(\frac{7\pi}{2}\right) = \cot\left(\frac{\pi}{2} + \frac{6\pi}{2}\right)$
$= \cot\left(\frac{\pi}{2} + 3\pi\right) = \cot\left(\frac{\pi}{2}\right) = 0$

25. $\csc\left(\frac{11\pi}{2}\right) = \csc\left(\frac{3\pi}{2} + \frac{8\pi}{2}\right)$
$= \csc\left(\frac{3\pi}{2} + 4\pi\right)$
$= \csc\left(\frac{3\pi}{2} + 2 \cdot 2\pi\right)$
$= \csc\left(\frac{3\pi}{2}\right)$
$= -1$

26. $\sec(8\pi) = \sec(0 + 8\pi)$
$= \sec(0 + 4 \cdot 2\pi) = \sec(0) = 1$

27. $\cos\left(-\frac{3\pi}{2}\right) = \cos\left(\frac{3\pi}{2}\right)$
$= \cos\left(\frac{\pi}{2} - \frac{4\pi}{2}\right)$
$= \cos\left(\frac{\pi}{2} + (-1) \cdot 2\pi\right)$
$= \cos\left(\frac{\pi}{2}\right)$
$= 0$

28. $\sin(-3\pi) = -\sin(3\pi)$
$= -\sin(\pi + 2\pi) = -\sin(\pi) = 0$

29. $\sec(-\pi) = \sec(\pi) = -1$

30. $\tan(-3\pi) = -\tan(3\pi)$
$= -\tan(0 + 3\pi) = -\tan(0) = 0$

31. $\sin 45^\circ + \cos 60^\circ = \frac{\sqrt{2}}{2} + \frac{1}{2} = \frac{1+\sqrt{2}}{2}$

32. $\sin 30^\circ - \cos 45^\circ = \frac{1}{2} - \frac{\sqrt{2}}{2} = \frac{1-\sqrt{2}}{2}$

33. $\sin 90^\circ + \tan 45^\circ = 1 + 1 = 2$

34. $\cos 180^\circ - \sin 180^\circ = -1 - 0 = -1$

35. $\sin 45^\circ \cos 45^\circ = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{2}{4} = \frac{1}{2}$

36. $\tan 45^\circ \cos 30^\circ = 1 \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

37. $\csc 45^\circ \tan 60^\circ = \sqrt{2} \cdot \sqrt{3} = \sqrt{6}$

38. $\sec 30^\circ \cot 45^\circ = \frac{2\sqrt{3}}{3} \cdot 1 = \frac{2\sqrt{3}}{3}$

39. $4\sin 90^\circ - 3\tan 180^\circ = 4 \cdot 1 - 3 \cdot 0 = 4$

40. $5\cos 90^\circ - 8\sin 270^\circ = 5 \cdot 0 - 8(-1) = 8$

41. $2\sin\frac{\pi}{3} - 3\tan\frac{\pi}{6} = 2 \cdot \frac{\sqrt{3}}{2} - 3 \cdot \frac{\sqrt{3}}{3} = \sqrt{3} - \sqrt{3} = 0$

42. $2\sin\frac{\pi}{4}+3\tan\frac{\pi}{4}=2\cdot\frac{\sqrt{2}}{2}+3\cdot 1=\sqrt{2}+3$

43. $2\sec\frac{\pi}{4}+4\cot\frac{\pi}{3}=2\cdot\sqrt{2}+4\cdot\frac{\sqrt{3}}{3}=2\sqrt{2}+\frac{4\sqrt{3}}{3}$

44. $3\csc\frac{\pi}{3}+\cot\frac{\pi}{4}=3\cdot\frac{2\sqrt{3}}{3}+1=2\sqrt{3}+1$

45. $\csc\frac{\pi}{2}+\cot\frac{\pi}{2}=1+0=1$

46. $\sec\pi-\csc\frac{\pi}{2}=-1-1=-2$

47. The point on the unit circle that corresponds to $\theta=\frac{2\pi}{3}=120°$ is $\left(-\frac{1}{2},\frac{\sqrt{3}}{2}\right)$.

$$\sin\frac{2\pi}{3}=\frac{\sqrt{3}}{2}$$

$$\cos\frac{2\pi}{3}=-\frac{1}{2}$$

$$\tan\frac{2\pi}{3}=\frac{\left(\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)}=\frac{\sqrt{3}}{2}\cdot\left(-\frac{2}{1}\right)=-\sqrt{3}$$

$$\csc\frac{2\pi}{3}=\frac{1}{\left(\frac{\sqrt{3}}{2}\right)}=\frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{2\sqrt{3}}{3}$$

$$\cos\frac{2\pi}{3}=\frac{1}{\left(-\frac{1}{2}\right)}=1\left(-\frac{2}{1}\right)=-2$$

$$\cot\frac{2\pi}{3}=\frac{\left(-\frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)}=-\frac{1}{2}\cdot\frac{2}{\sqrt{3}}=-\frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=-\frac{\sqrt{3}}{3}$$

48. The point on the unit circle that corresponds to $\theta=\frac{5\pi}{6}=150°$ is $\left(-\frac{\sqrt{3}}{2},\frac{1}{2}\right)$.

$$\sin\frac{5\pi}{6}=\frac{1}{2}$$

$$\cos\frac{5\pi}{6}=-\frac{\sqrt{3}}{2}$$

$$\tan\frac{5\pi}{6}=\frac{\left(\frac{1}{2}\right)}{\left(-\frac{\sqrt{3}}{2}\right)}=\frac{1}{2}\cdot\left(-\frac{2}{\sqrt{3}}\right)\cdot\frac{\sqrt{3}}{\sqrt{3}}=-\frac{\sqrt{3}}{3}$$

$$\csc\frac{5\pi}{6}=\frac{1}{\left(\frac{1}{2}\right)}=1\cdot\frac{2}{1}=2$$

$$\sec\frac{5\pi}{6}=\frac{1}{\left(-\frac{\sqrt{3}}{2}\right)}=1\cdot\left(-\frac{2}{\sqrt{3}}\right)\cdot\frac{\sqrt{3}}{\sqrt{3}}=-\frac{2\sqrt{3}}{3}$$

$$\cot\frac{5\pi}{6}=\frac{\left(-\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{2}\right)}=-\frac{\sqrt{3}}{2}\cdot\frac{2}{1}=-\sqrt{3}$$

49. The point on the unit circle that corresponds to $\theta=210°=\frac{7\pi}{6}$ is $\left(-\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$.

$$\sin 210°=-\frac{1}{2}$$

$$\cos 210°=-\frac{\sqrt{3}}{2}$$

$$\tan 210°=\frac{\left(-\frac{1}{2}\right)}{\left(-\frac{\sqrt{3}}{2}\right)}=-\frac{1}{2}\cdot\left(-\frac{2}{\sqrt{3}}\right)\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{3}}{3}$$

$$\csc 210°=\frac{1}{\left(-\frac{1}{2}\right)}=1\cdot\left(-\frac{2}{1}\right)=-2$$

$$\sec 210°=\frac{1}{\left(-\frac{\sqrt{3}}{2}\right)}=1\cdot\left(-\frac{2}{\sqrt{3}}\right)\cdot\frac{\sqrt{3}}{\sqrt{3}}=-\frac{2\sqrt{3}}{3}$$

$$\cot 210°=\frac{\left(-\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)}=-\frac{\sqrt{3}}{2}\cdot\left(-\frac{2}{1}\right)=\sqrt{3}$$

50. The point on the unit circle that corresponds to $\theta=240°=\frac{4\pi}{3}$ is $\left(-\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$.

$$\sin 240°=-\frac{\sqrt{3}}{2}$$

$$\cos 240°=-\frac{1}{2}$$

$$\tan 240^\circ = \frac{\left(-\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)} = -\frac{\sqrt{3}}{2}\cdot\left(-\frac{2}{1}\right) = \sqrt{3}$$

$$\csc 240^\circ = \frac{1}{\left(-\frac{\sqrt{3}}{2}\right)} = 1\cdot\left(-\frac{2}{\sqrt{3}}\right)\cdot\frac{\sqrt{3}}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$$

$$\sec 240^\circ = \frac{1}{\left(-\frac{1}{2}\right)} = 1\cdot\left(-\frac{2}{1}\right) = -2$$

$$\cot 240^\circ = \frac{\left(-\frac{1}{2}\right)}{\left(-\frac{\sqrt{3}}{2}\right)} = -\frac{1}{2}\cdot\left(-\frac{2}{\sqrt{3}}\right)\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

51. The point on the unit circle that corresponds to $\theta = \frac{3\pi}{4} = 135^\circ$ is $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

$$\sin\frac{3\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$$

$$\tan\frac{3\pi}{4} = \frac{\left(\frac{\sqrt{2}}{2}\right)}{\left(-\frac{\sqrt{2}}{2}\right)} = \frac{\sqrt{2}}{2}\cdot\left(-\frac{2}{\sqrt{2}}\right) = -1$$

$$\csc\frac{3\pi}{4} = \frac{1}{\left(\frac{\sqrt{2}}{2}\right)} = 1\cdot\frac{2}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

$$\sec\frac{3\pi}{4} = \frac{1}{\left(-\frac{\sqrt{2}}{2}\right)} = 1\cdot\left(-\frac{2}{\sqrt{2}}\right)\cdot\frac{\sqrt{2}}{\sqrt{2}} = -\frac{2\sqrt{2}}{2} = -\sqrt{2}$$

$$\cot\frac{3\pi}{4} = \frac{\left(-\frac{\sqrt{2}}{2}\right)}{\left(\frac{\sqrt{2}}{2}\right)} = -\frac{\sqrt{2}}{2}\cdot\frac{2}{\sqrt{2}} = -1$$

52. The point on the unit circle that corresponds to $\theta = \frac{11\pi}{4} = 495^\circ$ is $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

$$\sin\frac{11\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\cos\frac{11\pi}{4} = -\frac{\sqrt{2}}{2}$$

$$\tan\frac{11\pi}{4} = \frac{\left(\frac{\sqrt{2}}{2}\right)}{\left(-\frac{\sqrt{2}}{2}\right)} = \frac{\sqrt{2}}{2}\cdot\left(-\frac{2}{\sqrt{2}}\right) = -1$$

$$\csc\frac{11\pi}{4} = \frac{1}{\left(\frac{\sqrt{2}}{2}\right)} = 1\cdot\frac{2}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

$$\sec\frac{11\pi}{4} = \frac{1}{\left(-\frac{\sqrt{2}}{2}\right)} = 1\cdot\left(-\frac{2}{\sqrt{2}}\right)\cdot\frac{\sqrt{2}}{\sqrt{2}} = -\frac{2\sqrt{2}}{2} = -\sqrt{2}$$

$$\cot\frac{11\pi}{4} = \frac{\left(-\frac{\sqrt{2}}{2}\right)}{\left(\frac{\sqrt{2}}{2}\right)} = -\frac{\sqrt{2}}{2}\cdot\frac{2}{\sqrt{2}} = -1$$

53. The point on the unit circle that corresponds to $\theta = \frac{8\pi}{3} = 480^\circ$ is $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.

$$\sin\frac{8\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\cos\frac{8\pi}{3} = -\frac{1}{2}$$

$$\tan\frac{8\pi}{3} = \frac{\left(\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)} = \frac{\sqrt{3}}{2}\cdot\left(-\frac{2}{1}\right) = -\sqrt{3}$$

$$\csc\frac{8\pi}{3} = \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} = 1\cdot\frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\sec\frac{8\pi}{3} = \frac{1}{\left(-\frac{1}{2}\right)} = 1\cdot\left(-\frac{2}{1}\right) = -2$$

$$\cot\frac{8\pi}{3} = \frac{\left(-\frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)} = -\frac{1}{2}\cdot\frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

54. The point on the unit circle that corresponds to $\theta = \frac{13\pi}{6} = 390°$ is $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$.

$\sin\frac{13\pi}{6} = \frac{1}{2}$

$\cos\frac{13\pi}{6} = \frac{\sqrt{3}}{2}$

$\tan\frac{13\pi}{6} = \frac{\left(\frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{1}{2}\cdot\frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\csc\frac{13\pi}{6} = \frac{1}{\left(\frac{1}{2}\right)} = 1\cdot\frac{2}{1} = 2$

$\sec\frac{13\pi}{6} = \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} = 1\cdot\frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\cot\frac{13\pi}{6} = \frac{\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{2}\right)} = \frac{\sqrt{3}}{2}\cdot\frac{2}{1} = \sqrt{3}$

55. The point on the unit circle that corresponds to $\theta = 405° = \frac{9\pi}{4}$ is $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

$\sin 405° = \frac{\sqrt{2}}{2}$

$\cos 405° = \frac{\sqrt{2}}{2}$

$\tan 405° = \frac{\left(\frac{\sqrt{2}}{2}\right)}{\left(\frac{\sqrt{2}}{2}\right)} = \frac{\sqrt{2}}{2}\cdot\frac{2}{\sqrt{2}} = 1$

$\csc 405° = \frac{1}{\left(\frac{\sqrt{2}}{2}\right)} = 1\cdot\frac{2}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2}$

$\sec 405° = \frac{1}{\left(\frac{\sqrt{2}}{2}\right)} = 1\cdot\frac{2}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2}$

$\cot 405° = \frac{\left(\frac{\sqrt{2}}{2}\right)}{\left(\frac{\sqrt{2}}{2}\right)} = 1$

56. The point on the unit circle that corresponds to $\theta = 390° = \frac{13\pi}{6}$ is $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$.

$\sin 390° = \frac{1}{2}$

$\cos 390° = \frac{\sqrt{3}}{2}$

$\tan 390° = \frac{\left(\frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{1}{2}\cdot\frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\csc 390° = \frac{1}{\left(\frac{1}{2}\right)} = 1\cdot\frac{2}{1} = 2$

$\sec 390° = \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} = 1\cdot\frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\cot 390° = \frac{\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{2}\right)} = \frac{\sqrt{3}}{2}\cdot\frac{2}{1} = \sqrt{3}$

57. The point on the unit circle that corresponds to $\theta = -\frac{\pi}{6} = -30°$ is $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$.

$\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$

$\cos\left(-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

$\tan\left(-\frac{\pi}{6}\right) = \frac{\left(-\frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)} = -\frac{1}{2}\cdot\frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

$\csc\left(-\frac{\pi}{6}\right) = \frac{1}{\left(-\frac{1}{2}\right)} - 2$

$\sec\left(-\frac{\pi}{6}\right) = \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\cot\left(-\frac{\pi}{6}\right) = \frac{\left(\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)} = \left(\frac{\sqrt{3}}{2}\right)\cdot\left(-\frac{2}{1}\right) = -\sqrt{3}$

58. The point on the unit circle that corresponds to $\theta = -\frac{\pi}{3} = -60^\circ$ is $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$.

$\sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

$\cos\left(-\frac{\pi}{3}\right) = \frac{1}{2}$

$\tan\left(-\frac{\pi}{3}\right) = \frac{\left(-\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{2}\right)} = -\frac{\sqrt{3}}{2}\cdot\frac{2}{1} = -\sqrt{3}$

$\csc\left(-\frac{\pi}{3}\right) = \frac{1}{\left(-\frac{\sqrt{3}}{2}\right)} = 1\cdot\left(-\frac{2}{\sqrt{3}}\right)\cdot\frac{\sqrt{3}}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$

$\sec\left(-\frac{\pi}{3}\right) = \frac{1}{\left(\frac{1}{2}\right)} = 1\cdot\frac{2}{1} = 2$

$\cot\left(-\frac{\pi}{3}\right) = \frac{\left(\frac{1}{2}\right)}{\left(-\frac{\sqrt{3}}{2}\right)} = \frac{1}{2}\cdot\left(-\frac{2}{\sqrt{3}}\right)\cdot\frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

59. The point on the unit circle that corresponds to $\theta = -135^\circ = -\frac{3\pi}{4}$ is $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$.

$\sin(-135^\circ) = -\frac{\sqrt{2}}{2}$

$\cos(-135^\circ) = -\frac{\sqrt{2}}{2}$

$\tan(-135^\circ) = \frac{\left(-\frac{\sqrt{2}}{2}\right)}{\left(-\frac{\sqrt{2}}{2}\right)} = -\frac{\sqrt{2}}{2}\cdot\left(-\frac{2}{\sqrt{2}}\right) = 1$

$\csc(-135^\circ) = \frac{1}{\left(-\frac{\sqrt{2}}{2}\right)} = 1\cdot\left(-\frac{2}{\sqrt{2}}\right)\cdot\frac{\sqrt{2}}{\sqrt{2}} = -\sqrt{2}$

$\sec(-135^\circ) = \frac{1}{\left(-\frac{\sqrt{2}}{2}\right)} = 1\cdot\left(-\frac{2}{\sqrt{2}}\right)\cdot\frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2}$

$\cot(-135^\circ) = \frac{\left(-\frac{\sqrt{2}}{2}\right)}{\left(-\frac{\sqrt{2}}{2}\right)} = 1$

60. The point on the unit circle that corresponds to $\theta = -240^\circ = -\frac{4\pi}{3}$ is $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.

$\sin(-240^\circ) = \frac{\sqrt{3}}{2}$

$\cos(-240^\circ) = -\frac{1}{2}$

$\tan(-240^\circ) = \frac{\left(\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)} = \frac{\sqrt{3}}{2}\cdot\left(-\frac{2}{1}\right) = -\sqrt{3}$

$\csc(-240^\circ) = \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} = 1\cdot\left(\frac{2}{\sqrt{3}}\right)\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\sec(-240^\circ) = \frac{1}{\left(-\frac{1}{2}\right)} = 1\cdot\left(-\frac{2}{1}\right) = -2$

$\cot(-240^\circ) = \frac{\left(-\frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)} = -\frac{1}{2}\cdot\left(\frac{2}{\sqrt{3}}\right)\cdot\frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

61. The point on the unit circle that corresponds to $\theta = \frac{5\pi}{2} = 450^\circ$ is $(0, 1)$.

$\sin\frac{5\pi}{2} = 1$ $\qquad$ $\csc\frac{5\pi}{2} = \frac{1}{1} = 1$

$\cos\frac{5\pi}{2} = 0$ $\qquad$ $\sec\frac{5\pi}{2} = \frac{1}{0} = \text{undefined}$

$\tan\frac{5\pi}{2} = \frac{1}{0} = \text{undefined}$ $\qquad$ $\cot\frac{5\pi}{2} = \frac{0}{1} = 0$

62. The point on the unit circle that corresponds to $\theta = 5\pi = 900^\circ$ is $(-1, 0)$.

$\sin 5\pi = 0$ $\qquad$ $\csc 5\pi = \frac{1}{0} = \text{undefined}$

$\cos 5\pi = -1$ $\qquad$ $\sec 5\pi = \frac{1}{-1} = -1$

$\tan 5\pi = \frac{0}{-1} = 0$ $\qquad$ $\cot 5\pi = \frac{-1}{0} = \text{undefined}$

63. The point on the unit circle that corresponds to $\theta = -\dfrac{14\pi}{3} = -840°$ is $\left(-\dfrac{1}{2}, -\dfrac{\sqrt{3}}{2}\right)$.

$$\sin\left(-\frac{14\pi}{3}\right) = -\frac{\sqrt{3}}{2} \qquad \cos\left(-\frac{14\pi}{3}\right) = -\frac{1}{2}$$

$$\tan\left(-\frac{14\pi}{3}\right) = \frac{\left(-\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)} = -\frac{\sqrt{3}}{2}\cdot\left(-\frac{2}{1}\right) = \sqrt{3}$$

$$\csc\left(-\frac{14\pi}{3}\right) = \frac{1}{\left(-\frac{\sqrt{3}}{2}\right)} = 1\cdot\left(-\frac{2}{\sqrt{3}}\right)\cdot\frac{\sqrt{3}}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$$

$$\sec\left(-\frac{14\pi}{3}\right) = \frac{1}{\left(-\frac{1}{2}\right)} = 1\cdot\left(-\frac{2}{1}\right) = -2$$

$$\cot\left(-\frac{14\pi}{3}\right)\frac{\left(-\frac{1}{2}\right)}{\left(-\frac{\sqrt{3}}{2}\right)} = -\frac{1}{2}\cdot\left(-\frac{2}{\sqrt{3}}\right)\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

64. The point on the unit circle that corresponds to $\theta = -\dfrac{13\pi}{6} = -390°$ is $\left(\dfrac{\sqrt{3}}{2}, -\dfrac{1}{2}\right)$.

$$\sin\left(-\frac{13\pi}{6}\right) = -\frac{1}{2} \qquad \cos\left(-\frac{13\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

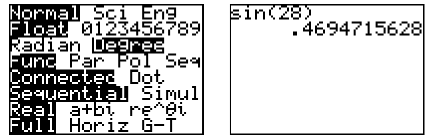
$$\tan\left(-\frac{13\pi}{6}\right) = \frac{\left(-\frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)} = -\frac{1}{2}\cdot\frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

$$\csc\left(-\frac{13\pi}{6}\right) = \frac{1}{\left(-\frac{1}{2}\right)} - 2$$

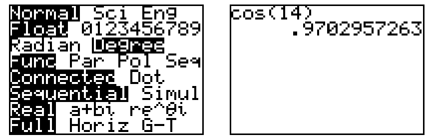
$$\sec\left(-\frac{13\pi}{6}\right) = \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\cot\left(-\frac{13\pi}{6}\right) = \frac{\left(\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)} = \left(\frac{\sqrt{3}}{2}\right)\cdot\left(-\frac{2}{1}\right) = -\sqrt{3}$$

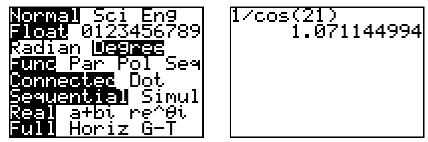
65. Set the calculator to degree mode: $\sin 28° \approx 0.47$.



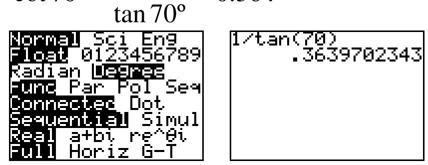
66. Set the calculator to degree mode: $\cos 14° \approx 0.97$.



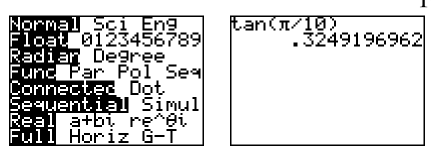
67. Set the calculator to degree mode: $\sec 21° = \dfrac{1}{\cos 21°} \approx 1.07$.



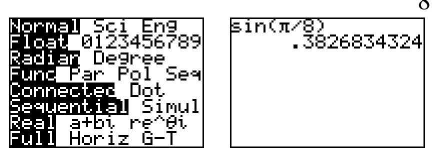
68. Set the calculator to degree mode: $\cot 70° = \dfrac{1}{\tan 70°} \approx 0.36$.



69. Set the calculator to radian mode: $\tan\dfrac{\pi}{10} \approx 0.32$.

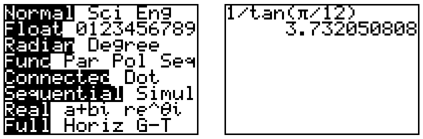


70. Set the calculator to radian mode: $\sin\dfrac{\pi}{8} \approx 0.38$.



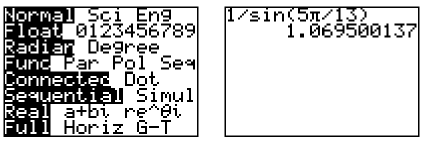
71. Set the calculator to radian mode:

$$\cot\frac{\pi}{12}=\frac{1}{\tan\frac{\pi}{12}}\approx 3.73.$$

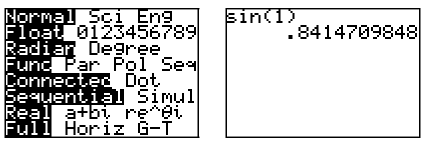


72. Set the calculator to radian mode:

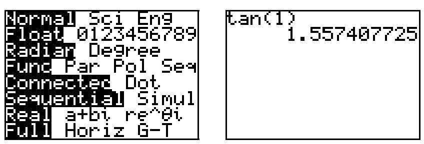
$$\csc\frac{5\pi}{13}=\frac{1}{\sin\frac{5\pi}{13}}\approx 1.07.$$



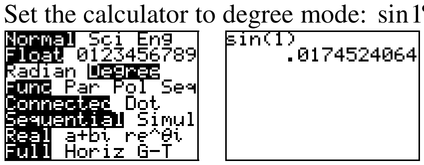
73. Set the calculator to radian mode: $\sin 1\approx 0.84$.



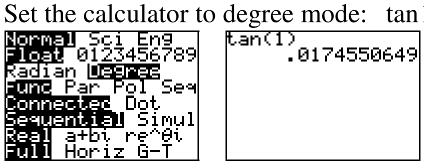
74. Set the calculator to radian mode: $\tan 1\approx 1.56$.



75. Set the calculator to degree mode: $\sin 1^\circ\approx 0.02$.



76. Set the calculator to degree mode: $\tan 1^\circ\approx 0.02$.



77. For the point $(-3,\ 4)$, $x=-3$, $y=4$,

$$r=\sqrt{x^2+y^2}=\sqrt{9+16}=\sqrt{25}=5$$

$\sin\theta=\frac{4}{5}$ $\qquad$ $\csc\theta=\frac{5}{4}$

$\cos\theta=-\frac{3}{5}$ $\qquad$ $\sec\theta=-\frac{5}{3}$

$\tan\theta=-\frac{4}{3}$ $\qquad$ $\cot\theta=-\frac{3}{4}$

78. For the point $(5,-12)$, $x=5$, $y=-12$,

$$r=\sqrt{x^2+y^2}=\sqrt{25+144}=\sqrt{169}=13$$

$\sin\theta=-\frac{12}{13}$ $\qquad$ $\csc\theta=-\frac{13}{12}$

$\cos\theta=\frac{5}{13}$ $\qquad$ $\sec\theta=\frac{13}{5}$

$\tan\theta=-\frac{12}{5}$ $\qquad$ $\cot\theta=-\frac{5}{12}$

79. For the point $(2,-3)$, $x=2$, $y=-3$,

$$r=\sqrt{x^2+y^2}=\sqrt{4+9}=\sqrt{13}$$

$\sin\theta=\frac{-3}{\sqrt{13}}\cdot\frac{\sqrt{13}}{\sqrt{13}}=-\frac{3\sqrt{13}}{13}$ $\qquad$ $\csc\theta=-\frac{\sqrt{13}}{3}$

$\cos\theta=\frac{2}{\sqrt{13}}\cdot\frac{\sqrt{13}}{\sqrt{13}}=\frac{2\sqrt{13}}{13}$ $\qquad$ $\sec\theta=\frac{\sqrt{13}}{2}$

$\tan\theta=-\frac{3}{2}$ $\qquad$ $\cot\theta=-\frac{2}{3}$

80. For the point $(-1,-2)$, $x=-1$, $y=-2$,

$$r=\sqrt{x^2+y^2}=\sqrt{1+4}=\sqrt{5}$$

$\sin\theta=\frac{-2}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}=-\frac{2\sqrt{5}}{5}$ $\qquad$ $\csc\theta=\frac{\sqrt{5}}{-2}=-\frac{\sqrt{5}}{2}$

$\cos\theta=\frac{-1}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}=-\frac{\sqrt{5}}{5}$ $\qquad$ $\sec\theta=\frac{\sqrt{5}}{-1}=-\sqrt{5}$

$\tan\theta=\frac{-2}{-1}=2$ $\qquad$ $\cot\theta=\frac{-1}{-2}=\frac{1}{2}$

81. For the point $(-2,-2)$, $x=-2$, $y=-2$,

$$r=\sqrt{x^2+y^2}=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}$$

$\sin\theta=\frac{-2}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=-\frac{\sqrt{2}}{2}$ $\qquad$ $\csc\theta=\frac{2\sqrt{2}}{-2}=-\sqrt{2}$

$\cos\theta=\frac{-2}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=-\frac{\sqrt{2}}{2}$ $\qquad$ $\sec\theta=\frac{2\sqrt{2}}{-2}=-\sqrt{2}$

$\tan\theta=\frac{-2}{-2}=1$ $\qquad$ $\cot\theta=\frac{-2}{-2}=1$

82. For the point $(-1,1)$, $x=-1$, $y=1$,

$r=\sqrt{x^2+y^2}=\sqrt{1+1}=\sqrt{2}$

$\sin\theta=\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{2}}{2}$ $\quad\csc\theta=\frac{\sqrt{2}}{1}=\sqrt{2}$

$\cos\theta=\frac{-1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=-\frac{\sqrt{2}}{2}$ $\quad\sec\theta=\frac{\sqrt{2}}{-1}=-\sqrt{2}$

$\tan\theta=\frac{1}{-1}=-1$ $\quad\cot\theta=\frac{-1}{1}=-1$

83. For the point $\left(\frac{1}{3},\frac{1}{4}\right)$, $x=\frac{1}{3}$, $y=\frac{1}{4}$,

$r=\sqrt{x^2+y^2}=\sqrt{\frac{1}{9}+\frac{1}{16}}=\sqrt{\frac{25}{144}}=\frac{5}{12}$

$\sin\theta=\frac{\frac{1}{4}}{\frac{5}{12}}=\frac{1}{4}\cdot\frac{12}{5}=\frac{3}{5}$ $\quad\csc\theta=\frac{\frac{5}{12}}{\frac{1}{4}}=\frac{5}{12}\cdot\frac{4}{1}=\frac{5}{3}$

$\cos\theta=\frac{\frac{1}{3}}{\frac{5}{12}}=\frac{1}{3}\cdot\frac{12}{5}=\frac{4}{5}$ $\quad\sec\theta=\frac{\frac{5}{12}}{\frac{1}{3}}=\frac{5}{12}\cdot\frac{3}{1}=\frac{5}{4}$

$\tan\theta=\frac{\frac{1}{4}}{\frac{1}{3}}=\frac{1}{4}\cdot\frac{3}{1}=\frac{3}{4}$ $\quad\cot\theta=\frac{\frac{1}{3}}{\frac{1}{4}}=\frac{1}{3}\cdot\frac{4}{1}=\frac{4}{3}$

84. For the point $(0.3,\ 0.4)$, $x=0.3$, $y=0.4$,

$r=\sqrt{x^2+y^2}=\sqrt{0.09+0.16}=\sqrt{0.25}=0.5$

$\sin\theta=\frac{0.4}{0.5}=\frac{4}{5}$ $\quad\csc\theta=\frac{0.5}{0.4}=\frac{5}{4}$

$\cos\theta=\frac{0.3}{0.5}=\frac{3}{5}$ $\quad\sec\theta=\frac{0.5}{0.3}=\frac{5}{3}$

$\tan\theta=\frac{0.4}{0.3}=\frac{4}{3}$ $\quad\cot\theta=\frac{0.3}{0.4}=\frac{3}{4}$

85. $\sin 45^\circ+\sin 135^\circ+\sin 225^\circ+\sin 315^\circ$

$=\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}+\left(-\frac{\sqrt{2}}{2}\right)+\left(-\frac{\sqrt{2}}{2}\right)$

$=0$

86. $\tan 60^\circ+\tan 150^\circ=\sqrt{3}+\left(-\frac{\sqrt{3}}{3}\right)$

$=\frac{3\sqrt{3}-\sqrt{3}}{3}=\frac{2\sqrt{3}}{3}$

87. $\sin 40^\circ+\sin 130^\circ+\sin 220^\circ+\sin 310^\circ$

$=\sin 40^\circ+\sin 130^\circ+\sin(40^\circ+180^\circ)+$

$\sin(130^\circ+180^\circ)$

$=\sin 40^\circ+\sin 130^\circ-\sin 40^\circ-\sin 130^\circ$

$=0$

88. $\tan 40^\circ+\tan 140^\circ=\tan 40^\circ+\tan(180^\circ-40^\circ)$

$=\tan 40^\circ-\tan 40^\circ$

$=0$

89. If $f(\theta)=\sin\theta=0.1$, then

$f(\theta+\pi)=\sin(\theta+\pi)=-0.1$.

90. If $f(\theta)=\cos\theta=0.3$, then

$f(\theta+\pi)=\cos(\theta+\pi)=-0.3$.

91. If $f(\theta)=\tan\theta=3$, then

$f(\theta+\pi)=\tan(\theta+\pi)=3$.

92. If $f(\theta)=\cot\theta=-2$, then

$f(\theta+\pi)=\cot(\theta+\pi)=-2$.

93. If $\sin\theta=\frac{1}{5}$, then $\csc\theta=\frac{1}{\left(\frac{1}{5}\right)}=1\cdot\frac{5}{1}=5$.

94. If $\cos\theta=\frac{2}{3}$, then $\sec\theta=\frac{1}{\left(\frac{2}{3}\right)}=1\cdot\frac{3}{2}=\frac{3}{2}$.

95. $f(60^\circ)=\sin(60^\circ)=\frac{\sqrt{3}}{2}$

96. $g(60^\circ)=\cos(60^\circ)=\frac{1}{2}$

97. $f\left(\frac{60^\circ}{2}\right)=\sin\left(\frac{60^\circ}{2}\right)=\sin(30^\circ)=\frac{1}{2}$

98. $g\left(\frac{60^\circ}{2}\right)=\cos\left(\frac{60^\circ}{2}\right)=\cos(30^\circ)=\frac{\sqrt{3}}{2}$

99. $\left[f(60^\circ)\right]^2=(\sin 60^\circ)^2=\left(\frac{\sqrt{3}}{2}\right)^2=\frac{3}{4}$

100. $\left[g(60°)\right]^2 = (\cos 60°)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

101. $f(2\cdot 60°) = \sin(2\cdot 60°) = \sin(120°) = \frac{\sqrt{3}}{2}$

102. $g(2\cdot 60°) = \cos(2\cdot 60°) = \cos(120°) = -\frac{1}{2}$

103. $2f(60°) = 2\sin(60°) = 2\cdot\frac{\sqrt{3}}{2} = \sqrt{3}$

104. $2g(60°) = 2\cos(60°) = 2\cdot\frac{1}{2} = 1$

105. $f(-60°) = \sin(-60°) = \sin(300°) = -\frac{\sqrt{3}}{2}$

106. $g(-60°) = \cos(-60°) = \cos(300°) = \frac{1}{2}$

107.
$$\begin{aligned}(f+g)(30°) &= f(30°) + g(30°)\\ &= \sin 30° + \cos 30°\\ &= \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}\end{aligned}$$

108.
$$\begin{aligned}(f-g)(60°) &= f(60°) - g(60°)\\ &= \sin 60° - \cos 60°\\ &= \frac{\sqrt{3}}{2} - \frac{1}{2} = \frac{\sqrt{3}-1}{2}\end{aligned}$$

109.
$$\begin{aligned}(f\cdot g)\left(\frac{3\pi}{4}\right) &= f\left(\frac{3\pi}{4}\right)\cdot g\left(\frac{3\pi}{4}\right)\\ &= \sin\left(\frac{3\pi}{4}\right)\cos\left(\frac{3\pi}{4}\right)\\ &= \frac{\sqrt{2}}{2}\cdot\left(-\frac{\sqrt{2}}{2}\right)\\ &= -\frac{\sqrt{4}}{4} = -\frac{2}{4} = -\frac{1}{2}\end{aligned}$$

110.
$$\begin{aligned}(f\cdot g)\left(\frac{4\pi}{3}\right) &= f\left(\frac{4\pi}{3}\right)\cdot g\left(\frac{4\pi}{3}\right)\\ &= \sin\left(\frac{4\pi}{3}\right)\cos\left(\frac{4\pi}{3}\right)\\ &= -\frac{\sqrt{3}}{2}\cdot\left(-\frac{1}{2}\right) = \frac{\sqrt{3}}{4}\end{aligned}$$

111.
$$\begin{aligned}(f\circ h)\left(\frac{\pi}{6}\right) &= f\left(h\left(\frac{\pi}{6}\right)\right)\\ &= \sin\left(2\left(\frac{\pi}{6}\right)\right)\\ &= \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}\end{aligned}$$

112.
$$\begin{aligned}(g\circ p)(60°) &= g(p(60°))\\ &= \cos\left(\frac{60°}{2}\right)\\ &= \cos 30° = \frac{\sqrt{3}}{2}\end{aligned}$$

113.
$$\begin{aligned}(p\circ g)(315°) &= p(g(315°))\\ &= \frac{\cos 315°}{2}\\ &= \frac{1}{2}\cos 315°\\ &= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4}\end{aligned}$$

114.
$$\begin{aligned}(h\circ f)\left(\frac{5\pi}{6}\right) &= h\left(f\left(\frac{5\pi}{6}\right)\right)\\ &= 2\left(\sin\left(\frac{5\pi}{6}\right)\right) = 2\cdot\frac{1}{2} = 1\end{aligned}$$

115. **a.** $f\left(\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

The point $\left(\frac{\pi}{4}, \frac{\sqrt{2}}{2}\right)$ is on the graph of *f*.

b. The point $\left(\frac{\sqrt{2}}{2}, \frac{\pi}{4}\right)$ is on the graph of f^{-1}.

c.
$$\begin{aligned}f\left(\frac{\pi}{4}+\frac{\pi}{4}\right) - 3 &= f\left(\frac{\pi}{2}\right) - 3\\ &= \sin\left(\frac{\pi}{2}\right) - 3\\ &= 1 - 3\\ &= -2\end{aligned}$$

The point $\left(\frac{\pi}{4}, -2\right)$ is on the graph of $y = f\left(x+\frac{\pi}{4}\right) - 3$.

116. a. $g\left(\frac{\pi}{6}\right)=\cos\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$

The point $\left(\frac{\pi}{6},\frac{\sqrt{3}}{2}\right)$ is on the graph of g.

b. The point $\left(\frac{\sqrt{3}}{2},\frac{\pi}{6}\right)$ is on the graph of g^{-1}.

c. $$2g\left(\frac{\pi}{6}-\frac{\pi}{6}\right)=2g(0)=2\cos(0)=2\cdot 1=2$$

Thus, the point $\left(\frac{\pi}{6},2\right)$ is on the graph of $y=2g\left(x-\frac{\pi}{6}\right)$.

117. Answers will vary. One set of possible answers is $-\frac{11\pi}{3},-\frac{5\pi}{3},\frac{\pi}{3},\frac{7\pi}{3},\frac{13\pi}{3}$.

118. Answers will vary. One ser of possible answers is $-\frac{13\pi}{4},-\frac{5\pi}{4},\frac{3\pi}{4},\frac{11\pi}{4},\frac{19\pi}{4}$

119.

θ	$\sin\theta$	$\frac{\sin\theta}{\theta}$
0.5	0.4794	0.9589
0.4	0.3894	0.9735
0.2	0.1987	0.9933
0.1	0.0998	0.9983
0.01	0.0100	1.0000
0.001	0.0010	1.0000
0.0001	0.0001	1.0000
0.00001	0.00001	1.0000

$f(\theta)=\frac{\sin\theta}{\theta}$ approaches 1 as θ approaches 0.

120.

θ	$\cos\theta-1$	$\frac{\cos\theta-1}{\theta}$
0.5	–0.1224	–0.2448
0.4	–0.0789	–0.1973
0.2	–0.0199	–0.0997
0.1	–0.0050	–0.0050
0.01	–0.00005	–0.0050
0.001	0.0000	–0.0005
0.0001	0.0000	–0.00005
0.00001	0.0000	–0.000005

$g(\theta)=\frac{\cos\theta-1}{\theta}$ approaches 0 as θ approaches 0.

121. Use the formula $R(\theta)=\frac{v_0^2\sin(2\theta)}{g}$ with $g=32.2\text{ft/sec}^2$; $\theta=45°$; $v_0=100$ ft/sec:

$$R(45°)=\frac{(100)^2\sin(2\cdot 45°)}{32.2}\approx 310.56\text{ feet}$$

Use the formula $H(\theta)=\frac{v_0^2(\sin\theta)^2}{2g}$ with $g=32.2\text{ft/sec}^2$; $\theta=45°$; $v_0=100$ ft/sec:

$$H(45°)=\frac{100^2(\sin 45°)^2}{2(32.2)}\approx 77.64\text{ feet}$$

122. Use the formula $R(\theta)=\frac{v_0^2\sin(2\theta)}{g}$ with $g=9.8\text{ m/sec}^2$; $\theta=30°$; $v_0=150$ m/sec:

$$R(30°)=\frac{150^2\sin(2\cdot 30°)}{9.8}\approx 1988.32\text{ m}$$

Use the formula $H(\theta)=\frac{v_0^2(\sin\theta)^2}{2g}$ with $g=9.8\text{ m/sec}^2$; $\theta=30°$; $v_0=150$ m/sec:

$$H(30°)=\frac{150^2(\sin 30°)^2}{2(9.8)}\approx 286.99\text{ m}$$

123. Use the formula $R(\theta)=\frac{v_0^2\sin(2\theta)}{g}$ with $g=9.8\text{ m/sec}^2$; $\theta=25°$; $v_0=500$ m/sec:

$$R(25°)=\frac{500^2\sin(2\cdot 25°)}{9.8}\approx 19{,}541.95\text{ m}$$

Use the formula $H(\theta)=\dfrac{v_0^{\,2}(\sin\theta)^2}{2g}$ with

$g=9.8\text{ m/sec}^2$; $\theta=25°$; $v_0=500\text{ m/sec}$:

$$H(25°)=\frac{500^2(\sin 25°)^2}{2(9.8)}\approx 2278.14\text{ m}$$

124. Use the formula $R(\theta)=\dfrac{v_0^{\,2}\sin(2\theta)}{g}$ with

$g=32.2\text{ft/sec}^2$; $\theta=50°$; $v_0=200\text{ ft/sec}$:

$$R(50°)=\frac{200^2\sin(2\cdot 50°)}{32.2}\approx 1223.36\text{ ft}$$

Use the formula $H(\theta)=\dfrac{v_0^{\,2}(\sin\theta)^2}{2g}$ with

$g=32.2\text{ft/sec}^2$; $\theta=50°$; $v_0=200\text{ ft/sec}$:

$$H(50°)=\frac{200^2(\sin 50°)^2}{2(32.2)}\approx 364.49\text{ ft}$$

125. Use the formula $t(\theta)=\sqrt{\dfrac{2a}{g\sin\theta\cos\theta}}$ with

$g=32\text{ ft/sec}^2$ and $a=10$ feet:

a. $$t(30)=\sqrt{\frac{2(10)}{32\sin 30°\cdot\cos 30°}}\approx 1.20\text{ seconds}$$

b. $$t(45)=\sqrt{\frac{2(10)}{32\sin 45°\cdot\cos 45°}}\approx 1.12\text{ seconds}$$

c. $$t(60)=\sqrt{\frac{2(10)}{32\sin 60°\cdot\cos 60°}}\approx 1.20\text{ seconds}$$

126. Use the formula

$x(\theta)=\cos\theta+\sqrt{16+0.5\cos(2\theta)}$.

$$\begin{aligned}x(30)&=\cos(30°)+\sqrt{16+0.5\cos(2\cdot 30°)}\\&=\cos(30°)+\sqrt{16+0.5\cos(60°)}\\&\approx 4.90\text{ cm}\end{aligned}$$

$$\begin{aligned}x(45)&=\cos(45°)+\sqrt{16+0.5\cos(2\cdot 45°)}\\&=\cos(45°)+\sqrt{16+0.5\cos(90°)}\\&\approx 4.71\text{ cm}\end{aligned}$$

127. Note: time on road $=\dfrac{\text{distance on road}}{\text{rate on road}}$

$$\begin{aligned}&=\frac{8-2x}{8}\\&=1-\frac{x}{4}\\&=1-\frac{\frac{1}{\tan\theta}}{4}\\&=1-\frac{1}{4\tan\theta}\end{aligned}$$

a. $$\begin{aligned}T(30°)&=1+\frac{2}{3\sin 30°}-\frac{1}{4\tan 30°}\\&=1+\frac{2}{3\cdot\frac{1}{2}}-\frac{1}{4\cdot\frac{1}{\sqrt{3}}}\\&=1+\frac{4}{3}-\frac{\sqrt{3}}{4}\approx 1.9\text{ hr}\end{aligned}$$

Sally is on the paved road for

$1-\dfrac{1}{4\tan 30°}\approx 0.57$ hr.

b. $$\begin{aligned}T(45°)&=1+\frac{2}{3\sin 45°}-\frac{1}{4\tan 45°}\\&=1+\frac{2}{3\cdot\frac{1}{\sqrt{2}}}-\frac{1}{4\cdot 1}\\&=1+\frac{2\sqrt{2}}{3}-\frac{1}{4}\approx 1.69\text{ hr}\end{aligned}$$

Sally is on the paved road for

$1-\dfrac{1}{4\tan 45°}=0.75$ hr.

c. $$\begin{aligned}T(60°)&=1+\frac{2}{3\sin 60°}-\frac{1}{4\tan 60°}\\&=1+\frac{2}{3\cdot\frac{\sqrt{3}}{2}}-\frac{1}{4\cdot\sqrt{3}}\\&=1+\frac{4}{3\sqrt{3}}-\frac{1}{4\sqrt{3}}\\&\approx 1.63\text{ hr}\end{aligned}$$

Sally is on the paved road for

$1-\dfrac{1}{4\tan 60°}\approx 0.86$ hr.

d. $T(90°)=1+\dfrac{2}{3\sin 90°}-\dfrac{1}{4\tan 90°}$.

But $\tan 90°$ is undefined, so we cannot use the function formula for this path. However, the distance would be 2 miles in the sand and 8 miles on the road. The total time would be: $\dfrac{2}{3}+1=\dfrac{5}{3}\approx 1.67$ hours. The path would be to leave the first house walking 1 mile in the sand straight to the road. Then turn and walk 8 miles on the road. Finally, turn and walk 1 mile in the sand to the second house.

128. When $\theta=30°$:

$$V(30°)=\frac{1}{3}\pi(2)^3\frac{(1+\sec 30°)^3}{(\tan 30°)^2}\approx 251.42 \text{ cm}^3$$

When $\theta=45°$:

$$V(45°)=\frac{1}{3}\pi(2)^3\frac{(1+\sec 45°)^3}{(\tan 45°)^2}\approx 117.88 \text{ cm}^3$$

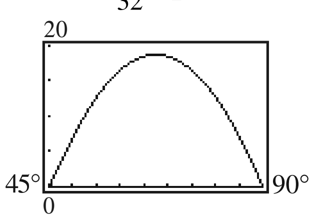
When $\theta=60°$:

$$V(60°)=\frac{1}{3}\pi(2)^3\frac{(1+\sec 60°)^3}{(\tan 60°)^2}\approx 75.40 \text{ cm}^3$$

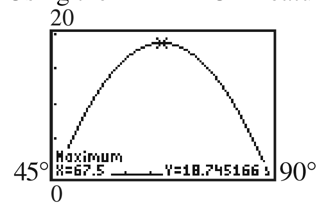
129. a.

$$\begin{aligned}R(60)&=\frac{32^2\sqrt{2}}{32}\left[\sin(2\cdot 60°)-\cos(2\cdot 60°)-1\right]\\&=\frac{32^2\sqrt{2}}{32}\left[\sin(120°)-\cos(120°)-1\right]\\&\approx 32\sqrt{2}\left(\frac{\sqrt{3}}{2}-\left(-\frac{1}{2}\right)-1\right)\\&\approx 16.56 \text{ ft}\end{aligned}$$

b. Let $Y_1=\dfrac{32^2\sqrt{2}}{32}\left[\sin(2x)-\cos(2x)-1\right]$



c. Using the MAXIMUM feature, we find:



R is largest when $\theta=67.5°$.

130. Slope of $M=\dfrac{\sin\theta-0}{\cos\theta-0}=\dfrac{\sin\theta}{\cos\theta}=\tan\theta$.

Since L is parallel to M, the slope of L is equal to the slope of M. Thus, the slope of $L=\tan\theta$.

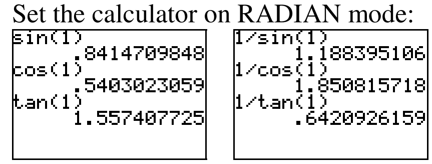
131. a. When $t=1$, the coordinate on the unit circle is approximately $(0.5,\ 0.8)$. Thus,

$\sin 1\approx 0.8$ $\qquad$ $\csc 1\approx\dfrac{1}{0.8}\approx 1.3$

$\cos 1\approx 0.5$ $\qquad$ $\sec 1\approx\dfrac{1}{0.5}=2.0$

$\tan 1\approx\dfrac{0.8}{0.5}=1.6$ $\qquad$ $\cot 1\approx\dfrac{0.5}{0.8}\approx 0.6$

Set the calculator on RADIAN mode:



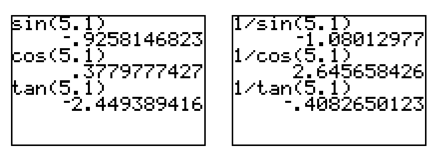
b. When $t=5.1$, the coordinate on the unit circle is approximately $(0.4,-0.9)$. Thus,

$\sin 5.1\approx -0.9$ $\qquad$ $\csc 5.1\approx\dfrac{1}{-0.9}\approx -1.1$

$\cos 5.1\approx 0.4$ $\qquad$ $\sec 5.1\approx\dfrac{1}{0.4}=2.5$

$\tan 5.1\approx\dfrac{-0.9}{0.4}\approx -2.3$ $\qquad$ $\cot 5.1\approx\dfrac{0.4}{-0.9}\approx -0.4$

Set the calculator on RADIAN mode:



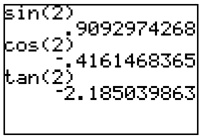
132. a. When $t=2$, the coordinate on the unit circle is approximately $(-0.4,\ 0.9)$. Thus,

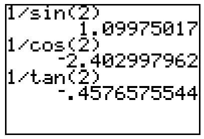
$\sin 2\approx 0.9$ $\qquad$ $\csc 2\approx\dfrac{1}{0.9}\approx 1.1$

$\cos 2 \approx -0.4$ $\qquad \sec 2 \approx \dfrac{1}{-0.4} = -2.5$

$\tan 2 \approx \dfrac{0.9}{-0.4} = -2.3$ $\qquad \cot 2 \approx \dfrac{-0.4}{0.9} \approx -0.4$

Set the calculator on RADIAN mode:





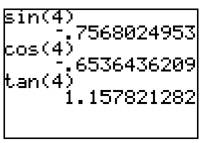
b. When $t = 4$, the coordinate on the unit circle is approximately $(-0.7, -0.8)$. Thus,

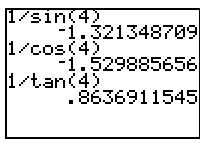
$\sin 4 \approx -0.8$ $\qquad \csc 4 \approx \dfrac{1}{-0.8} \approx -1.3$

$\cos 4 \approx -0.7$ $\qquad \sec 4 \approx \dfrac{1}{-0.7} \approx -1.4$

$\tan 4 \approx \dfrac{-0.8}{-0.7} \approx 1.1$ $\qquad \cot 4 \approx \dfrac{-0.7}{-0.8} \approx 0.9$

Set the calculator on RADIAN mode:





133 – 135. Answers will vary.

Section 6.3

1. All real numbers except $-\dfrac{1}{2}$; $\left\{x \mid x \neq -\dfrac{1}{2}\right\}$

2. even

3. False

4. True

5. 2π, π

6. All real number, except odd multiples of $\dfrac{\pi}{2}$

7. All real numbers between -1 and 1, inclusive. That is, $\{y \mid -1 \le y \le 1\}$ or $[-1, 1]$.

8. True

9. 1

10. False; $\sec\theta = \dfrac{1}{\cos\theta}$

11. $\sin 405° = \sin(360° + 45°) = \sin 45° = \dfrac{\sqrt{2}}{2}$

12. $\cos 420° = \cos(360° + 60°) = \cos 60° = \dfrac{1}{2}$

13. $\tan 405° = \tan(180° + 180° + 45°) = \tan 45° = 1$

14. $\sin 390° = \sin(360° + 30°) = \sin 30° = \dfrac{1}{2}$

15. $\csc 450° = \csc(360° + 90°) = \csc 90° = 1$

16. $\sec 540° = \sec(360° + 180°) = \sec 180° = -1$

17. $\cot 390° = \cot(180° + 180° + 30°) = \cot 30° = \sqrt{3}$

18. $\sec 420° = \sec(360° + 60°) = \sec 60° = 2$

19. $$\cos\frac{33\pi}{4} = \cos\left(\frac{\pi}{4} + 8\pi\right) = \cos\left(\frac{\pi}{4} + 4\cdot 2\pi\right) = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

20. $$\sin\frac{9\pi}{4} = \sin\left(\frac{\pi}{4} + 2\pi\right) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

21. $\tan(21\pi) = \tan(0 + 21\pi) = \tan(0) = 0$

22. $$\csc\frac{9\pi}{2} = \csc\left(\frac{\pi}{2} + 4\pi\right) = \csc\left(\frac{\pi}{2} + 2\cdot 2\pi\right) = \csc\frac{\pi}{2} = 1$$

23. $$\sec\frac{17\pi}{4} = \sec\left(\frac{\pi}{4} + 4\pi\right) = \sec\left(\frac{\pi}{4} + 2\cdot 2\pi\right) = \sec\frac{\pi}{4} = \sqrt{2}$$

24. $\cot\frac{17\pi}{4}=\cot\left(\frac{\pi}{4}+4\pi\right)=\cot\left(\frac{\pi}{4}+2\cdot 2\pi\right)$
$=\cot\frac{\pi}{4}$
$=1$

25. $\tan\frac{19\pi}{6}=\tan\left(\frac{\pi}{6}+3\pi\right)=\tan\frac{\pi}{6}=\frac{\sqrt{3}}{3}$

26. $\sec\frac{25\pi}{6}=\sec\left(\frac{\pi}{6}+4\pi\right)=\sec\left(\frac{\pi}{6}+2\cdot 2\pi\right)$
$=\sec\frac{\pi}{6}$
$=\frac{2\sqrt{3}}{3}$

27. Since $\sin\theta>0$ for points in quadrants I and II, and $\cos\theta<0$ for points in quadrants II and III, the angle θ lies in quadrant II.

28. Since $\sin\theta<0$ for points in quadrants III and IV, and $\cos\theta>0$ for points in quadrants I and IV, the angle θ lies in quadrant IV.

29. Since $\sin\theta<0$ for points in quadrants III and IV, and $\tan\theta<0$ for points in quadrants II and IV, the angle θ lies in quadrant IV.

30. Since $\cos\theta>0$ for points in quadrants I and IV, and $\tan\theta>0$ for points in quadrants I and III, the angle θ lies in quadrant I.

31. Since $\cos\theta>0$ for points in quadrants I and IV, and $\tan\theta<0$ for points in quadrants II and IV, the angle θ lies in quadrant IV.

32. Since $\cos\theta<0$ for points in quadrants II and III, and $\tan\theta>0$ for points in quadrants I and III, the angle θ lies in quadrant III.

33. Since $\sec\theta<0$ for points in quadrants II and III, and $\sin\theta>0$ for points in quadrants I and II, the angle θ lies in quadrant II.

34. Since $\csc\theta>0$ for points in quadrants I and II, and $\cos\theta<0$ for points in quadrants II and III, the angle θ lies in quadrant II.

35. $\sin\theta=-\frac{3}{5},\quad \cos\theta=\frac{4}{5}$

$$\tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{\left(-\frac{3}{5}\right)}{\left(\frac{4}{5}\right)}=-\frac{3}{5}\cdot\frac{5}{4}=-\frac{3}{4}$$

$$\csc\theta=\frac{1}{\sin\theta}=\frac{1}{\left(-\frac{3}{5}\right)}=-\frac{5}{3}$$

$$\sec\theta=\frac{1}{\cos\theta}=\frac{1}{\left(\frac{4}{5}\right)}=\frac{5}{4}$$

$$\cot\theta=\frac{1}{\tan\theta}=-\frac{4}{3}$$

36. $\sin\theta=\frac{4}{5},\quad \cos\theta=-\frac{3}{5}$

$$\tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{\left(\frac{4}{5}\right)}{\left(-\frac{3}{5}\right)}=\frac{4}{5}\cdot\left(-\frac{5}{3}\right)=-\frac{4}{3}$$

$$\csc\theta=\frac{1}{\sin\theta}=\frac{1}{\left(\frac{4}{5}\right)}=\frac{5}{4}$$

$$\sec\theta=\frac{1}{\cos\theta}=\frac{1}{\left(-\frac{3}{5}\right)}=-\frac{5}{3}$$

$$\cot\theta=\frac{1}{\tan\theta}=-\frac{3}{4}$$

37. $\sin\theta=\frac{2\sqrt{5}}{5},\quad \cos\theta=\frac{\sqrt{5}}{5}$

$$\tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{\left(\frac{2\sqrt{5}}{5}\right)}{\left(\frac{\sqrt{5}}{5}\right)}=\frac{2\sqrt{5}}{5}\cdot\frac{5}{\sqrt{5}}=2$$

$$\csc\theta=\frac{1}{\sin\theta}=\frac{1}{\left(\frac{2\sqrt{5}}{5}\right)}=1\cdot\frac{5}{2\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}=\frac{\sqrt{5}}{2}$$

$\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{1}{\left(\dfrac{\sqrt{5}}{5}\right)} = \dfrac{5}{\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}} = \sqrt{5}$

$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{1}{2}$

38. $\sin\theta = -\dfrac{\sqrt{5}}{5}, \quad \cos\theta = -\dfrac{2\sqrt{5}}{5}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\left(-\dfrac{\sqrt{5}}{5}\right)}{\left(-\dfrac{2\sqrt{5}}{5}\right)} = \left(-\dfrac{\sqrt{5}}{5}\right) \cdot \left(-\dfrac{5}{2\sqrt{5}}\right) = \dfrac{1}{2}$

$\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{1}{\left(-\dfrac{\sqrt{5}}{5}\right)} = 1 \cdot \left(-\dfrac{5}{\sqrt{5}}\right) \cdot \dfrac{\sqrt{5}}{\sqrt{5}} = -\sqrt{5}$

$\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{1}{\left(-\dfrac{2\sqrt{5}}{5}\right)} = \left(-\dfrac{5}{2\sqrt{5}}\right) \cdot \dfrac{\sqrt{5}}{\sqrt{5}} = -\dfrac{\sqrt{5}}{2}$

$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{1}{\left(\dfrac{1}{2}\right)} = 1 \cdot \dfrac{2}{1} = 2$

39. $\sin\theta = \dfrac{1}{2}, \quad \cos\theta = \dfrac{\sqrt{3}}{2}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\left(\dfrac{1}{2}\right)}{\left(\dfrac{\sqrt{3}}{2}\right)} = \dfrac{1}{2} \cdot \dfrac{2}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$

$\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{1}{\left(\dfrac{1}{2}\right)} = 1 \cdot \dfrac{2}{1} = 2$

$\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{1}{\left(\dfrac{\sqrt{3}}{2}\right)} = \dfrac{2}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{2\sqrt{3}}{3}$

$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{1}{\left(\dfrac{\sqrt{3}}{3}\right)} = \dfrac{3}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \sqrt{3}$

40. $\sin\theta = \dfrac{\sqrt{3}}{2}, \quad \cos\theta = \dfrac{1}{2}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\left(\dfrac{\sqrt{3}}{2}\right)}{\left(\dfrac{1}{2}\right)} = \dfrac{\sqrt{3}}{2} \cdot \dfrac{2}{1} = \sqrt{3}$

$\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{1}{\left(\dfrac{\sqrt{3}}{2}\right)} = 1 \cdot \dfrac{2}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{2\sqrt{3}}{3}$

$\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{1}{\left(\dfrac{1}{2}\right)} = 1 \cdot \dfrac{2}{1} = 2$

$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{1}{\sqrt{3}} = \dfrac{1}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$

41. $\sin\theta = -\dfrac{1}{3}, \quad \cos\theta = \dfrac{2\sqrt{2}}{3}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\left(-\dfrac{1}{3}\right)}{\left(\dfrac{2\sqrt{2}}{3}\right)} = -\dfrac{1}{3} \cdot \dfrac{3}{2\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = -\dfrac{\sqrt{2}}{4}$

$\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{1}{\left(-\dfrac{1}{3}\right)} = 1 \cdot \left(-\dfrac{3}{1}\right) = -3$

$\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{1}{\left(\dfrac{2\sqrt{2}}{3}\right)} = \dfrac{3}{2\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{3\sqrt{2}}{4}$

$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{1}{\left(-\dfrac{\sqrt{2}}{4}\right)} = -\dfrac{4}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = -2\sqrt{2}$

42. $\sin\theta = \dfrac{2\sqrt{2}}{3}, \quad \cos\theta = -\dfrac{1}{3}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\left(\dfrac{2\sqrt{2}}{3}\right)}{\left(-\dfrac{1}{3}\right)} = \dfrac{2\sqrt{2}}{3} \cdot -\dfrac{3}{1} = -2\sqrt{2}$

$\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{1}{\left(\dfrac{2\sqrt{2}}{3}\right)} = 1 \cdot \dfrac{3}{2\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{3\sqrt{2}}{4}$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\left(-\frac{1}{3}\right)} = 1\cdot\left(-\frac{3}{1}\right) = -3$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{-2\sqrt{2}} = -\frac{1}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{4}$$

43. $\sin\theta = \frac{12}{13}$, θ in quadrant II

Solve for $\cos\theta$: $\sin^2\theta + \cos^2\theta = 1$

$$\cos^2\theta = 1 - \sin^2\theta$$

$$\cos\theta = \pm\sqrt{1-\sin^2\theta}$$

Since θ is in quadrant II, $\cos\theta < 0$.

$$\cos\theta = -\sqrt{1-\sin^2\theta}$$

$$= -\sqrt{1-\left(\frac{12}{13}\right)^2} = -\sqrt{1-\frac{144}{169}} = -\sqrt{\frac{25}{169}} = -\frac{5}{13}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\left(\frac{12}{13}\right)}{\left(-\frac{5}{13}\right)} = \frac{12}{13}\cdot\left(-\frac{13}{5}\right) = -\frac{12}{5}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\left(\frac{12}{13}\right)} = \frac{13}{12}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\left(-\frac{5}{13}\right)} = -\frac{13}{5}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\left(-\frac{12}{5}\right)} = -\frac{5}{12}$$

44. $\cos\theta = \frac{3}{5}$, θ in quadrant IV

Solve for $\sin\theta$: $\sin^2\theta + \cos^2\theta = 1$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\sin\theta = \pm\sqrt{1-\cos^2\theta}$$

Since θ is in quadrant IV, $\sin\theta < 0$.

$$\sin\theta = -\sqrt{1-\cos^2\theta} = -\sqrt{1-\left(\frac{3}{5}\right)^2} = -\sqrt{\frac{16}{25}} = -\frac{4}{5}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\left(-\frac{4}{5}\right)}{\left(\frac{3}{5}\right)} = -\frac{4}{5}\cdot\frac{5}{3} = -\frac{4}{3}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\left(-\frac{4}{5}\right)} = -\frac{5}{4}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\left(\frac{3}{5}\right)} = \frac{5}{3}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\left(-\frac{4}{3}\right)} = -\frac{3}{4}$$

45. $\cos\theta = -\frac{4}{5}$, θ in quadrant III

Solve for $\sin\theta$: $\sin^2\theta + \cos^2\theta = 1$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\sin\theta = \pm\sqrt{1-\cos^2\theta}$$

Since θ is in quadrant III, $\sin\theta < 0$.

$$\sin\theta = -\sqrt{1-\cos^2\theta}$$

$$= -\sqrt{1-\left(-\frac{4}{5}\right)^2} = -\sqrt{1-\frac{16}{25}} = -\sqrt{\frac{9}{25}} = -\frac{3}{5}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\left(-\frac{3}{5}\right)}{\left(-\frac{4}{5}\right)} = -\frac{3}{5}\cdot\left(-\frac{5}{4}\right) = \frac{3}{4}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\left(-\frac{3}{5}\right)} = -\frac{5}{3}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\left(-\frac{4}{5}\right)} = -\frac{5}{4}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\left(\frac{3}{4}\right)} = \frac{4}{3}$$

46. $\sin\theta = -\frac{5}{13}$, θ in quadrant III

Solve for $\cos\theta$: $\sin^2\theta + \cos^2\theta = 1$

$$\cos^2\theta = 1 - \sin^2\theta$$

$$\cos\theta = \pm\sqrt{1-\sin^2\theta}$$

Since θ is in quadrant III, $\cos\theta < 0$.

$$\cos\theta = -\sqrt{1-\sin^2\theta} = -\sqrt{1-\left(-\frac{5}{13}\right)^2}$$

$$= -\sqrt{\frac{144}{169}} = -\frac{12}{13}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\left(-\frac{5}{13}\right)}{\left(-\frac{12}{13}\right)} = -\frac{5}{13}\cdot\left(-\frac{13}{12}\right) = \frac{5}{12}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\left(-\frac{5}{13}\right)} = -\frac{13}{5}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\left(-\frac{12}{13}\right)} = -\frac{13}{12}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\left(\frac{5}{12}\right)} = \frac{12}{5}$$

47. $\sin\theta = \frac{5}{13},\ 90° < \theta < 180°,\ \theta$ in quadrant II

Solve for $\cos\theta$: $\sin^2\theta + \cos^2\theta = 1$

$$\cos^2\theta = 1 - \sin^2\theta$$

$$\cos\theta = \pm\sqrt{1-\sin^2\theta}$$

Since θ is in quadrant II, $\cos\theta < 0$.

$$\cos\theta = -\sqrt{1-\sin^2\theta} = -\sqrt{1-\left(\frac{5}{13}\right)^2}$$

$$= -\sqrt{1-\frac{25}{169}} = -\sqrt{\frac{144}{169}} = -\frac{12}{13}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\left(\frac{5}{13}\right)}{\left(-\frac{12}{13}\right)} = \frac{5}{13}\cdot\left(-\frac{13}{12}\right) = -\frac{5}{12}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\left(\frac{5}{13}\right)} = \frac{13}{5}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\left(-\frac{12}{13}\right)} = -\frac{13}{12}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\left(-\frac{5}{12}\right)} = -\frac{12}{5}$$

48. $\cos\theta = \frac{4}{5},\ 270° < \theta < 360°;\ \theta$ in quadrant IV

Solve for $\sin\theta$: $\sin^2\theta + \cos^2\theta = 1$

$$\sin\theta = \pm\sqrt{1-\cos^2\theta}$$

Since θ is in quadrant IV, $\sin\theta < 0$.

$$\sin\theta = -\sqrt{1-\cos^2\theta} = -\sqrt{1-\left(\frac{4}{5}\right)^2} = -\sqrt{\frac{9}{25}} = -\frac{3}{5}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\left(-\frac{3}{5}\right)}{\left(\frac{4}{5}\right)} = -\frac{3}{5}\cdot\frac{5}{4} = -\frac{3}{4}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\left(-\frac{3}{5}\right)} = -\frac{5}{3}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\left(\frac{4}{5}\right)} = \frac{5}{4}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\left(-\frac{3}{4}\right)} = -\frac{4}{3}$$

49. $\cos\theta = -\frac{1}{3},\ \frac{\pi}{2} < \theta < \pi,\ \theta$ in quadrant II

Solve for $\sin\theta$: $\sin^2\theta + \cos^2\theta = 1$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\sin\theta = \pm\sqrt{1-\cos^2\theta}$$

Since θ is in quadrant II, $\sin\theta > 0$.

$$\sin\theta = \sqrt{1-\cos^2\theta}$$

$$= \sqrt{1-\left(-\frac{1}{3}\right)^2} = \sqrt{1-\frac{1}{9}} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\left(\frac{2\sqrt{2}}{3}\right)}{\left(-\frac{1}{3}\right)} = \frac{2\sqrt{2}}{3}\cdot\left(-\frac{3}{1}\right) = -2\sqrt{2}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\left(\frac{2\sqrt{2}}{3}\right)} = \frac{3}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{4}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\left(-\frac{1}{3}\right)} = -3$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{-2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{4}$$

50. $\sin\theta = -\frac{2}{3},\ \pi < \theta < \frac{3\pi}{2},\ \theta$ in quadrant III

Solve for $\cos\theta$: $\sin^2\theta + \cos^2\theta = 1$

$$\cos\theta = \pm\sqrt{1-\sin^2\theta}$$

Since θ is in quadrant III, $\cos\theta < 0$.

$$\cos\theta = -\sqrt{1-\sin^2\theta} = -\sqrt{1-\left(-\frac{2}{3}\right)^2}$$
$$= -\sqrt{\frac{5}{9}} = -\frac{\sqrt{5}}{3}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\left(-\frac{2}{3}\right)}{\left(-\frac{\sqrt{5}}{3}\right)}$$
$$= -\frac{2}{3}\cdot\left(-\frac{3}{\sqrt{5}}\right)\cdot\frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\left(-\frac{2}{3}\right)} = -\frac{3}{2}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\left(-\frac{\sqrt{5}}{3}\right)} = -\frac{3}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = -\frac{3\sqrt{5}}{5}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\left(\frac{2\sqrt{5}}{5}\right)} = \frac{5}{2\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{2}$$

51. $\sin\theta = \frac{2}{3}$, $\tan\theta < 0$, so θ is in quadrant II

Solve for $\cos\theta$: $\sin^2\theta + \cos^2\theta = 1$

$$\cos^2\theta = 1-\sin^2\theta$$
$$\cos\theta = \pm\sqrt{1-\sin^2\theta}$$

Since θ is in quadrant II, $\cos\theta < 0$.

$$\cos\theta = -\sqrt{1-\sin^2\theta}$$
$$= -\sqrt{1-\left(\frac{2}{3}\right)^2} = -\sqrt{1-\frac{4}{9}} = -\sqrt{\frac{5}{9}} = -\frac{\sqrt{5}}{3}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\left(\frac{2}{3}\right)}{\left(-\frac{\sqrt{5}}{3}\right)} = \frac{2}{3}\cdot\left(-\frac{3}{\sqrt{5}}\right) = -\frac{2\sqrt{5}}{5}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\left(\frac{2}{3}\right)} = \frac{3}{2}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\left(-\frac{\sqrt{5}}{3}\right)} = -\frac{3}{\sqrt{5}} = -\frac{3\sqrt{5}}{5}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\left(-\frac{2\sqrt{5}}{5}\right)} = -\frac{5}{2\sqrt{5}} = -\frac{\sqrt{5}}{2}$$

52. $\cos\theta = -\frac{1}{4}$, $\tan\theta > 0$

Since $\tan\theta = \frac{\sin\theta}{\cos\theta} > 0$ and $\cos\theta < 0$, $\sin\theta < 0$.

Solve for $\sin\theta$: $\sin^2\theta + \cos^2\theta = 1$

$$\sin\theta = \pm\sqrt{1-\cos^2\theta}$$

$$\sin\theta = -\sqrt{1-\cos^2\theta}$$
$$= -\sqrt{1-\left(-\frac{1}{4}\right)^2} = -\sqrt{1-\frac{1}{16}}$$
$$= -\sqrt{\frac{15}{16}} = -\frac{\sqrt{15}}{4}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\left(-\frac{\sqrt{15}}{4}\right)}{\left(-\frac{1}{4}\right)} = -\frac{\sqrt{15}}{4}\cdot\left(-\frac{4}{1}\right) = \sqrt{15}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\left(-\frac{\sqrt{15}}{4}\right)} = -\frac{4}{\sqrt{15}}\cdot\frac{\sqrt{15}}{\sqrt{15}} = -\frac{4\sqrt{15}}{15}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\left(-\frac{1}{4}\right)} = -\frac{4}{1} = -4$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\sqrt{15}}\cdot\frac{\sqrt{15}}{\sqrt{15}} = \frac{\sqrt{15}}{15}$$

53. $\sec\theta = 2,\ \sin\theta < 0$, so θ is in quadrant IV

Solve for $\cos\theta$: $\cos\theta = \dfrac{1}{\sec\theta} = \dfrac{1}{2}$

Solve for $\sin\theta$: $\sin^2\theta + \cos^2\theta = 1$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\sin\theta = \pm\sqrt{1-\cos^2\theta}$$

Since θ is in quadrant IV, $\sin\theta < 0$.

$$\sin\theta = -\sqrt{1-\cos^2\theta} = -\sqrt{1-\left(\frac{1}{2}\right)^2} = -\sqrt{1-\frac{1}{4}} = -\sqrt{\frac{3}{4}} = -\frac{\sqrt{3}}{2}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\left(-\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{2}\right)} = -\frac{\sqrt{3}}{2}\cdot\frac{2}{1} = -\sqrt{3}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\left(-\frac{\sqrt{3}}{2}\right)} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{-\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

54. $\csc\theta = 3,\ \cot\theta < 0$, so θ is in quadrant II

Solve for $\sin\theta$: $\sin\theta = \dfrac{1}{\csc\theta} = \dfrac{1}{3}$

Solve for $\cos\theta$: $\sin^2\theta + \cos^2\theta = 1$

$$\cos\theta = \pm\sqrt{1-\sin^2\theta}$$

Since θ is in quadrant II, $\cos\theta < 0$.

$$\cos\theta = -\sqrt{1-\sin^2\theta} = -\sqrt{1-\left(\frac{1}{3}\right)^2} = -\sqrt{1-\frac{1}{9}} = -\sqrt{\frac{8}{9}} = -\frac{2\sqrt{2}}{3}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\left(\frac{1}{3}\right)}{\left(-\frac{2\sqrt{2}}{3}\right)} = \frac{1}{3}\cdot\left(-\frac{3}{2\sqrt{2}}\right)\cdot\frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{4}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\left(-\frac{2\sqrt{2}}{3}\right)} = -\frac{3}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = -\frac{3\sqrt{2}}{4}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\left(-\frac{\sqrt{2}}{4}\right)} = -\frac{4}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = -2\sqrt{2}$$

55. $\tan\theta = \dfrac{3}{4},\ \sin\theta < 0$, so θ is in quadrant III

Solve for $\sec\theta$: $\sec^2\theta = 1 + \tan^2\theta$

$$\sec\theta = \pm\sqrt{1+\tan^2\theta}$$

Since θ is in quadrant III, $\sec\theta < 0$.

$$\sec\theta = -\sqrt{1+\tan^2\theta} = -\sqrt{1+\left(\frac{3}{4}\right)^2} = -\sqrt{1+\frac{9}{16}} = -\sqrt{\frac{25}{16}} = -\frac{5}{4}$$

$$\cos\theta = \frac{1}{\sec\theta} = -\frac{4}{5}$$

$$\sin\theta = -\sqrt{1-\cos^2\theta} = -\sqrt{1-\left(-\frac{4}{5}\right)^2} = -\sqrt{1-\frac{16}{25}} = -\sqrt{\frac{9}{25}} = -\frac{3}{5}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\left(-\frac{3}{5}\right)} = -\frac{5}{3}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\left(\frac{3}{4}\right)} = \frac{4}{3}$$

56. $\cot\theta = \dfrac{4}{3},\ \cos\theta < 0$, so θ is in quadrant III

$$\tan\theta = \frac{1}{\cot\theta} = \frac{1}{\left(\frac{4}{3}\right)} = \frac{3}{4}$$

Solve for $\sec\theta$: $\sec^2\theta = 1 + \tan^2\theta$

$$\sec\theta = \pm\sqrt{1+\tan^2\theta}$$

Since θ is in quadrant III, $\sec\theta < 0$.

$$\sec\theta = -\sqrt{1+\tan^2\theta} = -\sqrt{1+\left(\frac{3}{4}\right)^2} = -\sqrt{1+\frac{9}{16}} = -\sqrt{\frac{25}{16}} = -\frac{5}{4}$$

$$\cos\theta = \frac{1}{\sec\theta} = -\frac{4}{5}$$

$$\sin\theta = -\sqrt{1-\cos^2\theta}$$
$$= -\sqrt{1-\left(-\frac{4}{5}\right)^2} = -\sqrt{1-\frac{16}{25}} = -\sqrt{\frac{9}{25}} = -\frac{3}{5}$$
$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\left(-\frac{3}{5}\right)} = -\frac{5}{3}$$

57. $\tan\theta = -\frac{1}{3}$, $\sin\theta > 0$, so θ is in quadrant II

Solve for $\sec\theta$: $\sec^2\theta = 1+\tan^2\theta$

$$\sec\theta = \pm\sqrt{1+\tan^2\theta}$$

Since θ is in quadrant II, $\sec\theta < 0$.

$$\sec\theta = -\sqrt{1+\tan^2\theta}$$
$$= -\sqrt{1+\left(-\frac{1}{3}\right)^2} = -\sqrt{1+\frac{1}{9}} = -\sqrt{\frac{10}{9}} = -\frac{\sqrt{10}}{3}$$
$$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{\left(-\frac{\sqrt{10}}{3}\right)} = -\frac{3}{\sqrt{10}} = -\frac{3\sqrt{10}}{10}$$
$$\sin\theta = \sqrt{1-\cos^2\theta}$$
$$= \sqrt{1-\left(-\frac{3\sqrt{10}}{10}\right)^2} = \sqrt{1-\frac{90}{100}}$$
$$= \sqrt{\frac{10}{100}} = \frac{\sqrt{10}}{10}$$
$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\left(\frac{\sqrt{10}}{10}\right)} = \sqrt{10}$$
$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\left(-\frac{1}{3}\right)} = -3$$

58. $\sec\theta = -2$, $\tan\theta > 0$, so θ is in quadrant III

Solve for $\tan\theta$: $\sec^2\theta = 1+\tan^2\theta$

$$\tan\theta = \pm\sqrt{\sec^2\theta - 1}$$
$$\tan\theta = \sqrt{\sec^2\theta-1} = \sqrt{(-2)^2-1} = \sqrt{4-1} = \sqrt{3}$$
$$\cos\theta = \frac{1}{\sec\theta} = -\frac{1}{2}$$
$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\sin\theta = -\sqrt{1-\cos^2\theta}$$
$$= -\sqrt{1-\left(-\frac{1}{2}\right)^2} = -\sqrt{1-\frac{1}{4}} = -\sqrt{\frac{3}{4}} = -\frac{\sqrt{3}}{2}$$
$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\left(-\frac{\sqrt{3}}{2}\right)} = -\frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$$

59. $\sin(-60^\circ) = -\sin 60^\circ = -\frac{\sqrt{3}}{2}$

60. $\cos(-30^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$

61. $\tan(-30^\circ) = -\tan 30^\circ = -\frac{\sqrt{3}}{3}$

62. $\sin(-135^\circ) = -\sin 135^\circ = -\frac{\sqrt{2}}{2}$

63. $\sec(-60^\circ) = \sec 60^\circ = 2$

64. $\csc(-30^\circ) = -\csc 30^\circ = -2$

65. $\sin(-90^\circ) = -\sin 90^\circ = -1$

66. $\cos(-270^\circ) = \cos 270^\circ = 0$

67. $\tan\left(-\frac{\pi}{4}\right) = -\tan\frac{\pi}{4} = -1$

68. $\sin(-\pi) = -\sin\pi = 0$

69. $\cos\left(-\frac{\pi}{4}\right) = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

70. $\sin\left(-\frac{\pi}{3}\right) = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$

71. $\tan(-\pi) = -\tan\pi = 0$

72. $\sin\left(-\frac{3\pi}{2}\right) = -\sin\frac{3\pi}{2} = -(-1) = 1$

73. $\csc\left(-\frac{\pi}{4}\right) = -\csc\frac{\pi}{4} = -\sqrt{2}$

74. $\sec(-\pi) = \sec\pi = -1$

75. $\sec\left(-\frac{\pi}{6}\right)=\sec\frac{\pi}{6}=\frac{2\sqrt{3}}{3}$

76. $\csc\left(-\frac{\pi}{3}\right)=-\csc\frac{\pi}{3}=-\frac{2\sqrt{3}}{3}$

77. $\sin^2(40°)+\cos^2(40°)=1$

78. $\sec^2(18°)-\tan^2(18°)=1$

79. $\sin(80°)\csc(80°)=\sin(80°)\cdot\frac{1}{\sin(80°)}=1$

80. $\tan(10°)\cot(10°)=\tan(10°)\cdot\frac{1}{\tan(10°)}=1$

81. $\tan(40°)-\frac{\sin(40°)}{\cos(40°)}=\tan(40°)-\tan(40°)=0$

82. $\cot(20°)-\frac{\cos(20°)}{\sin(20°)}=\cot(20°)-\cot(20°)=0$

83.
$$\begin{aligned}\cos(400°)\cdot\sec(40°)&=\cos(40°+360°)\cdot\sec(40°)\\&=\cos(40°)\cdot\sec(40°)\\&=\cos(40°)\cdot\frac{1}{\cos(40°)}=1\end{aligned}$$

84.
$$\begin{aligned}\tan(200°)\cdot\cot(20°)&=\tan(20°+180°)\cdot\cot(20°)\\&=\tan(20°)\cdot\cot(20°)\\&=\tan(20°)\cdot\frac{1}{\tan(20°)}=1\end{aligned}$$

85.
$$\begin{aligned}\sin\left(-\frac{\pi}{12}\right)\csc\left(\frac{25\pi}{12}\right)&=-\sin\left(\frac{\pi}{12}\right)\csc\left(\frac{25\pi}{12}\right)\\&=-\sin\left(\frac{\pi}{12}\right)\csc\left(\frac{\pi}{12}+\frac{24\pi}{12}\right)\\&=-\sin\left(\frac{\pi}{12}\right)\csc\left(\frac{\pi}{12}+2\pi\right)\\&=-\sin\left(\frac{\pi}{12}\right)\csc\left(\frac{\pi}{12}\right)\\&=-\sin\left(\frac{\pi}{12}\right)\cdot\frac{1}{\sin\left(\frac{\pi}{12}\right)}=-1\end{aligned}$$

86.
$$\begin{aligned}\sec\left(-\frac{\pi}{18}\right)\cos\left(\frac{37\pi}{18}\right)&=\sec\left(\frac{\pi}{18}\right)\cos\left(\frac{37\pi}{18}\right)\\&=\sec\left(\frac{\pi}{18}\right)\cos\left(\frac{\pi}{18}+\frac{36\pi}{18}\right)\\&=\sec\left(\frac{\pi}{18}\right)\cos\left(\frac{\pi}{18}+2\pi\right)\\&=\sec\left(\frac{\pi}{18}\right)\cos\left(\frac{\pi}{18}\right)\\&=\sec\left(\frac{\pi}{18}\right)\cdot\frac{1}{\sec\left(\frac{\pi}{18}\right)}=1\end{aligned}$$

87.
$$\begin{aligned}&\frac{\sin(-20°)}{\cos(380°)}+\tan(200°)\\&=\frac{-\sin(20°)}{\cos(20°+360°)}+\tan(20°+180°)\\&=\frac{-\sin(20°)}{\cos(20°)}+\tan(20°)\\&=-\tan(20°)+\tan(20°)=0\end{aligned}$$

88.
$$\begin{aligned}&\frac{\sin(70°)}{\cos(-430°)}+\tan(-70°)\\&=\frac{\sin(70°)}{\cos(430°)}-\tan(70°)\\&=\frac{\sin(70°)}{\cos(70°+360°)}-\tan(70°)\\&=\frac{\sin(70°)}{\cos(70°)}-\tan(70°)\\&=\tan(70°)-\tan(70°)=0\end{aligned}$$

89. If $\sin\theta=0.3$, then
$\sin\theta+\sin(\theta+2\pi)+\sin(\theta+4\pi)$
$=0.3+0.3+0.3=0.9$

90. If $\cos\theta=0.2$, then
$\cos\theta+\cos(\theta+2\pi)+\cos(\theta+4\pi)$
$=-0.2+0.2+0.2=0.6$

91. If $\tan\theta=3$, then
$\tan\theta+\tan(\theta+\pi)+\tan(\theta+2\pi)$
$=3+3+3=9$

92. If $\cot\theta = -2$, then
$$\cot\theta + \cot(\theta - \pi) + \cot(\theta - 2\pi) = -2 + (-2) + (-2) = -6$$

93.
$$\begin{aligned}&\sin 1^\circ + \sin 2^\circ + \sin 3^\circ + ... + \sin 357^\circ + \sin 358^\circ + \sin 359^\circ\\ &= \sin 1^\circ + \sin 2^\circ + \sin 3^\circ + \cdots + \sin(360^\circ - 3^\circ) + \sin(360^\circ - 2^\circ) + \sin(360^\circ - 1^\circ)\\ &= \sin 1^\circ + \sin 2^\circ + \sin 3^\circ + \cdots + \sin(-3^\circ) + \sin(-2^\circ) + \sin(-1^\circ)\\ &= \sin 1^\circ + \sin 2^\circ + \sin 3^\circ + \cdots - \sin 3^\circ - \sin 2^\circ - \sin 1^\circ\\ &= \sin(180^\circ) = 0\end{aligned}$$

94.
$$\begin{aligned}&\cos 1^\circ + \cos 2^\circ + \cos 3^\circ + \cdots + \cos 357^\circ + \cos 358^\circ + \cos 359^\circ\\ &= \cos 1^\circ + \cos 2^\circ + \cos 3^\circ + ... + \cos(360^\circ - 3^\circ) + \cos(360^\circ - 2^\circ) + \cos(360^\circ - 1^\circ)\\ &= \cos 1^\circ + \cos 2^\circ + \cos 3^\circ + ... + \cos(-3^\circ) + \cos(-2^\circ) + \cos(-1^\circ)\\ &= \cos 1^\circ + \cos 2^\circ + \cos 3^\circ + ... + \cos 3^\circ + \cos 2^\circ + \cos 1^\circ\\ &= 2\cos 1^\circ + 2\cos 2^\circ + 2\cos 3^\circ + ... + 2\cos 178^\circ + 2\cos 179^\circ + \cos 180^\circ\\ &= 2\cos 1^\circ + 2\cos 2^\circ + 2\cos 3^\circ + ... + 2\cos(180^\circ - 2^\circ) + 2\cos(180^\circ - 1^\circ) + \cos(180^\circ)\\ &= 2\cos 1^\circ + 2\cos 2^\circ + 2\cos 3^\circ + ... - 2\cos 2^\circ - 2\cos 1^\circ + \cos 180^\circ\\ &= \cos 180^\circ = -1\end{aligned}$$

95. The domain of the sine function is the set of all real numbers.

96. The domain of the cosine function is the set of all real numbers.

97. $f(\theta) = \tan\theta$ is not defined for numbers that are odd multiples of $\frac{\pi}{2}$.

98. $f(\theta) = \cot\theta$ is not defined for numbers that are multiples of π.

99. $f(\theta) = \sec\theta$ is not defined for numbers that are odd multiples of $\frac{\pi}{2}$.

100. $f(\theta) = \csc\theta$ is not defined for numbers that are multiples of π.

101. The range of the sine function is the set of all real numbers between -1 and 1, inclusive.

102. The range of the cosine function is the set of all real numbers between -1 and 1, inclusive.

103. The range of the tangent function is the set of all real numbers.

104. The range of the cotangent function is the set of all real numbers.

105. The range of the secant function is the set of all real numbers greater than or equal to 1 and all real numbers less than or equal to -1.

106. The range of the cosecant function is the set of all real number greater than or equal to 1 and all real numbers less than or equal to -1.

107. The sine function is odd because $\sin(-\theta) = -\sin\theta$. Its graph is symmetric with respect to the origin.

108. The cosine function is even because $\cos(-\theta) = \cos\theta$. Its graph is symmetric with respect to the *y*-axis.

109. The tangent function is odd because $\tan(-\theta) = -\tan\theta$. Its graph is symmetric with respect to the origin.

110. The cotangent function is odd because $\cot(-\theta) = -\cot\theta$. Its graph is symmetric with respect to the origin.

111. The secant function is even because $\sec(-\theta) = \sec\theta$. Its graph is symmetric with respect to the *y*-axis.

112. The cosecant function is odd because $\csc(-\theta) = -\csc\theta$. Its graph is symmetric with respect to the origin.

113. **a.** $f(-a) = -f(a) = -\frac{1}{3}$

b.
$$\begin{aligned}&f(a) + f(a + 2\pi) + f(a + 4\pi)\\ &= f(a) + f(a) + f(a)\\ &= \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1\end{aligned}$$

114. a. $f(-a) = f(a) = \frac{1}{4}$

b. $f(a) + f(a+2\pi) + f(a-2\pi)$
$= f(a) + f(a) + f(a)$
$= \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$
$= \frac{3}{4}$

115. a. $f(-a) = -f(a) = -2$

b. $f(a) + f(a+\pi) + f(a+2\pi)$
$= f(a) + f(a) + f(a)$
$= 2 + 2 + 2 = 6$

116. a. $f(-a) = -f(a) = -(-3) = 3$

b. $f(a) + f(a+\pi) + f(a+4\pi)$
$= f(a) + f(a) + f(a)$
$= -3 + (-3) + (-3)$
$= -9$

117. a. $f(-a) = f(a) = -4$

b. $f(a) + f(a+2\pi) + f(a+4\pi)$
$= f(a) + f(a) + f(a)$
$= -4 + (-4) + (-4)$
$= -12$

118. a. $f(-a) = -f(a) = -2$

b. $f(a) + f(a+2\pi) + f(a+4\pi)$
$= f(a) + f(a) + f(a)$
$= 2 + 2 + 2$
$= 6$

119. Since $\tan\theta = \frac{500}{1500} = \frac{1}{3} = \frac{y}{x}$, then
$r^2 = x^2 + y^2 = 9 + 1 = 10$
$r = \sqrt{10}$
$\sin\theta = \frac{1}{\sqrt{1+9}} = \frac{1}{\sqrt{10}}$.
$$T = 5 - \frac{5}{\left(3 \cdot \frac{1}{3}\right)} + \frac{5}{\left(\frac{1}{\sqrt{10}}\right)}$$
$= 5 - 5 + 5\sqrt{10}$
$= 5\sqrt{10} \approx 15.8$ minutes

120. a. $\tan\theta = \frac{1}{4} = \frac{y}{x}$ for $0 < \theta < \frac{\pi}{2}$.
$r^2 = x^2 + y^2 = 16 + 1 = 17$
$r = \sqrt{17}$
Thus, $\sin\theta = \frac{1}{\sqrt{17}}$.
$$T(\theta) = 1 + \frac{2}{\left(3 \cdot \frac{1}{\sqrt{17}}\right)} - \frac{1}{\left(4 \cdot \frac{1}{4}\right)}$$
$= 1 + \frac{2\sqrt{17}}{3} - 1 = \frac{2\sqrt{17}}{3} \approx 2.75$ hours

b. Since $\tan\theta = \frac{1}{4}$, $x = 4$. Sally heads directly across the sand to the bridge, crosses the bridge, and heads directly across the sand to the other house.

c. θ must be larger than $14°$, or the road will not be reached and she cannot get across the river.

121. Let $P = (x, y)$ be the point on the unit circle that corresponds to an angle *t*. Consider the equation $\tan t = \frac{y}{x} = a$. Then $y = ax$. Now $x^2 + y^2 = 1$, so $x^2 + a^2x^2 = 1$. Thus, $x = \pm\frac{1}{\sqrt{1+a^2}}$ and $y = \pm\frac{a}{\sqrt{1+a^2}}$. That is, for any real number a, there is a point $P = (x, y)$ on the unit circle for which $\tan t = a$. In other words, $-\infty < \tan t < \infty$, and the range of the tangent function is the set of all real numbers.

122. Let $P = (x, y)$ be the point on the unit circle that corresponds to an angle *t*. Consider the equation $\cot t = \frac{x}{y} = a$. Then $x = ay$. Now $x^2 + y^2 = 1$, so $a^2y^2 + y^2 = 1$. Thus, $y = \pm\frac{1}{\sqrt{1+a^2}}$ and $x = \pm\frac{a}{\sqrt{1+a^2}}$. That is, for any real number a, there is a point $P = (x, y)$ on the unit circle for which $\cot t = a$. In other words, $-\infty < \cot t < \infty$, and the range of the tangent function is the set of all real numbers.

123. Suppose there is a number p, $0 < p < 2\pi$ for which $\sin(\theta + p) = \sin\theta$ for all θ. If $\theta = 0$, then $\sin(0 + p) = \sin p = \sin 0 = 0$; so that $p = \pi$. If $\theta = \frac{\pi}{2}$ then $\sin\left(\frac{\pi}{2} + p\right) = \sin\left(\frac{\pi}{2}\right)$. But $p = \pi$. Thus, $\sin\left(\frac{3\pi}{2}\right) = -1 = \sin\left(\frac{\pi}{2}\right) = 1$, or $-1 = 1$. This is impossible. The smallest positive number p for which $\sin(\theta + p) = \sin\theta$ for all θ must then be $p = 2\pi$.

124. Suppose there is a number p, $0 < p < 2\pi$, for which $\cos(\theta + p) = \cos\theta$ for all θ. If $\theta = \frac{\pi}{2}$, then $\cos\left(\frac{\pi}{2} + p\right) = \cos\left(\frac{\pi}{2}\right) = 0$; so that $p = \pi$. If $\theta = 0$, then $\cos(0 + p) = \cos(0)$. But $p = \pi$. Thus $\cos(\pi) = -1 = \cos(0) = 1$, or $-1 = 1$. This is impossible. The smallest positive number p for which $\cos(\theta + p) = \cos\theta$ for all θ must then be $p = 2\pi$.

125. $\sec\theta = \frac{1}{\cos\theta}$: Since $\cos\theta$ has period 2π, so does $\sec\theta$.

126. $\csc\theta = \frac{1}{\sin\theta}$: Since $\sin\theta$ has period 2π, so does $\csc\theta$.

127. If $P = (a,b)$ is the point on the unit circle corresponding to θ, then $Q = (-a,-b)$ is the point on the unit circle corresponding to $\theta + \pi$. Thus, $\tan(\theta + \pi) = \frac{-b}{-a} = \frac{b}{a} = \tan\theta$. If there exists a number p, $0 < p < \pi$, for which $\tan(\theta + p) = \tan\theta$ for all θ, then if $\theta = 0$, $\tan(p) = \tan(0) = 0$. But this means that p is a multiple of π. Since no multiple of π exists in the interval $(0, \pi)$, this is impossible. Therefore, the fundamental period of $f(\theta) = \tan\theta$ is π.

128. $\cot\theta = \frac{1}{\tan\theta}$: Since $\tan\theta$ has period π, so does $\cot\theta$.

129. Let $P = (a,b)$ be the point on the unit circle corresponding to θ. Then $\csc\theta = \frac{1}{b} = \frac{1}{\sin\theta}$

$$\sec\theta = \frac{1}{a} = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{a}{b} = \frac{1}{\left(\frac{b}{a}\right)} = \frac{1}{\tan\theta}$$

130. Let $P = (a,b)$ be the point on the unit circle corresponding to θ. Then

$$\tan\theta = \frac{b}{a} = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{a}{b} = \frac{\cos\theta}{\sin\theta}$$

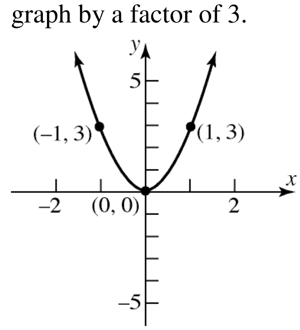
131.
$$\begin{aligned}
&(\sin\theta\cos\phi)^2 + (\sin\theta\sin\phi)^2 + \cos^2\theta \\
&= \sin^2\theta\cos^2\phi + \sin^2\theta\sin^2\phi + \cos^2\theta \\
&= \sin^2\theta(\cos^2\phi + \sin^2\phi) + \cos^2\theta \\
&= \sin^2\theta + \cos^2\theta \\
&= 1
\end{aligned}$$

132 – 136. Answers will vary.

Section 6.4

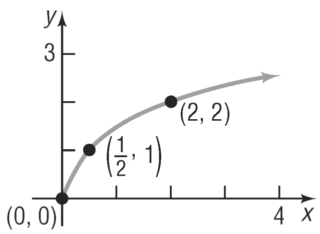
1. $y = 3x^2$

   Using the graph of $y = x^2$, vertically stretch the graph by a factor of 3.



2. $y = \sqrt{2x}$

Using the graph of $y = \sqrt{x}$, compress horizontally by a factor of $\frac{1}{2}$.



3. 1; $\frac{\pi}{2}$

4. 3; π

5. 3; $\frac{2\pi}{6} = \frac{\pi}{3}$

6. True

7. False; The period is $\frac{2\pi}{\pi} = 2$

8. True

9. a. The graph of $y = \sin x$ crosses the y-axis at the point (0, 0), so the y-intercept is 0.

b. The graph of $y = \sin x$ is increasing for $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

c. The largest value of $y = \sin x$ is 1.

d. $\sin x = 0$ when $x = 0, \pi, 2\pi$.

e. $\sin x = 1$ when $x = -\frac{3\pi}{2}, \frac{\pi}{2}$;
$\sin x = -1$ when $x = -\frac{\pi}{2}, \frac{3\pi}{2}$.

f. $\sin x = -\frac{1}{2}$ when $x = -\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

g. The x-intercepts of $\sin x$ are $\{x \mid x = k\pi, \text{ k an integer}\}$

10. a. The graph of $y = \cos x$ crosses the y-axis at the point (0, 1), so the y-intercept is 1.

b. The graph of $y = \cos x$ is decreasing for $0 < x < \pi$.

c. The smallest value of $y = \cos x$ is -1.

d. $\cos x = 0$ when $x = \frac{\pi}{2}, \frac{3\pi}{2}$

e. $\cos x = 1$ when $x = -2\pi, 0, 2\pi$;
$\cos x = -1$ when $x = -\pi, \pi$.

f. $\cos x = \frac{\sqrt{3}}{2}$ when $x = -\frac{11\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{11\pi}{6}$

g. The x-intercepts of $\cos x$ are
$\left\{x \mid x = \frac{(2k+1)\pi}{2}, \text{ k an integer}\right\}$

11. $y = 2\sin x$

This is in the form $y = A\sin(\omega x)$ where $A = 2$ and $\omega = 1$. Thus, the amplitude is $|A| = |2| = 2$ and the period is $T = \frac{2\pi}{\omega} = \frac{2\pi}{1} = 2\pi$.

12. $y = 3\cos x$

This is in the form $y = A\cos(\omega x)$ where $A = 3$ and $\omega = 1$. Thus, the amplitude is $|A| = |3| = 3$ and the period is $T = \frac{2\pi}{\omega} = \frac{2\pi}{1} = 2\pi$.

13. $y = -4\cos(2x)$

This is in the form $y = A\cos(\omega x)$ where $A = -4$ and $\omega = 2$. Thus, the amplitude is $|A| = |-4| = 4$ and the period is $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$.

14. $y = -\sin\left(\frac{1}{2}x\right)$

This is in the form $y = A\sin(\omega x)$ where $A = -1$ and $\omega = \frac{1}{2}$. Thus, the amplitude is $|A| = |-1| = 1$ and the period is $T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{1}{2}} = 4\pi$.

15. $y = 6\sin(\pi x)$

This is in the form $y = A\sin(\omega x)$ where $A = 6$

and $\omega = \pi$. Thus, the amplitude is $|A| = |6| = 6$ and the period is $T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi} = 2$.

16. $y = -3\cos(3x)$
This is in the form $y = A\cos(\omega x)$ where $A = -3$ and $\omega = 3$. Thus, the amplitude is $|A| = |-3| = 3$ and the period is $T = \frac{2\pi}{\omega} = \frac{2\pi}{3}$.

17. $y = -\frac{1}{2}\cos\left(\frac{3}{2}x\right)$
This is in the form $y = A\cos(\omega x)$ where $A = -\frac{1}{2}$ and $\omega = \frac{3}{2}$. Thus, the amplitude is $|A| = \left|-\frac{1}{2}\right| = \frac{1}{2}$ and the period is $T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{3}{2}} = \frac{4\pi}{3}$.

18. $y = \frac{4}{3}\sin\left(\frac{2}{3}x\right)$
This is in the form $y = A\sin(\omega x)$ where $A = \frac{4}{3}$ and $\omega = \frac{2}{3}$. Thus, the amplitude is $|A| = \left|\frac{4}{3}\right| = \frac{4}{3}$ and the period is $T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{2}{3}} = 3\pi$.

19. $y = \frac{5}{3}\sin\left(-\frac{2\pi}{3}x\right) = -\frac{5}{3}\sin\left(\frac{2\pi}{3}x\right)$
This is in the form $y = A\sin(\omega x)$ where $A = -\frac{5}{3}$ and $\omega = \frac{2\pi}{3}$. Thus, the amplitude is $|A| = \left|-\frac{5}{3}\right| = \frac{5}{3}$ and the period is $T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{2\pi}{3}} = 3$.

20. $y = \frac{9}{5}\cos\left(-\frac{3\pi}{2}x\right) = \frac{9}{5}\cos\left(\frac{3\pi}{2}x\right)$
This is in the form $y = A\cos(\omega x)$ where $A = \frac{9}{5}$ and $\omega = \frac{3\pi}{2}$. Thus, the amplitude is $|A| = \left|\frac{9}{5}\right| = \frac{9}{5}$ and the period is $T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{3\pi}{2}} = \frac{4}{3}$.

21. F

22. E

23. A

24. I

25. H

26. B

27. C

28. G

29. J

30. D

31. A

32. C

33. B

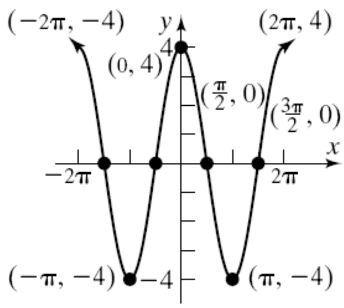
34. D

35. Comparing $y = 4\cos x$ to $y = A\cos(\omega x)$, we find $A = 4$ and $\omega = 1$. Therefore, the amplitude is $|4| = 4$ and the period is $\frac{2\pi}{1} = 2\pi$. Because the amplitude is 4, the graph of $y = 4\cos x$ will lie between -4 and 4 on the y-axis. Because the period is 2π, one cycle will begin at $x = 0$ and end at $x = 2\pi$. We divide the interval $[0, 2\pi]$ into four subintervals, each of length $\frac{2\pi}{4} = \frac{\pi}{2}$ by finding the following values:
0, $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$, and 2π
These values of x determine the x-coordinates of the five key points on the graph. To obtain the y-coordinates of the five key points for

$y = 4\cos x$, we multiply the *y*-coordinates of the five key points for $y = \cos x$ by $A = 4$. The five key points are

$(0,4)$, $\left(\dfrac{\pi}{2},0\right)$, $(\pi,-4)$, $\left(\dfrac{3\pi}{2},0\right)$, $(2\pi,4)$

We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown below.



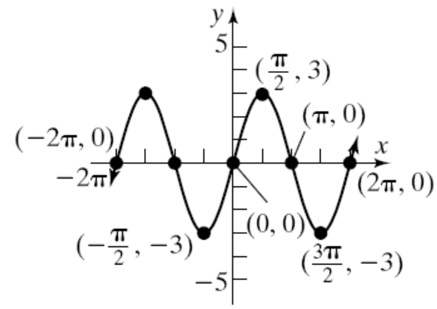
From the graph we can determine that the domain is all real numbers, $(-\infty,\infty)$ and the range is $[-4,4]$.

36. Comparing $y = 3\sin x$ to $y = A\sin(\omega x)$, we find $A = 3$ and $\omega = 1$. Therefore, the amplitude is $|3| = 3$ and the period is $\dfrac{2\pi}{1} = 2\pi$. Because the amplitude is 3, the graph of $y = 3\sin x$ will lie between -3 and 3 on the *y*-axis. Because the period is 2π, one cycle will begin at $x = 0$ and end at $x = 2\pi$. We divide the interval $[0,2\pi]$ into four subintervals, each of length $\dfrac{2\pi}{4} = \dfrac{\pi}{2}$ by finding the following values:

0, $\dfrac{\pi}{2}$, π, $\dfrac{3\pi}{2}$, and 2π

These values of *x* determine the *x*-coordinates of the five key points on the graph. To obtain the *y*-coordinates of the five key points for $y = 3\sin x$, we multiply the *y*-coordinates of the five key points for $y = \sin x$ by $A = 3$. The five key points are $(0,0)$, $\left(\dfrac{\pi}{2},3\right)$, $(\pi,0)$, $\left(\dfrac{3\pi}{2},-3\right)$, $(2\pi,0)$

We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown below.



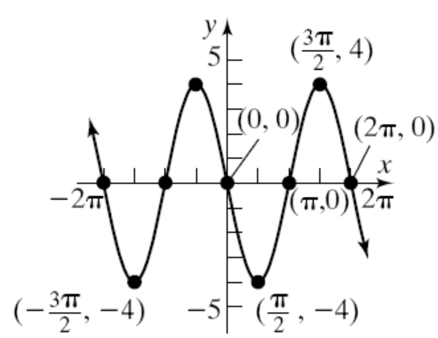
From the graph we can determine that the domain is all real numbers, $(-\infty,\infty)$ and the range is $[-3,3]$.

37. Comparing $y = -4\sin x$ to $y = A\sin(\omega x)$, we find $A = -4$ and $\omega = 1$. Therefore, the amplitude is $|-4| = 4$ and the period is $\dfrac{2\pi}{1} = 2\pi$. Because the amplitude is 4, the graph of $y = -4\sin x$ will lie between -4 and 4 on the *y*-axis. Because the period is 2π, one cycle will begin at $x = 0$ and end at $x = 2\pi$. We divide the interval $[0,2\pi]$ into four subintervals, each of length $\dfrac{2\pi}{4} = \dfrac{\pi}{2}$ by finding the following values: 0, $\dfrac{\pi}{2}$, π, $\dfrac{3\pi}{2}$, 2π

These values of *x* determine the *x*-coordinates of the five key points on the graph. To obtain the *y*-coordinates of the five key points for $y = -4\sin x$, we multiply the *y*-coordinates of the five key points for $y = \sin x$ by $A = -4$. The five key points are

$(0,0)$, $\left(\dfrac{\pi}{2},-4\right)$, $(\pi,0)$, $\left(\dfrac{3\pi}{2},4\right)$, $(2\pi,0)$

We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown below.



From the graph we can determine that the domain is all real numbers, $(-\infty,\infty)$ and the range is $[-4,4]$.

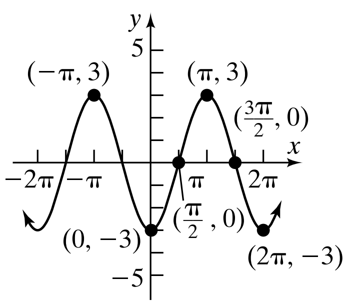
38. Comparing $y=-3\cos x$ to $y=A\cos(\omega x)$, we find $A=-3$ and $\omega=1$. Therefore, the amplitude is $|-3|=3$ and the period is $\frac{2\pi}{1}=2\pi$. Because the amplitude is 3, the graph of $y=-3\cos x$ will lie between -3 and 3 on the *y*-axis. Because the period is 2π, one cycle will begin at $x=0$ and end at $x=2\pi$. We divide the interval $[0,2\pi]$ into four subintervals, each of length $\frac{2\pi}{4}=\frac{\pi}{2}$ by finding the following values:

0, $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$, and 2π

These values of *x* determine the *x*-coordinates of the five key points on the graph. To obtain the *y*-coordinates of the five key points for $y=-3\cos x$, we multiply the *y*-coordinates of the five key points for $y=\cos x$ by $A=-3$. The five key points are

$(0,-3)$, $\left(\frac{\pi}{2},0\right)$, $(\pi,3)$, $\left(\frac{3\pi}{2},0\right)$, $(2\pi,-3)$

We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown below.



From the graph we can determine that the domain is all real numbers, $(-\infty,\infty)$ and the range is $[-3,3]$.

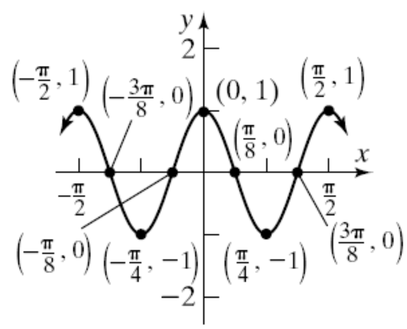
39. Comparing $y=\cos(4x)$ to $y=A\cos(\omega x)$, we find $A=1$ and $\omega=4$. Therefore, the amplitude is $|1|=1$ and the period is $\frac{2\pi}{4}=\frac{\pi}{2}$. Because the amplitude is 1, the graph of $y=\cos(4x)$ will lie between -1 and 1 on the *y*-axis. Because the period is $\frac{\pi}{2}$, one cycle will begin at $x=0$ and end at $x=\frac{\pi}{2}$. We divide the interval $\left[0,\frac{\pi}{2}\right]$ into four subintervals, each of length $\frac{\pi/2}{4}=\frac{\pi}{8}$ by finding the following values:

0, $\frac{\pi}{8}$, $\frac{\pi}{4}$, $\frac{3\pi}{8}$, and $\frac{\pi}{2}$

These values of *x* determine the *x*-coordinates of the five key points on the graph. The five key points are

$(0,1)$, $\left(\frac{\pi}{8},0\right)$, $\left(\frac{\pi}{4},-1\right)$, $\left(\frac{3\pi}{8},0\right)$, $\left(\frac{\pi}{2},1\right)$

We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown below.



From the graph we can determine that the domain is all real numbers, $(-\infty,\infty)$ and the range is $[-1,1]$.

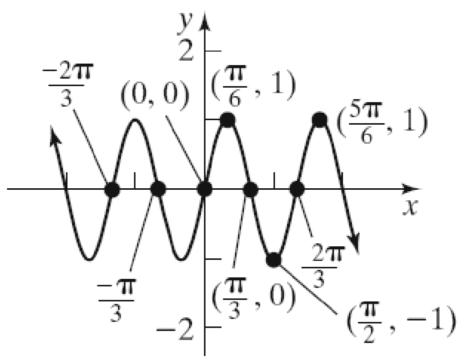
40. Comparing $y=\sin(3x)$ to $y=A\sin(\omega x)$, we find $A=1$ and $\omega=3$. Therefore, the amplitude is $|1|=1$ and the period is $\frac{2\pi}{3}$. Because the amplitude is 1, the graph of $y=\sin(3x)$ will lie between -1 and 1 on the *y*-axis. Because the period is $\frac{2\pi}{3}$, one cycle will begin at $x=0$ and end at $x=\frac{2\pi}{3}$. We divide the interval $\left[0,\frac{2\pi}{3}\right]$ into four subintervals, each of length $\frac{2\pi/3}{4}=\frac{\pi}{6}$ by finding the following values:

$0, \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}$, and $\frac{2\pi}{3}$

These values of x determine the x-coordinates of the five key points on the graph. The five key points are

$(0,0)$, $\left(\frac{\pi}{6},1\right)$, $\left(\frac{\pi}{3},0\right)$, $\left(\frac{\pi}{2},-1\right)$, $\left(\frac{2\pi}{3},0\right)$

We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown below.



From the graph we can determine that the domain is all real numbers, $(-\infty,\infty)$ and the range is $[-1,1]$.

41. Since sine is an odd function, we can plot the equivalent form $y=-\sin(2x)$.

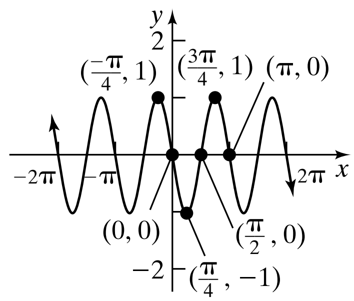
Comparing $y=-\sin(2x)$ to $y=A\sin(\omega x)$, we find $A=-1$ and $\omega=2$. Therefore, the amplitude is $|-1|=1$ and the period is $\frac{2\pi}{2}=\pi$. Because the amplitude is 1, the graph of $y=-\sin(2x)$ will lie between -1 and 1 on the y-axis. Because the period is π, one cycle will begin at $x=0$ and end at $x=\pi$. We divide the interval $[0,\pi]$ into four subintervals, each of length $\frac{\pi}{4}$ by finding the following values:

$0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$, and π

These values of x determine the x-coordinates of the five key points on the graph. To obtain the y-coordinates of the five key points for $y=-\sin(2x)$, we multiply the y-coordinates of the five key points for $y=\sin x$ by $A=-1$. The five key points are

$(0,0)$, $\left(\frac{\pi}{4},-1\right)$, $\left(\frac{\pi}{2},0\right)$, $\left(\frac{3\pi}{4},1\right)$, $(\pi,0)$

We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown below.



From the graph we can determine that the domain is all real numbers, $(-\infty,\infty)$ and the range is $[-1,1]$.

42. Since cosine is an even function, we can plot the equivalent form $y=\cos(2x)$.

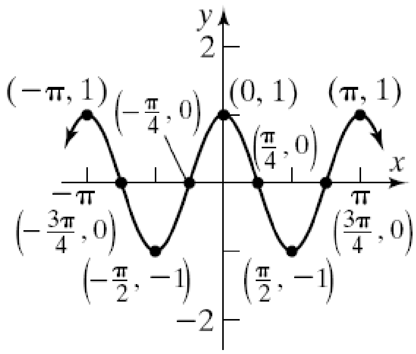
Comparing $y=\cos(2x)$ to $y=A\cos(\omega x)$, we find $A=1$ and $\omega=2$. Therefore, the amplitude is $|1|=1$ and the period is $\frac{2\pi}{2}=\pi$. Because the amplitude is 1, the graph of $y=\cos(2x)$ will lie between -1 and 1 on the y-axis. Because the period is π, one cycle will begin at $x=0$ and end at $x=\pi$. We divide the interval $[0,\pi]$ into four subintervals, each of length $\frac{\pi}{4}$ by finding the following values:

$0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$, and π

These values of x determine the x-coordinates of the five key points on the graph. To obtain the y-coordinates of the five key points for $y=\cos(2x)$, we multiply the y-coordinates of the five key points for $y=\cos x$ by $A=1$. The five key points are

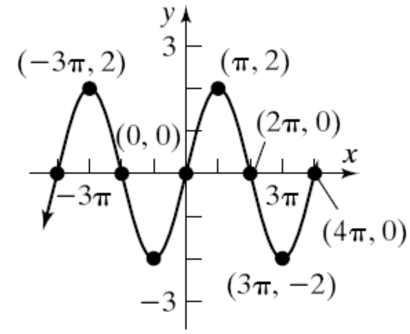
$(0,1)$, $\left(\frac{\pi}{4},0\right)$, $\left(\frac{\pi}{2},-1\right)$, $\left(\frac{3\pi}{4},0\right)$, $(\pi,1)$

We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown below.



From the graph we can determine that the domain is all real numbers, $(-\infty,\infty)$ and the range is $[-1,1]$.

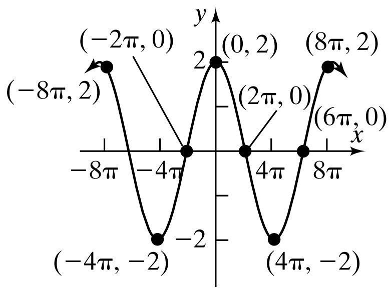
43. Comparing $y=2\sin\left(\frac{1}{2}x\right)$ to $y=A\sin(\omega x)$, we find $A=2$ and $\omega=\frac{1}{2}$. Therefore, the amplitude is $|2|=2$ and the period is $\frac{2\pi}{1/2}=4\pi$. Because the amplitude is 2, the graph of $y=2\sin\left(\frac{1}{2}x\right)$ will lie between -2 and 2 on the *y*-axis. Because the period is 4π, one cycle will begin at $x=0$ and end at $x=4\pi$. We divide the interval $[0,4\pi]$ into four subintervals, each of length $\frac{4\pi}{4}=\pi$ by finding the following values:
0, π, 2π, 3π, and 4π
These values of *x* determine the *x*-coordinates of the five key points on the graph. To obtain the *y*-coordinates of the five key points for $y=2\sin\left(\frac{1}{2}x\right)$, we multiply the *y*-coordinates of the five key points for $y=\sin x$ by $A=2$. The five key points are
$(0,0)$, $(\pi,2)$, $(2\pi,0)$, $(3\pi,-2)$, $(4\pi,0)$
We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown below.



From the graph we can determine that the domain is all real numbers, $(-\infty,\infty)$ and the range is $[-2,2]$.

44. Comparing $y=2\cos\left(\frac{1}{4}x\right)$ to $y=A\cos(\omega x)$, we find $A=2$ and $\omega=\frac{1}{4}$. Therefore, the amplitude is $|2|=2$ and the period is $\frac{2\pi}{1/4}=8\pi$. Because the amplitude is 2, the graph of $y=2\cos\left(\frac{1}{4}x\right)$ will lie between -2 and 2 on the *y*-axis. Because the period is 8π, one cycle will begin at $x=0$ and end at $x=8\pi$. We divide the interval $[0,8\pi]$ into four subintervals, each of length $\frac{8\pi}{4}=2\pi$ by finding the following values:
0, 2π, 4π, 6π, and 8π
These values of *x* determine the *x*-coordinates of the five key points on the graph. To obtain the *y*-coordinates of the five key points for $y=2\cos\left(\frac{1}{4}x\right)$, we multiply the *y*-coordinates of the five key points for $y=\cos x$ by $A=2$. The five key points are
$(0,2)$, $(2\pi,0)$, $(4\pi,-2)$, $(6\pi,0)$, $(8\pi,2)$
We plot these five points and fill in the graph of the curve. We then extend the graph in either

direction to obtain the graph shown below.



From the graph we can determine that the domain is all real numbers, $(-\infty,\infty)$ and the range is $[-2,2]$.

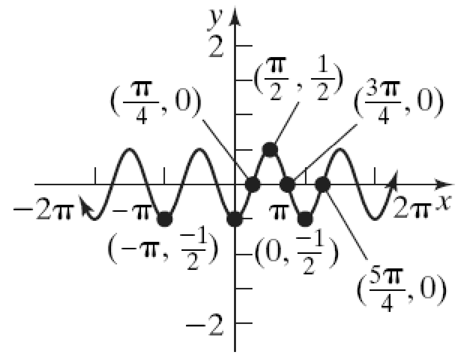
45. Comparing $y=-\frac{1}{2}\cos(2x)$ to $y=A\cos(\omega x)$, we find $A=-\frac{1}{2}$ and $\omega=2$. Therefore, the amplitude is $\left|-\frac{1}{2}\right|=\frac{1}{2}$ and the period is $\frac{2\pi}{2}=\pi$. Because the amplitude is $\frac{1}{2}$, the graph of $y=-\frac{1}{2}\cos(2x)$ will lie between $-\frac{1}{2}$ and $\frac{1}{2}$ on the *y*-axis. Because the period is π, one cycle will begin at $x=0$ and end at $x=\pi$. We divide the interval $[0,\pi]$ into four subintervals, each of length $\frac{\pi}{4}$ by finding the following values:

0, $\frac{\pi}{4}$, $\frac{\pi}{2}$, $\frac{3\pi}{4}$, and π

These values of *x* determine the *x*-coordinates of the five key points on the graph. To obtain the *y*-coordinates of the five key points for $y=-\frac{1}{2}\cos(2x)$, we multiply the *y*-coordinates of the five key points for $y=\cos x$ by $A=-\frac{1}{2}$.The five key points are

$\left(0,-\frac{1}{2}\right)$, $\left(\frac{\pi}{4},0\right)$, $\left(\frac{\pi}{2},\frac{1}{2}\right)$, $\left(\frac{3\pi}{4},0\right)$, $\left(\pi,-\frac{1}{2}\right)$

We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown below.



From the graph we can determine that the domain is all real numbers, $(-\infty,\infty)$ and the range is $\left[-\frac{1}{2},\frac{1}{2}\right]$.

46. Comparing $y=-4\sin\left(\frac{1}{8}x\right)$ to $y=A\sin(\omega x)$, we find $A=-4$ and $\omega=\frac{1}{8}$. Therefore, the amplitude is $|-4|=4$ and the period is $\frac{2\pi}{1/8}=16\pi$. Because the amplitude is 4, the graph of $y=-4\sin\left(\frac{1}{8}x\right)$ will lie between -4 and 4 on the *y*-axis. Because the period is 16π, one cycle will begin at $x=0$ and end at $x=16\pi$. We divide the interval $[0,16\pi]$ into four subintervals, each of length $\frac{16\pi}{4}=4\pi$ by finding the following values:

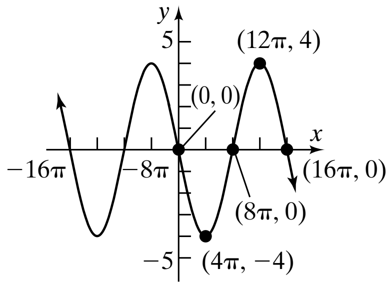
0, 4π, 8π, 12π, and 16π

These values of *x* determine the *x*-coordinates of the five key points on the graph. To obtain the *y*-coordinates of the five key points for $y=-4\sin\left(\frac{1}{8}x\right)$, we multiply the *y*-coordinates of the five key points for $y=\sin x$ by $A=-4$. The five key points are

$(0,0)$, $(4\pi,-4)$, $(8\pi,0)$, $(12\pi,4)$, $(16\pi,0)$

We plot these five points and fill in the graph of the curve. We then extend the graph in either

direction to obtain the graph shown below.



From the graph we can determine that the domain is all real numbers, $(-\infty,\infty)$ and the range is $[-4,4]$.

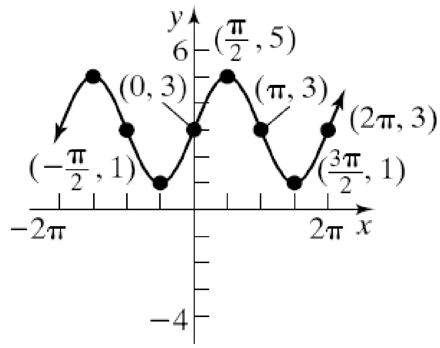
47. We begin by considering $y = 2\sin x$. Comparing $y = 2\sin x$ to $y = A\sin(\omega x)$, we find $A = 2$ and $\omega = 1$. Therefore, the amplitude is $|2| = 2$ and the period is $\frac{2\pi}{1} = 2\pi$. Because the amplitude is 2, the graph of $y = 2\sin x$ will lie between -2 and 2 on the *y*-axis. Because the period is 2π, one cycle will begin at $x = 0$ and end at $x = 2\pi$. We divide the interval $[0, 2\pi]$ into four subintervals, each of length $\frac{2\pi}{4} = \frac{\pi}{2}$ by finding the following values:

0, $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$, and 2π

These values of *x* determine the *x*-coordinates of the five key points on the graph. To obtain the *y*-coordinates of the five key points for $y = 2\sin x + 3$, we multiply the *y*-coordinates of the five key points for $y = \sin x$ by $A = 2$ and then add 3 units. Thus, the graph of $y = 2\sin x + 3$ will lie between 1 and 5 on the *y*-axis. The five key points are

$(0,3)$, $\left(\frac{\pi}{2},5\right)$, $(\pi,3)$, $\left(\frac{3\pi}{2},1\right)$, $(2\pi,3)$

We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown below.



From the graph we can determine that the domain is all real numbers, $(-\infty,\infty)$ and the range is $[1,5]$.

48. We begin by considering $y = 3\cos x$. Comparing $y = 3\cos x$ to $y = A\cos(\omega x)$, we find $A = 3$ and $\omega = 1$. Therefore, the amplitude is $|3| = 3$ and the period is $\frac{2\pi}{1} = 2\pi$. Because the amplitude is 3, the graph of $y = 3\cos x$ will lie between -3 and 3 on the *y*-axis. Because the period is 2π, one cycle will begin at $x = 0$ and end at $x = 2\pi$. We divide the interval $[0, 2\pi]$ into four subintervals, each of length $\frac{2\pi}{4} = \frac{\pi}{2}$ by finding the following values:

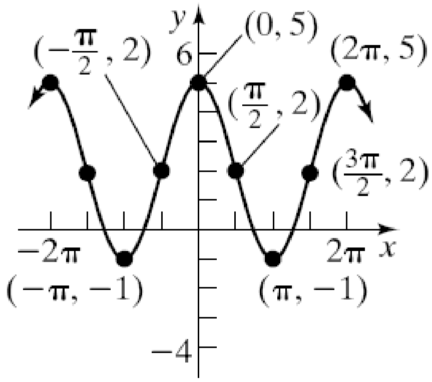
0, $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$, and 2π

These values of *x* determine the *x*-coordinates of the five key points on the graph. To obtain the *y*-coordinates of the five key points for $y = 3\cos x + 2$, we multiply the *y*-coordinates of the five key points for $y = \cos x$ by $A = 3$ and then add 2 units. Thus, the graph of $y = 3\cos x + 2$ will lie between -1 and 5 on the *y*-axis. The five key points are

$(0,5)$, $\left(\frac{\pi}{2},2\right)$, $(\pi,-1)$, $\left(\frac{3\pi}{2},2\right)$, $(2\pi,5)$

We plot these five points and fill in the graph of the curve. We then extend the graph in either

direction to obtain the graph shown below.



From the graph we can determine that the domain is all real numbers, $(-\infty,\infty)$ and the range is $[-1,5]$.

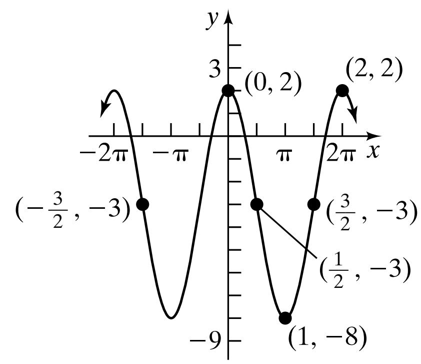
49. We begin by considering $y=5\cos(\pi x)$. Comparing $y=5\cos(\pi x)$ to $y=A\cos(\omega x)$, we find $A=5$ and $\omega=\pi$. Therefore, the amplitude is $|5|=5$ and the period is $\frac{2\pi}{\pi}=2$. Because the amplitude is 5, the graph of $y=5\cos(\pi x)$ will lie between -5 and 5 on the *y*-axis. Because the period is 2, one cycle will begin at $x=0$ and end at $x=2$. We divide the interval $[0,2]$ into four subintervals, each of length $\frac{2}{4}=\frac{1}{2}$ by finding the following values:

$0,\ \frac{1}{2},\ 1,\ \frac{3}{2}$, and 2

These values of *x* determine the *x*-coordinates of the five key points on the graph. To obtain the *y*-coordinates of the five key points for $y=5\cos(\pi x)-3$, we multiply the *y*-coordinates of the five key points for $y=\cos x$ by $A=5$ and then subtract 3 units. Thus, the graph of $y=5\cos(\pi x)-3$ will lie between -8 and 2 on the *y*-axis. The five key points are

$(0,2)$, $\left(\frac{1}{2},-3\right)$, $(1,-8)$, $\left(\frac{3}{2},-3\right)$, $(2,2)$

We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown below.



From the graph we can determine that the domain is all real numbers, $(-\infty,\infty)$ and the range is $[-8,2]$.

50. We begin by considering $y=4\sin\left(\frac{\pi}{2}x\right)$.

Comparing $y=4\sin\left(\frac{\pi}{2}x\right)$ to $y=A\sin(\omega x)$, we find $A=4$ and $\omega=\frac{\pi}{2}$. Therefore, the amplitude is $|4|=4$ and the period is $\frac{2\pi}{\pi/2}=4$. Because the amplitude is 4, the graph of $y=4\sin\left(\frac{\pi}{2}x\right)$ will lie between -4 and 4 on the *y*-axis. Because the period is 4, one cycle will begin at $x=0$ and end at $x=4$. We divide the interval $[0,4]$ into four subintervals, each of length $\frac{4}{4}=1$ by finding the following values:

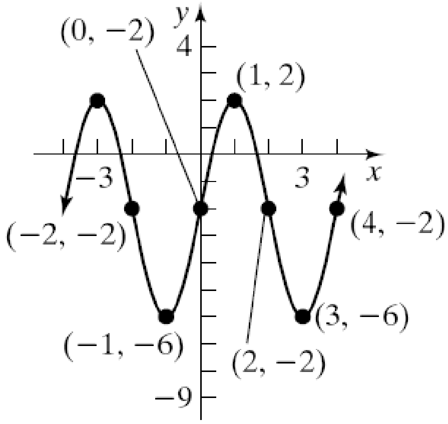
0, 1, 2, 3, and 4

These values of *x* determine the *x*-coordinates of the five key points on the graph. To obtain the *y*-coordinates of the five key points for $y=4\sin\left(\frac{\pi}{2}x\right)-2$, we multiply the *y*-coordinates of the five key points for $y=\sin x$ by $A=4$ and then subtract 2 units. Thus, the graph of $y=4\sin\left(\frac{\pi}{2}x\right)-2$ will lie between -6 and 2 on the *y*-axis. The five key points are

$(0,-2)$, $(1,2)$, $(2,-2)$, $(3,-6)$, $(4,-2)$

We plot these five points and fill in the graph of the curve. We then extend the graph in either

direction to obtain the graph shown below.



From the graph we can determine that the domain is all real numbers, $(-\infty, \infty)$ and the range is $[-6, 2]$.

51. We begin by considering $y = -6\sin\left(\frac{\pi}{3}x\right)$.

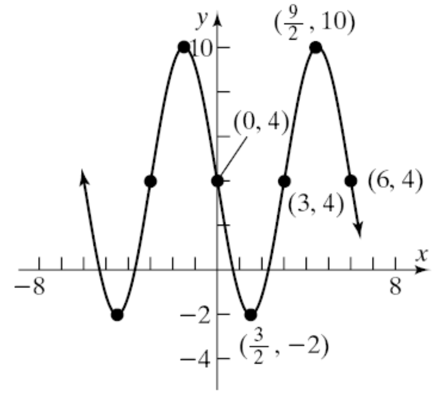
Comparing $y = -6\sin\left(\frac{\pi}{3}x\right)$ to $y = A\sin(\omega x)$, we find $A = -6$ and $\omega = \frac{\pi}{3}$. Therefore, the amplitude is $|-6| = 6$ and the period is $\frac{2\pi}{\pi/3} = 6$. Because the amplitude is 6, the graph of $y = 6\sin\left(\frac{\pi}{3}x\right)$ will lie between -6 and 6 on the y-axis. Because the period is 6, one cycle will begin at $x = 0$ and end at $x = 6$. We divide the interval $[0, 6]$ into four subintervals, each of length $\frac{6}{4} = \frac{3}{2}$ by finding the following values:

0, $\frac{3}{2}$, 3, $\frac{9}{2}$, and 6

These values of x determine the x-coordinates of the five key points on the graph. To obtain the y-coordinates of the five key points for $y = -6\sin\left(\frac{\pi}{3}x\right) + 4$, we multiply the y-coordinates of the five key points for $y = \sin x$ by $A = -6$ and then add 4 units. Thus, the graph of $y = -6\sin\left(\frac{\pi}{3}x\right) + 4$ will lie between -2 and 10 on the y-axis. The five key points are

$(0, 4)$, $\left(\frac{3}{2}, -2\right)$, $(3, 4)$, $\left(\frac{9}{2}, 10\right)$, $(6, 4)$

We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown below.



From the graph we can determine that the domain is all real numbers, $(-\infty, \infty)$ and the range is $[-2, 10]$.

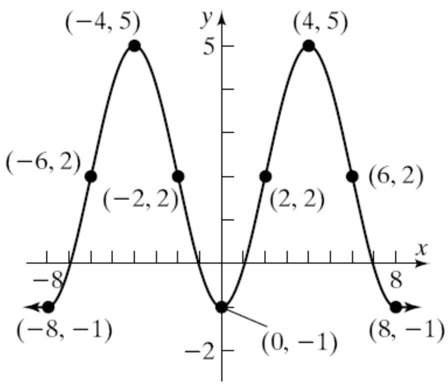
52. We begin by considering $y = -3\cos\left(\frac{\pi}{4}x\right)$.

Comparing $y = -3\cos\left(\frac{\pi}{4}x\right)$ to $y = A\cos(\omega x)$, we find $A = -3$ and $\omega = \frac{\pi}{4}$. Therefore, the amplitude is $|-3| = 3$ and the period is $\frac{2\pi}{\pi/4} = 8$. Because the amplitude is 3, the graph of $y = -3\cos\left(\frac{\pi}{4}x\right)$ will lie between -3 and 3 on the y-axis. Because the period is 8, one cycle will begin at $x = 0$ and end at $x = 8$. We divide the interval $[0, 8]$ into four subintervals, each of length $\frac{8}{4} = 2$ by finding the following values:

0, 2, 4, 6, and 8

These values of x determine the x-coordinates of the five key points on the graph. To obtain the y-coordinates of the five key points for $y = -3\cos\left(\frac{\pi}{4}x\right) + 2$, we multiply the y-coordinates of the five key points for $y = \cos x$ by $A = -3$ and then add 2 units. Thus, the graph

of $y=-3\cos\left(\frac{\pi}{4}x\right)+2$ will lie between -1 and 5 on the y-axis. The five key points are
$(0,-1)$, $(2,2)$, $(4,5)$, $(6,2)$, $(8,-1)$
We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown below.



From the graph we can determine that the domain is all real numbers, $(-\infty,\infty)$ and the range is $[-1,5]$.

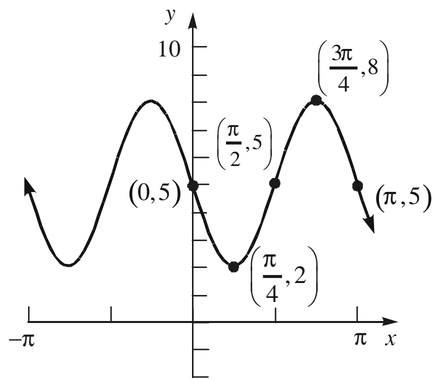
53. $y=5-3\sin(2x)=-3\sin(2x)+5$

We begin by considering $y=-3\sin(2x)$.

Comparing $y=-3\sin(2x)$ to $y=A\sin(\omega x)$, we find $A=-3$ and $\omega=2$. Therefore, the amplitude is $|-3|=3$ and the period is $\frac{2\pi}{2}=\pi$.
Because the amplitude is 3, the graph of $y=-3\sin(2x)$ will lie between -3 and 3 on the y-axis. Because the period is π, one cycle will begin at $x=0$ and end at $x=\pi$. We divide the interval $[0,\pi]$ into four subintervals, each of length $\frac{\pi}{4}$ by finding the following values:

0, $\frac{\pi}{4}$, $\frac{\pi}{2}$, $\frac{3\pi}{4}$, and π

These values of x determine the x-coordinates of the five key points on the graph. To obtain the y-coordinates of the five key points for $y=-3\sin(2x)+5$, we multiply the y-coordinates of the five key points for $y=\sin x$ by $A=-3$ and then add 5 units. Thus, the graph of $y=-3\sin(2x)+5$ will lie between 2 and 8 on the y-axis. The five key points are
$(0,5)$, $\left(\frac{\pi}{4},2\right)$, $\left(\frac{\pi}{2},5\right)$, $\left(\frac{3\pi}{4},8\right)$, $(\pi,5)$
We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown below.



From the graph we can determine that the domain is all real numbers, $(-\infty,\infty)$ and the range is $[2,8]$.

54. $y=2-4\cos(3x)=-4\cos(3x)+2$

We begin by considering $y=-4\cos(3x)$.

Comparing $y=-4\cos(3x)$ to $y=A\cos(\omega x)$, we find $A=-4$ and $\omega=3$. Therefore, the amplitude is $|-4|=4$ and the period is $\frac{2\pi}{3}$.
Because the amplitude is 4, the graph of $y=-4\cos(3x)$ will lie between -4 and 4 on the y-axis. Because the period is $\frac{2\pi}{3}$, one cycle will begin at $x=0$ and end at $x=\frac{2\pi}{3}$. We divide the interval $\left[0,\frac{2\pi}{3}\right]$ into four subintervals, each of length $\frac{2\pi/3}{4}=\frac{\pi}{6}$ by finding the following values:

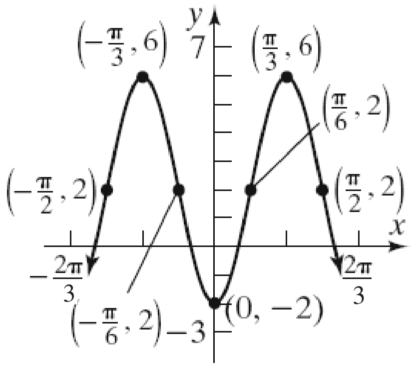
0, $\frac{\pi}{6}$, $\frac{\pi}{3}$, $\frac{\pi}{2}$, and $\frac{2\pi}{3}$

These values of x determine the x-coordinates of the five key points on the graph. To obtain the y-coordinates of the five key points for $y=-4\cos(3x)+2$, we multiply the y-

coordinates of the five key points for $y=\cos x$ by $A=-4$ and then adding 2 units. Thus, the graph of $y=-4\cos(3x)+2$ will lie between -2 and 6 on the y-axis. The five key points are

$(0,-2)$, $\left(\frac{\pi}{6},2\right)$, $\left(\frac{\pi}{3},6\right)$, $\left(\frac{\pi}{2},2\right)$, $\left(\frac{2\pi}{3},-2\right)$

We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown below.



From the graph we can determine that the domain is all real numbers, $(-\infty,\infty)$ and the range is $[-2,6]$.

55. Since sine is an odd function, we can plot the equivalent form $y=-\frac{5}{3}\sin\left(\frac{2\pi}{3}x\right)$.

Comparing $y=-\frac{5}{3}\sin\left(\frac{2\pi}{3}x\right)$ to

$y=A\sin(\omega x)$, we find $A=-\frac{5}{3}$ and $\omega=\frac{2\pi}{3}$.

Therefore, the amplitude is $\left|-\frac{5}{3}\right|=\frac{5}{3}$ and the

period is $\frac{2\pi}{2\pi/3}=3$. Because the amplitude is

$\frac{5}{3}$, the graph of $y=-\frac{5}{3}\sin\left(\frac{2\pi}{3}x\right)$ will lie

between $-\frac{5}{3}$ and $\frac{5}{3}$ on the y-axis. Because the period is 3, one cycle will begin at $x=0$ and end at $x=3$. We divide the interval $[0,3]$ into four subintervals, each of length $\frac{3}{4}$ by finding the following values:

0, $\frac{3}{4}$, $\frac{3}{2}$, $\frac{9}{4}$, and 3

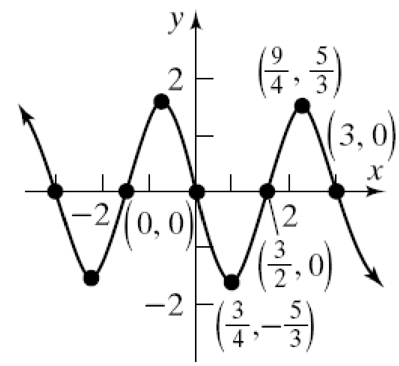
These values of x determine the x-coordinates of the five key points on the graph. To obtain the y-coordinates of the five key points for

$y=-\frac{5}{3}\sin\left(\frac{2\pi}{3}x\right)$, we multiply the y-coordinates of the five key points for $y=\sin x$

by $A=-\frac{5}{3}$. The five key points are

$(0,0)$, $\left(\frac{3}{4},-\frac{5}{3}\right)$, $\left(\frac{3}{2},0\right)$, $\left(\frac{9}{4},\frac{5}{3}\right)$, $(3,0)$

We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown below.



From the graph we can determine that the domain is all real numbers, $(-\infty,\infty)$ and the range is $\left[-\frac{5}{3},\frac{5}{3}\right]$.

56. Since cosine is an even function, we consider the equivalent form $y=\frac{9}{5}\cos\left(\frac{3\pi}{2}x\right)$. Comparing

$y=\frac{9}{5}\cos\left(\frac{3\pi}{2}x\right)$ to $y=A\cos(\omega x)$, we find

$A=\frac{9}{5}$ and $\omega=\frac{3\pi}{2}$. Therefore, the amplitude is

$\left|\frac{9}{5}\right|=\frac{9}{5}$ and the period is $\frac{2\pi}{3\pi/2}=\frac{4}{3}$. Because

the amplitude is $\frac{9}{5}$, the graph of

$y=\frac{9}{5}\cos\left(\frac{3\pi}{2}x\right)$ will lie between $-\frac{9}{5}$ and $\frac{9}{5}$

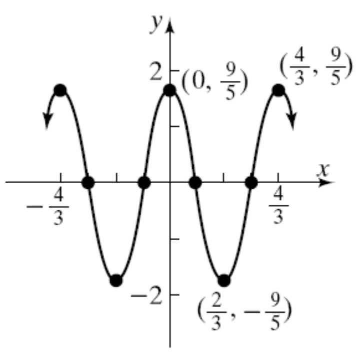
on the y-axis. Because the period is $\frac{4}{3}$, one cycle will begin at $x=0$ and end at $x=\frac{4}{3}$. We divide the interval $\left[0,\frac{4}{3}\right]$ into four subintervals, each of length $\frac{4/3}{4}=\frac{1}{3}$ by finding the following values:

0, $\frac{1}{3}$, $\frac{2}{3}$, 1, and $\frac{4}{3}$

These values of x determine the x-coordinates of the five key points on the graph. To obtain the y-coordinates of the five key points for $y=\frac{9}{5}\cos\left(\frac{3\pi}{2}x\right)$, we multiply the y-coordinates of the five key points for $y=\cos x$ by $A=\frac{9}{5}$. Thus, the graph of $y=\frac{9}{5}\cos\left(-\frac{3\pi}{2}x\right)$ will lie between $-\frac{9}{5}$ and $\frac{9}{5}$ on the y-axis. The five key points are

$\left(0,\frac{9}{5}\right)$, $\left(\frac{1}{3},0\right)$, $\left(\frac{2}{3},-\frac{9}{5}\right)$, $(1,0)$, $\left(\frac{4}{3},\frac{9}{5}\right)$

We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown below.



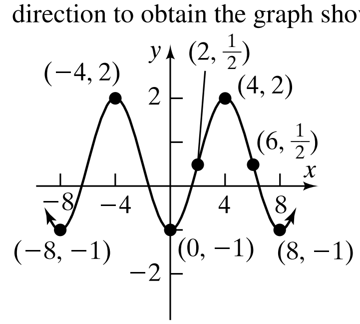
From the graph we can determine that the domain is all real numbers, $(-\infty,\infty)$ and the range is $\left[-\frac{9}{5},\frac{9}{5}\right]$.

57. We begin by considering $y=-\frac{3}{2}\cos\left(\frac{\pi}{4}x\right)$. Comparing $y=-\frac{3}{2}\cos\left(\frac{\pi}{4}x\right)$ to $y=A\cos(\omega x)$, we find $A=-\frac{3}{2}$ and $\omega=\frac{\pi}{4}$. Therefore, the amplitude is $\left|-\frac{3}{2}\right|=\frac{3}{2}$ and the period is $\frac{2\pi}{\pi/4}=8$. Because the amplitude is $\frac{3}{2}$, the graph of $y=-\frac{3}{2}\cos\left(\frac{\pi}{4}x\right)$ will lie between $-\frac{3}{2}$ and $\frac{3}{2}$ on the y-axis. Because the period is 8, one cycle will begin at $x=0$ and end at $x=8$. We divide the interval $[0,8]$ into four subintervals, each of length $\frac{8}{4}=2$ by finding the following values: 0, 2, 4, 6, and 8

These values of x determine the x-coordinates of the five key points on the graph. To obtain the y-coordinates of the five key points for $y=-\frac{3}{2}\cos\left(\frac{\pi}{4}x\right)+\frac{1}{2}$, we multiply the y-coordinates of the five key points for $y=\cos x$ by $A=-\frac{3}{2}$ and then add $\frac{1}{2}$ unit. Thus, the graph of $y=-\frac{3}{2}\cos\left(\frac{\pi}{4}x\right)+\frac{1}{2}$ will lie between -1 and 2 on the y-axis. The five key points are

$(0,-1)$, $\left(2,\frac{1}{2}\right)$, $(4,2)$, $\left(6,\frac{1}{2}\right)$, $(8,-1)$

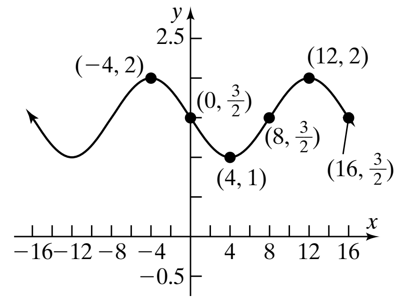
We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown below.



From the graph we can determine that the domain is all real numbers, $(-\infty,\infty)$ and the range is $[-1,2]$.

58. We begin by considering $y=-\frac{1}{2}\sin\left(\frac{\pi}{8}x\right)$.

Comparing $y=-\frac{1}{2}\sin\left(\frac{\pi}{8}x\right)$ to $y=A\sin(\omega x)$, we find $A=-\frac{1}{2}$ and $\omega=\frac{\pi}{8}$. Therefore, the amplitude is $\left|-\frac{1}{2}\right|=\frac{1}{2}$ and the period is $\frac{2\pi}{\pi/8}=16$. Because the amplitude is $\frac{1}{2}$, the graph of $y=-\frac{1}{2}\sin\left(\frac{\pi}{8}x\right)$ will lie between $-\frac{1}{2}$ and $\frac{1}{2}$ on the y-axis. Because the period is 16, one cycle will begin at $x=0$ and end at $x=16$. We divide the interval $[0,16]$ into four subintervals, each of length $\frac{16}{4}=4$ by finding the following values:
0, 4, 8, 12, and 16
These values of x determine the x-coordinates of the five key points on the graph. To obtain the y-coordinates of the five key points for $y=-\frac{1}{2}\sin\left(\frac{\pi}{8}x\right)+\frac{3}{2}$, we multiply the y-coordinates of the five key points for $y=\sin x$ by $A=-\frac{1}{2}$ and then add $\frac{3}{2}$ units. Thus, the graph of $y=-\frac{1}{2}\sin\left(\frac{\pi}{8}x\right)+\frac{3}{2}$ will lie between 1 and 2 on the y-axis. The five key points are
$\left(0,\frac{3}{2}\right)$, $(4,1)$, $\left(8,\frac{3}{2}\right)$, $(12,2)$, $\left(16,\frac{3}{2}\right)$
We plot these five points and fill in the graph of the curve. We then extend the graph in either direction to obtain the graph shown below.



From the graph we can determine that the domain is all real numbers, $(-\infty,\infty)$ and the range is $[1,2]$.

59. $|A|=3; T=\pi; \omega=\frac{2\pi}{T}=\frac{2\pi}{\pi}=2$

$y=\pm 3\sin(2x)$

60. $|A|=2; T=4\pi; \omega=\frac{2\pi}{T}=\frac{2\pi}{4\pi}=\frac{1}{2}$

$y=\pm 2\sin\left(\frac{1}{2}x\right)$

61. $|A|=3; T=2; \omega=\frac{2\pi}{T}=\frac{2\pi}{2}=\pi$

$y=\pm 3\sin(\pi x)$

62. $|A|=4;\quad T=1; \omega=\frac{2\pi}{T}=\frac{2\pi}{1}=2\pi$

$y=\pm 4\sin(2\pi x)$

63. The graph is a cosine graph with amplitude 5 and period 8.

Find ω: $\quad 8=\frac{2\pi}{\omega}$

$8\omega=2\pi$

$\omega=\frac{2\pi}{8}=\frac{\pi}{4}$

The equation is: $\quad y=5\cos\left(\frac{\pi}{4}x\right)$.

64. The graph is a sine graph with amplitude 4 and period 8π.

Find ω: $8\pi = \dfrac{2\pi}{\omega}$

$8\pi\omega = 2\pi$

$\omega = \dfrac{2\pi}{8\pi} = \dfrac{1}{4}$

The equation is: $y = 4\sin\left(\dfrac{1}{4}x\right)$.

65. The graph is a reflected cosine graph with amplitude 3 and period 4π.

Find ω: $4\pi = \dfrac{2\pi}{\omega}$

$4\pi\omega = 2\pi$

$\omega = \dfrac{2\pi}{4\pi} = \dfrac{1}{2}$

The equation is: $y = -3\cos\left(\dfrac{1}{2}x\right)$.

66. The graph is a reflected sine graph with amplitude 2 and period 4.

Find ω: $4 = \dfrac{2\pi}{\omega}$

$4\omega = 2\pi$

$\omega = \dfrac{2\pi}{4} = \dfrac{\pi}{2}$

The equation is: $y = -2\sin\left(\dfrac{\pi}{2}x\right)$.

67. The graph is a sine graph with amplitude $\dfrac{3}{4}$ and period 1.

Find ω: $1 = \dfrac{2\pi}{\omega}$

$\omega = 2\pi$

The equation is: $y = \dfrac{3}{4}\sin(2\pi x)$.

68. The graph is a reflected cosine graph with amplitude $\dfrac{5}{2}$ and period 2.

Find ω: $2 = \dfrac{2\pi}{\omega}$

$2\omega = 2\pi$

$\omega = \dfrac{2\pi}{2} = \pi$

The equation is: $y = -\dfrac{5}{2}\cos(\pi x)$.

69. The graph is a reflected sine graph with amplitude 1 and period $\dfrac{4\pi}{3}$.

Find ω: $\dfrac{4\pi}{3} = \dfrac{2\pi}{\omega}$

$4\pi\omega = 6\pi$

$\omega = \dfrac{6\pi}{4\pi} = \dfrac{3}{2}$

The equation is: $y = -\sin\left(\dfrac{3}{2}x\right)$.

70. The graph is a reflected cosine graph with amplitude π and period 2π.

Find ω: $2\pi = \dfrac{2\pi}{\omega}$

$2\pi\omega = 2\pi$

$\omega = \dfrac{2\pi}{2\pi} = 1$

The equation is: $y = -\pi\cos x$.

71. The graph is a reflected cosine graph, shifted up 1 unit, with amplitude 1 and period $\dfrac{3}{2}$.

Find ω: $\dfrac{3}{2} = \dfrac{2\pi}{\omega}$

$3\omega = 4\pi$

$\omega = \dfrac{4\pi}{3}$

The equation is: $y = -\cos\left(\dfrac{4\pi}{3}x\right) + 1$.

72. The graph is a reflected sine graph, shifted down 1 unit, with amplitude $\dfrac{1}{2}$ and period $\dfrac{4\pi}{3}$.

Find ω: $\dfrac{4\pi}{3} = \dfrac{2\pi}{\omega}$

$4\pi\omega = 6\pi$

$\omega = \dfrac{6\pi}{4\pi} = \dfrac{3}{2}$

The equation is: $y = -\dfrac{1}{2}\sin\left(\dfrac{3}{2}x\right) - 1$.

73. The graph is a sine graph with amplitude 3 and period 4.

Find ω: $4=\dfrac{2\pi}{\omega}$

$4\omega=2\pi$

$\omega=\dfrac{2\pi}{4}=\dfrac{\pi}{2}$

The equation is: $y=3\sin\left(\dfrac{\pi}{2}x\right)$.

74. The graph is a reflected cosine graph with amplitude 2 and period 2.

Find ω: $2=\dfrac{2\pi}{\omega}$

$2\omega=2\pi$

$\omega=\dfrac{2\pi}{2}=\pi$

The equation is: $y=-2\cos(\pi x)$.

75. The graph is a reflected cosine graph with amplitude 4 and period $\dfrac{2\pi}{3}$.

Find ω: $\dfrac{2\pi}{3}=\dfrac{2\pi}{\omega}$

$2\pi\omega=6\pi$

$\omega=\dfrac{6\pi}{2\pi}=3$

The equation is: $y=-4\cos(3x)$.

76. The graph is a sine graph with amplitude 4 and period π.

Find ω: $\pi=\dfrac{2\pi}{\omega}$

$\pi\omega=2\pi$

$\omega=\dfrac{2\pi}{\pi}=2$

The equation is: $y=4\sin(2x)$.

77. $$\frac{f(\pi/2)-f(0)}{\pi/2-0}=\frac{\sin(\pi/2)-\sin(0)}{\pi/2}=\frac{1-0}{\pi/2}=\frac{2}{\pi}$$

The average rate of change is $\dfrac{2}{\pi}$.

78. $$\frac{f(\pi/2)-f(0)}{\pi/2-0}=\frac{\cos(\pi/2)-\cos(0)}{\pi/2}=\frac{0-1}{\pi/2}=-\frac{2}{\pi}$$

The average rate of change is $-\dfrac{2}{\pi}$.

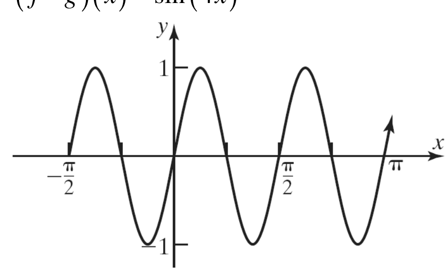
79. $$\frac{f(\pi/2)-f(0)}{\pi/2-0}=\frac{\sin\left(\frac{1}{2}\cdot\frac{\pi}{2}\right)-\sin\left(\frac{1}{2}\cdot 0\right)}{\pi/2}=\frac{\sin(\pi/4)-\sin(0)}{\pi/2}=\frac{\frac{\sqrt{2}}{2}}{\pi/2}=\frac{\sqrt{2}}{2}\cdot\frac{2}{\pi}=\frac{\sqrt{2}}{\pi}$$

The average rate of change is $\dfrac{\sqrt{2}}{\pi}$.

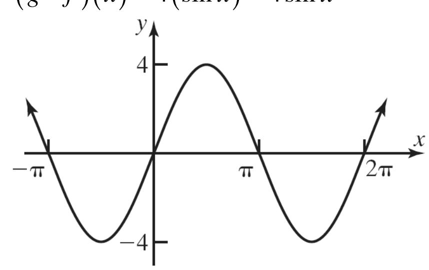
80. $$\frac{f(\pi/2)-f(0)}{\pi/2-0}=\frac{\cos\left(2\cdot\frac{\pi}{2}\right)-\cos(2\cdot 0)}{\pi/2}=\frac{\cos(\pi)-\cos(0)}{\pi/2}=\frac{-1-1}{\pi/2}=-2\cdot\frac{2}{\pi}=-\frac{4}{\pi}$$

The average rate of change is $-\dfrac{4}{\pi}$.

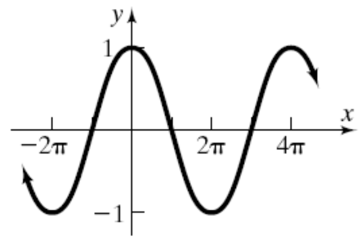
81. $(f\circ g)(x)=\sin(4x)$



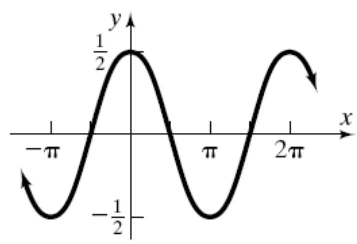
$(g\circ f)(x)=4(\sin x)=4\sin x$



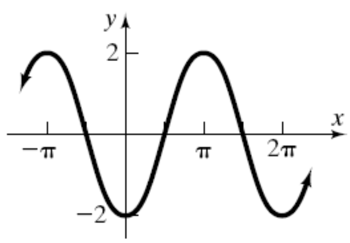
82. $(f \circ g)(x) = \cos\left(\frac{1}{2}x\right)$



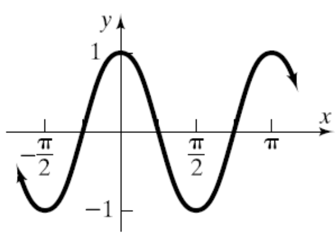
$(g \circ f)(x) = \frac{1}{2}(\cos x) = \frac{1}{2}\cos x$



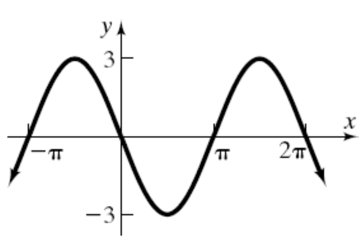
83. $(f \circ g)(x) = -2(\cos x) = -2\cos x$



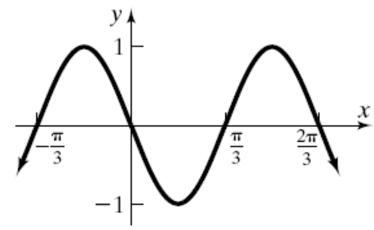
$(g \circ f)(x) = \cos(-2x)$



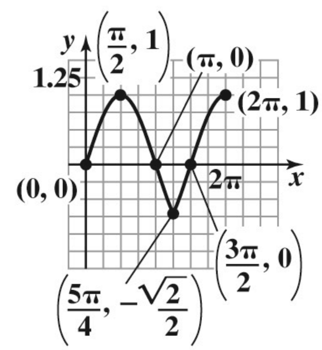
84. $(f \circ g)(x) = -3(\sin x) = -3\sin x$



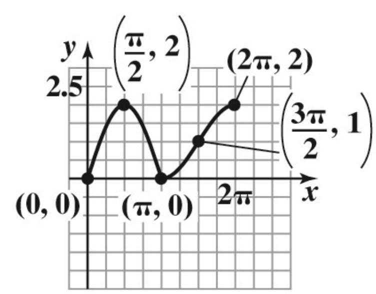
$(g \circ f)(x) = \sin(-3x)$



85.



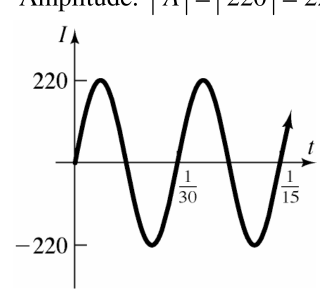
86.



87. $I(t) = 220\sin(60\pi t),\ t \geq 0$

Period: $T = \frac{2\pi}{\omega} = \frac{2\pi}{60\pi} = \frac{1}{30}$ second

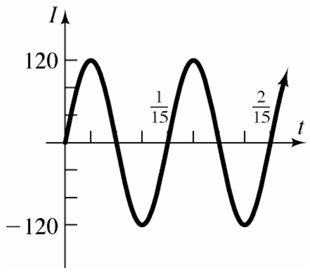
Amplitude: $|A| = |220| = 220$ amperes



88. $I(t) = 120\sin(30\pi t),\ t \ge 0$

Period: $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{30\pi} = \dfrac{1}{15}$ second

Amplitude: $|A| = |120| = 120$ amperes

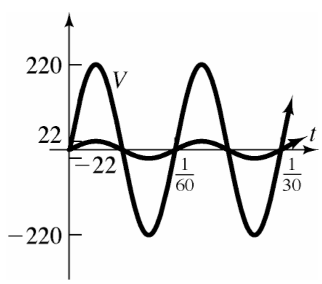


89. $V(t) = 220\sin(120\pi t)$

a. Amplitude: $|A| = |220| = 220$ volts

Period: $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{120\pi} = \dfrac{1}{60}$ second

b, e.



c. $V = IR$

$220\sin(120\pi t) = 10I$

$22\sin(120\pi t) = I$

$I(t) = 22\sin(120\pi t)$

d. Amplitude: $|A| = |22| = 22$ amperes

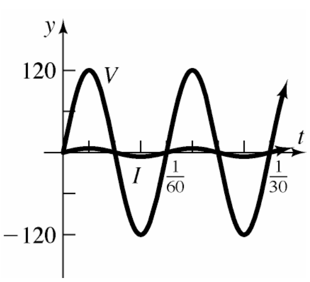
Period: $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{120\pi} = \dfrac{1}{60}$ second

90. $V(t) = 120\sin(120\pi t)$

a. Amplitude: $|A| = |120| = 120$ volts

Period: $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{120\pi} = \dfrac{1}{60}$ second

b, e.



c. $V = IR$

$120\sin(120\pi t) = 20I$

$6\sin(120\pi t) = I$

$I(t) = 6\sin(120\pi t)$

d. Amplitude: $|A| = |6| = 6$ amperes

Period: $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{120\pi} = \dfrac{1}{60}$ second

91. a.

$$P(t) = \frac{[V(t)]^2}{R} = \frac{\left[V_0 \sin(2\pi f t)\right]^2}{R} = \frac{V_0^2 \sin^2(2\pi f t)}{R} = \frac{V_0^2}{R}\sin^2(2\pi f t)$$

b. The graph is the reflected cosine graph translated up a distance equivalent to the amplitude. The period is $\dfrac{1}{2f}$, so $\omega = 4\pi f$.

The amplitude is $\dfrac{1}{2}\cdot\dfrac{V_0^2}{R} = \dfrac{V_0^2}{2R}$.

The equation is:

$$P(t) = -\frac{V_0^2}{2R}\cos(4\pi f t) + \frac{V_0^2}{2R} = \frac{V_0^2}{2R}\left[1 - \cos(4\pi f t)\right]$$

c. Comparing the formulas:

$$\sin^2(2\pi f t) = \frac{1}{2}\left(1 - \cos(4\pi f t)\right)$$

92. a. Since the tunnel is in the shape of one-half a sine cycle, the width of the tunnel at its base is one-half the period. Thus,

$T = \dfrac{2\pi}{\omega} = 2(28) = 56$ or $\omega = \dfrac{\pi}{28}$.

The tunnel has a maximum height of 15 feet so we have $A = 15$. Using the form $y = A\sin(\omega x)$, the equation for the sine curve that fits the opening is

$y = 15\sin\left(\dfrac{\pi x}{28}\right)$.

b. Since the shoulders are 7 feet wide and the road is 14 feet wide, the edges of the road correspond to $x = 7$ and $x = 21$.

$$15\sin\left(\frac{7\pi}{28}\right)=15\sin\left(\frac{\pi}{4}\right)=\frac{15\sqrt{2}}{2}\approx 10.6$$

$$15\sin\left(\frac{21\pi}{28}\right)=15\sin\left(\frac{3\pi}{4}\right)=\frac{15\sqrt{2}}{2}\approx 10.6$$

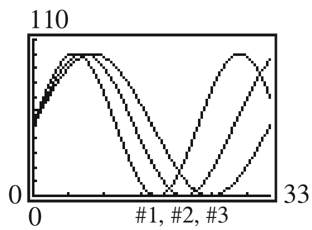
The tunnel is approximately 10.6 feet high at the edge of the road.

93. **a.** Physical potential: $\omega=\frac{2\pi}{23}$;

Emotional potential: $\omega=\frac{2\pi}{28}=\frac{\pi}{14}$;

Intellectual potential: $\omega=\frac{2\pi}{33}$

b.



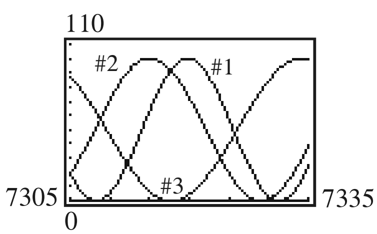
$\#1:\ P(t)=50\sin\left(\frac{2\pi}{23}t\right)+50$

$\#2:\ P(t)=50\sin\left(\frac{\pi}{14}t\right)+50$

$\#3:\ P(t)=50\sin\left(\frac{2\pi}{33}t\right)+50$

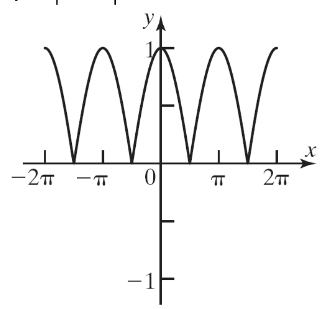
c. No.

d.

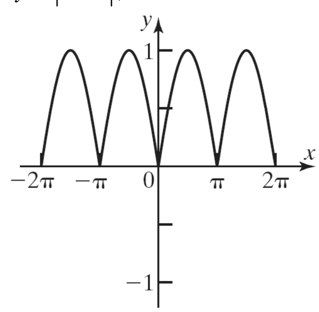


Physical potential peaks at 15 days after the 20th birthday, with minimums at the 3rd and 26th days. Emotional potential is 50% at the 17th day, with a maximum at the 10th day and a minimum at the 24th day. Intellectual potential starts fairly high, drops to a minimum at the 13th day, and rises to a maximum at the 29th day.

94. $y=|\cos x|,\quad -2\pi\le x\le 2\pi$



95. $y=|\sin x|,\quad -2\pi\le x\le 2\pi$



96. Answers may vary.
$\left(-\frac{7\pi}{6},\frac{1}{2}\right),\left(\frac{\pi}{6},\frac{1}{2}\right),\left(\frac{5\pi}{6},\frac{1}{2}\right),\left(\frac{13\pi}{6},\frac{1}{2}\right)$

97. Answers may vary.

$\left(-\frac{5\pi}{3},\frac{1}{2}\right),\left(-\frac{\pi}{3},\frac{1}{2}\right),\left(\frac{\pi}{3},\frac{1}{2}\right),\left(\frac{5\pi}{3},\frac{1}{2}\right)$

98. $2\sin x=-2$

$\sin x=-1$

Answers may vary.

$\left(-\frac{\pi}{2},-2\right),\left(\frac{3\pi}{2},-2\right),\left(\frac{7\pi}{2},-2\right),\left(\frac{11\pi}{2},-2\right)$

99. Answers may vary.
$\left(-\frac{3\pi}{4},1\right),\left(\frac{\pi}{4},1\right),\left(\frac{5\pi}{4},1\right),\left(\frac{9\pi}{4},1\right)$

100 – 104. Answers will vary.

105 – 108. Interactive Exercises.

Section 6.5

1. $x=4$

2. True

3. origin; x = odd multiples of $\frac{\pi}{2}$

4. y-axis; x = odd multiples of $\frac{\pi}{2}$

5. $y = \cos x$

6. True

7. The y-intercept of $y = \tan x$ is 0.

8. $y = \cot x$ has no y-intercept.

9. The y-intercept of $y = \sec x$ is 1.

10. $y = \csc x$ has no y-intercept.

11. $\sec x = 1$ when $x = -2\pi, 0, 2\pi$;
$\sec x = -1$ when $x = -\pi, \pi$

12. $\csc x = 1$ when $x = -\frac{3\pi}{2}, \frac{\pi}{2}$;
$\csc x = -1$ when $x = -\frac{\pi}{2}, \frac{3\pi}{2}$

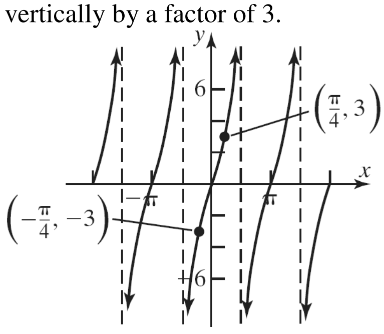
13. $y = \sec x$ has vertical asymptotes when $x = -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$.

14. $y = \csc x$ has vertical asymptotes when $x = -2\pi, -\pi, 0, \pi, 2\pi$.

15. $y = \tan x$ has vertical asymptotes when $x = -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$.

16. $y = \cot x$ has vertical asymptotes when $x = -2\pi, -\pi, 0, \pi, 2\pi$.

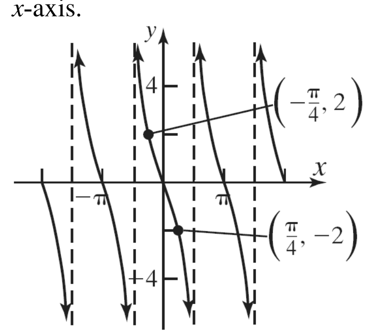
17. $y = 3\tan x$; The graph of $y = \tan x$ is stretched vertically by a factor of 3.



The domain is $\left\{x \middle| x \neq \frac{k\pi}{2}, k \text{ is an odd integer}\right\}$.

The range is the set of all real number or $(-\infty, \infty)$.

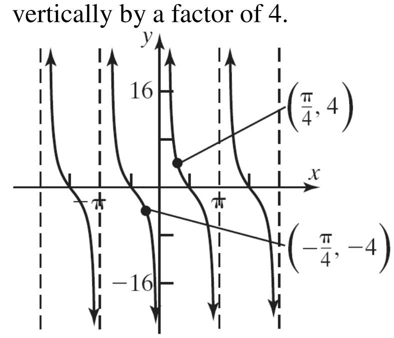
18. $y = -2\tan x$; The graph of $y = \tan x$ is stretched vertically by a factor of 2 and reflected about the x-axis.



The domain is $\left\{x \middle| x \neq \frac{k\pi}{2}, k \text{ is an odd integer}\right\}$.

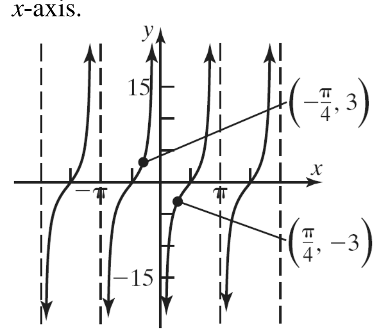
The range is the set of all real number or $(-\infty, \infty)$.

19. $y = 4\cot x$; The graph of $y = \cot x$ is stretched vertically by a factor of 4.



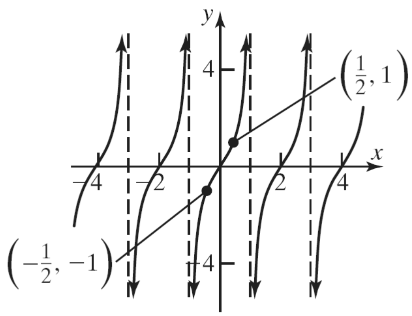
The domain is $\{x | x \neq k\pi, k \text{ is an integer}\}$. The range is the set of all real number or $(-\infty, \infty)$.

20. $y = -3\cot x$; The graph of $y = \cot x$ is stretched vertically by a factor of 3 and reflected about the x-axis.



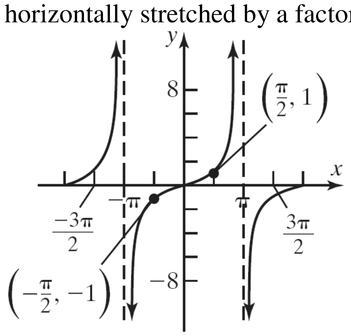
The domain is $\{x \mid x \neq k\pi,\ k \text{ is an integer}\}$. The range is the set of all real number or $(-\infty, \infty)$.

21. $y = \tan\left(\frac{\pi}{2}x\right)$; The graph of $y = \tan x$ is horizontally compressed by a factor of $\frac{2}{\pi}$.



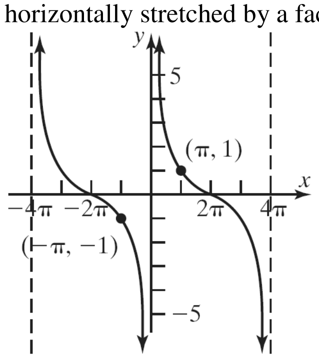
The domain is $\{x \mid x \text{ does not equal an odd integer}\}$. The range is the set of all real number or $(-\infty, \infty)$.

22. $y = \tan\left(\frac{1}{2}x\right)$; The graph of $y = \tan x$ is horizontally stretched by a factor of 2.



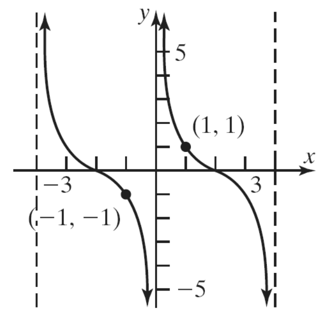
The domain is $\{x \mid x \neq k\pi,\ k \text{ is an integer}\}$. The range is the set of all real number or $(-\infty, \infty)$.

23. $y = \cot\left(\frac{1}{4}x\right)$; The graph of $y = \cot x$ is horizontally stretched by a factor of 4.



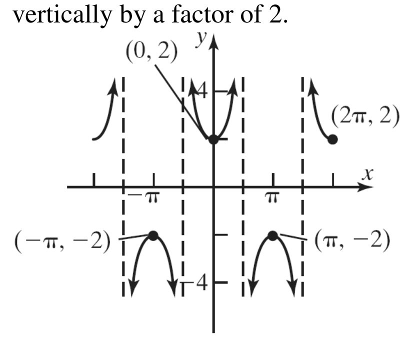
The domain is $\{x \mid x \neq 4k\pi,\ k \text{ is an integer}\}$. The range is the set of all real number or $(-\infty, \infty)$.

24. $y = \cot\left(\frac{\pi}{4}x\right)$; The graph of $y = \cot x$ is horizontally stretched by a factor of $\frac{4}{\pi}$.



The domain is $\{x \mid x \neq 4k,\ k \text{ is an integer}\}$. The range is the set of all real number or $(-\infty, \infty)$.

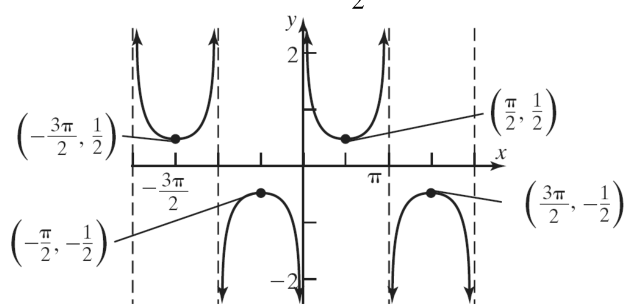
25. $y = 2\sec x$; The graph of $y = \sec x$ is stretched vertically by a factor of 2.



The domain is $\left\{x \,\middle|\, x \neq \frac{k\pi}{2},\ k \text{ is an odd integer}\right\}$.

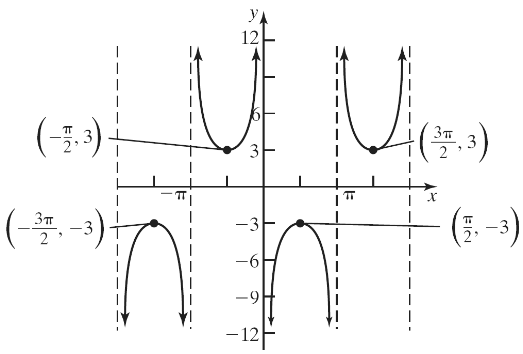
The range is $\{y \mid y \leq -2 \text{ or } y \geq 2\}$.

26. $y = \frac{1}{2}\csc x$; The graph of $y = \csc x$ is vertically compressed by a factor of $\frac{1}{2}$.



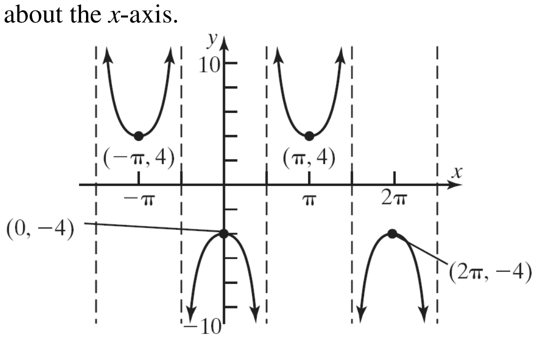
The domain is $\{x \mid x \neq k\pi, k \text{ is an integer}\}$. The range is $\left\{y \,\middle|\, y \leq -\frac{1}{2} \text{ or } y \geq \frac{1}{2}\right\}$.

27. $y = -3\csc x$; The graph of $y = \csc x$ is vertically stretched by a factor of 3 and reflected about the x-axis.



The domain is $\{x \mid x \neq k\pi, k \text{ is an integer}\}$. The range is $\{y \mid y \leq -3 \text{ or } y \geq 3\}$.

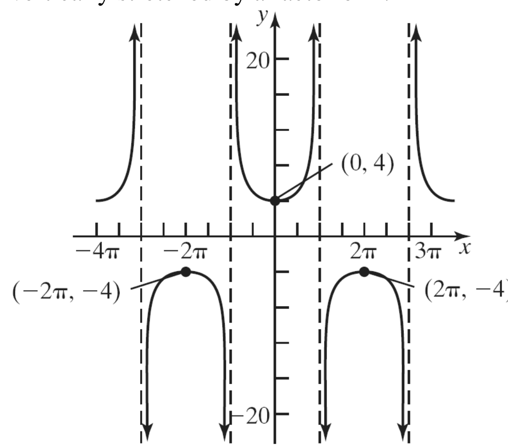
28. $y = -4\sec x$; The graph of $y = \sec x$ is vertically stretched by a factor of 4 and reflected about the x-axis.



The domain is $\left\{x \,\middle|\, x \neq \frac{k\pi}{2}, k \text{ is an odd integer}\right\}$.

The range is $\{y \mid y \leq -4 \text{ or } y \geq 4\}$.

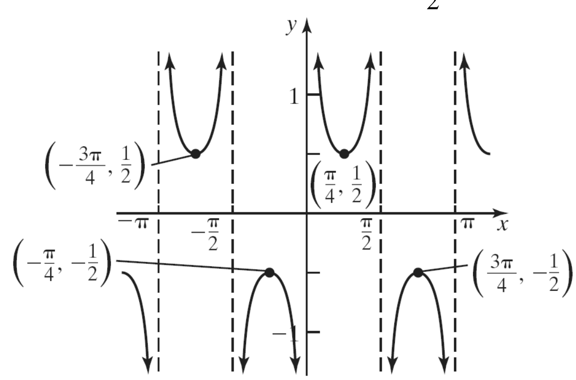
29. $y = 4\sec\left(\frac{1}{2}x\right)$; The graph of $y = \sec x$ is horizontally stretched by a factor of 2 and vertically stretched by a factor of 4.



The domain is $\{x \mid x \neq k\pi, k \text{ is an odd integer}\}$.

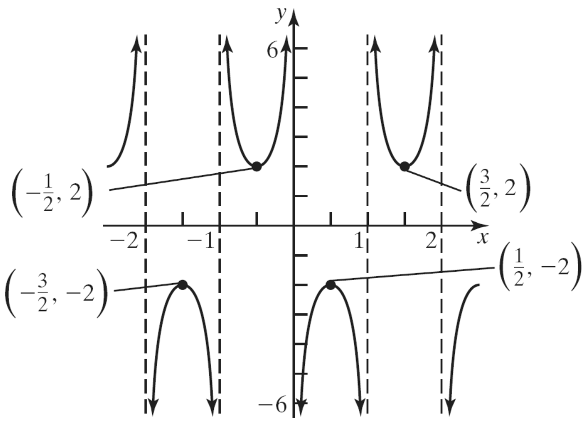
The range is $\{y \mid y \leq -4 \text{ or } y \geq 4\}$.

30. $y = \frac{1}{2}\csc(2x)$; The graph of $y = \csc x$ is horizontally compressed by a factor of $\frac{1}{2}$ and vertically compressed by a factor of $\frac{1}{2}$.



The domain is $\left\{x \,\middle|\, x \neq \frac{k\pi}{2}, k \text{ is an integer}\right\}$. The range is $\left\{y \,\middle|\, y \leq -\frac{1}{2} \text{ or } y \geq \frac{1}{2}\right\}$.

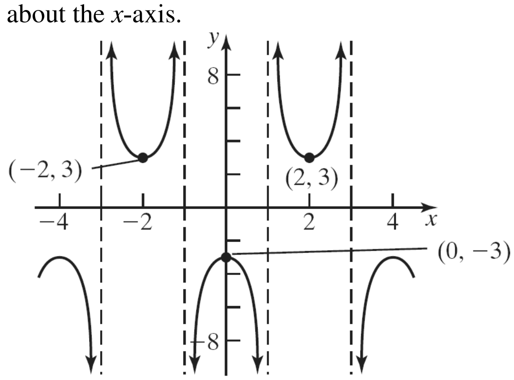
31. $y = -2\csc(\pi x)$; The graph of $y = \csc x$ is horizontally compressed by a factor of $\frac{1}{\pi}$, vertically stretched by a factor of 2, and reflected about the x-axis.



The domain is $\{x \mid x \text{ does not equal an integer}\}$.

The range is $\{y \mid y \le -2 \text{ or } y \ge 2\}$.

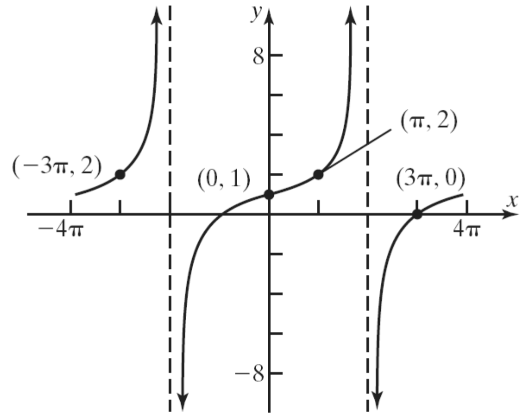
32. $y = -3\sec\left(\frac{\pi}{2}x\right)$; The graph of $y = \sec x$ is horizontally compressed by a factor of $\frac{2}{\pi}$, vertically stretched by a factor of 3, and reflected about the *x*-axis.



The domain is $\{x \mid x \text{ does not equal an odd integer}\}$.

The range is $\{y \mid y \le -3 \text{ or } y \ge 3\}$.

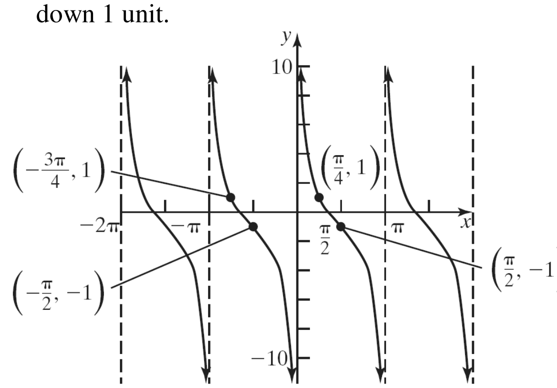
33. $y = \tan\left(\frac{1}{4}x\right) + 1$; The graph of $y = \tan x$ is horizontally stretched by a factor of 4 and shifted up 1 unit.



The domain is $\{x \mid x \ne 2k\pi,\ k \text{ is an odd integer}\}$.

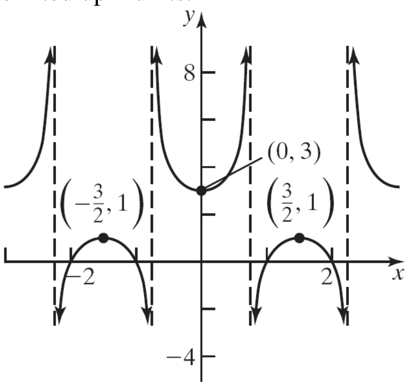
The range is the set of all real number or $(-\infty, \infty)$.

34. $y = 2\cot x - 1$; The graph of $y = \cot x$ is vertically stretched by a factor of 2 and shifted down 1 unit.



The domain is $\{x \mid x \ne k\pi,\ k \text{ is an integer}\}$. The range is the set of all real number or $(-\infty, \infty)$.

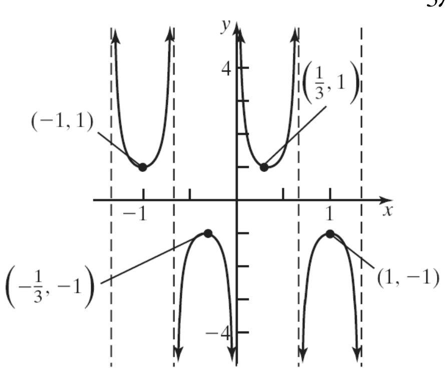
35. $y = \sec\left(\frac{2\pi}{3}x\right) + 2$; The graph of $y = \sec x$ is horizontally compressed by a factor of $\frac{3}{2\pi}$ and shifted up 2 units.



The domain is $\left\{x \middle| x \neq \frac{3}{4}k,\ k \text{ is an odd integer}\right\}$.

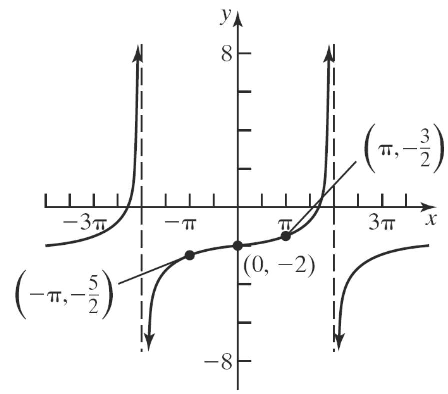
The range is $\left\{y \middle| y \leq 1 \text{ or } y \geq 3\right\}$.

36. $y = \csc\left(\frac{3\pi}{2}x\right)$; The graph of $y = \csc x$ is horizontally compressed by a factor of $\frac{2}{3\pi}$.



The domain is $\left\{x \middle| x \neq \frac{2}{3}k,\ k \text{ is an integer}\right\}$. The range is $\left\{y \middle| y \leq -1 \text{ or } y \geq 1\right\}$.

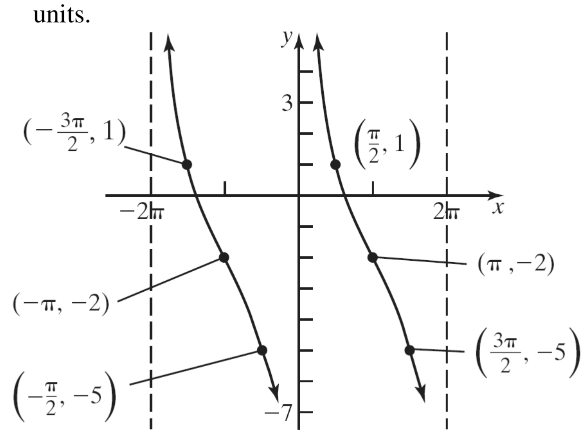
37. $y = \frac{1}{2}\tan\left(\frac{1}{4}x\right) - 2$; The graph of $y = \tan x$ is horizontally stretched by a factor of 4, vertically compressed by a factor of $\frac{1}{2}$, and shifted down 2 units.



The domain is $\left\{x \middle| x \neq 2\pi k,\ k \text{ is an odd integer}\right\}$.

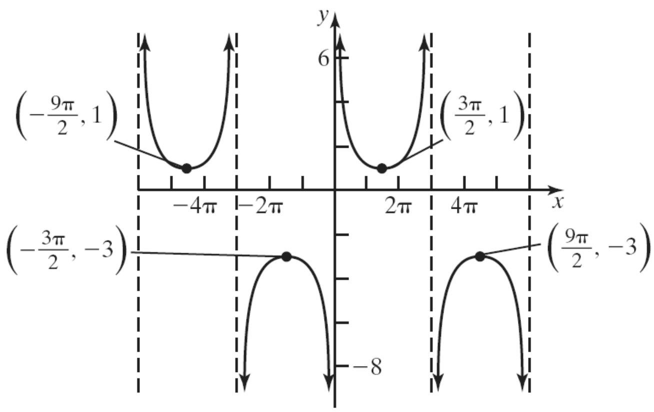
The range is the set of all real number or $(-\infty, \infty)$.

38. $y = 3\cot\left(\frac{1}{2}x\right) - 2$; The graph of $y = \cot x$ is horizontally stretched by a factor of 2, vertically stretched by a factor of 3, and shifted down 2 units.



The domain is $\left\{x \middle| x \neq 2\pi k,\ k \text{ is an integer}\right\}$. The range is the set of all real number or $(-\infty, \infty)$.

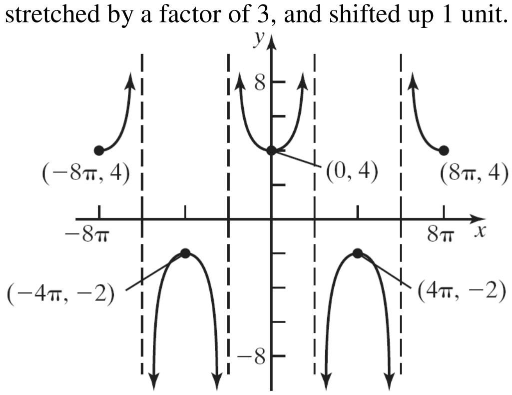
39. $y = 2\csc\left(\frac{1}{3}x\right) - 1$; The graph of $y = \csc x$ is horizontally stretched by a factor of 3, vertically stretched by a factor of 2, and shifted down 1 unit.



The domain is $\{x \mid x \neq 3\pi k,\ k \text{ is an integer}\}$.

The range is $\{y \mid y \leq -3 \text{ or } y \geq 1\}$.

40. $y = 3\sec\left(\frac{1}{4}x\right) + 1$; The graph of $y = \sec x$ is horizontally stretched by a factor of 4, vertically stretched by a factor of 3, and shifted up 1 unit.



The domain is $\{x \mid x \neq 2\pi k,\ k \text{ is an odd integer}\}$.

The range is $\{y \mid y \leq -2 \text{ or } y \geq 4\}$.

41. $$\frac{f\left(\frac{\pi}{6}\right) - f(0)}{\frac{\pi}{6} - 0} = \frac{\tan(\pi/6) - \tan(0)}{\pi/6} = \frac{\frac{\sqrt{3}}{3} - 0}{\pi/6}$$

$$= \frac{\sqrt{3}}{3} \cdot \frac{6}{\pi} = \frac{2\sqrt{3}}{\pi}$$

The average rate of change is $\frac{2\sqrt{3}}{\pi}$.

42. $$\frac{f\left(\frac{\pi}{6}\right) - f(0)}{\frac{\pi}{6} - 0} = \frac{\sec(\pi/6) - \sec(0)}{\pi/6} = \frac{\frac{2\sqrt{3}}{3} - 1}{\pi/6}$$

$$= \frac{2\sqrt{3} - 3}{3} \cdot \frac{6}{\pi} = \frac{2\sqrt{3}\left(2 - \sqrt{3}\right)}{\pi}$$

The average rate of change is $\frac{2\sqrt{3}\left(2 - \sqrt{3}\right)}{\pi}$.

43. $$\frac{f\left(\frac{\pi}{6}\right) - f(0)}{\frac{\pi}{6} - 0} = \frac{\tan(2 \cdot \pi/6) - \tan(2 \cdot 0)}{\pi/6}$$

$$= \frac{\sqrt{3} - 0}{\pi/6} = \frac{6\sqrt{3}}{\pi}$$

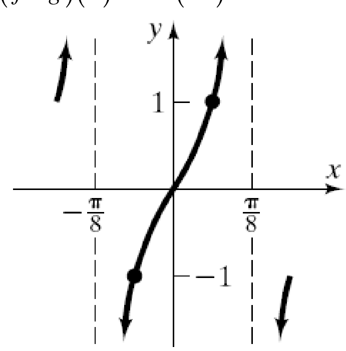
The average rate of change is $\frac{6\sqrt{3}}{\pi}$.

44. $$\frac{f\left(\frac{\pi}{6}\right) - f(0)}{\frac{\pi}{6} - 0} = \frac{\sec(2 \cdot \pi/6) - \sec(2 \cdot 0)}{\pi/6}$$

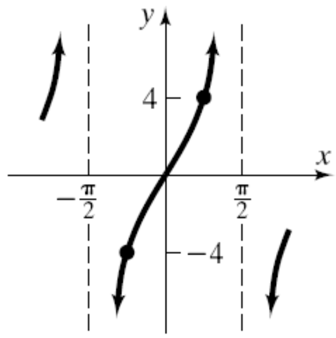
$$= \frac{2 - 1}{\pi/6} = \frac{6}{\pi}$$

The average rate of change is $\frac{6}{\pi}$.

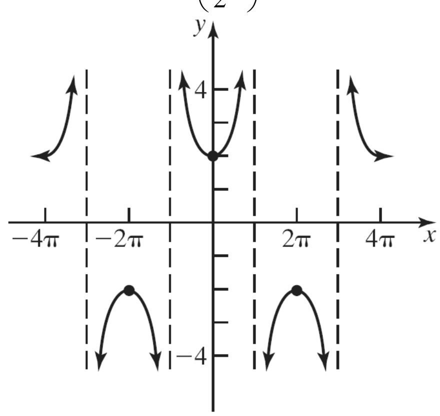
45. $(f \circ g)(x) = \tan(4x)$



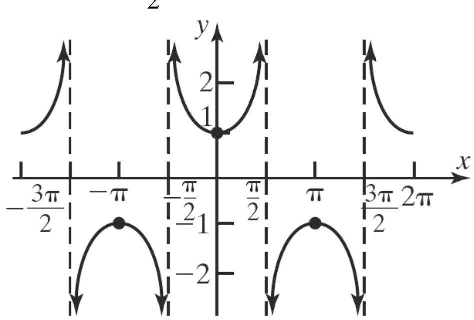
$(g \circ f)(x) = 4(\tan x) = 4\tan x$



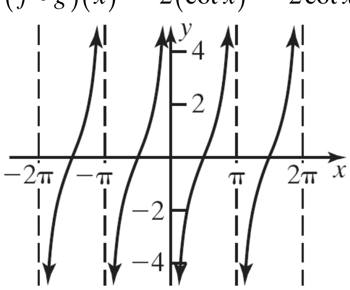
46. $(f \circ g)(x) = 2\sec\left(\frac{1}{2}x\right)$



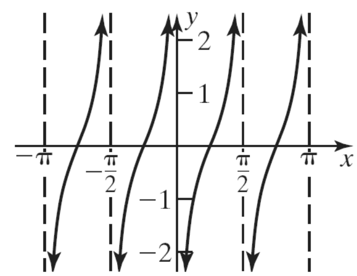
$(g \circ f)(x) = \frac{1}{2}(2\sec x) = \sec x$



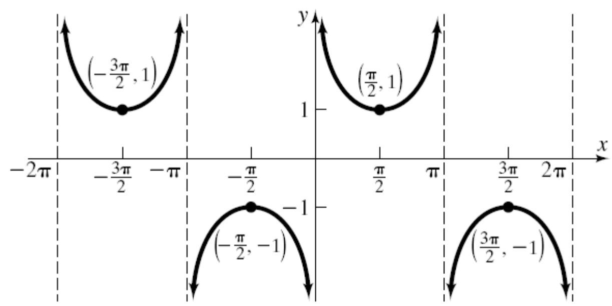
47. $(f \circ g)(x) = -2(\cot x) = -2\cot x$



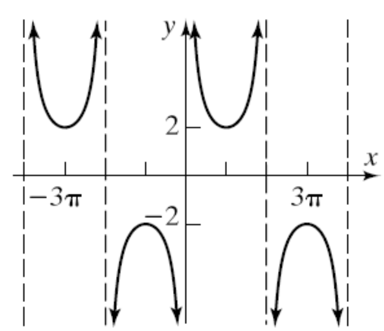
$(g \circ f)(x) = \cot(-2x)$



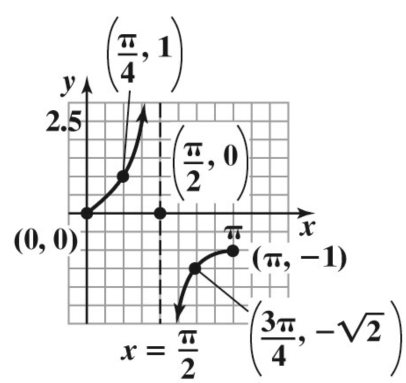
48. $(f \circ g)(x) = \frac{1}{2}(2\csc x) = \csc x$



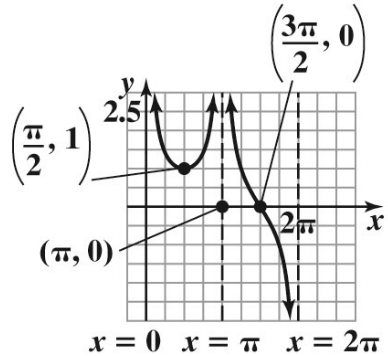
$(g \circ f)(x) = 2\csc\left(\frac{1}{2}x\right)$



49.



50.



51. a. Consider the length of the line segment in two sections, x, the portion across the hall that is 3 feet wide and y, the portion across that hall that is 4 feet wide. Then,

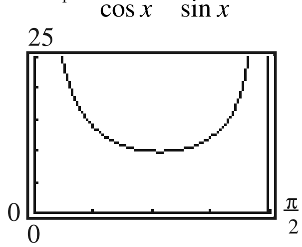
$$\cos\theta = \frac{3}{x} \qquad \text{and} \qquad \sin\theta = \frac{4}{y}$$

$$x = \frac{3}{\cos\theta} \qquad\qquad y = \frac{4}{\sin\theta}$$

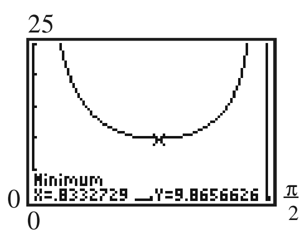
Thus,

$$L = x + y = \frac{3}{\cos\theta} + \frac{4}{\sin\theta} = 3\sec\theta + 4\csc\theta .$$

b. Let $Y_1 = \frac{3}{\cos x} + \frac{4}{\sin x}$.



c. Use MINIMUM to find the least value:

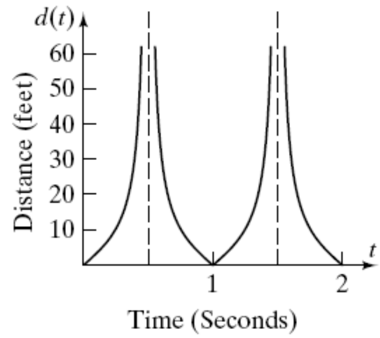


L is least when $\theta \approx 0.83$.

d. $L \approx \frac{3}{\cos(0.83)} + \frac{4}{\sin(0.83)} \approx 9.86$ feet.

Note that rounding up will result in a ladder that won't fit around the corner. Answers will vary.

52. a. $d(t) = |10\tan(\pi t)|$



b. $d(t) = |10\tan(\pi t)|$ is undefined at $t = \frac{1}{2}$ and $t = \frac{3}{2}$, or in general at $\left\{ t = \frac{k}{2} \;\middle|\; k \text{ is an odd integer} \right\}$. At these instances, the length of the beam of light approaches infinity. It is at these instances in the rotation of the beacon when the beam of light being cast on the wall changes from one side of the beacon to the other.

c.

t	$d(t) = 10\tan(\pi t)$
0	0
0.1	3.2492
0.2	7.2654
0.3	13.764
0.4	30.777

d.

$$\frac{d(0.1) - d(0)}{0.1 - 0} = \frac{3.2492 - 0}{0.1 - 0} \approx 32.492$$

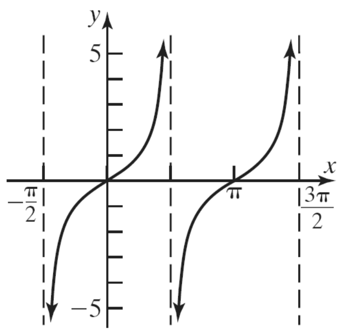
$$\frac{d(0.2) - d(0.1)}{0.2 - 0.1} = \frac{7.2654 - 3.2492}{0.2 - 0.1} \approx 40.162$$

$$\frac{d(0.3) - d(0.2)}{0.3 - 0.2} = \frac{13.764 - 7.2654}{0.3 - 0.2} \approx 64.986$$

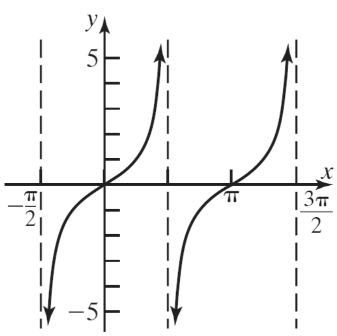
$$\frac{d(0.4) - d(0.3)}{0.4 - 0.3} = \frac{30.777 - 13.764}{0.4 - 0.3} \approx 170.13$$

e. The first differences represent the average rate of change of the beam of light against the wall, measured in feet per second. For example, between $t = 0$ seconds and $t = 0.1$ seconds, the average rate of change of the beam of light against the wall is 32.492 feet per second.

53.



$y = \tan x$



$y = -\cot\left(x + \frac{\pi}{2}\right)$

Yes, the two functions are equivalent.

Section 6.6

1. phase shift

2. False

3. $y = 4\sin(2x - \pi)$

   Amplitude: $|A| = |4| = 4$

   Period: $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{2} = \pi$

   Phase Shift: $\dfrac{\phi}{\omega} = \dfrac{\pi}{2}$

   Interval defining one cycle:

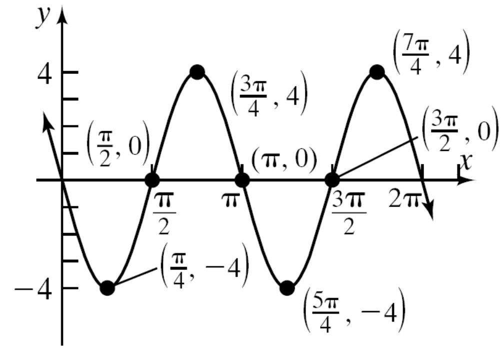
   $\left[\dfrac{\phi}{\omega}, \dfrac{\phi}{\omega} + T\right] = \left[\dfrac{\pi}{2}, \dfrac{3\pi}{2}\right]$

   Subinterval width:

   $\dfrac{T}{4} = \dfrac{\pi}{4}$

   Key points:

   $\left(\dfrac{\pi}{2}, 0\right), \left(\dfrac{3\pi}{4}, 4\right), (\pi, 0), \left(\dfrac{5\pi}{4}, -4\right), \left(\dfrac{3\pi}{2}, 0\right)$



4. $y = 3\sin(3x - \pi)$

   Amplitude: $|A| = |3| = 3$

   Period: $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{3}$

   Phase Shift: $\dfrac{\phi}{\omega} = \dfrac{\pi}{3}$

   Interval defining one cycle:

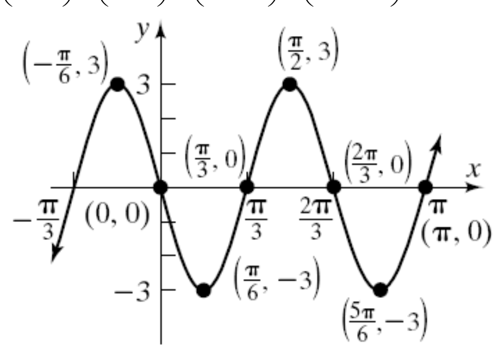
   $\left[\dfrac{\phi}{\omega}, \dfrac{\phi}{\omega} + T\right] = \left[\dfrac{\pi}{3}, \pi\right]$

   Subinterval width:

   $\dfrac{T}{4} = \dfrac{2\pi/3}{4} = \dfrac{\pi}{6}$

   Key points:

   $\left(\dfrac{\pi}{3}, 0\right), \left(\dfrac{\pi}{2}, 3\right), \left(\dfrac{2\pi}{3}, 0\right), \left(\dfrac{5\pi}{6}, -3\right), (\pi, 0)$



5. $y = 2\cos\left(3x + \frac{\pi}{2}\right)$

Amplitude: $|A| = |2| = 2$

Period: $T = \frac{2\pi}{\omega} = \frac{2\pi}{3}$

Phase Shift: $\frac{\phi}{\omega} = \frac{\left(-\frac{\pi}{2}\right)}{3} = -\frac{\pi}{6}$

Interval defining one cycle:

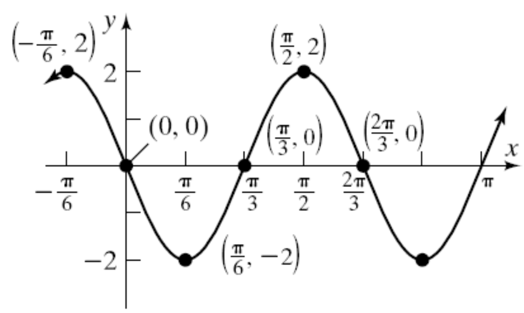
$\left[\frac{\phi}{\omega}, \frac{\phi}{\omega} + T\right] = \left[-\frac{\pi}{6}, \frac{\pi}{2}\right]$

Subinterval width:

$\frac{T}{4} = \frac{2\pi/3}{4} = \frac{\pi}{6}$

Key points:

$\left(-\frac{\pi}{6}, 2\right)$, $(0,0)$, $\left(\frac{\pi}{6}, -2\right)$, $\left(\frac{\pi}{3}, 0\right)$, $\left(\frac{\pi}{2}, 2\right)$



6. $y = 3\cos(2x + \pi)$

Amplitude: $|A| = |3| = 3$

Period: $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$

Phase Shift: $\frac{\phi}{\omega} = \frac{-\pi}{2} = -\frac{\pi}{2}$

Interval defining one cycle:

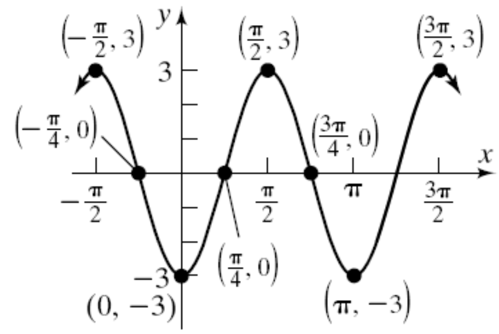
$\left[\frac{\phi}{\omega}, \frac{\phi}{\omega} + T\right] = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Subinterval width:

$\frac{T}{4} = \frac{\pi}{4}$

Key points:

$\left(-\frac{\pi}{2}, 3\right)$, $\left(-\frac{\pi}{4}, 0\right)$, $(0,-3)$, $\left(\frac{\pi}{4}, 0\right)$, $\left(\frac{\pi}{2}, 3\right)$



7. $y = -3\sin\left(2x + \frac{\pi}{2}\right)$

Amplitude: $|A| = |-3| = 3$

Period: $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$

Phase Shift: $\frac{\phi}{\omega} = \frac{-\frac{\pi}{2}}{2} = -\frac{\pi}{4}$

Interval defining one cycle:

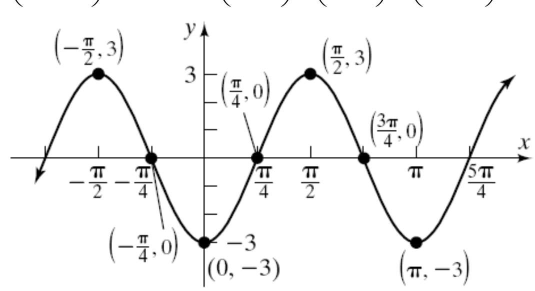
$\left[\frac{\phi}{\omega}, \frac{\phi}{\omega} + T\right] = \left[-\frac{\pi}{4}, \frac{3\pi}{4}\right]$

Subinterval width:

$\frac{T}{4} = \frac{\pi}{4}$

Key points:

$\left(-\frac{\pi}{4}, 0\right)$, $(0,-3)$, $\left(\frac{\pi}{4}, 0\right)$, $\left(\frac{\pi}{2}, 3\right)$, $\left(\frac{3\pi}{4}, 0\right)$



8. $y = -2\cos\left(2x - \frac{\pi}{2}\right)$

Amplitude: $|A| = |-2| = 2$

Period: $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$

Phase Shift: $\frac{\phi}{\omega} = \frac{\frac{\pi}{2}}{2} = \frac{\pi}{4}$

Interval defining one cycle:

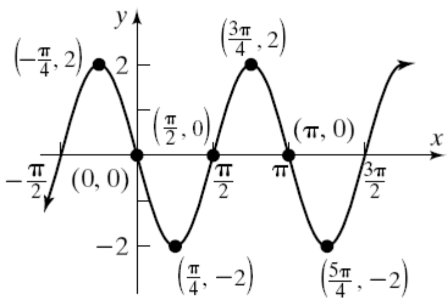
$$\left[\frac{\phi}{\omega}, \frac{\phi}{\omega} + T\right] = \left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$$

Subinterval width:

$$\frac{T}{4} = \frac{\pi}{4}$$

Key points:

$$\left(\frac{\pi}{4}, -2\right), \left(\frac{\pi}{2}, 0\right), \left(\frac{3\pi}{4}, 2\right), (\pi, 0), \left(\frac{5\pi}{4}, -2\right)$$



9. $y = 4\sin(\pi x + 2) - 5$

Amplitude: $|A| = |4| = 4$

Period: $T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi} = 2$

Phase Shift: $\frac{\phi}{\omega} = \frac{-2}{\pi} = -\frac{2}{\pi}$

Interval defining one cycle:

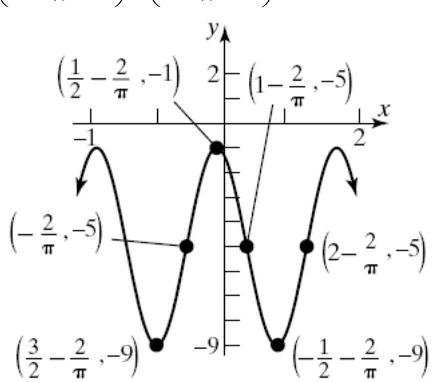
$$\left[\frac{\phi}{\omega}, \frac{\phi}{\omega} + T\right] = \left[-\frac{2}{\pi}, 2 - \frac{2}{\pi}\right]$$

Subinterval width:

$$\frac{T}{4} = \frac{2}{4} = \frac{1}{2}$$

Key points:

$$\left(-\frac{2}{\pi}, -5\right), \left(\frac{1}{2} - \frac{2}{\pi}, -1\right), \left(1 - \frac{2}{\pi}, -5\right),$$
$$\left(\frac{3}{2} - \frac{2}{\pi}, -9\right), \left(2 - \frac{2}{\pi}, -5\right)$$



10. $y = 2\cos(2\pi x + 4) + 4$

Amplitude: $|A| = |2| = 2$

Period: $T = \frac{2\pi}{\omega} = \frac{2\pi}{2\pi} = 1$

Phase Shift: $\frac{\phi}{\omega} = \frac{-4}{2\pi} = -\frac{2}{\pi}$

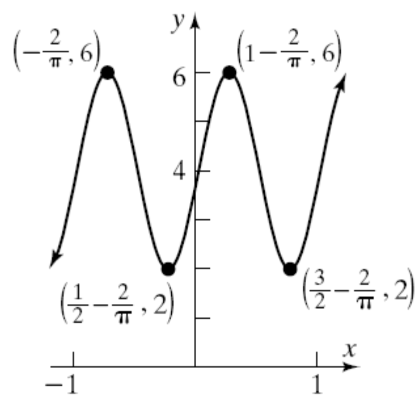
Interval defining one cycle:

$$\left[\frac{\phi}{\omega}, \frac{\phi}{\omega} + T\right] = \left[-\frac{2}{\pi}, 1 - \frac{2}{\pi}\right]$$

Subinterval width: $\frac{T}{4} = \frac{1}{4}$

Key points:

$$\left(-\frac{2}{\pi}, 6\right), \left(\frac{1}{4} - \frac{2}{\pi}, 4\right), \left(\frac{1}{2} - \frac{2}{\pi}, 2\right), \left(\frac{3}{4} - \frac{2}{\pi}, 4\right),$$
$$\left(1 - \frac{2}{\pi}, 6\right)$$



11. $y = 3\cos(\pi x - 2) + 5$

Amplitude: $|A| = |3| = 3$

Period: $T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi} = 2$

Phase Shift: $\frac{\phi}{\omega} = \frac{2}{\pi}$

Interval defining one cycle:

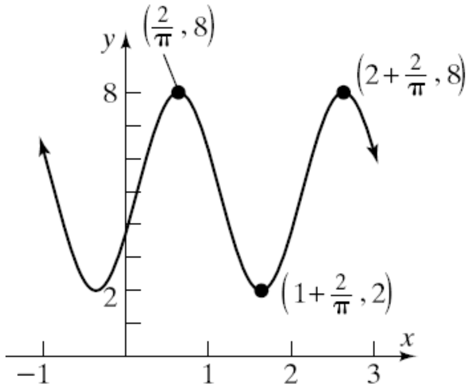
$$\left[\frac{\phi}{\omega}, \frac{\phi}{\omega} + T\right] = \left[\frac{2}{\pi}, 2 + \frac{2}{\pi}\right]$$

Subinterval width:

$$\frac{T}{4} = \frac{2}{4} = \frac{1}{2}$$

Key points:

$\left(\frac{2}{\pi},8\right)$, $\left(\frac{1}{2}+\frac{2}{\pi},5\right)$, $\left(1+\frac{2}{\pi},2\right)$, $\left(\frac{3}{2}+\frac{2}{\pi},5\right)$, $\left(2+\frac{2}{\pi},8\right)$



12. $y = 2\cos(2\pi x - 4) - 1$

Amplitude: $|A| = |2| = 2$

Period: $T = \frac{2\pi}{\omega} = \frac{2\pi}{2\pi} = 1$

Phase Shift: $\frac{\phi}{\omega} = \frac{4}{2\pi} = \frac{2}{\pi}$

Interval defining one cycle:

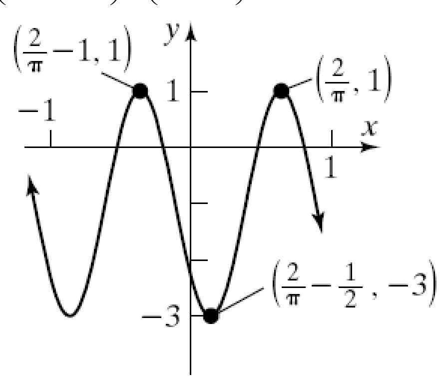
$\left[\frac{\phi}{\omega}, \frac{\phi}{\omega}+T\right] = \left[\frac{2}{\pi}, 1+\frac{2}{\pi}\right]$

Subinterval width:

$\frac{T}{4} = \frac{1}{4}$

Key points:

$\left(\frac{2}{\pi},1\right)$, $\left(\frac{1}{4}+\frac{2}{\pi},-1\right)$, $\left(\frac{1}{2}+\frac{2}{\pi},-3\right)$, $\left(\frac{3}{4}+\frac{2}{\pi},-1\right)$, $\left(1+\frac{2}{\pi},1\right)$



13. $y = -3\sin\left(-2x+\frac{\pi}{2}\right) = -3\sin\left(-\left(2x-\frac{\pi}{2}\right)\right)$

$= 3\sin\left(2x-\frac{\pi}{2}\right)$

Amplitude: $|A| = |3| = 3$

Period: $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$

Phase Shift: $\frac{\phi}{\omega} = \frac{\frac{\pi}{2}}{2} = \frac{\pi}{4}$

Interval defining one cycle:

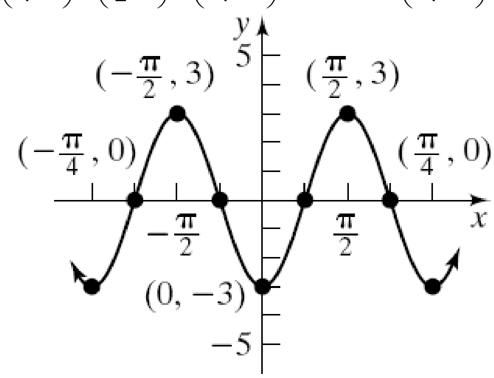
$\left[\frac{\phi}{\omega}, \frac{\phi}{\omega}+T\right] = \left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$

Subinterval width:

$\frac{T}{4} = \frac{\pi}{4}$

Key points:

$\left(\frac{\pi}{4},0\right)$, $\left(\frac{\pi}{2},3\right)$, $\left(\frac{3\pi}{4},0\right)$, $(\pi,-3)$, $\left(\frac{5\pi}{4},0\right)$



14. $y = -3\cos\left(-2x+\frac{\pi}{2}\right) = -3\cos\left(-\left(2x-\frac{\pi}{2}\right)\right)$

$= -3\cos\left(2x-\frac{\pi}{2}\right)$

Amplitude: $|A| = |-3| = 3$

Period: $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$

Phase Shift: $\frac{\phi}{\omega} = \frac{\frac{\pi}{2}}{2} = \frac{\pi}{4}$

Interval defining one cycle:

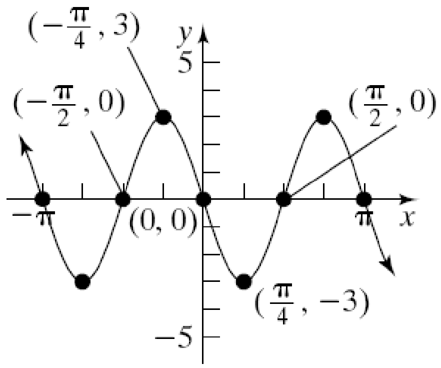
$\left[\frac{\phi}{\omega}, \frac{\phi}{\omega}+T\right] = \left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$

Subinterval width:

$\frac{T}{4}=\frac{\pi}{4}$

Key points:

$\left(\frac{\pi}{4},-3\right)$, $\left(\frac{\pi}{2},0\right)$, $\left(\frac{3\pi}{4},3\right)$, $(\pi,0)$, $\left(\frac{5\pi}{4},-3\right)$



15. $|A|=2;\quad T=\pi;\quad \frac{\phi}{\omega}=\frac{1}{2}$

$\omega=\frac{2\pi}{T}=\frac{2\pi}{\pi}=2 \qquad \frac{\phi}{\omega}=\frac{\phi}{2}=\frac{1}{2}$

$\phi=1$

Assuming A is positive, we have that

$y=A\sin(\omega x-\phi)=2\sin(2x-1)$

$=2\sin\left[2\left(x-\frac{1}{2}\right)\right]$

16. $|A|=3;\quad T=\frac{\pi}{2};\quad \frac{\phi}{\omega}=2$

$\omega=\frac{2\pi}{T}=\frac{2\pi}{\frac{\pi}{2}}=4 \qquad \frac{\phi}{\omega}=\frac{\phi}{4}=2$

$\phi=8$

Assuming A is positive, we have that

$y=A\sin(\omega x-\phi)=3\sin(4x-8)$

$=3\sin\left[4(x-2)\right]$

17. $|A|=3;\quad T=3\pi;\quad \frac{\phi}{\omega}=-\frac{1}{3}$

$\omega=\frac{2\pi}{T}=\frac{2\pi}{3\pi}=\frac{2}{3} \qquad \frac{\phi}{\omega}=\frac{\phi}{\frac{2}{3}}=-\frac{1}{3}$

$\phi=-\frac{1}{3}\cdot\frac{2}{3}=-\frac{2}{9}$

Assuming A is positive, we have that

$y=A\sin(\omega x-\phi)=3\sin\left(\frac{2}{3}x+\frac{2}{9}\right)$

$=3\sin\left[\frac{2}{3}\left(x+\frac{1}{3}\right)\right]$

18. $|A|=2;\quad T=\pi;\quad \frac{\phi}{\omega}=-2$

$\omega=\frac{2\pi}{T}=\frac{2\pi}{\pi}=2 \qquad \frac{\phi}{\omega}=\frac{\phi}{2}=-2$

$\phi=-4$

Assuming A is positive, we have that

$y=A\sin(\omega x-\phi)=2\sin(2x+4)$

$=2\sin\left[2(x+2)\right]$

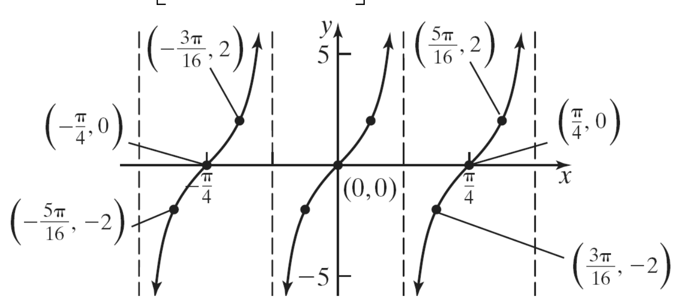
19. $y=2\tan(4x-\pi)$

Begin with the graph of $y=\tan x$ and apply the following transformations:

1) Shift right π units $\left[y=\tan(x-\pi)\right]$

2) Horizontally compress by a factor of $\frac{1}{4}$ $\left[y=\tan(4x-\pi)\right]$

3) Vertically stretch by a factor of 2 $\left[y=2\tan(4x-\pi)\right]$



20. $y=\frac{1}{2}\cot(2x-\pi)$

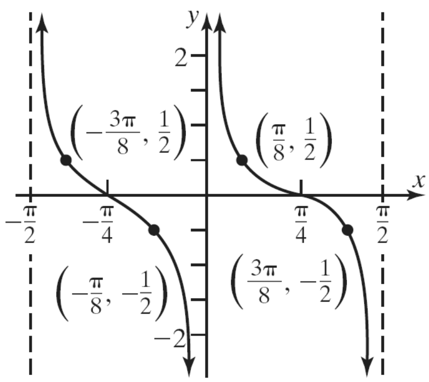
Begin with the graph of $y=\cot x$ and apply the following transformations:

1) Shift right π units $\left[y=\cot(x-\pi)\right]$

2) Horizontally compress by a factor of $\frac{1}{2}$ $\left[y=\cot(2x-\pi)\right]$

3) Vertically compress by a factor of $\frac{1}{2}$

$\left[y=\frac{1}{2}\cot(2x-\pi)\right]$



21. $y=3\csc\left(2x-\frac{\pi}{4}\right)$

Begin with the graph of $y=\csc x$ and apply the following transformations:

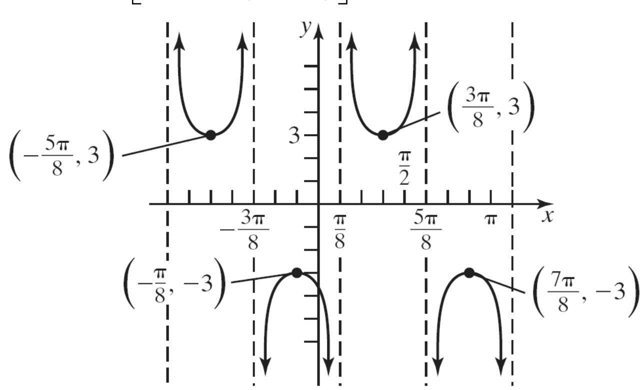
1) Shift right $\frac{\pi}{4}$ units $\left[y=\csc\left(x-\frac{\pi}{4}\right)\right]$

2) Horizontally compress by a factor of $\frac{1}{2}$

$\left[y=\csc\left(2x-\frac{\pi}{4}\right)\right]$

3) Vertically stretch by a factor of 3

$\left[y=3\csc\left(2x-\frac{\pi}{4}\right)\right]$



22. $y=\frac{1}{2}\sec(3x-\pi)$

Begin with the graph of $y=\sec x$ and apply the following transformations:

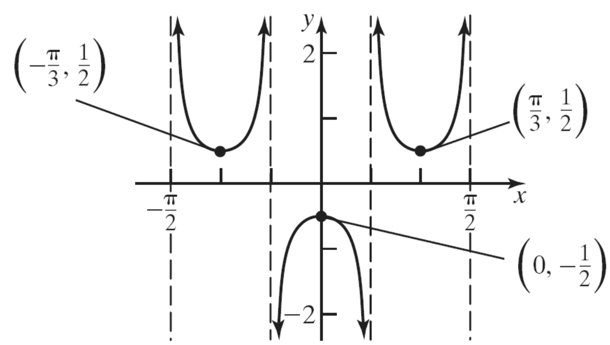
1) Shift right π units $\left[y=\sec(x-\pi)\right]$

2) Horizontally compress by a factor of $\frac{1}{3}$

$\left[y=\sec(3x-\pi)\right]$

3) Vertically compress by a factor of $\frac{1}{2}$

$\left[y=\frac{1}{2}\sec(3x-\pi)\right]$



23. $y=-\cot\left(2x+\frac{\pi}{2}\right)$

Begin with the graph of $y=\cot x$ and apply the following transformations:

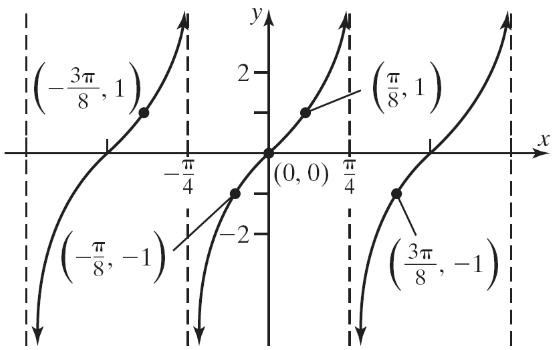
1) Shift left $\frac{\pi}{2}$ units $\left[y=\cot\left(x+\frac{\pi}{2}\right)\right]$

2) Horizontally compress by a factor of $\frac{1}{2}$

$\left[y=\cot\left(2x+\frac{\pi}{2}\right)\right]$

3) Reflect about the x-axis

$\left[y=-\cot\left(2x+\frac{\pi}{2}\right)\right]$



24. $y = -\tan\left(3x + \frac{\pi}{2}\right)$

Begin with the graph of $y = \tan x$ and apply the following transformations:

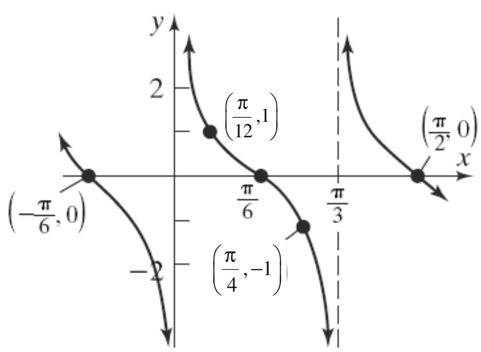
1) Shift left $\frac{\pi}{2}$ units $\left[y = \tan\left(x + \frac{\pi}{2}\right)\right]$

2) Horizontally compress by a factor of $\frac{1}{3}$

$\left[y = \tan\left(3x + \frac{\pi}{2}\right)\right]$

3) Reflect about the x-axis

$\left[y = -\tan\left(x + \frac{\pi}{2}\right)\right]$



25. $y = -\sec(2\pi x + \pi)$

Begin with the graph of $y = \sec x$ and apply the following transformations:

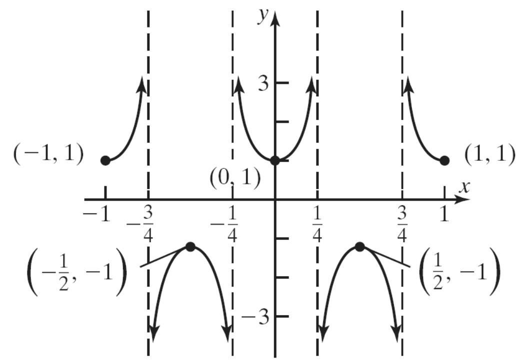
1) Shift left π units $\left[y = \sec(x + \pi)\right]$

2) Horizontally compress by a factor of $\frac{1}{2\pi}$

$\left[y = \sec(2\pi x + \pi)\right]$

3) Reflect about the x-axis

$\left[y = -\sec(2\pi x + \pi)\right]$



26. $y = -\csc\left(-\frac{1}{2}\pi x + \frac{\pi}{4}\right)$

Begin with the graph of $y = \csc x$ and apply the following transformations:

1) Shift left $\frac{\pi}{4}$ units $\left[y = \csc\left(x + \frac{\pi}{4}\right)\right]$

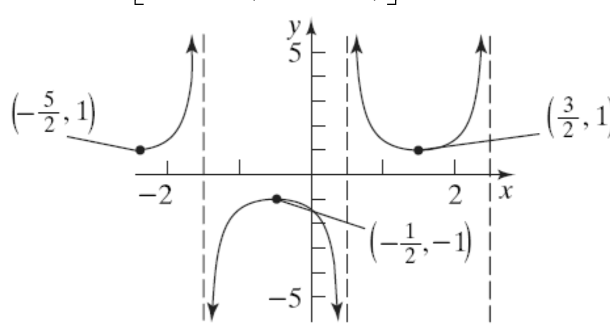
2) Reflect about the y-axis $\left[y = \csc\left(-x + \frac{\pi}{4}\right)\right]$

3) Horizontally compress by a factor of $\frac{2}{\pi}$

$\left[y = \csc\left(-\frac{1}{2}\pi x + \frac{\pi}{4}\right)\right]$

3) Reflect about the x-axis

$\left[y = -\csc\left(-\frac{1}{2}\pi x + \frac{\pi}{4}\right)\right]$

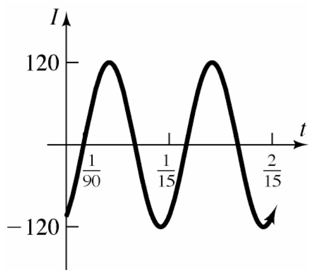


27. $I(t)=120\sin\left(30\pi t-\dfrac{\pi}{3}\right),\quad t\ge 0$

Period: $T=\dfrac{2\pi}{\omega}=\dfrac{2\pi}{30\pi}=\dfrac{1}{15}$ second

Amplitude: $|A|=|120|=120$ amperes

Phase Shift: $\dfrac{\phi}{\omega}=\dfrac{\frac{\pi}{3}}{30\pi}=\dfrac{1}{90}$ second

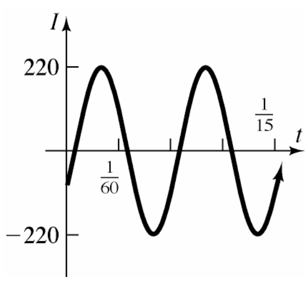


28. $I(t)=220\sin\left(60\pi t-\dfrac{\pi}{6}\right),\quad t\ge 0$

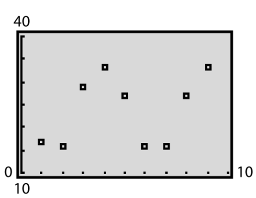
Period: $T=\dfrac{2\pi}{\omega}=\dfrac{2\pi}{60\pi}=\dfrac{1}{30}$ second

Amplitude: $|A|=|220|=220$ amperes

Phase Shift: $\dfrac{\phi}{\omega}=\dfrac{\frac{\pi}{6}}{60\pi}=\dfrac{1}{360}$ second



29. a.



b. Amplitude: $A=\dfrac{33-16}{2}=\dfrac{17}{2}=8.5$

Vertical Shift: $\dfrac{33+16}{2}=\dfrac{49}{2}=24.5$

$\omega=\dfrac{2\pi}{5}=\dfrac{2\pi}{5}$

Phase shift (use $y=16$, $x=6$):

$$16=8.5\sin\left(\frac{2\pi}{5}\cdot 6-\phi\right)+24.5$$

$$-8.5=8.5\sin\left(\frac{12\pi}{5}-\phi\right)$$

$$-1=\sin\left(\frac{12\pi}{5}-\phi\right)$$

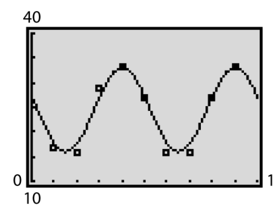
$$-\frac{\pi}{2}=\frac{12\pi}{5}-\phi$$

$$\phi=\frac{29\pi}{10}$$

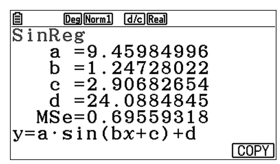
Thus, $y=8.5\sin\left(\dfrac{2\pi}{5}x-\dfrac{11\pi}{10}\right)+24.5$ or

$y=8.5\sin\left[\dfrac{2\pi}{5}\left(x-\dfrac{11}{4}\right)\right]+24.5$.

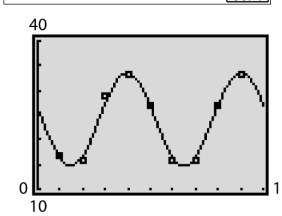
c.



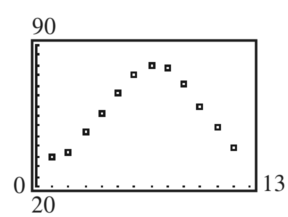
d. $y=9.46\sin(1.247x+2.906)+24.088$



e.



30. a.



b. Amplitude: $A=\dfrac{80.0-34.6}{2}=\dfrac{45.4}{2}=22.7$

Vertical Shift: $\dfrac{80.0+34.6}{2}=\dfrac{114.6}{2}=57.3$

$\omega=\dfrac{2\pi}{12}=\dfrac{\pi}{6}$

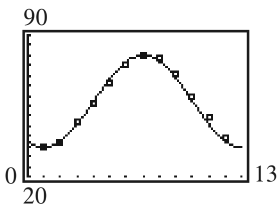
Phase shift (use $y=34.6$, $x=1$):

$$34.6 = 22.7\sin\left(\frac{\pi}{6}\cdot 1-\phi\right)+57.3$$
$$-22.7 = 22.7\sin\left(\frac{\pi}{6}-\phi\right)$$
$$-1 = \sin\left(\frac{\pi}{6}-\phi\right)$$
$$-\frac{\pi}{2} = \frac{\pi}{6}-\phi$$
$$\phi = \frac{2\pi}{3}$$

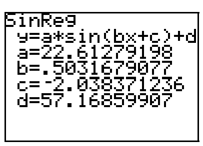
Thus, $y = 22.7\sin\left(\frac{\pi}{6}x-\frac{2\pi}{3}\right)+57.3$ or

$y = 22.7\sin\left[\frac{\pi}{6}(x-4)\right]+57.3$.

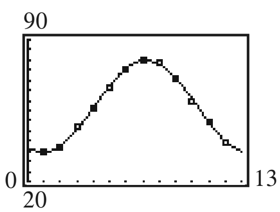
c.



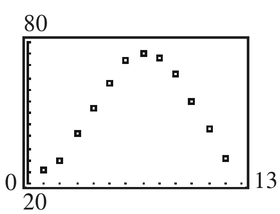
d. $y = 22.61\sin(0.503x-2.038)+57.17$



e.



31. a.



b. Amplitude: $A = \frac{75.4-25.5}{2} = \frac{49.9}{2} = 24.95$

Vertical Shift: $\frac{75.4+25.5}{2} = \frac{100.9}{2} = 50.45$

$\omega = \frac{2\pi}{12} = \frac{\pi}{6}$

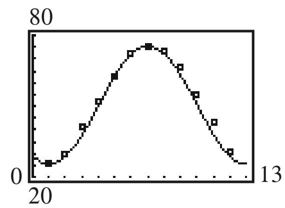
Phase shift (use $y = 25.5$, $x = 1$):

$$25.5 = 24.95\sin\left(\frac{\pi}{6}\cdot 1-\phi\right)+50.45$$
$$-24.95 = 24.95\sin\left(\frac{\pi}{6}-\phi\right)$$
$$-1 = \sin\left(\frac{\pi}{6}-\phi\right)$$
$$-\frac{\pi}{2} = \frac{\pi}{6}-\phi$$
$$\phi = \frac{2\pi}{3}$$

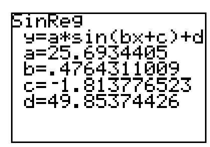
Thus, $y = 24.95\sin\left(\frac{\pi}{6}x-\frac{2\pi}{3}\right)+50.45$ or

$y = 24.95\sin\left[\frac{\pi}{6}(x-4)\right]+50.45$.

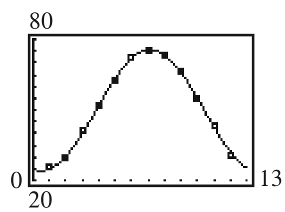
c.



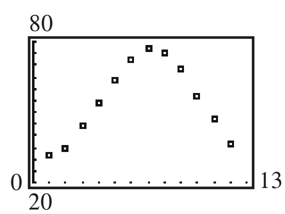
d. $y = 25.693\sin(0.476x-1.814)+49.854$



e.



32. a.



b. Amplitude: $A = \frac{77.0-31.8}{2} = \frac{45.2}{2} = 22.6$

Vertical Shift: $\frac{77.0+31.8}{2} = \frac{108.8}{2} = 54.4$

$\omega = \frac{2\pi}{12} = \frac{\pi}{6}$

Phase shift (use $y = 31.8$, $x = 1$):

$$31.8 = 22.6\sin\left(\frac{\pi}{6}\cdot 1-\phi\right)+54.4$$

$$-22.6 = 22.6\sin\left(\frac{\pi}{6}-\phi\right)$$

$$-1 = \sin\left(\frac{\pi}{6}-\phi\right)$$

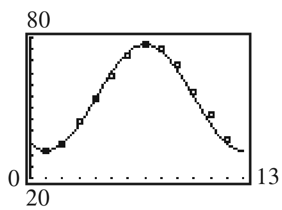
$$-\frac{\pi}{2} = \frac{\pi}{6}-\phi$$

$$\phi = \frac{2\pi}{3}$$

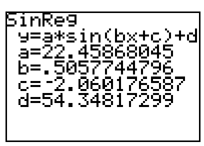
Thus, $y = 22.6\sin\left(\frac{\pi}{6}x-\frac{2\pi}{3}\right)+54.4$ or

$y = 22.6\sin\left[\frac{\pi}{6}(x-4)\right]+54.4$.

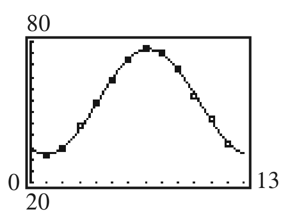
c.



d. $y = 22.46\sin(0.506x-2.060)+54.35$



e.



33. a. 11.5 + 12.4167 = 23.9167 hours which is at 11:55 PM.

b. Ampl: $A = \frac{5.84-(-0.37)}{2} = \frac{6.21}{2} = 3.105$

Vertical Shift: $\frac{5.84+(-0.37)}{2} = \frac{5.47}{2} = 2.735$

$\omega = \frac{2\pi}{12.4167} = \frac{\pi}{6.20835} = \frac{24\pi}{149}$

Phase shift (use $y = 5.84$, $x = 11.5$):

$$5.84 = 3.105\sin\left(\frac{24\pi}{149}\cdot 11.5-\phi\right)+2.735$$

$$3.105 = 3.105\sin\left(\frac{24\pi}{149}\cdot 11.5-\phi\right)$$

$$1 = \sin\left(\frac{276\pi}{149}-\phi\right)$$

$$\frac{\pi}{2} = \frac{276\pi}{149}-\phi$$

$$\phi \approx 4.2485$$

Thus,

$y = 3.105\sin\left(\frac{24\pi}{149}x-4.2485\right)+2.735$ or

$y = 3.105\sin\left[\frac{24\pi}{149}(x-8.3958)\right]+2.735$.

c. $y = 3.105\sin\left(\frac{24\pi}{149}(15)-4.2485\right)+2.735$

≈ 2.12 feet

34. a. 2.6167 + 12.4167 = 15.0334 hours which is at 3:02 PM.

b. Ampl: $A = \frac{11.09-(-2.49)}{2} = \frac{13.58}{2} = 6.79$

Vertical Shift: $\frac{11.09+(-2.49)}{2} = \frac{8.6}{2} = 4.3$

$\omega = \frac{2\pi}{12.4167} = \frac{\pi}{6.20835} = \frac{24\pi}{149}$

Phase shift (use $y = 11.09$, $x = 2.6167$):

$$11.09 = 6.79\sin\left(\frac{24\pi}{149}\cdot 2.6167-\phi\right)+4.3$$

$$6.79 = 6.79\sin\left(\frac{24\pi}{149}\cdot 2.6167-\phi\right)$$

$$1 = \sin\left(\frac{62.8008\pi}{149}-\phi\right)$$

$$\frac{\pi}{2} = \frac{62.8008\pi}{149}-\phi$$

$$\phi \approx -0.2467$$

Thus, $y = 6.79\sin\left(\frac{24\pi}{149}x+0.2467\right)+4.3$

or $y = 6.79\sin\left[\frac{24\pi}{149}(x+0.4875)\right]+4.3$.

c. $y = 6.79\sin\left(\frac{24\pi}{149}(18)+0.2467\right)+4.3$

≈ 4.77 feet

35. a. Amplitude: $A = \frac{13.75 - 10.55}{2} = 1.6$

Vertical Shift: $\frac{13.75 + 10.55}{2} = 12.15$

$\omega = \frac{2\pi}{365}$

Phase shift (use $y = 13.75$, $x = 172$):

$$13.75 = 1.6\sin\left(\frac{2\pi}{365}\cdot 172 - \phi\right) + 12.15$$
$$1.6 = 1.6\sin\left(\frac{2\pi}{365}\cdot 172 - \phi\right)$$
$$1 = \sin\left(\frac{344\pi}{365} - \phi\right)$$
$$\frac{\pi}{2} = \frac{344\pi}{365} - \phi$$
$$\phi \approx 1.3900$$

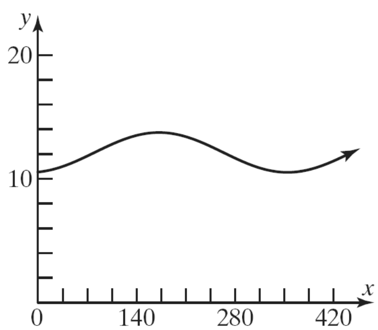
Thus, $y = 1.6\sin\left(\frac{2\pi}{365}x - 1.39\right) + 12.15$ or

$y = 1.6\sin\left[\frac{2\pi}{365}(x - 80.75)\right] + 12.15$.

b. $y = 1.6\sin\left[\frac{2\pi}{365}(91 - 80.75)\right] + 12.15$

≈ 12.43 hours

c.



d. The actual hours of sunlight on April 1, 2012 were 12.43 hours. This is the same as the predicted amount.

36. a. Amplitude: $A = \frac{15.30 - 9.10}{2} = 3.1$

Vertical Shift: $\frac{15.30 + 9.10}{2} = 12.2$

$\omega = \frac{2\pi}{365}$

Phase shift (use $y = 15.30$, $x = 172$):

$$15.30 = 3.1\sin\left(\frac{2\pi}{365}\cdot 172 - \phi\right) + 12.2$$
$$3.1 = 3.1\sin\left(\frac{2\pi}{365}\cdot 172 - \phi\right)$$
$$1 = \sin\left(\frac{344\pi}{365} - \phi\right)$$
$$\frac{\pi}{2} = \frac{344\pi}{365} - \phi$$
$$\phi \approx 1.39$$

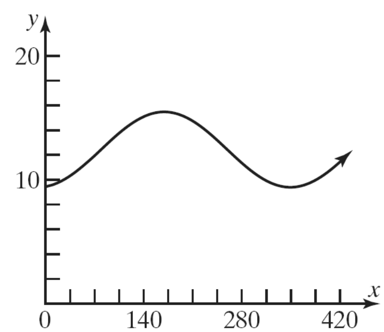
Thus, $y = 3.1\sin\left(\frac{2\pi}{365}x - 1.39\right) + 12.2$ or

$y = 3.1\sin\left[\frac{2\pi}{365}(x - 80.75)\right] + 12.2$.

b. $y = 3.1\sin\left(\frac{2\pi}{365}(91) - 1.39\right) + 12.2$

≈ 12.74 hours

c.



d. The actual hours of sunlight on April 1, 2012 were 12.72 hours. This is very close to the predicted amount of 12.74 hours.

37. a. Amplitude: $A = \frac{19.42 - 5.48}{2} = 6.97$

Vertical Shift: $\frac{19.42 + 5.48}{2} = 12.45$

$\omega = \frac{2\pi}{365}$

Phase shift (use $y = 19.42$, $x = 172$):

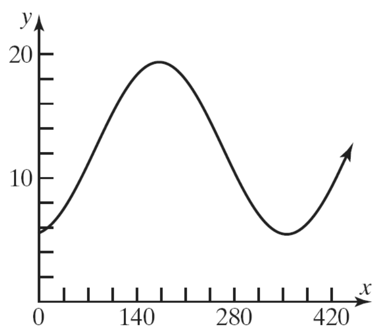
$$19.42 = 6.97\sin\left(\frac{2\pi}{365}\cdot 172 - \phi\right) + 12.45$$
$$6.975 = 6.975\sin\left(\frac{2\pi}{365}\cdot 172 - \phi\right)$$
$$1 = \sin\left(\frac{344\pi}{365} - \phi\right)$$
$$\frac{\pi}{2} = \frac{344\pi}{365} - \phi$$
$$\phi \approx 1.39$$

Thus, $y = 6.97\sin\left(\frac{2\pi}{365}x - 1.39\right) + 12.45$ or $y = 6.97\sin\left[\frac{2\pi}{365}(x - 80.75)\right] + 12.45$.

b. $y = 6.97\sin\left(\frac{2\pi}{365}(91) - 1.39\right) + 12.45$

≈ 13.67 hours

c.



d. The actual hours of sunlight on April 1, 2012 was 13.42 hours. This is close to the predicted amount of 13.67 hours.

38. a. Amplitude: $A = \frac{13.43 - 10.85}{2} = 1.29$

Vertical Shift: $\frac{13.43 + 10.85}{2} = 12.14$

$\omega = \frac{2\pi}{365}$

Phase shift (use $y = 13.43$, $x = 172$):

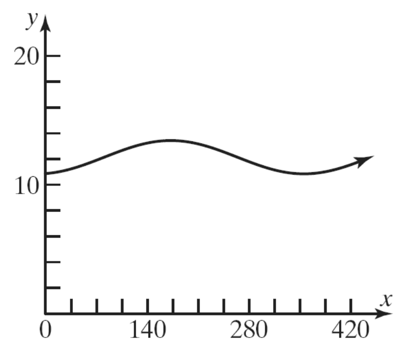
$$13.43 = 1.29\sin\left(\frac{2\pi}{365}\cdot 172 - \phi\right) + 12.14$$
$$1.29 = 1.29\sin\left(\frac{2\pi}{365}\cdot 172 - \phi\right)$$
$$1 = \sin\left(\frac{344\pi}{365} - \phi\right)$$
$$\frac{\pi}{2} = \frac{344\pi}{365} - \phi$$
$$\phi \approx 1.39$$

Thus, $y = 1.29\sin\left(\frac{2\pi}{365}x - 1.39\right) + 12.14$.

b. $y = 1.29\sin\left(\frac{2\pi}{365}(91) - 1.39\right) + 12.14$

≈ 12.37 hours

c.



d. The actual hours of sunlight on April 1, 2012 were 12.38 hours. This is very close to the predicted amount of 12.37 hours.

39 – 40. Answers will vary.

Chapter 6 Review Exercises

1. $135° = 135\cdot\frac{\pi}{180}$ radian $= \frac{3\pi}{4}$ radians

2. $18° = 18\cdot\frac{\pi}{180}$ radian $= \frac{\pi}{10}$ radian

3. $\frac{3\pi}{4} = \frac{3\pi}{4}\cdot\frac{180}{\pi}$ degrees $= 135°$

4. $-\frac{5\pi}{2} = -\frac{5\pi}{2}\cdot\frac{180}{\pi}$ degrees $= -450°$

5. $\tan\frac{\pi}{4} - \sin\frac{\pi}{6} = 1 - \frac{1}{2} = \frac{1}{2}$

6. $3\sin 45° - 4\tan\frac{\pi}{6} = 3\cdot\frac{\sqrt{2}}{2} - 4\cdot\frac{\sqrt{3}}{3} = \frac{3\sqrt{2}}{2} - \frac{4\sqrt{3}}{3}$

7. $6\cos\frac{3\pi}{4}+2\tan\left(-\frac{\pi}{3}\right)=6\left(-\frac{\sqrt{2}}{2}\right)+2\left(-\sqrt{3}\right)$

$=-3\sqrt{2}-2\sqrt{3}$

8. $\sec\left(-\frac{\pi}{3}\right)-\cot\left(-\frac{5\pi}{4}\right)=\sec\frac{\pi}{3}+\cot\frac{5\pi}{4}=2+1=3$

9. $\tan\pi+\sin\pi=0+0=0$

10. $\cos 540^\circ-\tan(-405^\circ)=-1-(-1)$

$=-1+1=0$

11. $\sin^2 20^\circ+\frac{1}{\sec^2 20^\circ}=\sin^2 20^\circ+\cos^2 20^\circ=1$

12. $\sec 50^\circ\cdot\cos 50^\circ=\frac{1}{\cos 50^\circ}\cdot\cos 50^\circ=1$

13. $\frac{\cos(-40^\circ)}{\cos 40^\circ}=\frac{\cos 40^\circ}{\cos 40^\circ}=1$

14. $\frac{\sin(-40^\circ)}{\sin 40^\circ}=\frac{-\sin 40^\circ}{\sin 40^\circ}=-1$

15. $\sin 400^\circ\cdot\sec(-50^\circ)=\sin 400^\circ\cdot\sec 50^\circ$

$=\sin(40^\circ+360^\circ)\cdot\frac{1}{\cos 50^\circ}$

$=\frac{\sin 40^\circ}{\cos 50^\circ}=\frac{\sin 40^\circ}{\sin(90^\circ-50^\circ)}$

$=\frac{\sin 40^\circ}{\sin 40^\circ}=1$

16. $\sin\theta=\frac{4}{5}$ and $0<\theta<\frac{\pi}{2}$, so θ lies in quadrant I.

Using the Pythagorean Identities:

$\cos^2\theta=1-\sin^2\theta$

$\cos^2\theta=1-\left(\frac{4}{5}\right)^2=1-\frac{16}{25}=\frac{9}{25}$

$\cos\theta=\pm\sqrt{\frac{9}{25}}=\pm\frac{3}{5}$

Note that $\cos\theta$ must be positive since θ lies in quadrant I. Thus, $\cos\theta=\frac{3}{5}$.

$\tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{\frac{4}{5}}{\frac{3}{5}}=\frac{4}{5}\cdot\frac{5}{3}=\frac{4}{3}$

$\csc\theta=\frac{1}{\sin\theta}=\frac{1}{\frac{4}{5}}=1\cdot\frac{5}{4}=\frac{5}{4}$

$\sec\theta=\frac{1}{\cos\theta}=\frac{1}{\frac{3}{5}}=1\cdot\frac{5}{3}=\frac{5}{3}$

$\cot\theta=\frac{1}{\tan\theta}=\frac{1}{\frac{4}{3}}=1\cdot\frac{3}{4}=\frac{3}{4}$

17. $\tan\theta=\frac{12}{5}$ and $\sin\theta<0$, so θ lies in quadrant III.

Using the Pythagorean Identities:

$\sec^2\theta=\tan^2\theta+1$

$\sec^2\theta=\left(\frac{12}{5}\right)^2+1=\frac{144}{25}+1=\frac{169}{25}$

$\sec\theta=\pm\sqrt{\frac{169}{25}}=\pm\frac{13}{5}$

Note that $\sec\theta$ must be negative since θ lies in quadrant III. Thus, $\sec\theta=-\frac{13}{5}$.

$\cos\theta=\frac{1}{\sec\theta}=\frac{1}{-\frac{13}{5}}=-\frac{5}{13}$

$\tan\theta=\frac{\sin\theta}{\cos\theta}$, so

$\sin\theta=(\tan\theta)(\cos\theta)=\frac{12}{5}\left(-\frac{5}{13}\right)=-\frac{12}{13}$

$\csc\theta=\frac{1}{\sin\theta}=\frac{1}{-\frac{12}{13}}=-\frac{13}{12}$

$\cot\theta=\frac{1}{\tan\theta}=\frac{1}{\frac{12}{5}}=\frac{5}{12}$

18. $\sec\theta=-\frac{5}{4}$ and $\tan\theta<0$, so θ lies in quadrant II.

Using the Pythagorean Identities:

$\tan^2\theta=\sec^2\theta-1$

$\tan^2\theta=\left(-\frac{5}{4}\right)^2-1=\frac{25}{16}-1=\frac{9}{16}$

$\tan\theta=\pm\sqrt{\frac{9}{16}}=\pm\frac{3}{4}$

Note that $\tan\theta<0$, so $\tan\theta=-\frac{3}{4}$.

$\cos\theta = \dfrac{1}{\sec\theta} = \dfrac{1}{-\frac{5}{4}} = -\dfrac{4}{5}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, so

$\sin\theta = (\tan\theta)(\cos\theta) = -\dfrac{3}{4}\left(-\dfrac{4}{5}\right) = \dfrac{3}{5}$.

$\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{1}{\frac{3}{5}} = \dfrac{5}{3}$

$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{1}{-\frac{3}{4}} = -\dfrac{4}{3}$

19. $\sin\theta = \dfrac{12}{13}$ and θ lies in quadrant II.

Using the Pythagorean Identities:

$\cos^2\theta = 1 - \sin^2\theta$

$\cos^2\theta = 1 - \left(\dfrac{12}{13}\right)^2 = 1 - \dfrac{144}{169} = \dfrac{25}{169}$

$\cos\theta = \pm\sqrt{\dfrac{25}{169}} = \pm\dfrac{5}{13}$

Note that $\cos\theta$ must be negative because θ lies in quadrant II. Thus, $\cos\theta = -\dfrac{5}{13}$.

$\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\frac{12}{13}}{-\frac{5}{13}} = \dfrac{12}{13}\left(-\dfrac{13}{5}\right) = -\dfrac{12}{5}$

$\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{1}{\frac{12}{13}} = \dfrac{13}{12}$

$\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{1}{-\frac{5}{13}} = -\dfrac{13}{5}$

$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{1}{-\frac{12}{5}} = -\dfrac{5}{12}$

20. $\sin\theta = -\dfrac{5}{13}$ and $\dfrac{3\pi}{2} < \theta < 2\pi$ (quadrant IV)

Using the Pythagorean Identities:

$\cos^2\theta = 1 - \sin^2\theta$

$\cos^2\theta = 1 - \left(-\dfrac{5}{13}\right)^2 = 1 - \dfrac{25}{169} = \dfrac{144}{169}$

$\cos\theta = \pm\sqrt{\dfrac{144}{169}} = \pm\dfrac{12}{13}$

Note that $\cos\theta$ must be positive because θ lies in quadrant IV. Thus, $\cos\theta = \dfrac{12}{13}$.

$\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{-\frac{5}{13}}{\frac{12}{13}} = -\dfrac{5}{13}\left(\dfrac{13}{12}\right) = -\dfrac{5}{12}$

$\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{1}{-\frac{5}{13}} = -\dfrac{13}{5}$

$\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{1}{\frac{12}{13}} = \dfrac{13}{12}$

$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{1}{-\frac{5}{12}} = -\dfrac{12}{5}$

21. $\tan\theta = \dfrac{1}{3}$ and $180° < \theta < 270°$ (quadrant III)

Using the Pythagorean Identities:

$\sec^2\theta = \tan^2\theta + 1$

$\sec^2\theta = \left(\dfrac{1}{3}\right)^2 + 1 = \dfrac{1}{9} + 1 = \dfrac{10}{9}$

$\sec\theta = \pm\sqrt{\dfrac{10}{9}} = \pm\dfrac{\sqrt{10}}{3}$

Note that $\sec\theta$ must be negative since θ lies in quadrant III. Thus, $\sec\theta = -\dfrac{\sqrt{10}}{3}$.

$\cos\theta = \dfrac{1}{\sec\theta} = \dfrac{1}{-\frac{\sqrt{10}}{3}} = -\dfrac{3}{\sqrt{10}}\cdot\dfrac{\sqrt{10}}{\sqrt{10}} = -\dfrac{3\sqrt{10}}{10}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, so

$\sin\theta = (\tan\theta)(\cos\theta) = \dfrac{1}{3}\left(-\dfrac{3\sqrt{10}}{10}\right) = -\dfrac{\sqrt{10}}{10}$

$\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{1}{-\frac{\sqrt{10}}{10}} = -\dfrac{10}{\sqrt{10}} = -\sqrt{10}$

$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{1}{\frac{1}{3}} = 3$

22. $\sec\theta = 3$ and $\dfrac{3\pi}{2} < \theta < 2\pi$ (quadrant IV)

Using the Pythagorean Identities:

$\tan^2\theta = \sec^2\theta - 1$

$\tan^2\theta = 3^2 - 1 = 9 - 1 = 8$

$\tan\theta = \pm\sqrt{8} = \pm 2\sqrt{2}$

Note that $\tan\theta$ must be negative since θ lies in quadrant IV. Thus,. $\tan\theta = -2\sqrt{2}$.

$\cos\theta = \dfrac{1}{\sec\theta} = \dfrac{1}{3}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, so

$\sin\theta = (\tan\theta)(\cos\theta) = -2\sqrt{2}\left(\dfrac{1}{3}\right) = -\dfrac{2\sqrt{2}}{3}$.

$\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{1}{-\frac{2\sqrt{2}}{3}} = -\dfrac{3}{2\sqrt{2}}\cdot\dfrac{\sqrt{2}}{2} = -\dfrac{3\sqrt{2}}{4}$

$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{1}{-2\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}} = -\dfrac{\sqrt{2}}{4}$

23. $\cot\theta = -2$ and $\dfrac{\pi}{2} < \theta < \pi$ (quadrant II)

Using the Pythagorean Identities:

$\csc^2\theta = 1 + \cot^2\theta$

$\csc^2\theta = 1 + (-2)^2 = 1 + 4 = 5$

$\csc\theta = \pm\sqrt{5}$

Note that $\csc\theta$ must be positive because θ lies in quadrant II. Thus, $\csc\theta = \sqrt{5}$.

$\sin\theta = \dfrac{1}{\csc\theta} = \dfrac{1}{\sqrt{5}}\cdot\dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{\sqrt{5}}{5}$

$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$, so

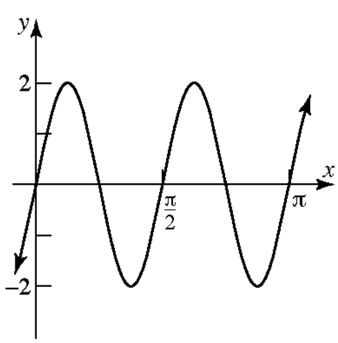
$\cos\theta = (\cot\theta)(\sin\theta) = -2\left(\dfrac{\sqrt{5}}{5}\right) = -\dfrac{2\sqrt{5}}{5}$.

$\tan\theta = \dfrac{1}{\cot\theta} = \dfrac{1}{-2} = -\dfrac{1}{2}$

$\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{1}{-\frac{2\sqrt{5}}{5}} = -\dfrac{5}{2\sqrt{5}} = -\dfrac{\sqrt{5}}{2}$

24. $y = 2\sin(4x)$

The graph of $y = \sin x$ is stretched vertically by a factor of 2 and compressed horizontally by a factor of $\dfrac{1}{4}$.

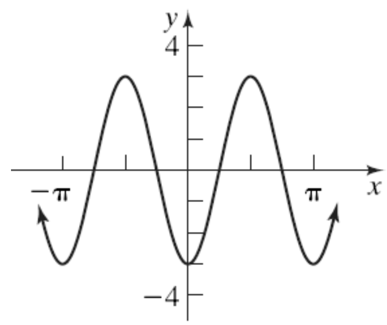


Domain: $(-\infty, \infty)$

Range: $[-2, 2]$

25. $y = -3\cos(2x)$

The graph of $y = \cos x$ is stretched vertically by a factor of 3, reflected across the x-axis, and compressed horizontally by a factor of $\dfrac{1}{2}$.

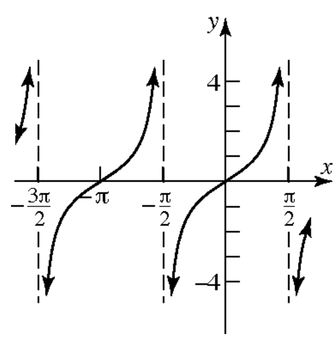


Domain: $(-\infty, \infty)$

Range: $[-3, 3]$

26. $y = \tan(x + \pi)$

The graph of $y = \tan x$ is shifted π units to the left.

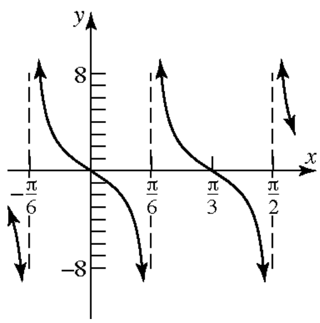


Domain: $\left\{x \mid x \neq \dfrac{k\pi}{2}, \text{k is an odd integer}\right\}$

Range: $(-\infty, \infty)$

27. $y = -2\tan(3x)$

The graph of $y = \tan x$ is stretched vertically by a factor of 2, reflected across the x-axis, and compressed horizontally by a factor of $\frac{1}{3}$.

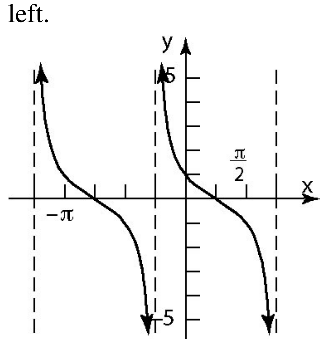


Domain: $\left\{x \mid x \neq \frac{\pi}{6} + k \cdot \frac{\pi}{3}, \text{k is an integer}\right\}$

Range: $(-\infty, \infty)$

28. $y = \cot\left(x + \frac{\pi}{4}\right)$

The graph of $y = \cot x$ is shifted $\frac{\pi}{4}$ units to the left.

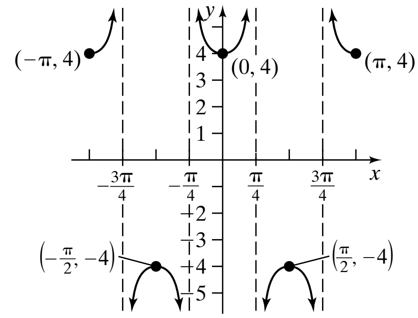


Domain: $\left\{x \mid x \neq -\frac{\pi}{4} + k\pi, \text{k is an integer}\right\}$

Range: $(-\infty, \infty)$

29. $y = 4\sec(2x)$

The graph of $y = \sec x$ is stretched vertically by a factor of 4 and compressed horizontally by a factor of $\frac{1}{2}$.

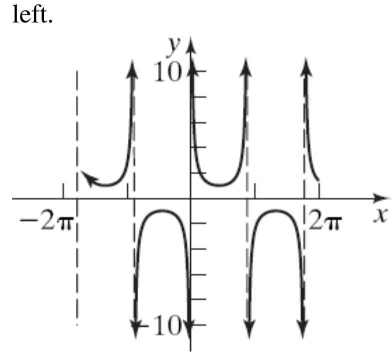


Domain: $\left\{x \mid x \neq \frac{k\pi}{4}, \text{k is an odd integer}\right\}$

Range: $\{y \mid y \leq -4 \text{ or } y \geq 4\}$

30. $y = \csc\left(x + \frac{\pi}{4}\right)$

The graph of $y = \csc x$ is shifted $\frac{\pi}{4}$ units to the left.

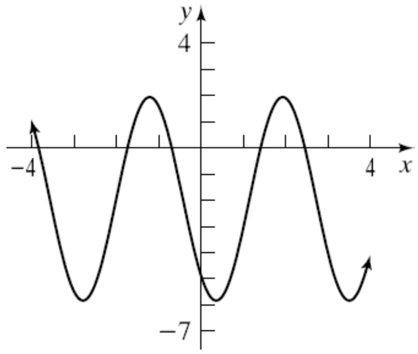


Domain: $\left\{x \mid x \neq -\frac{\pi}{4} + k\pi, \text{k is an integer}\right\}$

Range: $\{y \mid y \leq -1 \text{ or } y \geq 1\}$

31. $y = 4\sin(2x + 4) - 2$

The graph of $y = \sin x$ is shifted left 4 units, compressed horizontally by a factor of $\frac{1}{2}$, stretched vertically by a factor of 4, and shifted down 2 units.

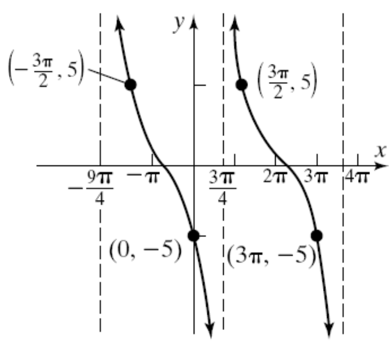


Domain: $(-\infty,\infty)$

Range: $[-6,2]$

32. $y=5\cot\left(\dfrac{x}{3}-\dfrac{\pi}{4}\right)$

The graph of $y=\cot x$ is shifted right $\frac{\pi}{4}$ units, stretched horizontally by a factor of 3, and stretched vertically by a factor of 5.



Domain: $\left\{x\,|\,x\neq\dfrac{3\pi}{4}+k\cdot3\pi,\text{k is an integer}\right\}$

Range: $(-\infty,\infty)$

33. $y=\sin(2x)$

Amplitude = $|\,1\,|=1$; Period = $\dfrac{2\pi}{2}=\pi$

34. $y=-2\cos(3\pi x)$

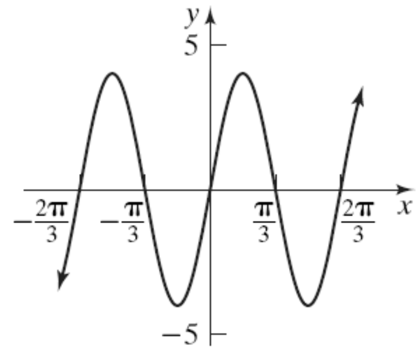
Amplitude = $|-2|=2$; Period = $\dfrac{2\pi}{3\pi}=\dfrac{2}{3}$

35. $y=4\sin(3x)$

Amplitude: $|A|=|4|=4$

Period: $T=\dfrac{2\pi}{\omega}=\dfrac{2\pi}{3}$

Phase Shift: $\dfrac{\phi}{\omega}=\dfrac{0}{3}=0$

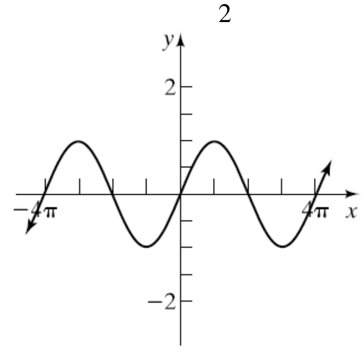


36. $y=-\cos\left(\dfrac{1}{2}x+\dfrac{\pi}{2}\right)$

Amplitude: $|A|=|-1|=1$

Period: $T=\dfrac{2\pi}{\omega}=\dfrac{2\pi}{\frac{1}{2}}=4\pi$

Phase Shift: $\dfrac{\phi}{\omega}=\dfrac{-\frac{\pi}{2}}{\frac{1}{2}}=-\pi$

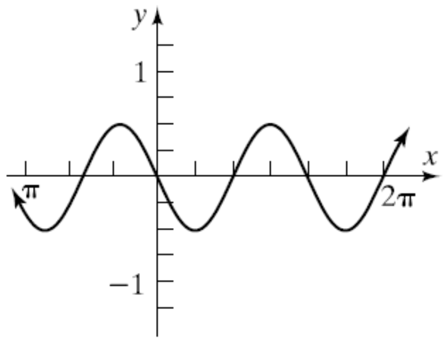


37. $y=\dfrac{1}{2}\sin\left(\dfrac{3}{2}x-\pi\right)$

Amplitude: $|A|=\left|\dfrac{1}{2}\right|=\dfrac{1}{2}$

Period: $T=\dfrac{2\pi}{\omega}=\dfrac{2\pi}{\frac{3}{2}}=\dfrac{4\pi}{3}$

Phase Shift: $\dfrac{\phi}{\omega}=\dfrac{\pi}{\frac{3}{2}}=\dfrac{2\pi}{3}$

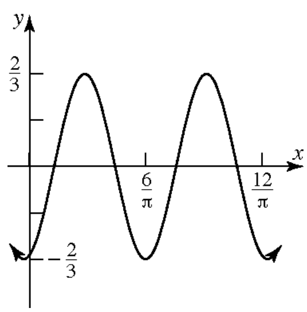


38. $y = -\frac{2}{3}\cos(\pi x - 6)$

Amplitude: $|A| = \left|-\frac{2}{3}\right| = \frac{2}{3}$

Period: $T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi} = 2$

Phase Shift: $\frac{\phi}{\omega} = \frac{6}{\pi}$



39. The graph is a cosine graph with amplitude 5 and period 8π.

Find ω: $8\pi = \frac{2\pi}{\omega}$

$8\pi\omega = 2\pi$

$\omega = \frac{2\pi}{8\pi} = \frac{1}{4}$

40. The graph is a reflected sine graph with amplitude 7 and period 8.

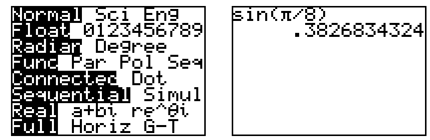
Find ω: $8 = \frac{2\pi}{\omega}$

$8\omega = 2\pi$

$\omega = \frac{2\pi}{8} = \frac{\pi}{4}$

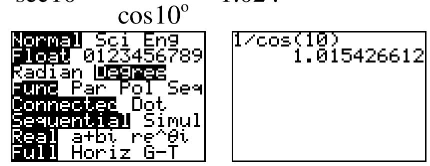
The equation is: $y = -7\sin\left(\frac{\pi}{4}x\right)$.

41. Set the calculator to radian mode: $\sin\frac{\pi}{8} \approx 0.38$.



42. Set the calculator to degree mode:

$\sec 10^\circ = \frac{1}{\cos 10^\circ} \approx 1.02$.



43. Terminal side of θ in Quadrant III implies

$\sin\theta < 0$ $\csc\theta < 0$

$\cos\theta < 0$ $\sec\theta < 0$

$\tan\theta > 0$ $\cot\theta > 0$

44. $\cos\theta > 0$, $\tan\theta < 0$; θ lies in quadrant IV.

45. $P = \left(-\frac{1}{3}, \frac{2\sqrt{2}}{3}\right)$

$\sin t = \frac{2\sqrt{2}}{3}$; $\csc t = \frac{1}{\left(\frac{2\sqrt{2}}{3}\right)} = \frac{3}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{4}$

$\cos t = -\frac{1}{3}$; $\sec t = \frac{1}{\left(-\frac{1}{3}\right)} = -3$

$\tan t = \frac{\left(\frac{2\sqrt{2}}{3}\right)}{\left(-\frac{1}{3}\right)} = \frac{2\sqrt{2}}{3} \cdot \left(-\frac{3}{1}\right) = -2\sqrt{2}$;

$\cot t = \frac{1}{-2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{4}$

46. The point $P = (-2, 5)$ is on a circle of radius $r = \sqrt{(-2)^2 + 5^2} = \sqrt{4+25} = \sqrt{29}$ with the center at the origin. So, we have $x = -2$, $y = 5$, and $r = \sqrt{29}$. Thus, $\sin t = \frac{y}{r} = \frac{5}{\sqrt{29}} = \frac{5\sqrt{29}}{29}$; $\cos t = \frac{x}{r} = \frac{-2}{\sqrt{29}} = -\frac{2\sqrt{29}}{29}$; $\tan t = \frac{y}{x} = -\frac{5}{2}$.

47. The domain of $y = \sec x$ is $\left\{x \,\middle|\, x \neq \text{odd multiple of } \dfrac{\pi}{2}\right\}$.

The range of $y = \sec x$ is $\{y \mid y \leq -1 \text{ or } y \geq 1\}$.

The period is 2π.

48. **a.** $32^\circ 20'35'' = 32 + \dfrac{20}{60} + \dfrac{35}{3600} \approx 32.34^\circ$

b. 63.18°

$0.18^\circ = (0.18)(60') = 10.8'$

$0.8' = (0.8)(60'') = 48''$

Thus, $63.18^\circ = 63^\circ 10'48''$

49. $r = 2$ feet, $\theta = 30^\circ$ or $\theta = \dfrac{\pi}{6}$

$$s = r\theta = 2 \cdot \frac{\pi}{6} = \frac{\pi}{3} \approx 1.047 \text{ feet}$$

$$A = \frac{1}{2} \cdot r^2\theta = \frac{1}{2} \cdot (2)^2 \cdot \frac{\pi}{6} = \frac{\pi}{3} \approx 1.047 \text{ square feet}$$

50. In 30 minutes: $r = 8$ inches, $\theta = 180^\circ$ or $\theta = \pi$

$s = r\theta = 8 \cdot \pi = 8\pi \approx 25.13$ inches

In 20 minutes: $r = 8$ inches, $\theta = 120^\circ$ or $\theta = \dfrac{2\pi}{3}$

$$s = r\theta = 8 \cdot \frac{2\pi}{3} = \frac{16\pi}{3} \approx 16.76 \text{ inches}$$

51. $v = 180$ mi/hr ; $d = \dfrac{1}{2}$ mile

$r = \dfrac{1}{4} = 0.25$ mile

$$\omega = \frac{v}{r} = \frac{180 \text{ mi/hr}}{0.25 \text{ mi}} = 720 \text{ rad/hr} = \frac{720 \text{ rad}}{\text{hr}} \cdot \frac{1 \text{ rev}}{2\pi \text{ rad}} = \frac{360 \text{ rev}}{\pi \text{ hr}} \approx 114.6 \text{ rev/hr}$$

52. Since there are two lights on opposite sides and the light is seen every 5 seconds, the beacon makes 1 revolution every 10 seconds:

$$\omega = \frac{1 \text{ rev}}{10 \text{ sec}} \cdot \frac{2\pi \text{ radians}}{1 \text{ rev}} = \frac{\pi}{5} \text{ radians/second}$$

53. $I(t) = 220\sin\left(30\pi t + \dfrac{\pi}{6}\right), \quad t \geq 0$

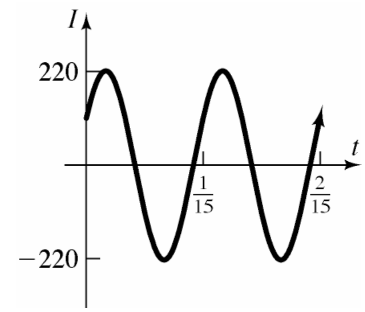
a. Period $= \dfrac{2\pi}{30\pi} = \dfrac{1}{15}$

b. The amplitude is 220.

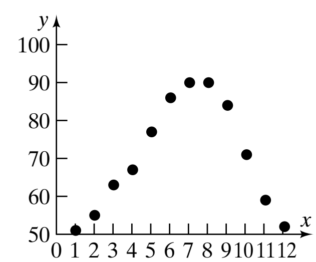
c. The phase shift is:

$$\frac{\phi}{\omega} = \frac{-\frac{\pi}{6}}{30\pi} = -\frac{\pi}{6} \cdot \frac{1}{30\pi} = -\frac{1}{180}$$

d.



54. **a.**



b. Amplitude: $A = \dfrac{90 - 51}{2} = \dfrac{39}{2} = 19.5$

Vertical Shift: $\dfrac{90+51}{2} = \dfrac{141}{2} = 70.5$

$\omega = \dfrac{2\pi}{12} = \dfrac{\pi}{6}$

Phase shift (use $y = 51$, $x = 1$):

$$51 = 19.5\sin\left(\frac{\pi}{6} \cdot 1 - \phi\right) + 70.5$$

$$-19.5 = 19.5\sin\left(\frac{\pi}{6} - \phi\right)$$

$$-1 = \sin\left(\frac{\pi}{6} - \phi\right)$$

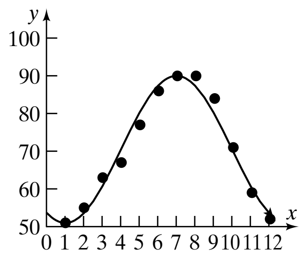
$$-\frac{\pi}{2} = \frac{\pi}{6} - \phi$$

$$\phi = \frac{2\pi}{3}$$

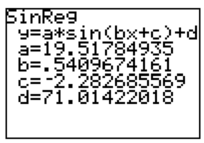
Thus, $y = 19.5\sin\left(\frac{\pi}{6}x - \frac{2\pi}{3}\right) + 70.5$, or

$y = 19.5\sin\left[\frac{\pi}{6}(x-4)\right] + 70.5$.

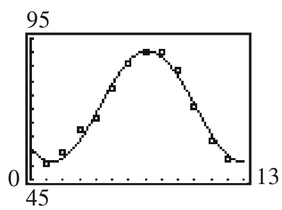
c.



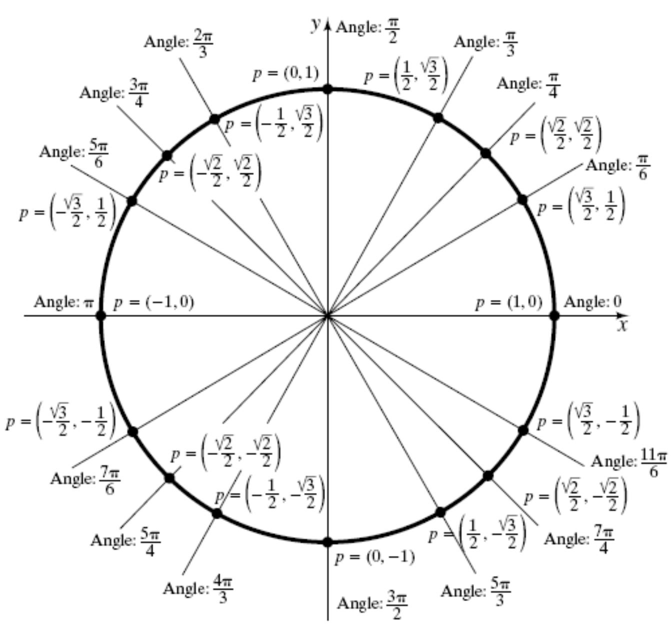
d. $y = 19.52\sin(0.54x - 2.28) + 71.01$



e.



55.



Chapter 6 Test

1. $260° = 260 \cdot 1 \text{ degree}$
$$= 260 \cdot \frac{\pi}{180} \text{ radian}$$
$$= \frac{260\pi}{180} \text{ radian} = \frac{13\pi}{9} \text{ radian}$$

2. $-400° = -400 \cdot 1 \text{ degree}$
$$= -400 \cdot \frac{\pi}{180} \text{ radian}$$
$$= -\frac{400\pi}{180} \text{ radian} = -\frac{20\pi}{9} \text{ radian}$$

3. $13° = 13 \cdot 1 \text{ degree} = 13 \cdot \frac{\pi}{180} \text{ radian} = \frac{13\pi}{180} \text{ radian}$

4. $-\frac{\pi}{8} \text{ radian} = -\frac{\pi}{8} \cdot 1 \text{ radian}$
$$= -\frac{\pi}{8} \cdot \frac{180}{\pi} \text{ degrees} = -22.5°$$

5. $\frac{9\pi}{2} \text{ radian} = \frac{9\pi}{2} \cdot 1 \text{ radian}$
$$= \frac{9\pi}{2} \cdot \frac{180}{\pi} \text{ degrees} = 810°$$

6. $\frac{3\pi}{4} \text{ radian} = \frac{3\pi}{4} \cdot 1 \text{ radian}$
$$= \frac{3\pi}{4} \cdot \frac{180}{\pi} \text{ degrees} = 135°$$

7. $\sin\frac{\pi}{6} = \frac{1}{2}$

8. $\cos\left(-\frac{5\pi}{4}\right) - \cos\left(\frac{3\pi}{4}\right) = \cos\left(-\frac{5\pi}{4} + 2\pi\right) - \cos\left(\frac{3\pi}{4}\right)$
$$= \cos\left(\frac{3\pi}{4}\right) - \cos\left(\frac{3\pi}{4}\right) = 0$$

9. $\cos(-120°) = \cos(120°) = -\frac{1}{2}$

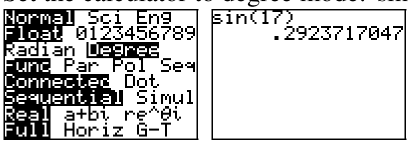
10. $\tan 330° = \tan(150° + 180°) = \tan(150°) = -\frac{\sqrt{3}}{3}$

11. $\sin\frac{\pi}{2} - \tan\frac{19\pi}{4} = \sin\frac{\pi}{2} - \tan\left(\frac{3\pi}{4} + 4\pi\right)$
$$= \sin\frac{\pi}{2} - \tan\left(\frac{3\pi}{4}\right) = 1 - (-1) = 2$$

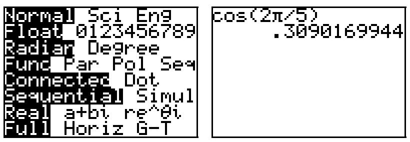
12. $2\sin^2 60° - 3\cos 45° = 2\left(\frac{\sqrt{3}}{2}\right)^2 - 3\left(\frac{\sqrt{2}}{2}\right)$

$= 2\left(\frac{3}{4}\right) - \frac{3\sqrt{2}}{2} = \frac{3}{2} - \frac{3\sqrt{2}}{2} = \frac{3\left(1-\sqrt{2}\right)}{2}$

13. Set the calculator to degree mode: $\sin 17° \approx 0.292$

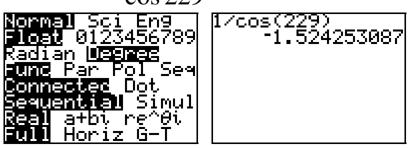


14. Set the calculator to radian mode: $\cos\frac{2\pi}{5} \approx 0.309$



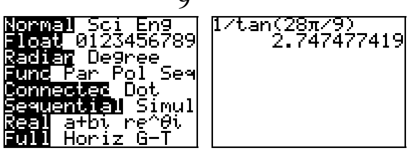
15. Set the calculator to degree mode:

$\sec 229° = \frac{1}{\cos 229°} \approx -1.524$



16. Set the calculator to radian mode:

$\cot\frac{28\pi}{9} = \frac{1}{\tan\frac{28\pi}{9}} \approx 2.747$



17. To remember the sign of each trig function, we primarily need to remember that $\sin\theta$ is positive in quadrants I and II, while $\cos\theta$ is positive in quadrants I and IV. The sign of the other four trig functions can be determined directly from sine and cosine by knowing $\tan\theta = \frac{\sin\theta}{\cos\theta}$, $\sec\theta = \frac{1}{\cos\theta}$, $\csc\theta = \frac{1}{\sin\theta}$, and $\cot\theta = \frac{\cos\theta}{\sin\theta}$.

	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\sec\theta$	$\csc\theta$	$\cot\theta$
θ in QI	+	+	+	+	+	+
θ in QII	+	−	−	−	+	−
θ in QIII	−	−	+	−	−	+
θ in QIV	−	+	−	+	−	−

18. Because $f(x) = \sin x$ is an odd function and since $f(a) = \sin a = \frac{3}{5}$, then $f(-a) = \sin(-a) = -\sin a = -\frac{3}{5}$.

19. $\sin\theta = \frac{5}{7}$ and θ in quadrant II.

Using the Pythagorean Identities:

$\cos^2\theta = 1 - \sin^2\theta = 1 - \left(\frac{5}{7}\right)^2 = 1 - \frac{25}{49} = \frac{24}{49}$

$\cos\theta = \pm\sqrt{\frac{24}{49}} = \pm\frac{2\sqrt{6}}{7}$

Note that $\cos\theta$ must be negative because θ lies in quadrant II. Thus, $\cos\theta = -\frac{2\sqrt{6}}{7}$.

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{5}{7}}{-\frac{2\sqrt{6}}{7}} = \frac{5}{7}\left(-\frac{7}{2\sqrt{6}}\right)\cdot\frac{\sqrt{6}}{\sqrt{6}} = -\frac{5\sqrt{6}}{12}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{5}{7}} = \frac{7}{5}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{-\frac{2\sqrt{6}}{7}} = -\frac{7}{2\sqrt{6}}\cdot\frac{\sqrt{6}}{\sqrt{6}} = -\frac{7\sqrt{6}}{12}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{-\frac{5\sqrt{6}}{12}} = -\frac{12}{5\sqrt{6}}\cdot\frac{\sqrt{6}}{\sqrt{6}} = -\frac{2\sqrt{6}}{5}$

20. $\cos\theta = \frac{2}{3}$ and $\frac{3\pi}{2} < \theta < 2\pi$ (in quadrant IV).

Using the Pythagorean Identities:

$\sin^2\theta = 1 - \cos^2\theta = 1 - \left(\frac{2}{3}\right)^2 = 1 - \frac{4}{9} = \frac{5}{9}$

$\sin\theta = \pm\sqrt{\frac{5}{9}} = \pm\frac{\sqrt{5}}{3}$

Note that $\sin\theta$ must be negative because θ lies in quadrant IV. Thus, $\sin\theta = -\frac{\sqrt{5}}{3}$.

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-\frac{\sqrt{5}}{3}}{\frac{2}{3}} = -\frac{\sqrt{5}}{3}\cdot\frac{3}{2} = -\frac{\sqrt{5}}{2}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{-\frac{\sqrt{5}}{3}} = -\frac{3}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = -\frac{3\sqrt{5}}{5}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{2}{3}} = \frac{3}{2}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{-\frac{\sqrt{5}}{2}} = -\frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$

21. $\tan\theta = -\frac{12}{5}$ and $\frac{\pi}{2} < \theta < \pi$ (in quadrant II)

Using the Pythagorean Identities:

$\sec^2\theta = \tan^2\theta + 1 = \left(-\frac{12}{5}\right)^2 + 1 = \frac{144}{25} + 1 = \frac{169}{25}$

$\sec\theta = \pm\sqrt{\frac{169}{25}} = \pm\frac{13}{5}$

Note that $\sec\theta$ must be negative since θ lies in quadrant II. Thus, $\sec\theta = -\frac{13}{5}$.

$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{-\frac{13}{5}} = -\frac{5}{13}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$, so

$\sin\theta = (\tan\theta)(\cos\theta) = -\frac{12}{5}\left(-\frac{5}{13}\right) = \frac{12}{13}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{12}{13}} = \frac{13}{12}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{-\frac{12}{5}} = -\frac{5}{12}$

22. The point $(2,7)$ lies in quadrant I with $x = 2$ and $y = 7$. Since $x^2 + y^2 = r^2$, we have $r = \sqrt{2^2 + 7^2} = \sqrt{53}$. So,

$\sin\theta = \frac{y}{r} = \frac{7}{\sqrt{53}} = \frac{7}{\sqrt{53}} \cdot \frac{\sqrt{53}}{\sqrt{53}} = \frac{7\sqrt{53}}{53}$.

23. The point $(-5,11)$ lies in quadrant II with $x = -5$ and $y = 11$. Since $x^2 + y^2 = r^2$, we have $r = \sqrt{(-5)^2 + 11^2} = \sqrt{146}$. So,

$\cos\theta = \frac{x}{r} = \frac{-5}{\sqrt{146}} = \frac{-5}{\sqrt{146}} \cdot \frac{\sqrt{146}}{\sqrt{146}} = -\frac{5\sqrt{146}}{146}$.

24. The point $(6,-3)$ lies in quadrant IV with $x = 6$ and $y = -3$. Since $x^2 + y^2 = r^2$, we have

$r = \sqrt{6^2 + (-3)^2} = \sqrt{45} = 3\sqrt{5}$. So,

$\tan\theta = \frac{y}{x} = \frac{-3}{6} = -\frac{1}{2}$

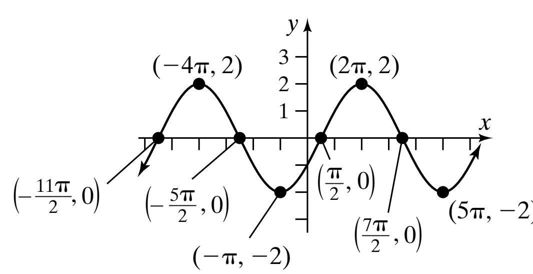
25. Comparing $y = 2\sin\left(\frac{x}{3} - \frac{\pi}{6}\right)$ to $y = A\sin(\omega x - \phi)$, we see that $A = 2$, $\omega = \frac{1}{3}$, and $\phi = \frac{\pi}{6}$. The graph is a sine curve with amplitude $|A| = 2$, period $T = \frac{2\pi}{\omega} = \frac{2\pi}{1/3} = 6\pi$, and phase shift $= \frac{\phi}{\omega} = \frac{\frac{\pi}{6}}{1/3} = \frac{\pi}{2}$. The graph of $y = 2\sin\left(\frac{x}{3} - \frac{\pi}{6}\right)$ will lie between -2 and 2 on the y-axis. One period will begin at $x = \frac{\phi}{\omega} = \frac{\pi}{2}$ and end at $x = \frac{2\pi}{\omega} + \frac{\phi}{\omega} = 6\pi + \frac{\pi}{2} = \frac{13\pi}{2}$. We divide the interval $\left[\frac{\pi}{2}, \frac{13\pi}{2}\right]$ into four subintervals, each of length $\frac{6\pi}{4} = \frac{3\pi}{2}$.

$\left[\frac{\pi}{2}, 2\pi\right], \left[2\pi, \frac{7\pi}{2}\right], \left[\frac{7\pi}{2}, 5\pi\right], \left[5\pi, \frac{13\pi}{2}\right]$

The five key points on the graph are

$\left(\frac{\pi}{2}, 0\right), (2\pi, 2), \left(\frac{7\pi}{2}, 0\right), (5\pi, -2), \left(\frac{13\pi}{2}, 0\right)$

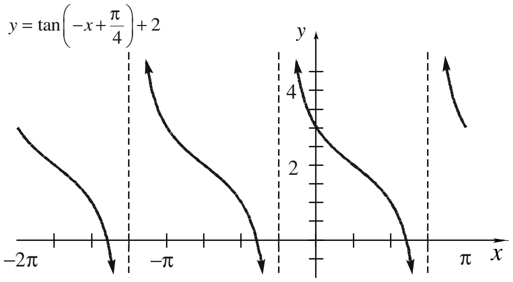
We plot these five points and fill in the graph of the sine function. The graph can then be extended in both directions.



26. $y = \tan\left(-x + \frac{\pi}{4}\right) + 2$

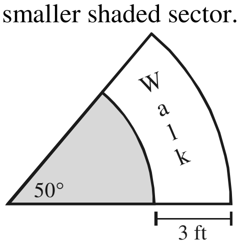
Begin with the graph of $y = \tan x$, and shift it $\frac{\pi}{4}$ units to the left to obtain the graph of

$y=\tan\left(x+\frac{\pi}{4}\right)$. Next, reflect this graph about the y-axis to obtain the graph of $y=\tan\left(-x+\frac{\pi}{4}\right)$. Finally, shift the graph up 2 units to obtain the graph of $y=\tan\left(-x+\frac{\pi}{4}\right)+2$.



27. For a sinusoidal graph of the form $y=A\sin(\omega x-\phi)$, the amplitude is given by $|A|$, the period is given by $\frac{2\pi}{\omega}$, and the phase shift is given by $\frac{\phi}{\omega}$. Therefore, we have $A=-3$, $\omega=3$, and $\phi=3\left(-\frac{\pi}{4}\right)=-\frac{3\pi}{4}$. The equation for the graph is $y=-3\sin\left(3x+\frac{3\pi}{4}\right)$.

28. The area of the walk is the difference between the area of the larger sector and the area of the smaller shaded sector.



The area of the walk is given by

$$A=\frac{1}{2}R^2\theta-\frac{1}{2}r^2\theta,$$
$$=\frac{\theta}{2}\left(R^2-r^2\right)$$

where R is the radius of the larger sector and r is the radius of the smaller sector. The larger radius is 3 feet longer than the smaller radius because the walk is to be 3 feet wide. Therefore,

$R=r+3$, and

$$A=\frac{\theta}{2}\left((r+3)^2-r^2\right)$$
$$=\frac{\theta}{2}\left(r^2+6r+9-r^2\right)$$
$$=\frac{\theta}{2}(6r+9)$$

The shaded sector has an arc length of 25 feet and a central angle of $50^\circ=\frac{5\pi}{18}$ radians. The radius of this sector is $r=\frac{s}{\theta}=\frac{25}{\frac{5\pi}{18}}=\frac{90}{\pi}$ feet.

Thus, the area of the walk is given by

$$A=\frac{\frac{5\pi}{18}}{2}\left(6\left(\frac{90}{\pi}\right)+9\right)=\frac{5\pi}{36}\left(\frac{540}{\pi}+9\right)$$
$$=75+\frac{5\pi}{4}\text{ ft}^2\approx 78.93\text{ ft}^2$$

29. To throw the hammer 83.19 meters, we need

$$s=\frac{{v_0}^2}{g}$$
$$83.19\text{ m}=\frac{{v_0}^2}{9.8\text{ m/s}^2}$$
$${v_0}^2=815.262\text{ m}^2/s^2$$
$$v_0=28.553\text{ m/s}$$

Linear speed and angular speed are related according to the formula $v=r\cdot\omega$. The radius is $r=190\text{ cm}=1.9\text{ m}$. Thus, we have

$$28.553=r\cdot\omega$$
$$28.553=(1.9)\omega$$
$$\omega=15.028\text{ radians per second}$$
$$\omega=15.028\frac{\text{radians}}{\text{sec}}\cdot\frac{60\text{ sec}}{1\text{ min}}\cdot\frac{1\text{ revolution}}{2\pi\text{ radians}}$$
$$\approx 143.5\text{ revolutions per minute (rpm)}$$

To throw the hammer 83.19 meters, Adrian must have been swinging it at a rate of 143.5 rpm upon release.

Chapter 6 Cumulative Review

1.
$$2x^2+x-1=0$$
$$(2x-1)(x+1)=0$$
$$x=\frac{1}{2}\text{ or }x=-1$$

The solution set is $\left\{-1,\frac{1}{2}\right\}$.

2. Slope = −3 , containing (–2,5)

Using $y - y_1 = m(x - x_1)$

$y - 5 = -3(x - (-2))$

$y - 5 = -3(x + 2)$

$y - 5 = -3x - 6$

$y = -3x - 1$

3. radius = 4, center (0,–2)

Using $(x-h)^2 + (y-k)^2 = r^2$

$(x-0)^2 + (y-(-2))^2 = 4^2$

$x^2 + (y+2)^2 = 16$

4. $2x - 3y = 12$

This equation yields a line.

$2x - 3y = 12$

$-3y = -2x + 12$

$y = \frac{2}{3}x - 4$

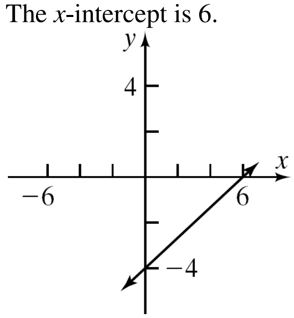
The slope is $m = \frac{2}{3}$ and the y-intercept is -4.

Let $y = 0$: $2x - 3(0) = 12$

$2x = 12$

$x = 6$

The x-intercept is 6.



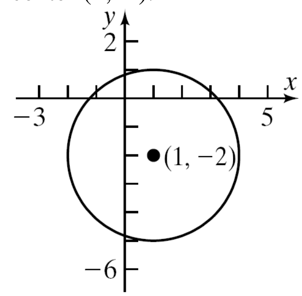
5. $x^2 + y^2 - 2x + 4y - 4 = 0$

$x^2 - 2x + 1 + y^2 + 4y + 4 = 4 + 1 + 4$

$(x-1)^2 + (y+2)^2 = 9$

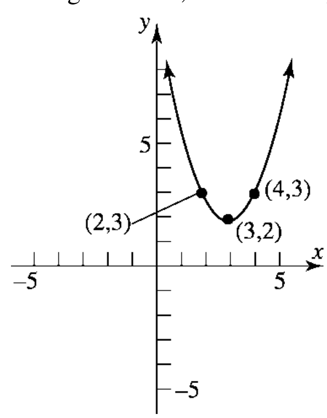
$(x-1)^2 + (y+2)^2 = 3^2$

This equation yields a circle with radius 3 and center (1,–2).

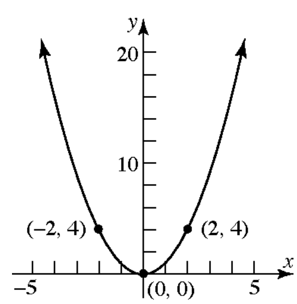


6. $y = (x-3)^2 + 2$

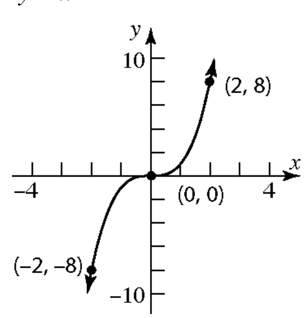
Using the graph of $y = x^2$, horizontally shift to the right 3 units, and vertically shift up 2 units.



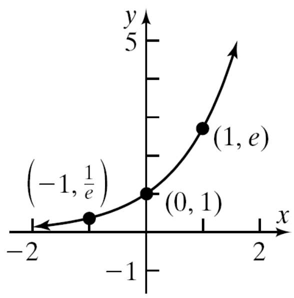
7. a. $y = x^2$



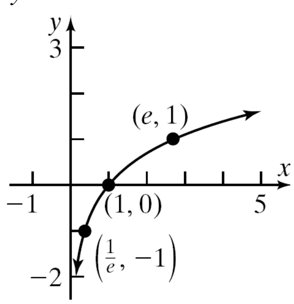
b. $y = x^3$



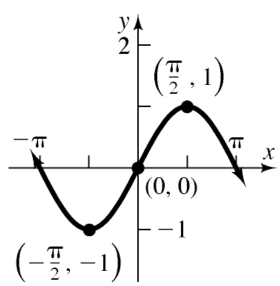
c. $y = e^x$



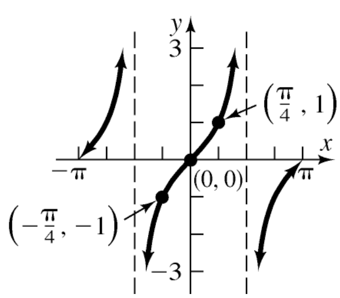
d. $y = \ln x$



e. $y = \sin x$



f. $y = \tan x$



8. $f(x) = 3x - 2$

$$y = 3x - 2$$
$$x = 3y - 2 \quad \text{Inverse}$$
$$x + 2 = 3y$$
$$\frac{x+2}{3} = y$$
$$f^{-1}(x) = \frac{x+2}{3} = \frac{1}{3}(x+2)$$

9. Since $(\sin\theta)^2 + (\cos\theta)^2 = 1$, then

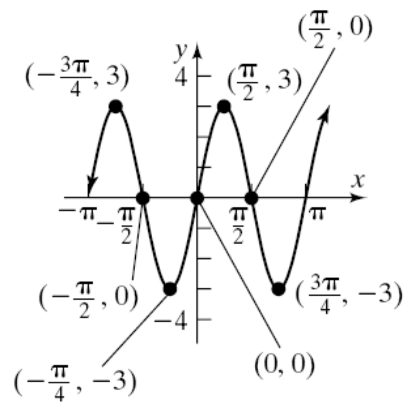
$$(\sin 14^\circ)^2 + (\cos 14^\circ)^2 - 3 = 1 - 3 = -2$$

10. $y = 3\sin(2x)$

Amplitude: $|A| = |3| = 3$

Period: $T = \frac{2\pi}{2} = \pi$

Phase Shift: $\frac{\phi}{\omega} = \frac{0}{2} = 0$



11. $\tan\frac{\pi}{4} - 3\cos\frac{\pi}{6} + \csc\frac{\pi}{6} = 1 - 3\left(\frac{\sqrt{3}}{2}\right) + 2$

$$= 3 - \frac{3\sqrt{3}}{2}$$
$$= \frac{6 - 3\sqrt{3}}{2}$$

12. We need a function of the form $y = A \cdot b^x$, with $b > 0,\ b \neq 1$. The graph contains the points $(0, 2)$ and $(1, 6)$. Therefore, $2 = A \cdot b^0$.

$$2 = A \cdot 1$$
$$A = 2$$

And $y = 2b^x$

$6 = 2b^1$

$b = 3$

So we have the function $y = 2\cdot 3^x$.

13. The graph is a cosine graph with amplitude 3 and period 12.

Find ω: $12 = \frac{2\pi}{\omega}$

$12\omega = 2\pi$

$\omega = \frac{2\pi}{12} = \frac{\pi}{6}$

The equation is: $y = 3\cos\left(\frac{\pi}{6}x\right)$.

14. a. Given points $(-2, 3)$ and $(1,-6)$, we compute the slope as follows:

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6-3}{1-(-2)} = \frac{-9}{3} = -3$$

Using $y - y_1 = m(x - x_1)$:

$y - 3 = -3(x-(-2))$

$y - 3 = -3(x+2)$

$y = -3x - 6 + 3$

$y = -3x - 3$

The linear function is $f(x) = -3x - 3$.

Slope: $m = -3$;

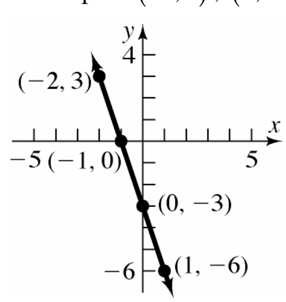
y-intercept: $f(0) = -3(0) - 3 = -3$

x-intercept: $0 = -3x - 3$

$3x = -3$

$x = -1$

Intercepts: $(-1,0)$, $(0,-3)$



b. Given that the graph of $f(x) = ax^2 + bx + c$ has vertex (1, –6) and passes through the point (–2,3), we can conclude $-\frac{b}{2a} = 1$, $f(-2) = 3$, and $f(1) = -6$.

Notice that $-\frac{b}{2a} = 1$

$b = -2a$

Also note that

$f(-2) = 3$

$a(-2)^2 + b(-2) + c = 3$

$4a - 2b + c = 3$

$f(1) = -6$

$a(1)^2 + b(1) + c = -6$

$a + b + c = -6$

Replacing b with $-2a$ in these equations yields: $4a - 2(-2a) + c = 3$

$c = 3 - 8a$

and $a - 2a + c = -6$

$c = -6 + a$

So $3 - 8a = -6 + a$

$9 = 9a \Rightarrow a = 1$

Thus, $b = -2a = -2(1) = -2$

and $c = 3 - 8a = 3 - 8(1) = -5$

Therefore, we have the function

$f(x) = x^2 - 2x - 5 = (x-1)^2 - 6$.

y-intercept: $f(0) = 0^2 - 2(0) - 5 = -5$

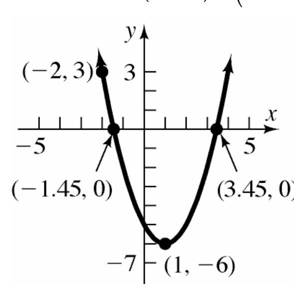
x-intercepts: $0 = x^2 - 2x - 5$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-5)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4+20}}{2} = \frac{2 \pm \sqrt{24}}{2} = \frac{2 \pm 2\sqrt{6}}{2}$$

$= 1 \pm \sqrt{6} \approx -1.45$ or 3.45

Intercepts: $(0,-5)$, $(1-\sqrt{6},0)$, $(1+\sqrt{6},0)$



c. If $f(x) = ae^x$ contains the points (–2,3) and (1,–6), we would have the equations $f(-2) = ae^{-2} = 3$ and $f(1) = ae^1 = -6$.

Note that $ae^{-2} = 3$

$$a = \frac{3}{e^{-2}} = 3e^2$$

But $ae^1 = -6 \Rightarrow a = -\frac{6}{e}$

Since $3e^2 \neq -\frac{6}{e}$, there is no exponential function of the form $f(x) = ae^x$ that contains the points (–2,3) and (1,–6).

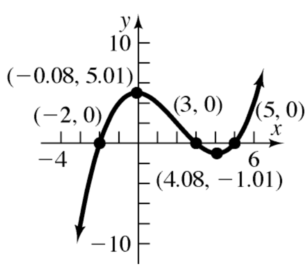
15. a. A polynomial function of degree 3 whose x-intercepts are –2, 3, and 5 will have the form $f(x) = a(x+2)(x-3)(x-5)$, since the x-intercepts correspond to the zeros of the function. Given a y-intercept of 5, we have

$$f(0) = 5$$
$$a(0+2)(0-3)(0-5) = 5$$

$$30a = 5 \Rightarrow a = \frac{5}{30} = \frac{1}{6}$$

Therefore, we have the function

$f(x) = \frac{1}{6}(x+2)(x-3)(x-5)$.

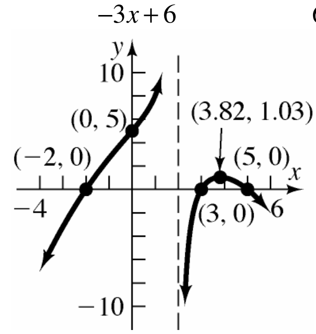


b. A rational function whose x-intercepts are -2, 3, and 5 and that has the line $x = 2$ as a vertical asymptote will have the form $f(x) = \frac{a(x+2)(x-3)(x-5)}{x-2}$, since the x-intercepts correspond to the zeros of the numerator, and the vertical asymptote corresponds to the zero of the denominator. Given a y-intercept of 5, we have

$$f(0) = 5$$
$$\frac{a(0+2)(0-3)(0-5)}{0-2} = 5$$
$$30a = -10 \Rightarrow a = -\frac{1}{3}$$

Therefore, we have the function

$$f(x) = \frac{-\frac{1}{3}(x+2)(x-3)(x-5)}{x-2}$$
$$= \frac{(x+2)(x^2-8x+15)}{-3(x-2)}$$
$$= \frac{(x^3-6x^2-x+30)}{-3(x-2)}$$
$$= \frac{x^3-6x^2-x+30}{-3x+6} = \frac{x^3-6x^2-x+30}{6-3x}$$



Chapter 6 Projects

Project I – Internet Based Project

Project II

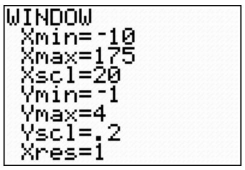
1. November 15: High tide: 11:18 am and 11:15 pm
   November 19: low tide: 7:17 am and 8:38 pm

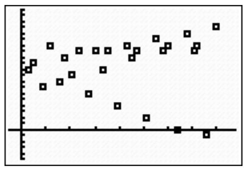
2. The low tide was below sea level. It is measured against calm water at sea level.

3.

Nov	Low Tide			Low Tide			High Tide			High Tide		
	Time	Ht (ft)	t	Time	Ht (ft)	t	Time	Ht (ft)	t	Time	Ht (ft)	t
14 0-24	6:26a	2.0	6.43	4:38p	1.4	16.63	9:29a	2.2	9.48	11:14p	2.8	23.23
15	6:22a	1.6	30.37	5:34p	1.8	41.57	11:18a	2.4	35.3	11:15p	2.6	47.25

24-48												
16 48-72	6:28a	1.2	54.47	6:25p	2.0	66.42	12:37p	2.6	60.62	11:16p	2.6	71.27
17 72-96	6:40a	0.8	78.67	7:12p	2.4	91.2	1:38p	2.8	85.63	11:16p	2.6	95.27
18 96-120	6:56a	0.4	102.93	7:57p	2.6	115.95	2:27p	3.0	110.45	11:14p	2.8	119.23
19 120-144	7:17a	0.0	127.28	8:38p	2.6	140.63	3:10p	3.2	135.17	11:05p	2.8	143.08
20 144-168	7:43a	-0.2	151.72				3:52p	3.4	159.87			





4. The data seems to take on a sinusoidal shape (oscillates). The period is approximately 12 hours. The amplitude varies each day:
Nov 14: 0.1, 0.7
Nov 15: 0.4, 0.4
Nov 16: 0.7, 0.3
Nov 17: 1.0, 0.1
Nov 18: 1.3, 0.1
Nov 19: 1.6, 0.1
Nov 20: 1.8

5. Average of the amplitudes: 0.66
Period : 12
Average of vertical shifts: 2.15 (approximately)
There is no phase shift. However, keeping in mind the vertical shift, the amplitude

$y = A\sin(Bx) + D$

$A = 0.66 \qquad 12 = \dfrac{2\pi}{B} \qquad D = 2.15$

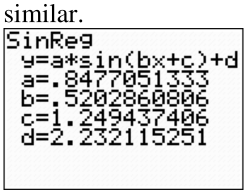
$B = \dfrac{\pi}{6} \approx 0.52$

Thus, $y = 0.66\sin(0.52x) + 2.15$

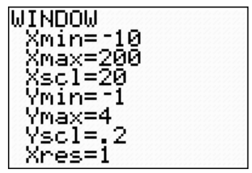
(Answers may vary)

6. $y = 0.848\sin(0.52x + 1.25) + 2.23$

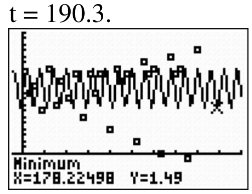
The two functions are not the same, but they are similar.

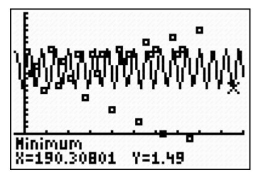


7. Find the high and low tides on November 21 which are the min and max that lie between $t = 168$ and $t = 192$. Looking at the graph of the equation for part (5) and using MAX/MIN for values between $t = 168$ and $t = 192$:

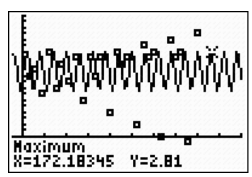


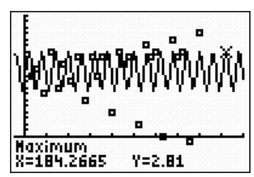
Low tides of 1.49 feet when t = 178.2 and t = 190.3.





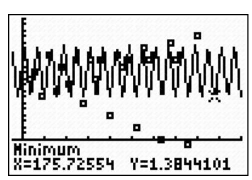
High tides of 2.81 feet occur when t = 172.2 and t = 184.3.

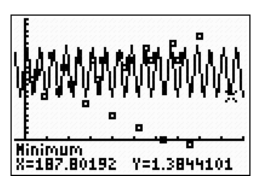




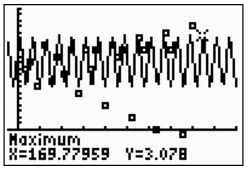
Looking at the graph for the equation in part (6) and using MAX/MIN for values between t = 168 and t = 192:

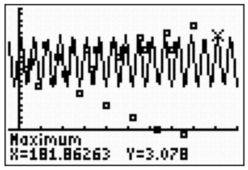
A low tide of 1.38 feet occurs when t = 175.7 and $t = 187.8$.



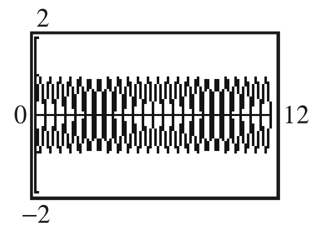


A high tide of 3.08 feet occurs when t = 169.8 and $t = 181.9$.





8. The low and high tides vary because of the moon phase. The moon has a gravitational pull on the water on Earth.



Project III

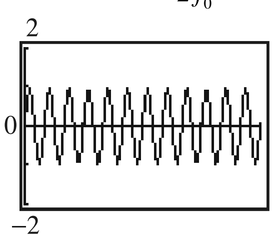
1. $s(t) = 1\sin\left(2\pi f_0 t\right)$

2. $T_0 = \dfrac{2\pi}{2\pi f_0} = \dfrac{1}{f_0}$

3.

t	0	$\frac{1}{4f_0}$	$\frac{1}{2f_0}$	$\frac{3}{4f_0}$	$\frac{1}{f_0}$
$s(t)$	0	1	0	−1	0

4. Let $f_0 = 1 = 1$. Let $0 \le x \le 12$, with $\Delta x = 0.5$. Label the graph as $0 \le x \le 12T_0$, and each tick mark is at $\Delta x = \dfrac{1}{2f_0}$.



$12T_0 = \dfrac{12}{f_0}$

5. $t = \dfrac{1}{4f_0}$, $t = \dfrac{5}{4f_0}$, $t = \dfrac{9}{4f_0}$, … , $t = \dfrac{45}{4f_0}$

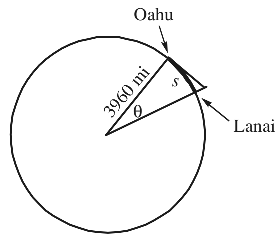
6. M = 0 1 0 $\rightarrow$ P = 0 π 0

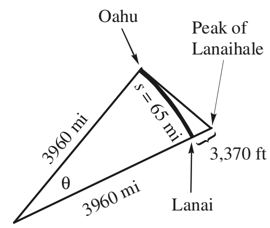
7. $S_0(t) = 1\sin(2\pi f_0 t + 0)$, $S_1(t) = 1\sin(2\pi f_0 t + \pi)$

8. $[0, 4T_0]$ S_0
$[4T_0, 8T_0]$ S_1
$[8T_0, 12T_0]$ S_0

Project IV

1. Lanai:





2. $s = r\theta$

$\theta = \dfrac{s}{r} = \dfrac{65}{3960} = 0.0164$

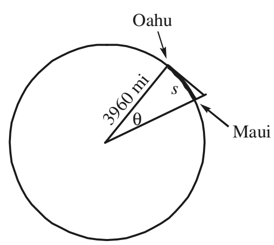
3. $\dfrac{3960}{3960 + h} = \cos(0.164)$

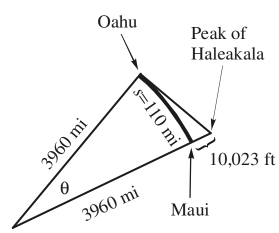
$3960 = 0.9999(3960 + h)$

$h = 0.396$ miles

$0.396 \times 5280 = 2090$ feet

4. Maui:





$\theta = \dfrac{s}{r} = \dfrac{110}{3960} = 0.0278$

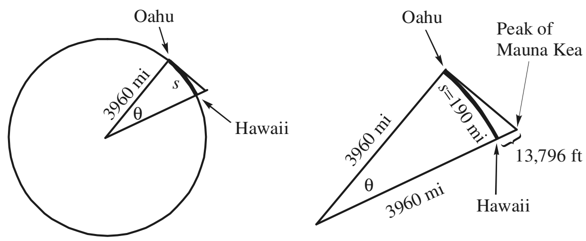
$\dfrac{3960}{3960 + h} = \cos(0.278)$

$3960 = 0.9996(3960 + h)$

$h = 1.584$ miles

$h = 1.584 \times 5280 = 8364$ feet

Hawaii:



$$\theta = \frac{s}{r} = \frac{190}{3960} = 0.0480$$

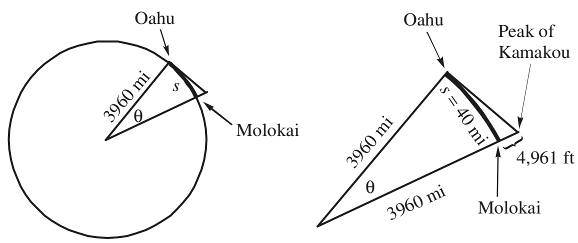
$$\frac{3960}{3960+h} = \cos(0.480)$$

$$3960 = 0.9988(3960+h)$$

$$h = 4.752 \text{ miles}$$

$$h = 4.752 \times 5280 = 25,091 \text{ feet}$$

Molokai:



$$\theta = \frac{s}{r} = \frac{40}{3960} = 0.0101$$

$$\frac{3960}{3960+h} = \cos(0.0101)$$

$$3960 = 0.9999(3960+h)$$

$$h = 0.346 \text{ miles}$$

$$h = 0.346x5280 = 2090 \text{ feet}$$

5. Kamakou, Haleakala, and Lanaihale are all visible from Oahu.

Project V

Answers will vary.