Chapter 5

Exponential and Logarithmic Functions

Section 5.1

1. $f(3) = -4(3)^2 + 5(3)$
$= -4(9) + 15$
$= -36 + 15$
$= -21$

2. $f(3x) = 4 - 2(3x)^2$
$= 4 - 2(9x^2)$
$= 4 - 18x^2$

3. $f(x) = \dfrac{x^2 - 1}{x^2 - 25}$
$x^2 - 25 \neq 0$
$(x+5)(x-5) \neq 0$
$x \neq -5, \quad x \neq 5$
Domain: $\{x \mid x \neq -5,\ x \neq 5\}$

4. composite function; $f(g(x))$

5. False: $f(g(x)) = (f \circ g)(x)$

6. False. The domain of $(f \circ g)(x)$ is a subset of the domain of $g(x)$.

7. **a.** $(f \circ g)(1) = f(g(1)) = f(0) = -1$

b. $(f \circ g)(-1) = f(g(-1)) = f(0) = -1$

c. $(g \circ f)(-1) = g(f(-1)) = g(-3) = 8$

d. $(g \circ f)(0) = g(f(0)) = g(-1) = 0$

e. $(g \circ g)(-2) = g(g(-2)) = g(3) = 8$

f. $(f \circ f)(-1) = f(f(-1)) = f(-3) = -7$

8. **a.** $(f \circ g)(1) = f(g(1)) = f(0) = 5$

b. $(f \circ g)(2) = f(g(2)) = f(-3) = 11$

c. $(g \circ f)(2) = g(f(2)) = g(1) = 0$

d. $(g \circ f)(3) = g(f(3)) = g(-1) = 0$

e. $(g \circ g)(1) = g(g(1)) = g(0) = 1$

f. $(f \circ f)(3) = f(f(3)) = f(-1) = 7$

9. **a.** $(g \circ f)(-1) = g(f(-1)) = g(1) = 4$

b. $(g \circ f)(0) = g(f(0)) = g(0) = 5$

c. $(f \circ g)(-1) = f(g(-1)) = f(3) = -1$

d. $(f \circ g)(4) = f(g(4)) = f(2) = -2$

10. **a.** $(g \circ f)(1) = g(f(1)) = g(-1) = 3$

b. $(g \circ f)(5) = g(f(5)) = g(1) = 4$

c. $(f \circ g)(0) = f(g(0)) = f(5) = 1$

d. $(f \circ g)(2) = f(g(2)) = f(2) = -2$

11. $f(x) = 2x \qquad g(x) = 3x^2 + 1$

a. $(f \circ g)(4) = f(g(4))$
$= f(3(4)^2 + 1)$
$= f(49)$
$= 2(49)$
$= 98$

b. $(g \circ f)(2) = g(f(2))$
$= g(2 \cdot 2)$
$= g(4)$
$= 3(4)^2 + 1$
$= 48 + 1$
$= 49$

c. $(f \circ f)(1) = f(f(1))$
$= f(2(1))$
$= f(2)$
$= 2(2)$
$= 4$

d. $(g \circ g)(0) = g(g(0))$
$$= g\left(3(0)^2 + 1\right)$$
$$= g(1)$$
$$= 3(1)^2 + 1$$
$$= 4$$

12. $f(x) = 3x + 2 \qquad g(x) = 2x^2 - 1$

a. $(f \circ g)(4) = f(g(4))$
$$= f\left(2(4)^2 - 1\right)$$
$$= f(31)$$
$$= 3(31) + 2$$
$$= 95$$

b. $(g \circ f)(2) = g(f(2))$
$$= g(3(2) + 2)$$
$$= g(8)$$
$$= 2(8)^2 - 1$$
$$= 128 - 1$$
$$= 127$$

c. $(f \circ f)(1) = f(f(1))$
$$= f\left(3(1) + 2\right)$$
$$= f(5)$$
$$= 3(5) + 2$$
$$= 17$$

d. $(g \circ g)(0) = g(g(0))$
$$= g\left(2(0)^2 - 1\right)$$
$$= g(-1)$$
$$= 2(-1)^2 - 1$$
$$= 1$$

13. $f(x) = 4x^2 - 3 \qquad g(x) = 3 - \frac{1}{2}x^2$

a. $(f \circ g)(4) = f(g(4))$
$$= f\left(3 - \frac{1}{2}(4)^2\right)$$
$$= f(-5)$$
$$= 4(-5)^2 - 3$$
$$= 97$$

b. $(g \circ f)(2) = g(f(2))$
$$= g(4(2)^2 - 3)$$
$$= g(13)$$
$$= 3 - \frac{1}{2}(13)^2$$
$$= 3 - \frac{169}{2}$$
$$= -\frac{163}{2}$$

c. $(f \circ f)(1) = f(f(1))$
$$= f(4(1)^2 - 3)$$
$$= f(1)$$
$$= 4(1)^2 - 3$$
$$= 1$$

d. $(g \circ g)(0) = g(g(0))$
$$= g\left(3 - \frac{1}{2}(0)^2\right)$$
$$= g(3)$$
$$= 3 - \frac{1}{2}(3)^2$$
$$= 3 - \frac{9}{2}$$
$$= -\frac{3}{2}$$

14. $f(x) = 2x^2 \qquad g(x) = 1 - 3x^2$

a. $(f \circ g)(4) = f(g(4))$
$$= f\left(1 - 3(4)^2\right)$$
$$= f(-47)$$
$$= 2(-47)^2$$
$$= 4418$$

b. $(g \circ f)(2) = g(f(2))$
$$= g(2(2)^2)$$
$$= g(8)$$
$$= 1 - 3(8)^2$$
$$= 1 - 192$$
$$= -191$$

c. $(f \circ f)(1) = f(f(1))$
$$= f\left(2(1)^2\right)$$
$$= f(2)$$
$$= 2(2)^2$$
$$= 8$$

d. $$\begin{aligned}(g\circ g)(0) &= g(g(0))\\ &= g\left(1-3(0)^2\right)\\ &= g(1)\\ &= 1-3(1)^2\\ &= 1-3\\ &= -2\end{aligned}$$

15. $f(x)=\sqrt{x}\qquad g(x)=2x$

a. $$\begin{aligned}(f\circ g)(4) &= f(g(4))\\ &= f\left(2(4)\right)\\ &= f(8)\\ &= \sqrt{8}\\ &= 2\sqrt{2}\end{aligned}$$

b. $$\begin{aligned}(g\circ f)(2) &= g(f(2))\\ &= g\left(\sqrt{2}\right)\\ &= 2\sqrt{2}\end{aligned}$$

c. $$\begin{aligned}(f\circ f)(1) &= f(f(1))\\ &= f\left(\sqrt{1}\right)\\ &= f(1)\\ &= \sqrt{1}\\ &= 1\end{aligned}$$

d. $$\begin{aligned}(g\circ g)(0) &= g(g(0))\\ &= g\left(2(0)\right)\\ &= g(0)\\ &= 2(0)\\ &= 0\end{aligned}$$

16. $f(x)=\sqrt{x+1}\qquad g(x)=3x$

a. $$\begin{aligned}(f\circ g)(4) &= f(g(4))\\ &= f\left(3(4)\right)\\ &= f(12)\\ &= \sqrt{12+1}\\ &= \sqrt{13}\end{aligned}$$

b. $$\begin{aligned}(g\circ f)(2) &= g(f(2))\\ &= g\left(\sqrt{2+1}\right)\\ &= g\left(\sqrt{3}\right)\\ &= 3\sqrt{3}\end{aligned}$$

c. $$\begin{aligned}(f\circ f)(1) &= f(f(1))\\ &= f\left(\sqrt{1+1}\right)\\ &= f\left(\sqrt{2}\right)\\ &= \sqrt{\sqrt{2}+1}\end{aligned}$$

d. $$\begin{aligned}(g\circ g)(0) &= g(g(0))\\ &= g\left(3(0)\right)\\ &= g(0)\\ &= 3(0)\\ &= 0\end{aligned}$$

17. $f(x)=\left|x\right|\qquad g(x)=\dfrac{1}{x^2+1}$

a. $$\begin{aligned}(f\circ g)(4) &= f(g(4))\\ &= f\left(\frac{1}{4^2+1}\right)\\ &= f\left(\frac{1}{17}\right)\\ &= \left|\frac{1}{17}\right|\\ &= \frac{1}{17}\end{aligned}$$

b. $$\begin{aligned}(g\circ f)(2) &= g(f(2))\\ &= g\left(\left|2\right|\right)\\ &= g(2)\\ &= \frac{1}{2^2+1}\\ &= \frac{1}{5}\end{aligned}$$

c. $$\begin{aligned}(f\circ f)(1) &= f(f(1))\\ &= f\left(\left|1\right|\right)\\ &= f(1)\\ &= \left|1\right|\\ &= 1\end{aligned}$$

d. $$\begin{aligned}(g\circ g)(0) &= g(g(0))\\ &= g\left(\frac{1}{0^2+1}\right)\\ &= g(1)\\ &= \frac{1}{1^2+1}\\ &= \frac{1}{2}\end{aligned}$$

18. $f(x)=|x-2| \qquad g(x)=\dfrac{3}{x^2+2}$

a. $$\begin{aligned}(f\circ g)(4)&=f(g(4))\\&=f\left(\frac{3}{4^2+2}\right)\\&=f\left(\frac{3}{18}\right)\\&=f\left(\frac{1}{6}\right)\\&=\left|\frac{1}{6}-2\right|\\&=\left|-\frac{11}{6}\right|\\&=\frac{11}{6}\end{aligned}$$

b. $$\begin{aligned}(g\circ f)(2)&=g(f(2))\\&=g(|2-2|)\\&=g(0)\\&=\frac{3}{0^2+2}\\&=\frac{3}{2}\end{aligned}$$

c. $$\begin{aligned}(f\circ f)(1)&=f(f(1))\\&=f(|1-2|)\\&=f(1)\\&=|1-2|\\&=1\end{aligned}$$

d. $$\begin{aligned}(g\circ g)(0)&=g(g(0))\\&=g\left(\frac{3}{0^2+2}\right)\\&=g\left(\frac{3}{2}\right)\\&=\frac{3}{\left(\frac{3}{2}\right)^2+2}\\&=\frac{3}{\frac{17}{4}}\\&=\frac{12}{17}\end{aligned}$$

19. $f(x)=\dfrac{3}{x+1} \qquad g(x)=\sqrt[3]{x}$

a. $$\begin{aligned}(f\circ g)(4)&=f(g(4))\\&=f\left(\sqrt[3]{4}\right)\\&=\frac{3}{\sqrt[3]{4}+1}\end{aligned}$$

b. $$\begin{aligned}(g\circ f)(2)&=g(f(2))\\&=g\left(\frac{3}{2+1}\right)\\&=g\left(\frac{3}{3}\right)\\&=g(1)\\&=\sqrt[3]{1}\\&=1\end{aligned}$$

c. $$\begin{aligned}(f\circ f)(1)&=f(f(1))\\&=f\left(\frac{3}{1+1}\right)\\&=f\left(\frac{3}{2}\right)\\&=\frac{3}{\frac{3}{2}+1}\\&=\frac{3}{\frac{5}{2}}\\&=\frac{6}{5}\end{aligned}$$

d. $$\begin{aligned}(g\circ g)(0)&=g(g(0))\\&=g\left(\sqrt[3]{0}\right)\\&=g(0)\\&=\sqrt[3]{0}\\&=0\end{aligned}$$

20. $f(x) = x^{3/2}$ $\quad g(x) = \dfrac{2}{x+1}$

a.
$$\begin{aligned}(f \circ g)(4) &= f(g(4)) \\ &= f\left(\frac{2}{4+1}\right) \\ &= f\left(\frac{2}{5}\right) \\ &= \left(\frac{2}{5}\right)^{3/2} \\ &= \sqrt{\left(\frac{2}{5}\right)^3} \\ &= \sqrt{\frac{8}{125}} \\ &= \frac{2\sqrt{2}}{5\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} \\ &= \frac{2\sqrt{10}}{25}\end{aligned}$$

b.
$$\begin{aligned}(g \circ f)(2) &= g(f(2)) \\ &= g\left(2^{3/2}\right) \\ &= g\left(\sqrt{2^3}\right) \\ &= g\left(2\sqrt{2}\right) \\ &= \frac{2}{2\sqrt{2}+1} \quad \text{or} \quad \frac{4\sqrt{2}-2}{7}\end{aligned}$$

c.
$$\begin{aligned}(f \circ f)(1) &= f(f(1)) \\ &= f\left(1^{3/2}\right) \\ &= f(1) \\ &= 1^{3/2} \\ &= 1\end{aligned}$$

d.
$$\begin{aligned}(g \circ g)(0) &= g(g(0)) \\ &= g\left(\frac{2}{0+1}\right) \\ &= g(2) \\ &= \frac{2}{2+1} \\ &= \frac{2}{3}\end{aligned}$$

21. The domain of g is $\{x \mid x \neq 0\}$. The domain of f is $\{x \mid x \neq 1\}$. Thus, $g(x) \neq 1$, so we solve:

$$\begin{aligned}g(x) &= 1 \\ \frac{2}{x} &= 1 \\ x &= 2\end{aligned}$$

Thus, $x \neq 2$; so the domain of $f \circ g$ is $\{x \mid x \neq 0, x \neq 2\}$.

22. The domain of g is $\{x \mid x \neq 0\}$. The domain of f is $\{x \mid x \neq -3\}$. Thus, $g(x) \neq -3$, so we solve:

$$\begin{aligned}g(x) &= -3 \\ -\frac{2}{x} &= -3 \\ x &= \frac{2}{3}\end{aligned}$$

Thus, $x \neq \frac{2}{3}$; so the domain of $f \circ g$ is $\left\{x \,\middle|\, x \neq 0, x \neq \frac{2}{3}\right\}$.

23. The domain of g is $\{x \mid x \neq 0\}$. The domain of f is $\{x \mid x \neq 1\}$. Thus, $g(x) \neq 1$, so we solve:

$$\begin{aligned}g(x) &= 1 \\ -\frac{4}{x} &= 1 \\ x &= -4\end{aligned}$$

Thus, $x \neq -4$; so the domain of $f \circ g$ is $\{x \mid x \neq -4, x \neq 0\}$.

24. The domain of g is $\{x \mid x \neq 0\}$. The domain of f is $\{x \mid x \neq -3\}$. Thus, $g(x) \neq -3$, so we solve:

$$\begin{aligned}g(x) &= -3 \\ \frac{2}{x} &= -3 \\ x &= -\frac{2}{3}\end{aligned}$$

Thus, $x \neq -\frac{2}{3}$; so the domain of $f \circ g$ is $\left\{x \,\middle|\, x \neq -\frac{2}{3}, x \neq 0\right\}$.

25. The domain of g is $\{x \mid x \text{ is any real number}\}$. The domain of f is $\{x \mid x \geq 0\}$. Thus, $g(x) \geq 0$, so we solve:

$$2x+3 \geq 0$$
$$x \geq -\frac{3}{2}$$

Thus, the domain of $f \circ g$ is $\left\{x \,\middle|\, x \geq -\frac{3}{2}\right\}$.

26. The domain of g is $\{x \mid x \leq 1\}$. The domain of f is $\{x \mid x \text{ is any real number}\}$. Thus, the domain of $f \circ g$ is $\{x \mid x \leq 1\}$.

27. The domain of g is $\{x \mid x \geq 1\}$. The domain of f is $\{x \mid x \text{ is any real number}\}$. Thus, the domain of $f \circ g$ is $\{x \mid x \geq 1\}$.

28. The domain of g is $\{x \mid x \geq 2\}$. The domain of f is $\{x \mid x \text{ is any real number}\}$. Thus, the domain of $f \circ g$ is $\{x \mid x \geq 2\}$.

29. $f(x) = 2x+3 \qquad g(x) = 3x$

The domain of f is $\{x \mid x \text{ is any real number}\}$.

The domain of g is $\{x \mid x \text{ is any real number}\}$.

a. $$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f(3x) \\ &= 2(3x)+3 \\ &= 6x+3\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

b. $$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= g(2x+3) \\ &= 3(2x+3) \\ &= 6x+9\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

c. $$\begin{aligned}(f \circ f)(x) &= f(f(x)) \\ &= f(2x+3) \\ &= 2(2x+3)+3 \\ &= 4x+6+3 \\ &= 4x+9\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

d. $$\begin{aligned}(g \circ g)(x) &= g(g(x)) \\ &= g(3x) \\ &= 3(3x) \\ &= 9x\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

30. $f(x) = -x \qquad g(x) = 2x-4$

The domain of f is $\{x \mid x \text{ is any real number}\}$.

The domain of g is $\{x \mid x \text{ is any real number}\}$.

a. $$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f(2x-4) \\ &= -(2x-4) \\ &= -2x+4\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

b. $$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= g(-x) \\ &= 2(-x)-4 \\ &= -2x-4\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

c. $$\begin{aligned}(f \circ f)(x) &= f(f(x)) \\ &= f(-x) \\ &= -(-x) \\ &= x\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

d. $$\begin{aligned}(g \circ g)(x) &= g(g(x)) \\ &= g(2x-4) \\ &= 2(2x-4)-4 \\ &= 4x-8-4 \\ &= 4x-12\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

31. $f(x)=3x+1 \qquad g(x)=x^2$

The domain of *f* is $\{x \mid x \text{ is any real number}\}$.

The domain of *g* is $\{x \mid x \text{ is any real number}\}$.

a. $$\begin{aligned}(f\circ g)(x)&=f(g(x))\\&=f\left(x^2\right)\\&=3x^2+1\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

b. $$\begin{aligned}(g\circ f)(x)&=g(f(x))\\&=g(3x+1)\\&=(3x+1)^2\\&=9x^2+6x+1\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

c. $$\begin{aligned}(f\circ f)(x)&=f(f(x))\\&=f(3x+1)\\&=3(3x+1)+1\\&=9x+3+1\\&=9x+4\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

d. $$\begin{aligned}(g\circ g)(x)&=g(g(x))\\&=g\left(x^2\right)\\&=\left(x^2\right)^2\\&=x^4\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

32. $f(x)=x+1 \qquad g(x)=x^2+4$

The domain of *f* is $\{x \mid x \text{ is any real number}\}$.

The domain of *g* is $\{x \mid x \text{ is any real number}\}$.

a. $$\begin{aligned}(f\circ g)(x)&=f(g(x))\\&=f\left(x^2+4\right)\\&=x^2+4+1\\&=x^2+5\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

b. $$\begin{aligned}(g\circ f)(x)&=g(f(x))\\&=g(x+1)\\&=(x+1)^2+4\\&=x^2+2x+1+4\\&=x^2+2x+5\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

c. $$\begin{aligned}(f\circ f)(x)&=f(f(x))\\&=f(x+1)\\&=(x+1)+1\\&=x+2\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

d. $$\begin{aligned}(g\circ g)(x)&=g(g(x))\\&=g\left(x^2+4\right)\\&=\left(x^2+4\right)^2+4\\&=x^4+8x^2+16+4\\&=x^4+8x^2+20\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

33. $f(x)=x^2 \qquad g(x)=x^2+4$

The domain of *f* is $\{x \mid x \text{ is any real number}\}$.

The domain of *g* is $\{x \mid x \text{ is any real number}\}$.

a. $$\begin{aligned}(f\circ g)(x)&=f(g(x))\\&=f\left(x^2+4\right)\\&=\left(x^2+4\right)^2\\&=x^4+8x^2+16\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

b. $$\begin{aligned}(g\circ f)(x)&=g(f(x))\\&=g\left(x^2\right)\\&=\left(x^2\right)^2+4\\&=x^4+4\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

c. $$\begin{aligned}(f\circ f)(x)&=f(f(x))\\&=f\left(x^2\right)\\&=\left(x^2\right)^2\\&=x^4\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

d. $$\begin{aligned}(g\circ g)(x)&=g(g(x))\\&=g\left(x^2+4\right)\\&=\left(x^2+4\right)^2+4\\&=x^4+8x^2+16+4\\&=x^4+8x^2+20\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

34. $f(x)=x^2+1 \qquad g(x)=2x^2+3$

The domain of f is $\{x \mid x \text{ is any real number}\}$.

The domain of g is $\{x \mid x \text{ is any real number}\}$.

a. $$\begin{aligned}(f\circ g)(x)&=f(g(x))\\&=f\left(2x^2+3\right)\\&=\left(2x^2+3\right)^2+1\\&=4x^4+12x^2+9+1\\&=4x^4+12x^2+10\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

b. $$\begin{aligned}(g\circ f)(x)&=g(f(x))\\&=g\left(x^2+1\right)\\&=2\left(x^2+1\right)^2+3\\&=2\left(x^4+2x^2+1\right)+3\\&=2x^4+4x^2+2+3\\&=2x^4+4x^2+5\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

c. $$\begin{aligned}(f\circ f)(x)&=f(f(x))\\&=f\left(x^2+1\right)\\&=\left(x^2+1\right)^2+1\\&=x^4+2x^2+1+1\\&=x^4+2x^2+2\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

d. $$\begin{aligned}(g\circ g)(x)&=g(g(x))\\&=g\left(2x^2+3\right)\\&=2\left(2x^2+3\right)^2+3\\&=2\left(4x^4+12x^2+9\right)+3\\&=8x^4+24x^2+18+3\\&=8x^4+24x^2+21\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

35. $f(x)=\dfrac{3}{x-1} \qquad g(x)=\dfrac{2}{x}$

The domain of f is $\{x \mid x\neq 1\}$. The domain of g is $\{x \mid x\neq 0\}$.

a. $$\begin{aligned}(f\circ g)(x)&=f(g(x))\\&=f\left(\frac{2}{x}\right)\\&=\frac{3}{\frac{2}{x}-1}\\&=\frac{3}{\frac{2-x}{x}}\\&=\frac{3x}{2-x}\end{aligned}$$

Domain $\{x \mid x\neq 0,\ x\neq 2\}$.

b. $$\begin{aligned}(g\circ f)(x)&=g(f(x))\\&=g\left(\frac{3}{x-1}\right)\\&=\frac{2}{\frac{3}{x-1}}\\&=\frac{2(x-1)}{3}\end{aligned}$$

Domain $\{x \mid x\neq 1\}$

c. $$\begin{aligned}(f\circ f)(x)&=f(f(x))\\&=f\left(\frac{3}{x-1}\right)\\&=\frac{3}{\frac{3}{x-1}-1}=\frac{3}{\frac{3-(x-1)}{x-1}}\\&=\frac{3(x-1)}{4-x}\end{aligned}$$

Domain $\{x \mid x\neq 1,\ x\neq 4\}$.

d. $$(g\circ g)(x)=g(g(x))=g\left(\frac{2}{x}\right)=\frac{2}{\frac{2}{x}}=\frac{2x}{2}=x$$

Domain $\{x \mid x\neq 0\}$.

36. $f(x)=\frac{1}{x+3}$ $g(x)=-\frac{2}{x}$

The domain of *f* is $\{x \mid x \neq -3\}$. The domain of g is $\{x \mid x \neq 0\}$.

a. $$(f \circ g)(x) = f(g(x)) = f\left(-\frac{2}{x}\right) = \frac{1}{-\frac{2}{x}+3} = \frac{1}{\frac{-2+3x}{x}} = \frac{x}{-2+3x} \text{ or } \frac{x}{3x-2}$$

Domain $\left\{x \,\middle|\, x \neq 0,\, x \neq \frac{2}{3}\right\}$.

b. $$(g \circ f)(x) = g(f(x)) = g\left(\frac{1}{x+3}\right) = -\frac{2}{\frac{1}{x+3}} = \frac{-2(x+3)}{1} = -2(x+3)$$

Domain $\{x \mid x \neq -3\}$.

c. $$(f \circ f)(x) = f(f(x)) = f\left(\frac{1}{x+3}\right) = \frac{1}{\frac{1}{x+3}+3} = \frac{1}{\frac{1+3x+9}{x+3}} = \frac{x+3}{3x+10}$$

Domain $\left\{x \,\middle|\, x \neq -\frac{10}{3},\, x \neq -3\right\}$.

d. $$(g \circ g)(x) = g(g(x)) = g\left(-\frac{2}{x}\right) = -\frac{2}{-\frac{2}{x}} = -\frac{2x}{-2} = x$$

Domain $\{x \mid x \neq 0\}$.

37. $f(x)=\frac{x}{x-1}$ $g(x)=-\frac{4}{x}$

The domain of *f* is $\{x \mid x \neq 1\}$. The domain of g is $\{x \mid x \neq 0\}$.

a. $$(f \circ g)(x) = f(g(x)) = f\left(-\frac{4}{x}\right) = \frac{-\frac{4}{x}}{-\frac{4}{x}-1} = \frac{-\frac{4}{x}}{\frac{-4-x}{x}} = \frac{-4}{-4-x} = \frac{4}{4+x}$$

Domain $\{x \mid x \neq -4,\, x \neq 0\}$.

b. $$(g \circ f)(x) = g(f(x)) = g\left(\frac{x}{x-1}\right) = -\frac{4}{\frac{x}{x-1}} = \frac{-4(x-1)}{x}$$

Domain $\{x \mid x \neq 0,\, x \neq 1\}$.

c. $$(f \circ f)(x) = f(f(x)) = f\left(\frac{x}{x-1}\right) = \frac{\frac{x}{x-1}}{\frac{x}{x-1}-1} = \frac{\frac{x}{x-1}}{\frac{x-(x-1)}{x-1}} = \frac{\frac{x}{x-1}}{\frac{1}{x-1}} = x$$

Domain $\{x \mid x \neq 1\}$.

d. $$(g \circ g)(x) = g(g(x)) = g\left(\frac{-4}{x}\right) = -\frac{4}{-\frac{4}{x}} = \frac{-4x}{-4} = x$$

Domain $\{x \mid x \neq 0\}$.

38. $f(x)=\dfrac{x}{x+3}\qquad g(x)=\dfrac{2}{x}$

The domain of *f* is $\{x \mid x \neq -3\}$. The domain of *g* is $\{x \mid x \neq 0\}$.

a. $$\begin{aligned}(f\circ g)(x) &= f(g(x))\\ &= f\left(\frac{2}{x}\right)\\ &= \frac{\frac{2}{x}}{\frac{2}{x}+3}=\frac{\frac{2}{x}}{\frac{2+3x}{x}}\\ &= \frac{2}{2+3x}\end{aligned}$$

Domain $\left\{x \,\middle|\, x \neq -\frac{2}{3},\ x \neq 0\right\}$.

b. $$\begin{aligned}(g\circ f)(x) &= g(f(x))\\ &= g\left(\frac{x}{x+3}\right)\\ &= \frac{2}{\frac{x}{x+3}}\\ &= \frac{2(x+3)}{x}\end{aligned}$$

Domain $\{x \mid x \neq -3,\ x \neq 0\}$.

c. $$\begin{aligned}(f\circ f)(x) &= f(f(x))\\ &= f\left(\frac{x}{x+3}\right)\\ &= \frac{\frac{x}{x+3}}{\frac{x}{x+3}+3}=\frac{\frac{x}{x+3}}{\frac{4x+9}{x+3}}\\ &= \frac{x}{4x+9}\end{aligned}$$

Domain $\left\{x \,\middle|\, x \neq -3, x \neq -\frac{9}{4}\right\}$.

d. $$(g\circ g)(x)=g(g(x))=g\left(\frac{2}{x}\right)=\frac{2}{\frac{2}{x}}=\frac{2x}{2}=x$$

Domain $\{x \mid x \neq 0\}$.

39. $f(x)=\sqrt{x}\qquad g(x)=2x+3$

The domain of *f* is $\{x \mid x \geq 0\}$. The domain of *g* is $\{x \mid x \text{ is any real number}\}$.

a. $(f\circ g)(x)=f(g(x))=f(2x+3)=\sqrt{2x+3}$

Domain $\left\{x \,\middle|\, x \geq -\frac{3}{2}\right\}$.

b. $(g\circ f)(x)=g(f(x))=g\left(\sqrt{x}\right)=2\sqrt{x}+3$

Domain $\{x \mid x \geq 0\}$.

c. $$\begin{aligned}(f\circ f)(x) &= f(f(x))\\ &= f\left(\sqrt{x}\right)\\ &= \sqrt{\sqrt{x}}\\ &= \left(x^{1/2}\right)^{1/2}\\ &= x^{1/4}\\ &= \sqrt[4]{x}\end{aligned}$$

Domain $\{x \mid x \geq 0\}$.

d. $$\begin{aligned}(g\circ g)(x) &= g(g(x))\\ &= g(2x+3)\\ &= 2(2x+3)+3\\ &= 4x+6+3\\ &= 4x+9\end{aligned}$$

Domain $\{x \mid x \text{ is any real number}\}$.

40. $f(x)=\sqrt{x-2}\qquad g(x)=1-2x$

The domain of *f* is $\{x \mid x \geq 2\}$. The domain of *g* is $\{x \mid x \text{ is any real number}\}$.

a. $$\begin{aligned}(f\circ g)(x) &= f(g(x))\\ &= f(1-2x)\\ &= \sqrt{1-2x-2}\\ &= \sqrt{-2x-1}\end{aligned}$$

Domain $\left\{x \,\middle|\, x \leq -\frac{1}{2}\right\}$.

b. $$\begin{aligned}(g\circ f)(x) &= g(f(x))\\ &= g\left(\sqrt{x-2}\right)\\ &= 1-2\sqrt{x-2}\end{aligned}$$

Domain $\{x \mid x \geq 2\}$.

c. $(f \circ f)(x) = f(f(x))$
$= f\left(\sqrt{x-2}\right)$
$= \sqrt{\sqrt{x-2}-2}$

Now, $\sqrt{x-2}-2 \geq 0$
$\sqrt{x-2} \geq 2$
$x-2 \geq 4$
$x \geq 6$

Domain $\{x \mid x \geq 6\}$.

d. $(g \circ g)(x) = g(g(x))$
$= g(1-2x)$
$= 1-2(1-2x)$
$= 1-2+4x$
$= 4x-1$

Domain $\{x \mid x \text{ is any real number}\}$.

41. $f(x) = x^2+1 \qquad g(x) = \sqrt{x-1}$

The domain of *f* is $\{x \mid x \text{ is any real number}\}$.

The domain of *g* is $\{x \mid x \geq 1\}$.

a. $(f \circ g)(x) = f(g(x))$
$= f\left(\sqrt{x-1}\right)$
$= \left(\sqrt{x-1}\right)^2+1$
$= x-1+1$
$= x$

Domain $\{x \mid x \geq 1\}$.

b. $(g \circ f)(x) = g(f(x))$
$= g\left(x^2+1\right)$
$= \sqrt{x^2+1-1}$
$= \sqrt{x^2}$
$= |x|$

Domain $\{x \mid x \text{ is any real number}\}$.

c. $(f \circ f)(x) = f(f(x))$
$= f\left(x^2+1\right)$
$= \left(x^2+1\right)^2+1$
$= x^4+2x^2+1+1$
$= x^4+2x^2+2$

Domain $\{x \mid x \text{ is any real number}\}$.

d. $(g \circ g)(x) = g(g(x))$
$= g\left(\sqrt{x-1}\right) = \sqrt{\sqrt{x-1}-1}$

Now, $\sqrt{x-1}-1 \geq 0$
$\sqrt{x-1} \geq 1$
$x-1 \geq 1$
$x \geq 2$

Domain $\{x \mid x \geq 2\}$.

42. $f(x) = x^2+4 \qquad g(x) = \sqrt{x-2}$

The domain of *f* is $\{x \mid x \text{ is any real number}\}$.

The domain of *g* is $\{x \mid x \geq 2\}$.

a. $(f \circ g)(x) = f(g(x))$
$= f\left(\sqrt{x-2}\right)$
$= \left(\sqrt{x-2}\right)^2+4$
$= x-2+4$
$= x+2$

Domain $\{x \mid x \geq 2\}$.

b. $(g \circ f)(x) = g(f(x))$
$= g\left(x^2+4\right)$
$= \sqrt{x^2+4-2}$
$= \sqrt{x^2+2}$

Domain $\{x \mid x \text{ is any real number}\}$.

c. $(f \circ f)(x) = f(f(x))$
$= f\left(x^2+4\right)$
$= \left(x^2+4\right)^2+4$
$= x^4+8x^2+16+4$
$= x^4+8x^2+20$

Domain $\{x \mid x \text{ is any real number}\}$.

d. $(g \circ g)(x) = g(g(x))$
$= g\left(\sqrt{x-2}\right) = \sqrt{\sqrt{x-2}-2}$

Now, $\sqrt{x-2}-2 \geq 0$
$\sqrt{x-2} \geq 2$
$x-2 \geq 4$
$x \geq 6$

Domain $\{x \mid x \geq 6\}$.

43. $f(x)=\dfrac{x-5}{x+1} \qquad g(x)=\dfrac{x+2}{x-3}$

The domain of *f* is $\{x \mid x \neq -1\}$. The domain of *g* is $\{x \mid x \neq 3\}$.

a. $(f \circ g)(x) = f(g(x)) = f\left(\dfrac{x+2}{x-3}\right)$

$$= \frac{\frac{x+2}{x-3}-5}{\frac{x+2}{x-3}+1} = \frac{\left(\frac{x+2}{x-3}-5\right)(x-3)}{\left(\frac{x+2}{x-3}+1\right)(x-3)}$$

$$= \frac{x+2-5(x-3)}{x+2+1(x-3)} = \frac{x+2-5x+15}{x+2+x-3}$$

$$= \frac{-4x+17}{2x-1} \text{ or } -\frac{4x-17}{2x-1}$$

Now, $2x-1 \neq 0$, so $x \neq \frac{1}{2}$. Also, from the domain of *g*, we know $x \neq 3$.

Domain of $f \circ g$: $\left\{x \,\middle|\, x \neq \frac{1}{2}, x \neq 3\right\}$.

b. $(g \circ f)(x) = g(f(x)) = g\left(\dfrac{x-5}{x+1}\right)$

$$= \frac{\frac{x-5}{x+1}+2}{\frac{x-5}{x+1}-3} = \frac{\left(\frac{x-5}{x+1}+2\right)(x+1)}{\left(\frac{x-5}{x+1}-3\right)(x+1)}$$

$$= \frac{x-5+2(x+1)}{x-5-3(x+1)} = \frac{x-5+2x+2}{x-5-3x-3}$$

$$= \frac{3x-3}{-2x-8} \text{ or } -\frac{3x-3}{2x+8}$$

Now, $-2x-8 \neq 0$, so $x \neq -4$. Also, from the domain of *f*, we know $x \neq -1$.

Domain of $g \circ f$: $\{x \mid x \neq -4, x \neq -1\}$.

c. $(f \circ f)(x) = f(f(x)) = f\left(\dfrac{x-5}{x+1}\right)$

$$= \frac{\frac{x-5}{x+1}-5}{\frac{x-5}{x+1}+1} = \frac{\left(\frac{x-5}{x+1}-5\right)(x+1)}{\left(\frac{x-5}{x+1}+1\right)(x+1)}$$

$$= \frac{x-5-5(x+1)}{x-5+1(x+1)} = \frac{x-5-5x-5}{x-5+x+1}$$

$$= \frac{-4x-10}{2x-4} = \frac{-2(2x+5)}{2(x-2)} = -\frac{2x+5}{x-2}$$

Now, $x-2 \neq 0$, so $x \neq 2$. Also, from the domain of *f*, we know $x \neq -1$.

Domain of $f \circ f$: $\{x \mid x \neq -1, x \neq 2\}$.

d. $(g \circ g)(x) = g(g(x)) = g\left(\dfrac{x+2}{x-3}\right)$

$$= \frac{\frac{x+2}{x-3}+2}{\frac{x+2}{x-3}-3} = \frac{\left(\frac{x+2}{x-3}+2\right)(x-3)}{\left(\frac{x+2}{x-3}-3\right)(x-3)}$$

$$= \frac{x+2+2(x-3)}{x+2-3(x-3)} = \frac{x+2+2x-6}{x+2-3x+9}$$

$$= \frac{3x-4}{-2x+11} \text{ or } -\frac{3x-4}{2x-11}$$

Now, $-2x+11 \neq 0$, so $x \neq \frac{11}{2}$. Also, from the domain of *g*, we know $x \neq 3$.

Domain of $g \circ g$: $\left\{x \,\middle|\, x \neq \frac{11}{2}, x \neq 3\right\}$.

44. $f(x)=\dfrac{2x-1}{x-2} \qquad g(x)=\dfrac{x+4}{2x-5}$

The domain of *f* is $\{x \mid x \neq 2\}$. The domain of *g* is $\left\{x \,\middle|\, x \neq \frac{5}{2}\right\}$.

a. $(f \circ g)(x) = f(g(x)) = f\left(\dfrac{x+4}{2x-5}\right)$

$$= \frac{2\left(\frac{x+4}{2x-5}\right)-1}{\frac{x+4}{2x-5}-2}$$

$$= \frac{\left(2\left(\frac{x+4}{2x-5}\right)-1\right)(2x-5)}{\left(\frac{x+4}{2x-5}-2\right)(2x-5)}$$

$$= \frac{2(x+4)-1(2x-5)}{x+4-2(2x-5)}$$

$$= \frac{2x+8-2x+5}{x+4-4x+10}$$

$$= \frac{13}{-3x+14} \text{ or } -\frac{13}{3x-14}$$

Now, $-3x+14 \neq 0$, so $x \neq \frac{14}{3}$. Also, from the domain of *g*, we know $x \neq \frac{5}{2}$.

Domain of $f \circ g$: $\left\{x \,\middle|\, x \neq \frac{5}{2}, x \neq \frac{14}{3}\right\}$.

b. $(g \circ f)(x) = g(f(x)) = g\left(\frac{2x-1}{x-2}\right)$

$$= \frac{\frac{2x-1}{x-2}+4}{2\left(\frac{2x-1}{x-2}\right)-5}$$

$$= \frac{\left(\frac{2x-1}{x-2}+4\right)(x-2)}{\left(2\left(\frac{2x-1}{x-2}\right)-5\right)(x-2)}$$

$$= \frac{2x-1+4(x-2)}{2(2x-1)-5(x-2)}$$

$$= \frac{2x-1+4x-8}{4x-2-5x+10}$$

$$= \frac{6x-9}{-x+8} \text{ or } -\frac{6x-9}{x-8}$$

Now, $-x+8 \neq 0$, so $x \neq 8$. Also, from the domain of *f*, we know $x \neq 2$.

Domain of $f \circ g$: $\{x \mid x \neq 2, x \neq 8\}$.

c. $(f \circ f)(x) = f(f(x)) = f\left(\frac{2x-1}{x-2}\right)$

$$= \frac{2\left(\frac{2x-1}{x-2}\right)-1}{\frac{2x-1}{x-2}-2}$$

$$= \frac{\left(2\left(\frac{2x-1}{x-2}\right)-1\right)(x-2)}{\left(\frac{2x-1}{x-2}-2\right)(x-2)}$$

$$= \frac{2(2x-1)-1(x-2)}{2x-1-2(x-2)}$$

$$= \frac{4x-2-x+2}{2x-1-2x+4} = \frac{3x}{3} = x$$

From the domain of *f*, we know $x \neq 2$.

Domain of $f \circ f$: $\{x \mid x \neq 2\}$.

d. $(g \circ g)(x) = g(g(x)) = g\left(\frac{x+4}{2x-5}\right)$

$$= \frac{\frac{x+4}{2x-5}+4}{2\left(\frac{x+4}{2x-5}\right)-5}$$

$$= \frac{\left(\frac{x+4}{2x-5}+4\right)(2x-5)}{\left(2\left(\frac{x+4}{2x-5}\right)-5\right)(2x-5)}$$

$$= \frac{x+4+4(2x-5)}{2(x+4)-5(2x-5)}$$

$$= \frac{x+4+8x-20}{2x+8-10x+25}$$

$$= \frac{9x-16}{-8x+33} \text{ or } -\frac{9x-16}{8x-33}$$

Now, $8x-33 \neq 0$, so $x \neq \frac{33}{8}$. Also, from the domain of *g*, we know $x \neq \frac{5}{2}$.

Domain of $f \circ g$: $\left\{x \,\middle|\, x \neq \frac{5}{2}, x \neq \frac{33}{8}\right\}$.

45. $(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{2}x\right) = 2\left(\frac{1}{2}x\right) = x$

$(g \circ f)(x) = g(f(x)) = g(2x) = \frac{1}{2}(2x) = x$

46. $(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{4}x\right) = 4\left(\frac{1}{4}x\right) = x$

$(g \circ f)(x) = g(f(x)) = g(4x) = \frac{1}{4}(4x) = x$

47. $(f \circ g)(x) = f(g(x)) = f\left(\sqrt[3]{x}\right) = \left(\sqrt[3]{x}\right)^3 = x$

$(g \circ f)(x) = g(f(x)) = g\left(x^3\right) = \sqrt[3]{x^3} = x$

48. $(f \circ g)(x) = f(g(x)) = f\left(x-5\right) = x-5+5 = x$

$(g \circ f)(x) = g(f(x)) = g\left(x+5\right) = x+5-5 = x$

49. $(f \circ g)(x) = f(g(x))$
$$= f\left(\frac{1}{2}(x+6)\right)$$
$$= 2\left(\frac{1}{2}(x+6)\right) - 6$$
$$= x + 6 - 6$$
$$= x$$
$(g \circ f)(x) = g(f(x))$
$$= g\left(2x-6\right)$$
$$= \frac{1}{2}\left((2x-6)+6\right)$$
$$= \frac{1}{2}(2x)$$
$$= x$$

50. $(f \circ g)(x) = f(g(x))$
$$= f\left(\frac{1}{3}(4-x)\right)$$
$$= 4 - 3\left(\frac{1}{3}(4-x)\right)$$
$$= 4 - 4 + x$$
$$= x$$
$(g \circ f)(x) = g(f(x))$
$$= g\left(4-3x\right)$$
$$= \frac{1}{3}\left(4-(4-3x)\right)$$
$$= \frac{1}{3}(3x)$$
$$= x$$

51. $(f \circ g)(x) = f(g(x))$
$$= f\left(\frac{1}{a}(x-b)\right)$$
$$= a\left(\frac{1}{a}(x-b)\right) + b$$
$$= x - b + b$$
$$= x$$
$(g \circ f)(x) = g(f(x))$
$$= g\left(ax+b\right)$$
$$= \frac{1}{a}\left((ax+b)-b\right)$$
$$= \frac{1}{a}(ax)$$
$$= x$$

52. $(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{x}\right) = \frac{1}{\frac{1}{x}} = 1 \cdot \frac{x}{1} = x$

$(g \circ f)(x) = g(f(x)) = g\left(\frac{1}{x}\right) = \frac{1}{\frac{1}{x}} = 1 \cdot \frac{x}{1} = x$

53. $H(x) = (2x+3)^4$
Answers may vary. One possibility is
$f(x) = x^4, \quad g(x) = 2x+3$

54. $H(x) = \left(1+x^2\right)^3$
Answers may vary. One possibility is
$f(x) = x^3, \quad g(x) = 1+x^2$

55. $H(x) = \sqrt{x^2+1}$
Answers may vary. One possibility is
$f(x) = \sqrt{x}, \quad g(x) = x^2+1$

56. $H(x) = \sqrt{1-x^2}$
Answers may vary. One possibility is
$f(x) = \sqrt{x}, \quad g(x) = 1-x^2$

57. $H(x) = \left|2x+1\right|$
Answers may vary. One possibility is
$f(x) = \left|x\right|, \quad g(x) = 2x+1$

58. $H(x) = \left|2x^2+3\right|$
Answer may vary. One possibility is
$f(x) = \left|x\right|, \quad g(x) = 2x^2+3$

59. $f(x) = 2x^3 - 3x^2 + 4x - 1 \qquad g(x) = 2$
$(f \circ g)(x) = f(g(x))$
$$= f(2)$$
$$= 2(2)^3 - 3(2)^2 + 4(2) - 1$$
$$= 16 - 12 + 8 - 1$$
$$= 11$$
$(g \circ f)(x) = g(f(x)) = g\left(2x^3 - 3x^2 + 4x - 1\right) = 2$

60. $f(x)=\dfrac{x+1}{x-1},\ x\neq 1$

$$\begin{aligned}(f\circ f)(x)&=f(f(x))\\&=f\left(\frac{x+1}{x-1}\right)\\&=\frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1}\\&=\frac{\frac{x+1+x-1}{x-1}}{\frac{x+1-(x-1)}{x-1}}\\&=\frac{\frac{2x}{x-1}}{\frac{2}{x-1}}\\&=\frac{2x}{x-1}\cdot\frac{x-1}{2}\\&=x,\quad x\neq 1\end{aligned}$$

61. $f(x)=2x^2+5 \qquad g(x)=3x+a$

$(f\circ g)(x)=f(g(x))=f(3x+a)=2(3x+a)^2+5$

When $x=0$, $(f\circ g)(0)=23$.

Solving:
$$\begin{aligned}2(3\cdot 0+a)^2+5&=23\\2a^2+5&=23\\2a^2-18&=0\\2(a+3)(a-3)&=0\\a=-3 \quad\text{or}\quad a&=3\end{aligned}$$

62. $f(x)=3x^2-7 \qquad g(x)=2x+a$

$(f\circ g)(x)=f(g(x))=f(2x+a)=3(2x+a)^2-7$

When $x=0$, $(f\circ g)(0)=68$.

Solving:
$$\begin{aligned}3(2\cdot 0+a)^2-7&=68\\3a^2-7&=68\\3a^2-75&=0\\3(a+5)(a-5)&=0\\a=-5 \text{ or } a&=5\end{aligned}$$

63. a.
$$\begin{aligned}(f\circ g)(x)&=f(g(x))\\&=f(cx+d)\\&=a(cx+d)+b\\&=acx+ad+b\end{aligned}$$

b.
$$\begin{aligned}(g\circ f)(x)&=g(f(x))\\&=g(ax+b)\\&=c(ax+b)+d\\&=acx+bc+d\end{aligned}$$

c. Since the domain of *f* is the set of all real numbers and the domain of *g* is the set of all real numbers, the domains of both $f\circ g$ and $g\circ f$ are all real numbers.

d.
$$\begin{aligned}(f\circ g)(x)&=(g\circ f)(x)\\acx+ad+b&=acx+bc+d\\ad+b&=bc+d\end{aligned}$$
Thus, $f\circ g=g\circ f$ when $ad+b=bc+d$.

64. a.
$$\begin{aligned}(f\circ g)(x)&=f(g(x))\\&=f(mx)\\&=\frac{a(mx)+b}{c(mx)+d}\\&=\frac{amx+b}{cmx+d}\end{aligned}$$

b.
$$\begin{aligned}(g\circ f)(x)&=g(f(x))\\&=g\left(\frac{ax+b}{cx+d}\right)\\&=m\left(\frac{ax+b}{cx+d}\right)\\&=\frac{m(ax+b)}{cx+d}\end{aligned}$$

c. To find the domain of $f\circ g$, we first recognize that the domain of *g* is the set of all real numbers. This means that the only restrictions are those that cause zero in the denominator of the final result in part (a).
$$\begin{aligned}cmx+d&\neq 0\\cmx&\neq -d\\x&\neq -\frac{d}{cm}\end{aligned}$$
Thus the domain of $f\circ g$ is $\left\{x \,\middle|\, x\neq -\dfrac{d}{cm}\right\}$.

To find the domain of $g\circ f$, we first recognize that the domain of *f* is $\left\{x \,\middle|\, x\neq -\dfrac{d}{c}\right\}$ and the domain of *g* is the set of all real numbers. Thus, the domain of $g\circ f$ is also $\left\{x \,\middle|\, x\neq -\dfrac{d}{c}\right\}$.

d.
$$(f \circ g)(x) = (g \circ f)(x)$$
$$\frac{amx+b}{cmx+d} = \frac{m(ax+b)}{cx+d}$$
$$\frac{amx+b}{cmx+d} = \frac{amx+bm}{cx+d}$$
$$(amx+bm)(cmx+d) = (amx+b)(cx+d)$$

Now, this equation will only be true if $m=1$. Thus, $f \circ g = g \circ f$ when $m=1$.

65. $S(r) = 4\pi r^2 \qquad r(t) = \frac{2}{3}t^3,\ t \ge 0$

$$S(r(t)) = S\left(\frac{2}{3}t^3\right) = 4\pi\left(\frac{2}{3}t^3\right)^2 = 4\pi\left(\frac{4}{9}t^6\right) = \frac{16}{9}\pi t^6$$

Thus, $S(t) = \frac{16}{9}\pi t^6$.

66. $V(r) = \frac{4}{3}\pi r^3 \qquad r(t) = \frac{2}{3}t^3,\ t \ge 0$

$$V(r(t)) = V\left(\frac{2}{3}t^3\right) = \frac{4}{3}\pi\left(\frac{2}{3}t^3\right)^3 = \frac{4}{3}\pi\left(\frac{8}{27}t^9\right) = \frac{32}{81}\pi t^9$$

Thus, $V(t) = \frac{32}{81}\pi t^9$.

67. $N(t) = 100t - 5t^2,\ 0 \le t \le 10$

$C(N) = 15{,}000 + 8000N$

$$C(N(t)) = C\left(100t - 5t^2\right) = 15{,}000 + 8000\left(100t - 5t^2\right) = 15{,}000 + 800{,}000t - 40{,}000t^2$$

Thus, $C(t) = 15{,}000 + 800{,}000t - 40{,}000t^2$.

68. $A(r) = \pi r^2 \qquad r(t) = 200\sqrt{t}$

$$A(r(t)) = A\left(200\sqrt{t}\right) = \pi\left(200\sqrt{t}\right)^2 = 40{,}000\pi t$$

Thus, $A(t) = 40{,}000\pi t$.

69. $p = -\frac{1}{4}x + 100, \quad 0 \le x \le 400$

$$\frac{1}{4}x = 100 - p$$
$$x = 4(100 - p)$$

$$C = \frac{\sqrt{x}}{25} + 600 = \frac{\sqrt{4(100-p)}}{25} + 600 = \frac{2\sqrt{100-p}}{25} + 600, \quad 0 \le p \le 100$$

Thus, $C(p) = \frac{2\sqrt{100-p}}{25} + 600, \quad 0 \le p \le 100$.

70. $p = -\frac{1}{5}x + 200, \quad 0 \le x \le 1000$

$$\frac{1}{5}x = 200 - p$$
$$x = 5(200 - p)$$

$$C = \frac{\sqrt{x}}{10} + 400 = \frac{\sqrt{5(200-p)}}{10} + 400 = \frac{\sqrt{1000-5p}}{10} + 400, \quad 0 \le p \le 200$$

Thus, $C(p) = \frac{\sqrt{1000-5p}}{10} + 400, \quad 0 \le p \le 200$.

71. $V = \pi r^2 h \qquad h = 2r$

$$V(r) = \pi r^2 (2r) = 2\pi r^3$$

72. $V = \frac{1}{3}\pi r^2 h \quad h = 2r$

$$V(r) = \frac{1}{3}\pi r^2 (2r) = \frac{2}{3}\pi r^3$$

73. $f(x)$ = the number of Euros bought for x dollars; $g(x)$ = the number of yen bought for x Euros

a. $f(x) = 0.7045x$

b. $g(x) = 114.9278x$

c. $$\begin{aligned}(g \circ f)(x) &= g(f(x)) = g(0.7045x)\\ &= 114.9278(0.7045x)\\ &= 80.9666351x\end{aligned}$$

d. $$\begin{aligned}(g \circ f)(1000) &= 80.9666351(1000)\\ &= 80{,}966.635 \text{ yen}\end{aligned}$$

On March 22, 2001, you can exchange \$1000 for euros and then use the euros to purchase 80,966.635 yen.

74. a. Given $C(F) = \frac{5}{9}(F-32)$ and $K(C) = C + 273$, we need to find $K(C(F))$.

$$\begin{aligned}K(C(F)) &= \left[\frac{5}{9}(F-32)\right] + 273\\ &= \frac{5}{9}(F-32) + 273\\ &= \frac{5}{9}F - \frac{160}{9} + 273\\ &= \frac{5}{9}F + \frac{2297}{9} \quad \text{or} \quad \frac{5F+2297}{9}\end{aligned}$$

b. $K(C(80)) = \dfrac{5(80)+2297}{9} \approx 299.7$ kelvins

75. a. $f(p) = p - 200$

b. $g(p) = 0.80p$

c. $$\begin{aligned}(f \circ g)(p) &= f(g(p))\\ &= (0.80p) - 200\\ &= 0.80p - 200\end{aligned}$$

This represents the final price when the rebate is issued on the sale price.

$$\begin{aligned}(g \circ f)(p) &= g(f(p))\\ &= 0.80(p-200) = 0.80p - 160\end{aligned}$$

This represents the final price when the sale price is calculated after the rebate is given.

Appling the 20% first is a better deal since a larger portion will be removed up front.

76. Given that f and g are odd functions, we know that $f(-x) = -f(x)$ and $g(-x) = -g(x)$ for all x in the domain of f and g, respectively. The composite function $(f \circ g)(x) = f(g(x))$ has the following property:

$$\begin{aligned}(f \circ g)(-x) &= f(g(-x))\\ &= f(-g(x)) \quad \text{since } g \text{ is odd}\\ &= -f(g(x)) \quad \text{since } f \text{ is odd}\\ &= -(f \circ g)(x)\end{aligned}$$

Thus, $f \circ g$ is an odd function.

77. Given that f is odd and g is even, we know that $f(-x) = -f(x)$ and $g(-x) = g(x)$ for all x in the domain of f and g, respectively. The composite function $(f \circ g)(x) = f(g(x))$ has the following property:

$$\begin{aligned}(f \circ g)(-x) &= f(g(-x))\\ &= f(g(x)) \quad \text{since } g \text{ is even}\\ &= (f \circ g)(x)\end{aligned}$$

Thus, $f \circ g$ is an even function.

The composite function $(g \circ f)(x) = g(f(x))$ has the following property:

$$\begin{aligned}(g \circ f)(-x) &= g(f(-x))\\ &= g(-f(x)) \quad \text{since } f \text{ is odd}\\ &= g(f(x)) \quad \text{since } g \text{ is even}\\ &= (g \circ f)(x)\end{aligned}$$

Thus, $g \circ f$ is an even function.

Section 5.2

1. The set of ordered pairs is a function because there are no ordered pairs with the same first element and different second elements.

2. The function $f(x) = x^2$ is increasing on the interval $(0, \infty)$. It is decreasing on the interval $(-\infty, 0)$.

3. The function is not defined when $x^2 + 3x - 18 = 0$.

Solve: $$\begin{aligned}x^2 + 3x - 18 &= 0\\ (x+6)(x-3) &= 0\end{aligned}$$

$x = -6$ or $x = 3$

The domain is $\{x \mid x \neq -6,\ x \neq 3\}$.

4. $$\frac{\frac{1}{x}+1}{\frac{1}{x^2}-1}=\frac{\frac{1}{x}+\frac{x}{x}}{\frac{1}{x^2}-\frac{x^2}{x^2}}=\frac{\frac{1+x}{x}}{\frac{1-x^2}{x^2}}$$
$$=\left(\frac{1+x}{x}\right)\left(\frac{x^2}{1-x^2}\right)$$
$$=\left(\frac{1+x}{x}\right)\left(\frac{x^2}{(1-x)(1+x)}\right)$$
$$=\frac{x}{(1-x)};\ x\neq 0,1,-1$$

5. $f(x_1)\neq f(x_2)$

6. one-to-one

7. 3

8. $y=x$

9. $[4,\infty)$

10. True

11. The function is one-to-one because there are no two distinct inputs that correspond to the same output.

12. The function is one-to-one because there are no two distinct inputs that correspond to the same output.

13. The function is not one-to-one because there are two different inputs, 20 Hours and 50 Hours, that correspond to the same output, $200.

14. The function is not one-to-one because there are two different inputs, John and Chuck, that correspond to the same output, Phoebe.

15. The function is not one-to-one because there are two distinct inputs, 2 and -3, that correspond to the same output.

16. The function is one-to-one because there are no two distinct inputs that correspond to the same output.

17. The function is one-to-one because there are no two distinct inputs that correspond to the same output.

18. The function is one-to-one because there are no two distinct inputs that correspond to the same output.

19. The function *f* is one-to-one because every horizontal line intersects the graph at exactly one point.

20. The function *f* is one-to-one because every horizontal line intersects the graph at exactly one point.

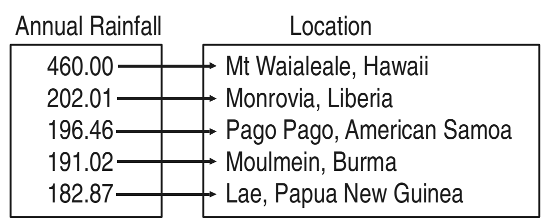
21. The function *f* is not one-to-one because there are horizontal lines (for example, $y=1$) that intersect the graph at more than one point.

22. The function *f* is not one-to-one because there are horizontal lines (for example, $y=1$) that intersect the graph at more than one point.

23. The function *f* is one-to-one because every horizontal line intersects the graph at exactly one point.

24. The function *f* is not one-to-one because the horizontal line $y=2$ intersects the graph at more than one point.

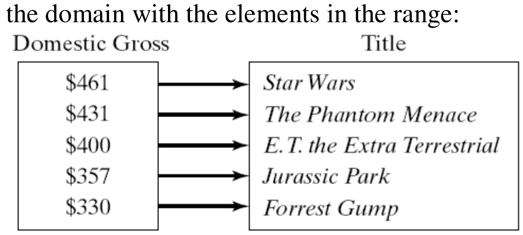
25. To find the inverse, interchange the elements in the domain with the elements in the range:



Domain: {460.00, 202.01, 196.46, 191.02, 182.87}

Range: {Mt Waialeale, Monrovia, Pago Pago, Moulmein, Lea}

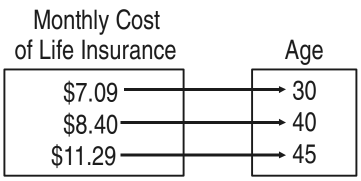
26. To find the inverse, interchange the elements in the domain with the elements in the range:



Domain: {$461, $431, $400, $357, $330}
(in millions)

Range: {*Star Wars*, *The Phantom Menace*, *E.T. the Extra Terrestrial*, *Jurassic Park*, *Forrest Gump*}

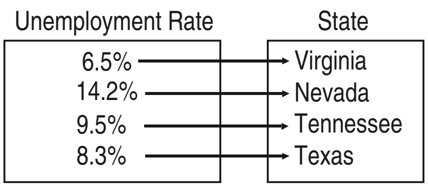
27. To find the inverse, interchange the elements in the domain with the elements in the range:



Domain: {\$7.09, \$8.40, \$11.29}

Range: {30, 40, 45}

28. To find the inverse, interchange the elements in the domain with the elements in the range:



Domain: {6.5%, 14.2%, 9.5%, 8.3%}

Range: {Virginia, Nevada, Tennessee, Texas}

29. Interchange the entries in each ordered pair: $\{(5,-3),\ (9,-2),\ (2,-1),\ (11,0),\ (-5,1)\}$

Domain: $\{5,\ 9,\ 2,\ 11, -5\}$

Range: $\{-3,-2,-1,\ 0,\ 1\}$

30. Interchange the entries in each ordered pair: $\{(2,-2),\ (6,-1),\ (8,0),\ (-3,1),\ (9,2)\}$

Domain: $\{2,\ 6,\ 8,-3,\ 9\}$

Range: $\{-2,-1,\ 0,\ 1,\ 2\}$

31. Interchange the entries in each ordered pair: $\{(1,-2),\ (2,-3),\ (0,-10),\ (9,1),\ (4,2)\}$

Domain: $\{1,\ 2,\ 0,\ 9,\ 4\}$

Range: $\{-2,-3,-10,\ 1,\ 2\}$

32. Interchange the entries in each ordered pair: $\{(-8,-2),\ (-1,-1),\ (0,0),\ (1,1),\ (8,2)\}$

Domain: $\{-8,-1,\ 0,\ 1,\ 8\}$

Range: $\{-2,-1,\ 0,\ 1,\ 2\}$

33. $f(x) = 3x + 4; \qquad g(x) = \frac{1}{3}(x-4)$

$$\begin{aligned} f\left(g(x)\right) &= f\left(\frac{1}{3}(x-4)\right) \\ &= 3\left(\frac{1}{3}(x-4)\right) + 4 \\ &= (x-4) + 4 \\ &= x \end{aligned}$$

$$\begin{aligned} g\left(f(x)\right) &= g(3x+4) \\ &= \frac{1}{3}\left((3x+4) - 4\right) \\ &= \frac{1}{3}(3x) \\ &= x \end{aligned}$$

Thus, f and g are inverses of each other.

34. $f(x) = 3 - 2x; \qquad g(x) = -\frac{1}{2}(x-3)$

$$\begin{aligned} f\left(g(x)\right) &= f\left(-\frac{1}{2}(x-3)\right) \\ &= 3 - 2\left(-\frac{1}{2}(x-3)\right) \\ &= 3 + (x-3) \\ &= x \end{aligned}$$

$$\begin{aligned} g\left(f(x)\right) &= g(3-2x) \\ &= -\frac{1}{2}\left((3-2x) - 3\right) \\ &= -\frac{1}{2}(-2x) \\ &= x \end{aligned}$$

Thus, f and g are inverses of each other.

35. $f(x) = 4x - 8; \qquad g(x) = \frac{x}{4} + 2$

$$\begin{aligned} f\left(g(x)\right) &= f\left(\frac{x}{4} + 2\right) \\ &= 4\left(\frac{x}{4} + 2\right) - 8 \\ &= x + 8 - 8 \\ &= x \end{aligned}$$

$$\begin{aligned} g(f(x)) &= g(4x-8) \\ &= \frac{4x-8}{4}+2 \\ &= x-2+2 \\ &= x \end{aligned}$$

Thus, *f* and *g* are inverses of each other.

36. $f(x)=2x+6; \qquad g(x)=\frac{1}{2}x-3$

$$\begin{aligned} f(g(x)) &= f\left(\frac{1}{2}x-3\right) \\ &= 2\left(\frac{1}{2}x-3\right)+6 = x-6+6 \\ &= x \end{aligned}$$

$$\begin{aligned} g(f(x)) &= g(2x+6) \\ &= \frac{1}{2}(2x+6)-3 = x+3-3 \\ &= x \end{aligned}$$

Thus, *f* and *g* are inverses of each other.

37. $f(x)=x^3-8; \qquad g(x)=\sqrt[3]{x+8}$

$$\begin{aligned} f(g(x)) &= f\left(\sqrt[3]{x+8}\right) \\ &= \left(\sqrt[3]{x+8}\right)^3-8 \\ &= x+8-8 \\ &= x \end{aligned}$$

$$\begin{aligned} g(f(x)) &= g(x^3-8) \\ &= \sqrt[3]{(x^3-8)+8} \\ &= \sqrt[3]{x^3} \\ &= x \end{aligned}$$

Thus, *f* and *g* are inverses of each other.

38. $f(x)=(x-2)^2,\ x\ge 2; \qquad g(x)=\sqrt{x}+2$

$$\begin{aligned} f(g(x)) &= f\left(\sqrt{x}+2\right) \\ &= \left(\sqrt{x}+2-2\right)^2 \\ &= \left(\sqrt{x}\right)^2 \\ &= x \end{aligned}$$

$$\begin{aligned} g(f(x)) &= g\left((x-2)^2\right) \\ &= \sqrt{(x-2)^2}+2 \\ &= x-2+2 \\ &= x \end{aligned}$$

Thus, *f* and *g* are inverses of each other.

39. $f(x)=\frac{1}{x}; \qquad g(x)=\frac{1}{x}$

$$f(g(x)) = f\left(\frac{1}{x}\right) = \frac{1}{\frac{1}{x}} = 1\cdot\frac{x}{1} = x$$

$$g(f(x)) = g\left(\frac{1}{x}\right) = \frac{1}{\frac{1}{x}} = 1\cdot\frac{x}{1} = x$$

Thus, *f* and *g* are inverses of each other.

40. $f(x)=x; \qquad g(x)=x$

$$f(g(x)) = f(x) = x$$

$$g(f(x)) = g(x) = x$$

Thus, *f* and *g* are inverses of each other.

41. $f(x)=\frac{2x+3}{x+4}; \quad g(x)=\frac{4x-3}{2-x}$

$$\begin{aligned} f(g(x)) &= f\left(\frac{4x-3}{2-x}\right),\ x\ne 2 \\ &= \frac{2\left(\frac{4x-3}{2-x}\right)+3}{\frac{4x-3}{2-x}+4} \\ &= \frac{\left(2\left(\frac{4x-3}{2-x}\right)+3\right)(2-x)}{\left(\frac{4x-3}{2-x}+4\right)(2-x)} \\ &= \frac{2(4x-3)+3(2-x)}{4x-3+4(2-x)} \\ &= \frac{8x-6+6-3x}{4x-3+8-4x} \\ &= \frac{5x}{5} \\ &= x \end{aligned}$$

$$g(f(x)) = g\left(\frac{2x+3}{x+4}\right), \quad x \neq -4$$
$$= \frac{4\left(\frac{2x+3}{x+4}\right)-3}{2-\frac{2x+3}{x+4}}$$
$$= \frac{\left(4\left(\frac{2x+3}{x+4}\right)-3\right)(x+4)}{\left(2-\frac{2x+3}{x+4}\right)(x+4)}$$
$$= \frac{4(2x+3)-3(x+4)}{2(x+4)-(2x+3)}$$
$$= \frac{8x+12-3x-12}{2x+8-2x-3}$$
$$= \frac{5x}{5}$$
$$= x$$

Thus, f and g are inverses of each other.

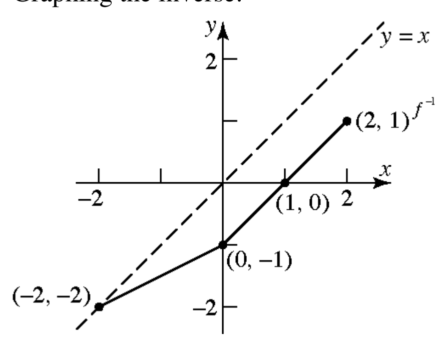
42. $f(x) = \frac{x-5}{2x+3}$; $\quad g(x) = \frac{3x+5}{1-2x}$

$$f(g(x)) = f\left(\frac{3x+5}{1-2x}\right), \quad x \neq \frac{1}{2}$$
$$= \frac{\frac{3x+5}{1-2x}-5}{2\left(\frac{3x+5}{1-2x}\right)+3}$$
$$= \frac{\left(\frac{3x+5}{1-2x}-5\right)(1-2x)}{\left(2\left(\frac{3x+5}{1-2x}\right)+3\right)(1-2x)}$$
$$= \frac{3x+5-5(1-2x)}{2(3x+5)+3(1-2x)}$$
$$= \frac{3x+5-5+10x}{6x+10+3-6x}$$
$$= \frac{13x}{13}$$
$$= x$$

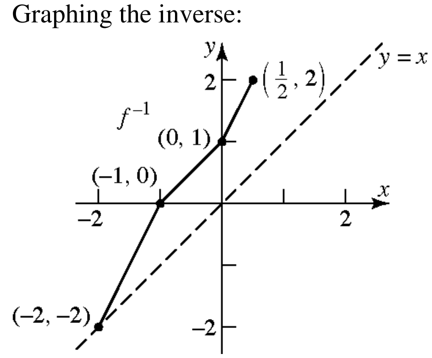
$$g(f(x)) = g\left(\frac{x-5}{2x+3}\right), \quad x \neq -\frac{3}{2}$$
$$= \frac{3\left(\frac{x-5}{2x+3}\right)+5}{1-2\left(\frac{x-5}{2x+3}\right)}$$
$$= \frac{\left(3\left(\frac{x-5}{2x+3}\right)+5\right)(2x+3)}{\left(1-2\left(\frac{x-5}{2x+3}\right)\right)(2x+3)}$$
$$= \frac{3(x-5)+5(2x+3)}{1(2x+3)-2(x-5)}$$
$$= \frac{3x-15+10x+15}{2x+3-2x+10}$$
$$= \frac{13x}{13}$$
$$= x$$

Thus, f and g are inverses of each other.

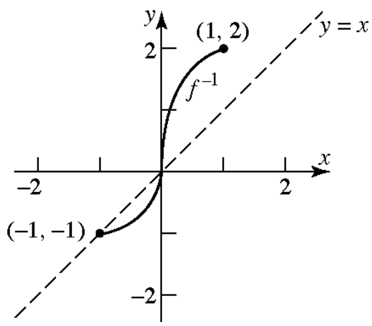
43. Graphing the inverse:



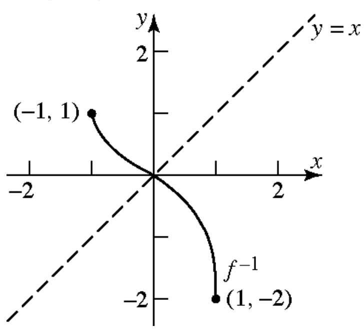
44. Graphing the inverse:



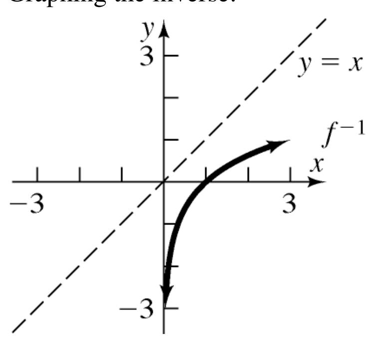
45. Graphing the inverse:



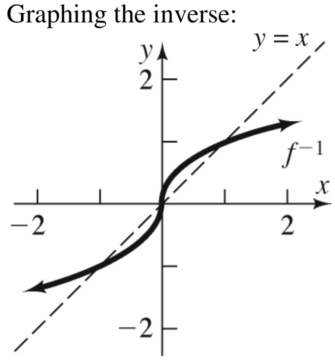
46. Graphing the inverse:



47. Graphing the inverse:



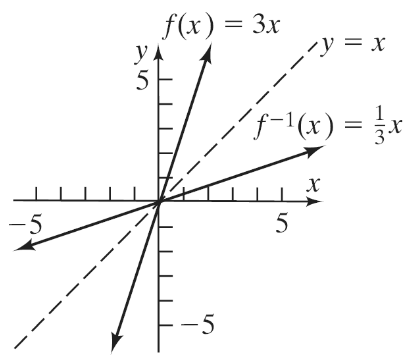
48. Graphing the inverse:



49.
$$f(x) = 3x$$
$$y = 3x$$
$$x = 3y \quad \text{Inverse}$$
$$y = \frac{x}{3}$$
$$f^{-1}(x) = \frac{1}{3}x$$

Verifying: $f\left(f^{-1}(x)\right) = f\left(\frac{1}{3}x\right) = 3\left(\frac{1}{3}x\right) = x$

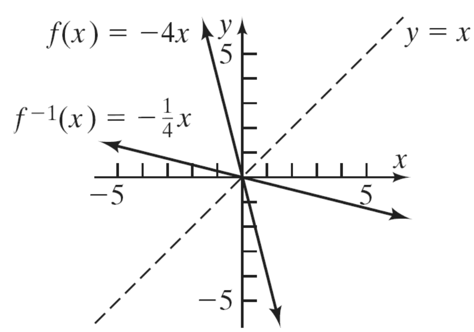
$$f^{-1}\left(f(x)\right) = f^{-1}(3x) = \frac{1}{3}(3x) = x$$



50.
$$f(x) = -4x$$
$$y = -4x$$
$$x = -4y \quad \text{Inverse}$$
$$y = \frac{x}{-4}$$
$$f^{-1}(x) = -\frac{1}{4}x$$

Verifying:

$$f\left(f^{-1}(x)\right) = f\left(-\frac{1}{4}x\right) = -4\left(-\frac{1}{4}x\right) = x$$
$$f^{-1}\left(f(x)\right) = f^{-1}(-4x) = -\frac{1}{4}(-4x) = x$$

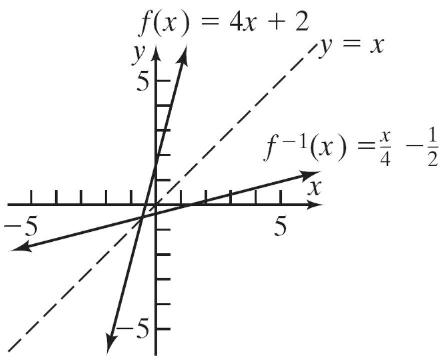


51. $f(x)=4x+2$

$$y=4x+2$$
$$x=4y+2 \quad \text{Inverse}$$
$$4y=x-2$$
$$y=\frac{x-2}{4}$$
$$y=\frac{x}{4}-\frac{1}{2}$$
$$f^{-1}(x)=\frac{x}{4}-\frac{1}{2}$$

Verifying:

$$f\left(f^{-1}(x)\right)=f\left(\frac{x}{4}-\frac{1}{2}\right)=4\left(\frac{x}{4}-\frac{1}{2}\right)+2$$
$$=x-2+2=x$$
$$f^{-1}\left(f(x)\right)=f^{-1}(4x+2)=\frac{4x+2}{4}-\frac{1}{2}$$
$$=x+\frac{1}{2}-\frac{1}{2}=x$$

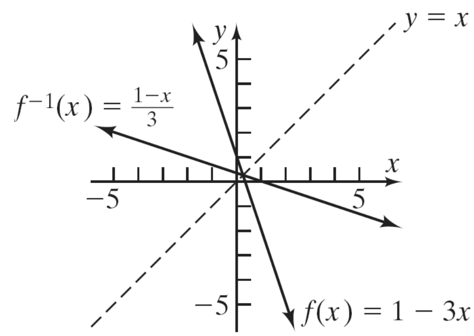


52. $f(x)=1-3x$

$$y=1-3x$$
$$x=1-3y \quad \text{Inverse}$$
$$3y=1-x$$
$$y=\frac{1-x}{3}$$
$$f^{-1}(x)=\frac{1-x}{3}$$

Verifying:

$$f\left(f^{-1}(x)\right)=f\left(\frac{1-x}{3}\right)=1-3\left(\frac{1-x}{3}\right)$$
$$=1-(1-x)=x$$
$$f^{-1}\left(f(x)\right)=f^{-1}(1-3x)=\frac{1-(1-3x)}{3}=\frac{3x}{3}=x$$

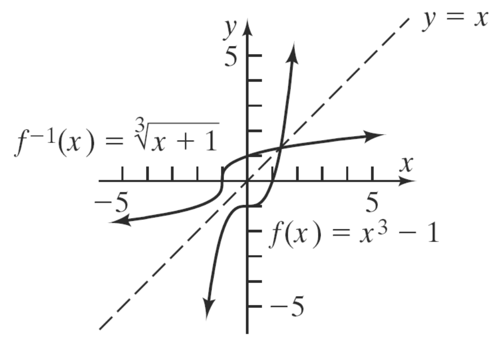


53. $f(x)=x^3-1$

$$y=x^3-1$$
$$x=y^3-1 \quad \text{Inverse}$$
$$y^3=x+1$$
$$y=\sqrt[3]{x+1}$$
$$f^{-1}(x)=\sqrt[3]{x+1}$$

Verifying:

$$f\left(f^{-1}(x)\right)=f\left(\sqrt[3]{x+1}\right)=\left(\sqrt[3]{x+1}\right)^3-1$$
$$=x+1-1=x$$
$$f^{-1}\left(f(x)\right)=f^{-1}\left(x^3-1\right)=\sqrt[3]{\left(x^3-1\right)+1}$$
$$=\sqrt[3]{x^3}=x$$

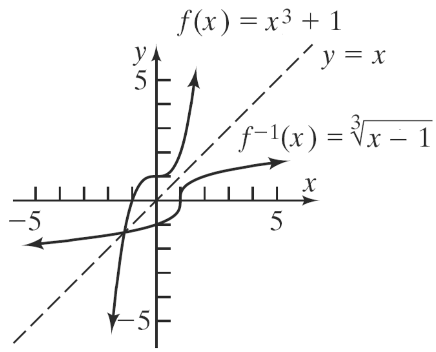


54. $f(x)=x^3+1$

$$y=x^3+1$$
$$x=y^3+1 \quad \text{Inverse}$$
$$y^3=x-1$$
$$y=\sqrt[3]{x-1}$$
$$f^{-1}(x)=\sqrt[3]{x-1}$$

Verifying:

$$f\left(f^{-1}(x)\right) = f\left(\sqrt[3]{x-1}\right) = \left(\sqrt[3]{x-1}\right)^3 + 1$$
$$= x - 1 + 1 = x$$
$$f^{-1}\left(f(x)\right) = f^{-1}\left(x^3+1\right) = \sqrt[3]{\left(x^3+1\right)-1}$$
$$= \sqrt[3]{x^3} = x$$

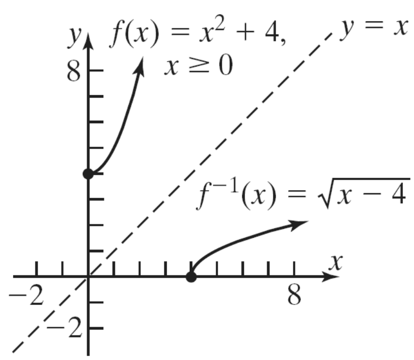


55. $f(x) = x^2 + 4,\ \ x \ge 0$

$$y = x^2 + 4,\ \ x \ge 0$$
$$x = y^2 + 4,\ y \ge 0 \quad \text{Inverse}$$
$$y^2 = x - 4,\quad x \ge 4$$
$$y = \sqrt{x-4},\ \ x \ge 4$$
$$f^{-1}(x) = \sqrt{x-4},\ \ x \ge 4$$

Verifying: $f\left(f^{-1}(x)\right) = f\left(\sqrt{x-4}\right)$

$$= \left(\sqrt{x-4}\right)^2 + 4$$
$$= x - 4 + 4 = x$$
$$f^{-1}\left(f(x)\right) = f^{-1}\left(x^2+4\right)$$
$$= \sqrt{\left(x^2+4\right)-4}$$
$$= \sqrt{x^2} = |x|$$
$$= x,\ x \ge 0$$

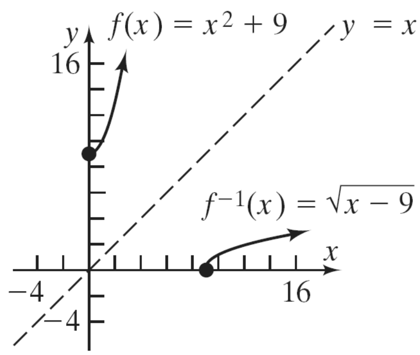


56. $f(x) = x^2 + 9,\ \ x \ge 0$

$$y = x^2 + 9,\ \ x \ge 0$$
$$x = y^2 + 9,\ \ y \ge 0 \quad \text{Inverse}$$
$$y^2 = x - 9,\quad x \ge 9$$
$$y = \sqrt{x-9},\ \ x \ge 9$$
$$f^{-1}(x) = \sqrt{x-9},\ \ x \ge 9$$

Verifying: $f\left(f^{-1}(x)\right) = f\left(\sqrt{x-9}\right)$

$$= \left(\sqrt{x-9}\right)^2 + 9$$
$$= x - 9 + 9$$
$$= x$$
$$f^{-1}\left(f(x)\right) = f^{-1}\left(x^2+9\right)$$
$$= \sqrt{\left(x^2+9\right)-9}$$
$$= \sqrt{x^2}$$
$$= |x|$$
$$= x,\ x \ge 0$$



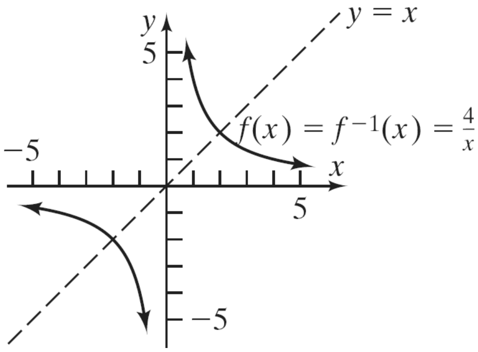
57. $f(x) = \dfrac{4}{x}$

$$y = \frac{4}{x}$$
$$x = \frac{4}{y} \quad \text{Inverse}$$
$$xy = 4$$
$$y = \frac{4}{x}$$
$$f^{-1}(x) = \frac{4}{x}$$

Verifying:

$$f\left(f^{-1}(x)\right) = f\left(\frac{4}{x}\right) = \frac{4}{\frac{4}{x}} = 4 \cdot \left(\frac{x}{4}\right) = x$$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{4}{x}\right) = \frac{4}{\frac{4}{x}} = 4\cdot\left(\frac{x}{4}\right) = x$$



58. $f(x) = -\frac{3}{x}$

$$y = -\frac{3}{x}$$

$$x = -\frac{3}{y} \quad \text{Inverse}$$

$$xy = -3$$

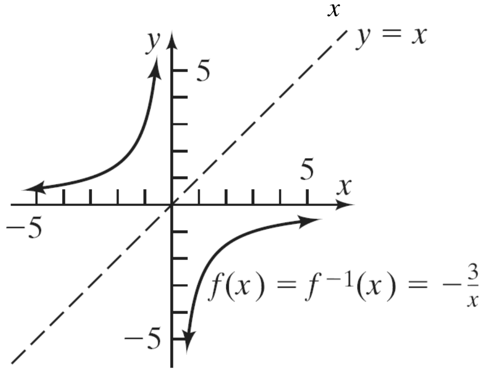
$$y = -\frac{3}{x}$$

$$f^{-1}(x) = -\frac{3}{x}$$

Verifying:

$$f\left(f^{-1}(x)\right) = f\left(-\frac{3}{x}\right) = -\frac{3}{-\frac{3}{x}} = -3\cdot\left(-\frac{x}{3}\right) = x$$

$$f^{-1}(f(x)) = f^{-1}\left(-\frac{3}{x}\right) = -\frac{3}{-\frac{3}{x}} = -3\cdot\left(-\frac{x}{3}\right) = x$$



59. $f(x) = \frac{1}{x-2}$

$$y = \frac{1}{x-2}$$

$$x = \frac{1}{y-2} \quad \text{Inverse}$$

$$xy - 2x = 1$$

$$xy = 2x + 1$$

$$y = \frac{2x+1}{x}$$

$$f^{-1}(x) = \frac{2x+1}{x}$$

Verifying:

$$f\left(f^{-1}(x)\right) = f\left(\frac{2x+1}{x}\right) = \frac{1}{\frac{2x+1}{x} - 2}$$

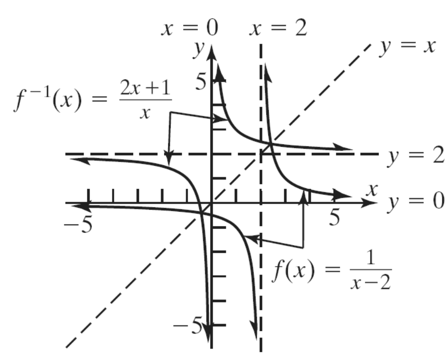
$$= \frac{1\cdot x}{\left(\frac{2x+1}{x} - 2\right)x} = \frac{x}{2x+1-2x}$$

$$= \frac{x}{1} = x$$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{1}{x-2}\right) = \frac{2\left(\frac{1}{x-2}\right)+1}{\frac{1}{x-2}}$$

$$= \frac{\left(2\left(\frac{1}{x-2}\right)+1\right)(x-2)}{\left(\frac{1}{x-2}\right)(x-2)}$$

$$= \frac{2+(x-2)}{1} = \frac{x}{1} = x$$



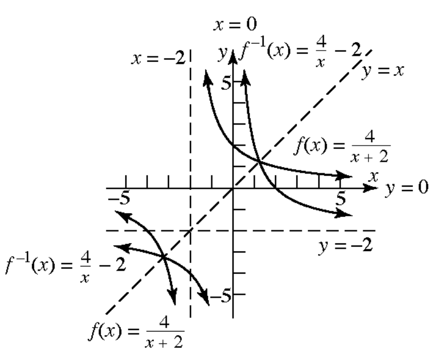
60.

$$f(x)=\frac{4}{x+2}$$
$$y=\frac{4}{x+2}$$
$$x=\frac{4}{y+2} \quad \text{Inverse}$$
$$x(y+2)=4$$
$$xy+2x=4$$
$$xy=4-2x$$
$$y=\frac{4-2x}{x}$$
$$f^{-1}(x)=\frac{4-2x}{x} \text{ or } f^{-1}(x)=\frac{4}{x}-2$$

Verifying:

$$f\left(f^{-1}(x)\right)=f\left(\frac{4-2x}{x}\right)=\frac{4}{\frac{4-2x}{x}+2}$$
$$=\frac{4\cdot x}{\left(\frac{4-2x}{x}+2\right)x}=\frac{4x}{4-2x+2x}=\frac{4x}{4}=x$$

$$f^{-1}\left(f(x)\right)=f^{-1}\left(\frac{4}{x+2}\right)=\frac{4-2\left(\frac{4}{x+2}\right)}{\frac{4}{x+2}}$$
$$=\frac{\left(4-2\left(\frac{4}{x+2}\right)\right)(x+2)}{\left(\frac{4}{x+2}\right)(x+2)}=\frac{4(x+2)-2(4)}{4}$$
$$=\frac{4x+8-8}{4}=\frac{4x}{4}=x$$



61.

$$f(x)=\frac{2}{3+x}$$
$$y=\frac{2}{3+x}$$
$$x=\frac{2}{3+y} \quad \text{Inverse}$$
$$x(3+y)=2$$
$$3x+xy=2$$
$$xy=2-3x$$
$$y=\frac{2-3x}{x}$$
$$f^{-1}(x)=\frac{2-3x}{x}$$

Verifying:

$$f\left(f^{-1}(x)\right)=f\left(\frac{2-3x}{x}\right)=\frac{2}{3+\frac{2-3x}{x}}$$
$$=\frac{2\cdot x}{\left(3+\frac{2-3x}{x}\right)x}=\frac{2x}{3x+2-3x}$$
$$=\frac{2x}{2}=x$$

$$f^{-1}\left(f(x)\right)=f^{-1}\left(\frac{2}{3+x}\right)=\frac{2-3\left(\frac{2}{3+x}\right)}{\frac{2}{3+x}}$$
$$=\frac{\left(2-3\left(\frac{2}{3+x}\right)\right)(3+x)}{\left(\frac{2}{3+x}\right)(3+x)}$$
$$=\frac{2(3+x)-3(2)}{2}=\frac{6+2x-6}{2}$$
$$=\frac{2x}{2}=x$$

62. $f(x)=\dfrac{4}{2-x}$

$$y=\frac{4}{2-x}$$
$$x=\frac{4}{2-y} \quad \text{Inverse}$$
$$x(2-y)=4$$
$$2x-xy=4$$
$$xy=2x-4$$
$$y=\frac{2x-4}{x}$$
$$y=2-\frac{4}{x}$$
$$f^{-1}(x)=2-\frac{4}{x}$$

Verifying:

$$f\left(f^{-1}(x)\right)=f\left(2-\frac{4}{x}\right)=\frac{4}{2-\left(2-\frac{4}{x}\right)}$$
$$=\frac{4}{2-2+\frac{4}{x}}=\frac{4}{\frac{4}{x}}=4\cdot\frac{x}{4}=x$$
$$f^{-1}\left(f(x)\right)=f^{-1}\left(\frac{4}{2-x}\right)=2-\frac{4}{\left(\frac{4}{2-x}\right)}$$
$$=2-4\left(\frac{2-x}{4}\right)=2-(2-x)$$
$$=2-2+x=x$$

63. $f(x)=\dfrac{3x}{x+2}$

$$y=\frac{3x}{x+2}$$
$$x=\frac{3y}{y+2} \quad \text{Inverse}$$
$$x(y+2)=3y$$
$$xy+2x=3y$$
$$xy-3y=-2x$$
$$y(x-3)=-2x$$
$$y=\frac{-2x}{x-3}$$
$$f^{-1}(x)=\frac{-2x}{x-3}$$

Verifying:

$$f\left(f^{-1}(x)\right)=f\left(\frac{-2x}{x-3}\right)$$
$$=\frac{3\left(\frac{-2x}{x-3}\right)}{\frac{-2x}{x-3}+2}=\frac{\left(3\left(\frac{-2x}{x-3}\right)\right)(x-3)}{\left(\frac{-2x}{x-3}+2\right)(x-3)}$$
$$=\frac{-6x}{-2x+2x-6}=\frac{-6x}{-6}=x$$
$$f^{-1}\left(f(x)\right)=f^{-1}\left(\frac{3x}{x+2}\right)$$
$$=\frac{-2\left(\frac{3x}{x+2}\right)}{\frac{3x}{x+2}-3}=\frac{\left(-2\left(\frac{3x}{x+2}\right)\right)(x+2)}{\left(\frac{3x}{x+2}-3\right)(x+2)}$$
$$=\frac{-6x}{3x-3x-6}=\frac{-6x}{-6}=x$$

64. $f(x)=-\dfrac{2x}{x-1}$

$$y=-\frac{2x}{x-1}$$
$$x=-\frac{2y}{y-1} \quad \text{Inverse}$$
$$x(y-1)=-2y$$
$$xy+-x=-2y$$
$$xy+2y=x$$
$$y(x+2)=x$$
$$y=\frac{x}{x+2}$$
$$f^{-1}(x)=\frac{x}{x+2}$$

Verifying:

$$f\left(f^{-1}(x)\right)=f\left(\frac{x}{x+2}\right)=-\frac{2\left(\frac{x}{x+2}\right)}{\frac{x}{x+2}-1}$$
$$=-\frac{\left(2\left(\frac{x}{x+2}\right)\right)(x+2)}{\left(\frac{x}{x+2}-1\right)(x+2)}$$
$$=\frac{-2x}{x-(x+2)}$$
$$=\frac{-2x}{-2}$$
$$=x$$

$$f^{-1}\left(f(x)\right)=f^{-1}\left(\frac{-2x}{x-1}\right)=\frac{-\frac{2x}{x-1}}{-\frac{2x}{x-1}+2}$$
$$=\frac{\left(-\frac{2x}{x-1}\right)(x-1)}{\left(-\frac{2x}{x-1}+2\right)(x-1)}$$
$$=\frac{-2x}{-2x+2x-2}$$
$$=\frac{-2x}{-2}$$
$$=x$$

65. $f(x)=\dfrac{2x}{3x-1}$

$$y=\frac{2x}{3x-1}$$
$$x=\frac{2y}{3y-1}\quad\text{Inverse}$$
$$3xy-x=2y$$
$$3xy-2y=x$$
$$y(3x-2)=x$$
$$y=\frac{x}{3x-2}$$
$$f^{-1}(x)=\frac{x}{3x-2}$$

Verifying:

$$f\left(f^{-1}(x)\right)=f\left(\frac{x}{3x-2}\right)=\frac{2\left(\frac{x}{3x-2}\right)}{3\left(\frac{x}{3x-2}\right)-1}$$
$$=\frac{\left(2\left(\frac{x}{3x-2}\right)\right)(3x-2)}{\left(3\left(\frac{x}{3x-2}\right)-1\right)(3x-2)}$$
$$=\frac{2x}{3x-(3x-2)}=\frac{2x}{2}=x$$

$$f^{-1}\left(f(x)\right)=f\left(\frac{2x}{3x-1}\right)=\frac{\frac{2x}{3x-1}}{3\left(\frac{2x}{3x-1}\right)-2}$$
$$=\frac{\left(\frac{2x}{3x-1}\right)(3x-1)}{\left(3\left(\frac{2x}{3x-1}\right)-2\right)(3x-1)}$$
$$=\frac{2x}{3(2x)-2(3x-1)}$$
$$=\frac{2x}{6x-6x+2}=\frac{2x}{2}=x$$

66. $f(x)=-\dfrac{3x+1}{x}$

$$y=-\frac{3x+1}{x}$$
$$x=-\frac{3y+1}{y}\quad\text{Inverse}$$
$$xy=-(3y+1)$$
$$xy=-3y-1$$
$$xy+3y=-1$$
$$y(x+3)=-1$$
$$y=\frac{-1}{x+3}$$
$$f^{-1}(x)=\frac{-1}{x+3}$$

Verifying:

$$f\left(f^{-1}(x)\right)=f\left(\frac{-1}{x+3}\right)$$
$$=\frac{3\left(-\frac{1}{x+3}\right)+1}{-\left(-\frac{1}{x+3}\right)}=\frac{\frac{-3}{x+3}+1}{\frac{1}{x+3}}$$
$$=\left(\frac{-3}{x+3}+1\right)\cdot\frac{x+3}{1}$$
$$=-3+(x+3)$$
$$=x$$

$$f^{-1}\left(f(x)\right)=f^{-1}\left(\frac{3x+1}{-x}\right)$$
$$=\frac{-1}{\left(\frac{3x+1}{-x}\right)+3}=\frac{1}{\frac{3x+1}{x}-3}$$
$$=\frac{1\cdot x}{\left(\frac{3x+1}{x}-3\right)x}=\frac{x}{3x+1-3x}$$
$$=\frac{x}{1}=x$$

67.
$$f(x)=\frac{3x+4}{2x-3}$$
$$y=\frac{3x+4}{2x-3}$$
$$x=\frac{3y+4}{2y-3}\quad\text{Inverse}$$
$$x(2y-3)=3y+4$$
$$2xy-3x=3y+4$$
$$2xy-3y=3x+4$$
$$y(2x-3)=3x+4$$
$$y=\frac{3x+4}{2x-3}$$
$$f^{-1}(x)=\frac{3x+4}{2x-3}$$

Verifying:

$$f\left(f^{-1}(x)\right)=f\left(\frac{3x+4}{2x-3}\right)=\frac{3\left(\frac{3x+4}{2x-3}\right)+4}{2\left(\frac{3x+4}{2x-3}\right)-3}$$
$$=\frac{\left(3\left(\frac{3x+4}{2x-3}\right)+4\right)(2x-3)}{\left(2\left(\frac{3x+4}{2x-3}\right)-3\right)(2x-3)}$$
$$=\frac{3(3x+4)+4(2x-3)}{2(3x+4)-3(2x-3)}$$
$$=\frac{9x+12+8x-12}{6x+8-6x+9}=\frac{17x}{17}=x$$

$$f^{-1}\left(f(x)\right)=f^{-1}\left(\frac{3x+4}{2x-3}\right)=\frac{3\left(\frac{3x+4}{2x-3}\right)+4}{2\left(\frac{3x+4}{2x-3}\right)-3}$$
$$=\frac{\left(3\left(\frac{3x+4}{2x-3}\right)+4\right)(2x-3)}{\left(2\left(\frac{3x+4}{2x-3}\right)-3\right)(2x-3)}$$
$$=\frac{3(3x+4)+4(2x-3)}{2(3x+4)-3(2x-3)}$$
$$=\frac{9x+12+8x-12}{6x+8-6x+9}=\frac{17x}{17}=x$$

68.
$$f(x)=\frac{2x-3}{x+4}$$
$$y=\frac{2x-3}{x+4}$$
$$x=\frac{2y-3}{y+4}\quad\text{Inverse}$$
$$x(y+4)=2y-3$$
$$xy+4x=2y-3$$
$$xy-2y=-4x-3$$
$$y(x-2)=-(4x+3)$$
$$y=\frac{-(4x+3)}{x-2}=\frac{4x+3}{2-x}$$
$$f^{-1}(x)=\frac{4x+3}{2-x}$$

Verifying:

$$f\left(f^{-1}(x)\right) = f\left(\frac{4x+3}{2-x}\right) = \frac{2\left(\frac{4x+3}{2-x}\right)-3}{\frac{4x+3}{2-x}+4}$$
$$= \frac{2(4x+3)-3(2-x)}{4x+3+4(2-x)}$$
$$= \frac{8x+6-6+3x}{4x+3+8-4x}$$
$$= \frac{11x}{11}$$
$$= x$$

$$f^{-1}\left(f(x)\right) = f^{-1}\left(\frac{2x-3}{x+4}\right)$$
$$= \frac{4\left(\frac{2x-3}{x+4}\right)+3}{2-\frac{2x-3}{x+4}}$$
$$= \frac{4(2x-3)+3(x+4)}{2(x+4)-(2x-3)}$$
$$= \frac{8x-12+3x+12}{2x+8-2x+3}$$
$$= \frac{11x}{11}$$
$$= x$$

Verifying:

$$f\left(f^{-1}(x)\right) = f\left(\frac{-2x+3}{x-2}\right) = \frac{2\left(\frac{-2x+3}{x-2}\right)+3}{\frac{-2x+3}{x-2}+2}$$
$$= \frac{\left(2\left(\frac{-2x+3}{x-2}\right)+3\right)(x-2)}{\left(\frac{-2x+3}{x-2}+2\right)(x-2)}$$
$$= \frac{2(-2x+3)+3(x-2)}{-2x+3+2(x-2)}$$
$$= \frac{-4x+6+3x-6}{-2x+3+2x-4} = \frac{-x}{-1} = x$$

$$f^{-1}\left(f(x)\right) = f^{-1}\left(\frac{2x+3}{x+2}\right) = \frac{-2\left(\frac{2x+3}{x+2}\right)+3}{\frac{2x+3}{x+2}-2}$$
$$= \frac{\left(-2\left(\frac{2x+3}{x+2}\right)+3\right)(x+2)}{\left(\frac{2x+3}{x+2}-2\right)(x+2)}$$
$$= \frac{-2(2x+3)+3(x+2)}{2x+3-2(x+2)}$$
$$= \frac{-4x-6+3x+6}{2x+3-2x-4} = \frac{-x}{-1} = x$$

69. $f(x) = \frac{2x+3}{x+2}$

$$y = \frac{2x+3}{x+2}$$
$$x = \frac{2y+3}{y+2} \quad \text{Inverse}$$
$$xy+2x = 2y+3$$
$$xy-2y = -2x+3$$
$$y(x-2) = -2x+3$$
$$y = \frac{-2x+3}{x-2}$$
$$f^{-1}(x) = \frac{-2x+3}{x-2}$$

70. $f(x) = \frac{-3x-4}{x-2}$

$$y = \frac{-3x-4}{x-2}$$
$$x = \frac{-3y-4}{y-2} \quad \text{Inverse}$$
$$x(y-2) = -3y-4$$
$$xy-2x = -3y-4$$
$$xy+3y = 2x-4$$
$$y(x+3) = 2x-4$$
$$y = \frac{2x-4}{x+3}$$
$$f^{-1}(x) = \frac{2x-4}{x+3}$$

Verifying:

$$f\left(f^{-1}(x)\right)=f\left(\frac{2x-4}{x+3}\right)$$
$$=\frac{-3\left(\frac{2x-4}{x+3}\right)-4}{\frac{2x-4}{x+3}-2}$$
$$=\frac{-3(2x-4)-4(x+3)}{2x-4-2(x+3)}$$
$$=\frac{-6x+12-4x-12}{2x-4-2x-6}$$
$$=\frac{-10x}{-10}$$
$$=x$$

$$f^{-1}\left(f(x)\right)=f^{-1}\left(\frac{-3x-4}{x-2}\right)$$
$$=\frac{2\left(\frac{-3x-4}{x-2}\right)-4}{\frac{-3x-4}{x-2}+3}$$
$$=\frac{2(-3x-4)-4(x-2)}{-3x-4+3(x-2)}$$
$$=\frac{-6x-8-4x+8}{-3x-4+3x-6}$$
$$=\frac{-10x}{-10}$$
$$=x$$

71. $f(x)=\frac{x^2-4}{2x^2},\ x>0$

$$y=\frac{x^2-4}{2x^2},\quad x>0$$
$$x=\frac{y^2-4}{2y^2},\quad y>0 \quad \text{Inverse}$$
$$2xy^2=y^2-4,\quad x<\frac{1}{2}$$
$$2xy^2-y^2=-4,\quad x<\frac{1}{2}$$
$$y^2\left(2x-1\right)=-4,\quad x<\frac{1}{2}$$
$$y^2\left(1-2x\right)=4,\quad x<\frac{1}{2}$$
$$y^2=\frac{4}{1-2x},\quad x<\frac{1}{2}$$
$$y=\sqrt{\frac{4}{1-2x}},\quad x<\frac{1}{2}$$
$$y=\frac{2}{\sqrt{1-2x}},\quad x<\frac{1}{2}$$
$$f^{-1}\left(x\right)=\frac{2}{\sqrt{1-2x}},\quad x<\frac{1}{2}$$

Verifying:

$$f\left(f^{-1}(x)\right)=f\left(\frac{2}{\sqrt{1-2x}}\right)=\frac{\left(\frac{2}{\sqrt{1-2x}}\right)^2-4}{2\left(\frac{2}{\sqrt{1-2x}}\right)^2}$$
$$=\frac{\frac{4}{1-2x}-4}{2\left(\frac{4}{1-2x}\right)}=\frac{\left(\frac{4}{1-2x}-4\right)(1-2x)}{\left(2\left(\frac{4}{1-2x}\right)\right)(1-2x)}$$
$$=\frac{4-4(1-2x)}{2(4)}=\frac{4-4+8x}{8}=\frac{8x}{8}=x$$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{x^2-4}{2x^2}\right) = \frac{2}{\sqrt{1-2\left(\frac{x^2-4}{2x^2}\right)}}$$

$$= \frac{2}{\sqrt{1-\frac{x^2-4}{x^2}}} = \frac{2}{\sqrt{1-1+\frac{4}{x^2}}}$$

$$= \frac{2}{\sqrt{\frac{4}{x^2}}} = \frac{2}{\frac{2}{|x|}} = 2\cdot\frac{|x|}{2}$$

$$= |x| = x, \quad x>0$$

72.
$$f(x) = \frac{x^2+3}{3x^2}, \qquad x>0$$

$$y = \frac{x^2+3}{3x^2}, \qquad x>0$$

$$x = \frac{y^2+3}{3y^2}, \qquad y>0 \quad \text{Inverse}$$

$$3xy^2 = y^2+3, \qquad x>\frac{1}{3}$$

$$3xy^2 - y^2 = 3, \qquad x>\frac{1}{3}$$

$$y^2(3x-1) = 3, \qquad x>\frac{1}{3}$$

$$y^2 = \frac{3}{3x-1}, \qquad x>\frac{1}{3}$$

$$y = \sqrt{\frac{3}{3x-1}}, \quad x>\frac{1}{3}$$

$$f^{-1}(x) = \sqrt{\frac{3}{3x-1}}, \quad x>\frac{1}{3}$$

Verifying:

$$f(f^{-1}(x)) = f\left(\sqrt{\frac{3}{3x-1}}\right) = \frac{\left(\sqrt{\frac{3}{3x-1}}\right)^2+3}{3\left(\sqrt{\frac{3}{3x-1}}\right)^2}$$

$$= \frac{\frac{3}{3x-1}+3}{3\left(\frac{3}{3x-1}\right)} = \frac{\left(\frac{3}{3x-1}+3\right)(3x-1)}{\left(3\left(\frac{3}{3x-1}\right)\right)(3x-1)}$$

$$= \frac{3+3(3x-1)}{3(3)}$$

$$= \frac{3+9x-3}{9}$$

$$= \frac{9x}{9}$$

$$= x$$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{x^2+3}{3x^2}\right) = \sqrt{\frac{3}{3\left(\frac{x^2+3}{3x^2}\right)-1}}$$

$$= \sqrt{\frac{3}{\frac{x^2+3}{x^2}-1}} = \sqrt{\frac{3}{1+\frac{3}{x^2}-1}}$$

$$= \sqrt{\frac{3}{\frac{3}{x^2}}} = \sqrt{(3)\left(\frac{x^2}{3}\right)} = \sqrt{x^2}$$

$$= |x| = x, \quad x>0$$

73. **a.** Because the ordered pair $(-1,0)$ is on the graph, $f(-1)=0$.

b. Because the ordered pair $(1,2)$ is on the graph, $f(1)=2$.

c. Because the ordered pair $(0,1)$ is on the graph, $f^{-1}(1)=0$.

d. Because the ordered pair $(1,2)$ is on the graph, $f^{-1}(2)=1$.

74. **a.** Because the ordered pair $\left(2,\frac{1}{2}\right)$ is on the graph, $f(2)=\frac{1}{2}$.

b. Because the ordered pair $(1,0)$ is on the graph, $f(1)=0$.

c. Because the ordered pair $(1,0)$ is on the graph, $f^{-1}(0)=1$.

d. Because the ordered pair $(0,-1)$ is on the graph, $f^{-1}(-1)=0$.

75. Since $f(7)=13$, we have $f^{-1}(13)=7$; the input of the function is the output of the inverse when the output of the function is the input of the inverse.

76. Since $g(-5)=3$, we have $g^{-1}(3)=-5$; the input of the function is the output of the inverse when the output of the function is the input of the inverse.

77. Since the domain of a function is the range of the inverse, and the range of the function is the domain of the inverse, we get the following for f^{-1}:
Domain: $[-2,\infty)$ Range: $[5,\infty)$

78. Since the domain of a function is the range of the inverse, and the range of the function is the domain of the inverse, we get the following for f^{-1}:
Domain: $[5,\infty)$ Range: $[0,\infty)$

79. Since the domain of a function is the range of the inverse, and the range of the function is the domain of the inverse, we get the following for g^{-1}:
Domain: $[0,\infty)$ Range: $(-\infty,0]$

80. Since the domain of a function is the range of the inverse, and the range of the function is the domain of the inverse, we get the following for g^{-1}:
Domain: $(0,8)$ Range: $[0,15]$

81. Since $f(x)$ is increasing on the interval $(0,5)$, it is one-to-one on the interval and has an inverse, $f^{-1}(x)$. In addition, we can say that $f^{-1}(x)$ is increasing on the interval $(f(0),f(5))$.

82. Since $f(x)$ is decreasing on the interval $(0,5)$, it is one-to-one on the interval and has an inverse, $f^{-1}(x)$. In addition, we can say that $f^{-1}(x)$ is decreasing on the interval $(f(5),f(0))$.

83.
$$f(x)=mx+b,\quad m\neq 0$$
$$y=mx+b$$
$$x=my+b \quad \text{Inverse}$$
$$x-b=my$$
$$y=\frac{1}{m}(x-b)$$
$$f^{-1}(x)=\frac{1}{m}(x-b),\quad m\neq 0$$

84.
$$f(x)=\sqrt{r^2-x^2},\quad 0\le x\le r$$
$$y=\sqrt{r^2-x^2}$$
$$x=\sqrt{r^2-y^2} \quad \text{Inverse}$$
$$x^2=r^2-y^2$$
$$y^2=r^2-x^2$$
$$y=\sqrt{r^2-x^2}$$
$$f^{-1}(x)=\sqrt{r^2-x^2},\quad 0\le x\le r$$

85. If (a,b) is on the graph of *f*, then (b,a) is on the graph of f^{-1}. Since the graph of f^{-1} lies in quadrant I, both coordinates of (a,b) are positive, which means that both coordinates of (b,a) are positive. Thus, the graph of f^{-1} must lie in quadrant I.

86. If (a,b) is on the graph of *f*, then (b,a) is on the graph of f^{-1}. Since the graph of *f* lies in quadrant II, *a* must be negative and *b* must be positive. Thus, (b,a) must be a point in quadrant IV, which means the graph of f^{-1} lies in quadrant IV.

87. Answers may vary. One possibility follows:
$f(x)=|x|,\ x\ge 0$ is one-to-one.
Thus, $f(x)=x,\ x\ge 0$
$$y=x,\ x\ge 0$$
$$f^{-1}(x)=x,\ x\ge 0$$

88. Answers may vary. One possibility follows:
$f(x)=x^4,\ x\ge 0$ is one-to-one.

Thus,
$$\begin{aligned} f(x)&=x^4, \quad x\ge 0\\ y&=x^4, \quad x\ge 0\\ x&=y^4 \quad \text{Inverse}\\ y&=\sqrt[4]{x}, \quad x\ge 0\\ f^{-1}(x)&=\sqrt[4]{x}, \quad x\ge 0 \end{aligned}$$

89. a.
$$\begin{aligned} d&=6.97r-90.39\\ d+90.39&=6.97r\\ \frac{d+90.39}{6.97}&=r \end{aligned}$$
Therefore, we would write
$$r(d)=\frac{d+90.39}{6.97}$$

b.
$$\begin{aligned} r(d(r))&=\frac{(6.97r-90.39)+90.39}{6.97}\\ &=\frac{6.97r+90.39-90.39}{6.97}=\frac{6.97r}{6.97}\\ &=r \end{aligned}$$
$$\begin{aligned} d(r(d))&=6.97\left(\frac{d+90.39}{6.97}\right)-90.39\\ &=d+90.39-90.39\\ &=d \end{aligned}$$

c. $r(300)=\dfrac{300+90.39}{6.97}\approx 56.01$

If the distance required to stop was 300 feet, the speed of the car was roughly 56 miles per hour.

90. a.
$$\begin{aligned} H(C)&=2.15C-10.53\\ H&=2.15C-10.53\\ H+10.53&=2.15C\\ \frac{H+10.53}{2.15}&=C\\ C(H)&=\frac{H+10.53}{2.15} \end{aligned}$$

b.
$$\begin{aligned} H(C(H))&=2.15\left(\frac{H+10.53}{2.15}\right)-10.53\\ &=H+10.53-10.53\\ &=H \end{aligned}$$
$$\begin{aligned} C(H(C))&=\frac{(2.15C-10.53)+10.53}{2.15}\\ &=\frac{2.15C-10.53+10.53}{2.15}\\ &=\frac{2.15C}{2.15}=C \end{aligned}$$

c. $C(26)=\dfrac{26+10.53}{2.15}\approx 16.99$

The head circumference of a child who is 26 inches tall is about 17 inches.

91. a. 6 feet = 72 inches
$$\begin{aligned} W(72)&=50+2.3(72-60)\\ &=50+2.3(12)=50+27.6=77.6 \end{aligned}$$
The ideal weight of a 6-foot male is 77.6 kilograms.

b.
$$\begin{aligned} W&=50+2.3(h-60)\\ W-50&=2.3h-138\\ W+88&=2.3h\\ \frac{W+88}{2.3}&=h \end{aligned}$$
Therefore, we would write
$$h(W)=\frac{W+88}{2.3}$$

c.
$$\begin{aligned} h(W(h))&=\frac{(50+2.3(h-60))+88}{2.3}\\ &=\frac{50+2.3h-138+88}{2.3}=\frac{2.3h}{2.3}=h \end{aligned}$$
$$\begin{aligned} W(h(W))&=50+2.3\left(\frac{W+88}{2.3}-60\right)\\ &=50+W+88-138=W \end{aligned}$$

d. $h(80)=\dfrac{80+88}{2.3}=\dfrac{168}{2.3}\approx 73.04$

The height of a male who is at his ideal weight of 80 kg is roughly 73 inches.

92. a.
$$\begin{aligned} F&=\frac{9}{5}C+32\\ F-32&=\frac{9}{5}C\\ \frac{5}{9}(F-32)&=C \end{aligned}$$
Therefore, we would write
$$C(F)=\frac{5}{9}(F-32)$$

b. $C\left(F\left(C\right)\right)=\frac{5}{9}\left(\left(\frac{9}{5}C+32\right)-32\right)$
$=\frac{5}{9}\cdot\frac{9}{5}C=C$
$F\left(C\left(F\right)\right)=\frac{9}{5}\left(\frac{5}{9}\left(F-32\right)\right)+32$
$=F-32+32=F$

c. $C\left(70\right)=\frac{5}{9}\left(70-32\right)=\frac{5}{9}\left(38\right)\approx 21.1°\text{C}$

93. a. From the restriction given in the problem statement, the domain is $\{g\,|\,34,500\le g\le 83,600\}$ or $[34500,83600]$.

b. $T\left(34,500\right)=4750+0.25\left(34,500-34,500\right)$
$=4750$
$T\left(83,600\right)=4750+0.25\left(83,600-34,500\right)$
$=17,025$
Since T is linear and increasing, we have that the range is $\{T\,|\,4750\le T\le 17,025\}$ or $[4750,17025]$.

c. $T=4750+0.25\left(g-34,500\right)$
$T-4750=0.25\left(g-34,500\right)$
$\frac{T-4750}{0.25}=g-34,500$
$\frac{T-4750}{0.25}+34,500=g$
Therefore, we would write
$g\left(T\right)=\frac{T-4750}{0.25}+34,500$
Domain: $\{T\,|\,4750\le T\le 17,025\}$
Range: $\{g\,|\,34,500\le g\le 83,600\}$

94. a. From the restriction given in the problem statement, the domain is $\{g\,|\,17,000\le g\le 69,000\}$ or $[17000,\ 69000]$.

b. $T\left(17,000\right)=1700+0.15\left(17,000-17,000\right)$
$=1700$
$T\left(69,000\right)=1700+0.15\left(69,000-17,000\right)$
$=9500$
Since T is linear and increasing, we have that the range is $\{T\,|\,1700\le T\le 9500\}$ or $[1700,\ 9500]$.

c.
$T=1700+0.15\left(g-17,000\right)$
$T-1700=0.15\left(g-17,000\right)$
$\frac{T-1700}{0.15}=g-17,000$
$\frac{T-1700}{0.15}+17,000=g$

We would write $g\left(T\right)=\frac{T-1700}{0.15}+17,000$.
Domain: $\{T\,|\,1700\le T\le 9500\}$
Range: $\{g\,|\,17,000\le g\le 69,000\}$

95. a. The graph of H is symmetric about the y-axis. Since t represents the number of seconds *after* the rock begins to fall, we know that $t\ge 0$. The graph is strictly decreasing over its domain, so it is one-to-one.

b. $H=100-4.9t^2$
$H+4.9t^2=100$
$4.9t^2=100-H$
$t^2=\frac{100-H}{4.9}$
$t=\sqrt{\frac{100-H}{4.9}}$

Therefore, we would write $t\left(H\right)=\sqrt{\frac{100-H}{4.9}}$.
(Note: we only need the principal square root since we know $t\ge 0$)

$H\left(t\left(H\right)\right)=100-4.9\left(\sqrt{\frac{100-H}{4.9}}\right)^2$
$=100-4.9\left(\frac{100-H}{4.9}\right)$
$=100-100+H$
$=H$
$t\left(H\left(t\right)\right)=\sqrt{\frac{100-\left(100-4.9t^2\right)}{4.9}}$
$=\sqrt{\frac{4.9t^2}{4.9}}=\sqrt{t^2}=t \quad (\text{since } t\ge 0)$

c. $t\left(80\right)=\sqrt{\frac{100-80}{4.9}}\approx 2.02$
It will take the rock about 2.02 seconds to fall 80 meters.

96. **a.** $T(l) = 2\pi\sqrt{\dfrac{l}{32.2}}$

$$T = 2\pi\sqrt{\frac{l}{32.2}}$$
$$\frac{T}{2\pi} = \sqrt{\frac{l}{32.2}}$$
$$\frac{T^2}{4\pi^2} = \frac{l}{32.2}$$
$$l = \frac{32.2T^2}{4\pi^2}$$
$$l(T) = \frac{8.05T^2}{\pi^2} = 8.05\left(\frac{T}{\pi}\right)^2, \quad T > 0$$

b. $l(3) = 8.05\left(\dfrac{3}{\pi}\right)^2 \approx 7.34$

A pendulum whose period is 3 seconds will be about 7.34 feet long.

97. $f(x) = \dfrac{ax+b}{cx+d}$

$$y = \frac{ax+b}{cx+d}$$
$$x = \frac{ay+b}{cy+d} \quad \text{Inverse}$$
$$x(cy+d) = ay+b$$
$$cxy+dx = ay+b$$
$$cxy-ay = b-dx$$
$$y(cx-a) = b-dx$$
$$y = \frac{b-dx}{cx-a}$$
$$f^{-1}(x) = \frac{-dx+b}{cx-a}$$

Now, $f = f^{-1}$ provided that $\dfrac{ax+b}{cx+d} = \dfrac{-dx+b}{cx-a}$.

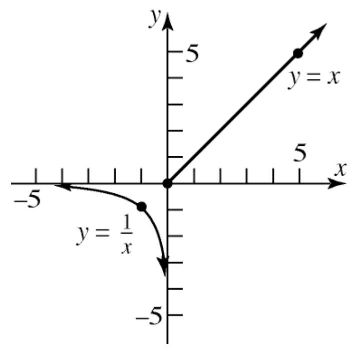
This is only true if $a = -d$.

98. Yes. In order for a one-to-one function and its inverse to be equal, its graph must be symmetric about the line $y = x$. One such example is the function $f(x) = \dfrac{1}{x}$.

99. Answers will vary.

100. Answers will vary. One example is

$$f(x) = \begin{cases} \dfrac{1}{x}, & \text{if } x < 0 \\ x, & \text{if } x \ge 0 \end{cases}$$



This function is one-to-one since the graph passes the Horizontal Line Test. However, the function is neither increasing nor decreasing on its domain.

101. No, not every odd function is one-to-one. For example, $f(x) = x^3 - x$ is an odd function, but it is not one-to-one.

102. $C^{-1}(800,000)$ represents the number of cars manufactured for \$800,000.

103. If a horizontal line passes through two points on a graph of a function, then the y value associated with that horizontal line will be assigned to two different x values which violates the definition of one-to-one.

Section 5.3

1. $4^3 = 64$; $8^{2/3} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$; $3^{-2} = \dfrac{1}{3^2} = \dfrac{1}{9}$

2.
$$x^2 + 3x - 4 = 0$$
$$(x+4)(x-1) = 0$$
$$x+4 = 0 \text{ or } x-1 = 0$$
The solution set is $\{-4, 1\}$.

3. False. To obtain the graph of $y = (x-2)^3$, we would shift the graph of $y = x^3$ to the *right* 2 units.

4. $\dfrac{f(4)-f(0)}{4-0}=\dfrac{[3(4)-5]-[3(0)-5]}{4}$
$=\dfrac{(12-5)-(0-5)}{4}$
$=\dfrac{7-(-5)}{4}$
$=\dfrac{12}{4}$
$=3$

5. True

6. exponential function; growth factor; initial value

7. a

8. True

9. False. The range will be $\{x \mid x>0\}$ or $(0, \infty)$.

10. True

11. $\left(-1,\dfrac{1}{a}\right),\ (0,1),\ (1,a)$

12. 1

13. 4

14. False ; for example, the point $(1,3)$ is on the first graph and $\left(1,\frac{1}{3}\right)$ is on the other.

15. **a.** $3^{2.2}\approx 11.212$

b. $3^{2.23}\approx 11.587$

c. $3^{2.236}\approx 11.664$

d. $3^{\sqrt{5}}\approx 11.665$

16. **a.** $5^{1.7}\approx 15.426$

b. $5^{1.73}\approx 16.189$

c. $5^{1.732}\approx 16.241$

d. $5^{\sqrt{3}}\approx 16.242$

17. **a.** $2^{3.14}\approx 8.815$

b. $2^{3.141}\approx 8.821$

c. $2^{3.1415}\approx 8.824$

d. $2^{\pi}\approx 8.825$

18. **a.** $2^{2.7}\approx 6.498$

b. $2^{2.71}\approx 6.543$

c. $2^{2.718}\approx 6.580$

d. $2^{e}\approx 6.581$

19. **a.** $3.1^{2.7}\approx 21.217$

b. $3.14^{2.71}\approx 22.217$

c. $3.141^{2.718}\approx 22.440$

d. $\pi^{e}\approx 22.459$

20. **a.** $2.7^{3.1}\approx 21.738$

b. $2.71^{3.14}\approx 22.884$

c. $2.718^{3.141}\approx 23.119$

d. $e^{\pi}\approx 23.141$

21. $e^{1.2}\approx 3.320$

22. $e^{-1.3}\approx 0.273$

23. $e^{-0.85}\approx 0.427$

24. $e^{2.1}\approx 8.166$

25.

x	$y=f(x)$	$\dfrac{\Delta y}{\Delta x}$	$\dfrac{f(x+1)}{f(x)}$
-1	3		$\dfrac{6}{3}=2$
0	6	$\dfrac{6-3}{0-(-1)}=3$	$\dfrac{12}{6}=2$
1	12	$\dfrac{12-6}{1-0}=6$	$\dfrac{18}{12}=\dfrac{3}{2}$
2	18		
3	30		

Not a linear function since the average rate of change is not constant.

Not an exponential function since the ratio of consecutive terms is not constant.

26.

x	$y=g(x)$	$\dfrac{\Delta y}{\Delta x}$	$\dfrac{g(x+1)}{g(x)}$
-1	2		$\dfrac{5}{2}$

0	5	$\frac{5-2}{0-(-1)}=3$	$\frac{8}{5}$
1	8	$\frac{8-5}{1-0}=3$	
2	11	$\frac{11-8}{2-1}=3$	
3	14	$\frac{14-11}{3-2}=3$	

Not an exponential function since the ratio of consecutive terms is not constant.

The average rate of change is a constant, 3. Therefore, this is a linear function. In a linear function the average rate of change is the slope *m*. So, $m=3$. When $x=0$ we have $y=5$ so the *y*-intercept is $b=5$. The linear function that models this data is $g(x)=mx+b=3x+5$.

27.

x	$y=H(x)$	$\frac{\Delta y}{\Delta x}$	$\frac{H(x+1)}{H(x)}$
–1	$\frac{1}{4}$		$\frac{1}{(1/4)}=4$
0	1	$\frac{1-\frac{1}{4}}{0-(-1)}=\frac{3}{4}$	$\frac{4}{1}=4$
1	4	$\frac{4-1}{1-0}=3$	$\frac{16}{4}=4$
2	16		$\frac{64}{16}=4$
3	64		

Not a linear function since the average rate of change is not constant.

The ratio of consecutive outputs is a constant, 4. This is an exponential function with growth factor $a=4$. The initial value of the exponential function is $C=1$. Therefore, the exponential function that models the data is $H(x)=Ca^x=1\cdot(4)^x=4^x$.

28.

x	$y=F(x)$	$\frac{\Delta y}{\Delta x}$	$\frac{F(x+1)}{F(x)}$
–1	$\frac{2}{3}$		$\frac{1}{(2/3)}=\frac{3}{2}$
0	1	$\frac{1-\frac{2}{3}}{0-(-1)}=\frac{1}{3}$	$\frac{(3/2)}{1}=\frac{3}{2}$
1	$\frac{3}{2}$	$\frac{\frac{3}{2}-1}{1-0}=\frac{1}{2}$	$\frac{(9/4)}{(3/2)}=\frac{3}{2}$
2	$\frac{9}{4}$		$\frac{(27/8)}{(9/4)}=\frac{3}{2}$
3	$\frac{27}{8}$		

Not a linear function since the average rate of change is not constant.

The ratio of consecutive outputs is a constant, $\frac{3}{2}$. This is an exponential function with growth factor $a=\frac{3}{2}$. The initial value of the exponential function is $C=1$. Therefore, the exponential function that models the data is

$F(x)=Ca^x=1\cdot\left(\frac{3}{2}\right)^x=\left(\frac{3}{2}\right)^x$.

29.

x	$y=f(x)$	$\frac{\Delta y}{\Delta x}$	$\frac{f(x+1)}{f(x)}$
–1	$\frac{3}{2}$		$\frac{3}{(3/2)}=2$
0	3	$\frac{3-\frac{3}{2}}{0-(-1)}=\frac{3}{2}$	$\frac{6}{3}=2$
1	6	$\frac{6-3}{1-0}=3$	$\frac{12}{6}=2$
2	12		$\frac{24}{12}=2$
3	24		

Not a linear function since the average rate of change is not constant.

The ratio of consecutive outputs is a constant, 2. This is an exponential function with growth factor $a=2$. The initial value of the exponential function is $C=3$. Therefore, the exponential function that models the data is $f(x)=Ca^x=3\cdot(2)^x=3\cdot 2^x$.

30.

x	$y=g(x)$	$\frac{\Delta y}{\Delta x}$	$\frac{g(x+1)}{g(x)}$
–1	6		$\frac{1}{6}$

0	1	$\frac{1-6}{0-(-1)}=-5$	$\frac{0}{1}=0$
1	0	$\frac{0-1}{1-0}=-1$	
2	3		
3	10		

Not a linear function since the average rate of change is not constant.

Not an exponential function since the ratio of consecutive terms is not constant.

31.

x	$y=H(x)$	$\frac{\Delta y}{\Delta x}$	$\frac{H(x+1)}{H(x)}$
–1	2		$\frac{4}{2}=2$
0	4	$\frac{4-2}{0-(-1)}=2$	$\frac{6}{4}=\frac{3}{2}$
1	6	$\frac{6-4}{1-0}=2$	
2	8	$\frac{8-6}{2-1}=2$	
3	10	$\frac{10-8}{3-2}=2$	

Not an exponential function since the ratio of consecutive terms is not constant.

The average rate of change is a constant, 2. Therefore, this is a linear function. In a linear function the average rate of change is the slope m. So, $m=2$. When $x=0$ we have $y=4$ so the y-intercept is $b=4$. The linear function that models this data is $H(x)=mx+b=2x+4$.

32.

x	$y=F(x)$	$\frac{\Delta y}{\Delta x}$	$\frac{F(x+1)}{F(x)}$
–1	$\frac{1}{2}$		$\frac{(1/4)}{(1/2)}=\frac{1}{2}$
0	$\frac{1}{4}$	$\frac{\frac{1}{4}-\frac{1}{2}}{0-(-1)}=-\frac{1}{4}$	$\frac{(1/8)}{(1/4)}=\frac{1}{2}$
1	$\frac{1}{8}$	$\frac{\frac{1}{8}-\frac{1}{4}}{1-0}=\frac{1}{8}$	$\frac{(1/16)}{(1/8)}=\frac{1}{2}$
2	$\frac{1}{16}$		$\frac{(1/32)}{(1/16)}=\frac{1}{2}$
3	$\frac{1}{32}$		

Not a linear function since the average rate of change is not constant.

The ratio of consecutive outputs is a constant, $\frac{1}{2}$. This is an exponential function with growth factor $a=\frac{1}{2}$. The initial value of the exponential function is $C=\frac{1}{4}$. Therefore, the exponential function that models the data is $F(x)=Ca^x=\frac{1}{4}\left(\frac{1}{2}\right)^x$.

33. *B*

34. F

35. *D*

36. *H*

37. A

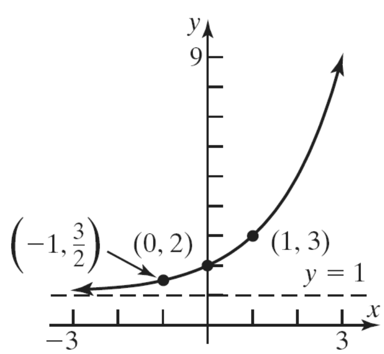
38. *C*

39. *E*

40. *G*

41. $f(x)=2^x+1$

Using the graph of $y=2^x$, shift the graph up 1 unit.
Domain: All real numbers
Range: $\{y \mid y>1\}$ or $(1,\infty)$
Horizontal Asymptote: $y=1$

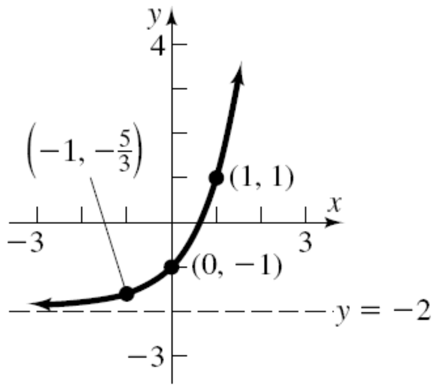


42. $f(x)=3^x-2$

Using the graph of $y=3^x$, shift the graph down 2 units.
Domain: All real numbers

Range: $\{y \mid y > -2\}$ or $(-2, \infty)$

Horizontal Asymptote: $y = -2$



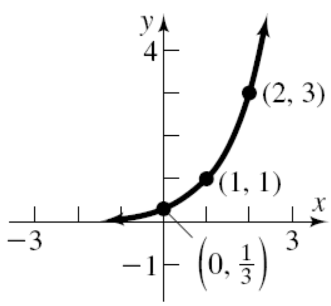
43. $f(x) = 3^{x-1}$

Using the graph of $y = 3^x$, shift the graph right 1 unit.

Domain: All real numbers

Range: $\{y \mid y > 0\}$ or $(0, \infty)$

Horizontal Asymptote: $y = 0$



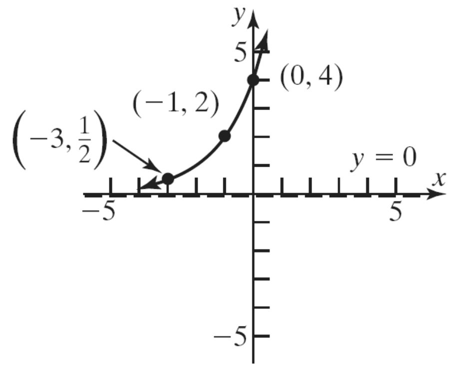
44. $f(x) = 2^{x+2}$

Using the graph of $y = 2^x$, shift the graph left 2 units.

Domain: All real numbers

Range: $\{y \mid y > 0\}$ or $(0, \infty)$

Horizontal Asymptote: $y = 0$



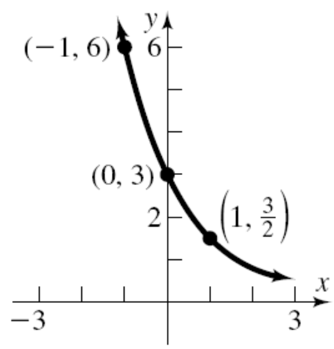
45. $f(x) = 3 \cdot \left(\frac{1}{2}\right)^x$

Using the graph of $y = \left(\frac{1}{2}\right)^x$, vertically stretch the graph by a factor of 3. That is, for each point on the graph, multiply the *y*-coordinate by 3.

Domain: All real numbers

Range: $\{y \mid y > 0\}$ or $(0, \infty)$

Horizontal Asymptote: $y = 0$



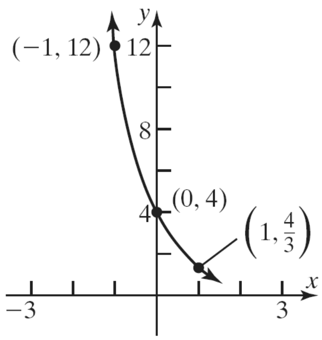
46. $f(x) = 4 \cdot \left(\frac{1}{3}\right)^x$

Using the graph of $y = \left(\frac{1}{3}\right)^x$, vertically stretch the graph by a factor of 4. That is, for each point on the graph, multiply the *y*-coordinate by 4.

Domain: All real numbers

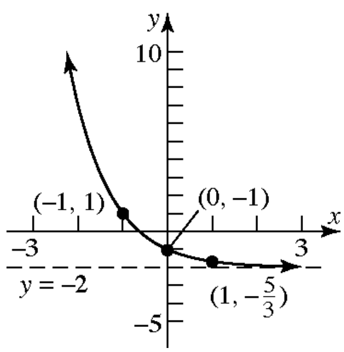
Range: $\{y \mid y > 0\}$ or $(0, \infty)$

Horizontal Asymptote: $y = 0$



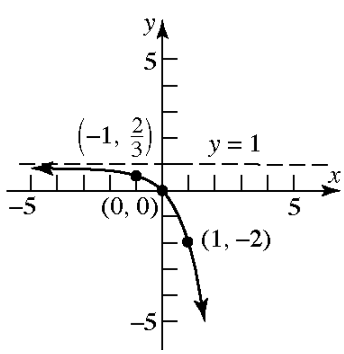
47. $f(x) = 3^{-x} - 2$

Using the graph of $y = 3^x$, reflect the graph about the y-axis, and shift down 2 units.
Domain: All real numbers
Range: $\{y \mid y > -2\}$ or $(-2, \infty)$
Horizontal Asymptote: $y = -2$



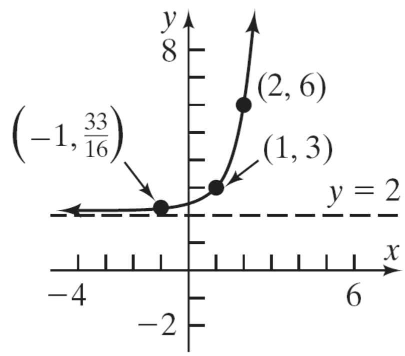
48. $f(x) = -3^x + 1$

Using the graph of $y = 3^x$, reflect the graph about the x-axis, and shift up 1 unit.
Domain: All real numbers
Range: $\{y \mid y < 1\}$ or $(-\infty, 1)$
Horizontal Asymptote: $y = 1$



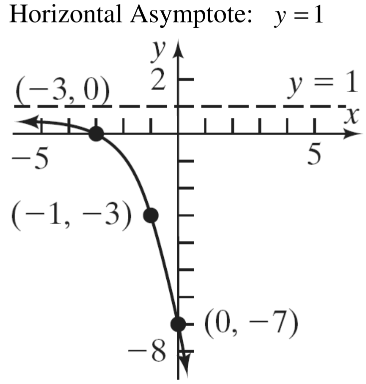
49. $f(x) = 2 + 4^{x-1}$

Using the graph of $y = 4^x$, shift the graph to the right one unit and up 2 units.
Domain: All real numbers
Range: $\{y \mid y > 2\}$ or $(2, \infty)$
Horizontal Asymptote: $y = 2$



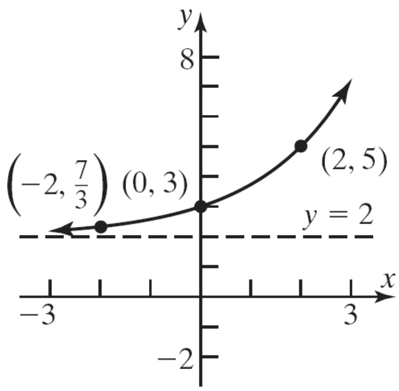
50. $f(x) = 1 - 2^{x+3}$

Using the graph of $y = 2^x$, shift the graph to the left 3 units, reflect about the x-axis, and shift up 1 unit.
Domain: All real numbers
Range: $\{y \mid y < 1\}$ or $(-\infty, 1)$
Horizontal Asymptote: $y = 1$



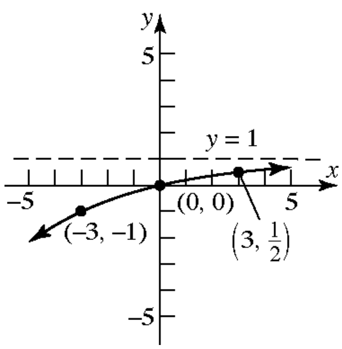
51. $f(x) = 2 + 3^{x/2}$

Using the graph of $y = 3^x$, stretch the graph horizontally by a factor of 2, and shift up 2 units.
Domain: All real numbers
Range: $\{y \mid y > 2\}$ or $(2, \infty)$
Horizontal Asymptote: $y = 2$



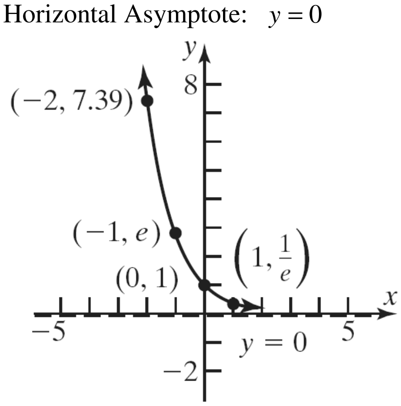
52. $f(x) = 1 - 2^{-x/3}$

Using the graph of $y = 2^x$, stretch the graph horizontally by a factor of 3, reflect about the *y*-axis, reflect about the *x*-axis, and shift up 1 unit.
Domain: All real numbers
Range: $\{y \mid y < 1\}$ or $(-\infty, 1)$
Horizontal Asymptote: $y = 1$



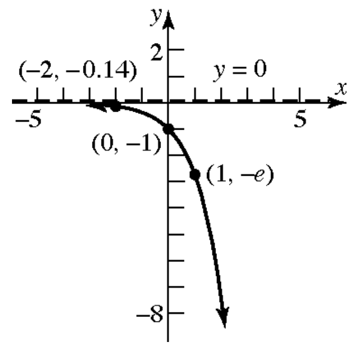
53. $f(x) = e^{-x}$

Using the graph of $y = e^x$, reflect the graph about the *y*-axis.
Domain: All real numbers
Range: $\{y \mid y > 0\}$ or $(0, \infty)$
Horizontal Asymptote: $y = 0$



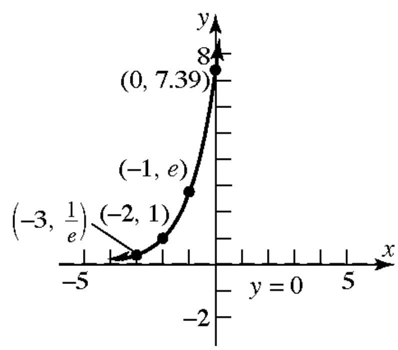
54. $f(x) = -e^x$

Using the graph of $y = e^x$, reflect the graph about the *x*-axis.
Domain: All real numbers
Range: $\{y \mid y < 0\}$ or $(-\infty, 0)$
Horizontal Asymptote: $y = 0$



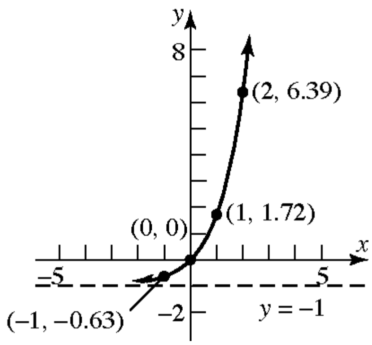
55. $f(x) = e^{x+2}$

Using the graph of $y = e^x$, shift the graph 2 units to the left.
Domain: All real numbers
Range: $\{y \mid y > 0\}$ or $(0, \infty)$
Horizontal Asymptote: $y = 0$



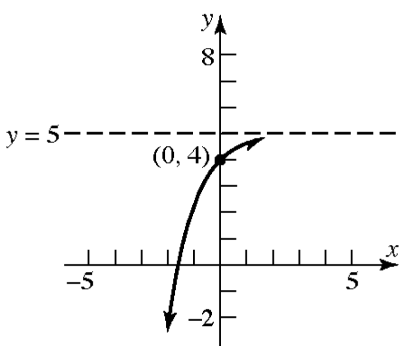
56. $f(x) = e^x - 1$

Using the graph of $y = e^x$, shift the graph down 1 unit.
Domain: All real numbers
Range: $\{y \mid y > -1\}$ or $(-1, \infty)$
Horizontal Asymptote: $y = -1$



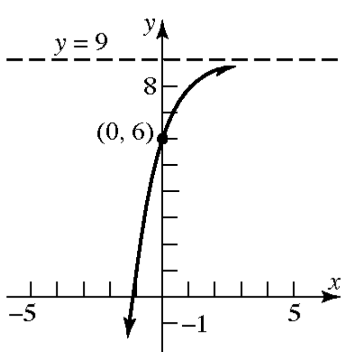
57. $f(x) = 5 - e^{-x}$

Using the graph of $y = e^{x}$, reflect the graph about the y-axis, reflect about the x-axis, and shift up 5 units.
Domain: All real numbers
Range: $\{y \mid y < 5\}$ or $(-\infty, 5)$
Horizontal Asymptote: $y = 5$



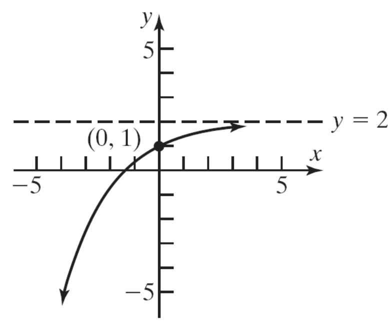
58. $f(x) = 9 - 3e^{-x}$

Using the graph of $y = e^{x}$, reflect the graph about the y-axis, stretch vertically by a factor of 3, reflect about the x-axis, and shift up 9 units.
Domain: All real numbers
Range: $\{y \mid y < 9\}$ or $(-\infty, 9)$
Horizontal Asymptote: $y = 9$



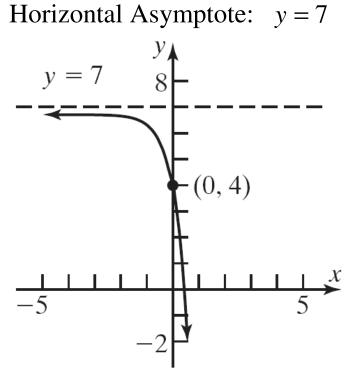
59. $f(x) = 2 - e^{-x/2}$

Using the graph of $y = e^{x}$, reflect the graph about the y-axis, stretch horizontally by a factor of 2, reflect about the x-axis, and shift up 2 units.
Domain: All real numbers
Range: $\{y \mid y < 2\}$ or $(-\infty, 2)$
Horizontal Asymptote: $y = 2$



60. $f(x) = 7 - 3e^{-2x}$

Using the graph of $y = e^{x}$, reflect the graph about the y-axis, shrink horizontally by a factor of $\frac{1}{2}$, stretch vertically by a factor of 3, reflect about the x-axis, and shift up 7 units.
Domain: All real numbers
Range: $\{y \mid y < 7\}$ or $(-\infty, 7)$
Horizontal Asymptote: $y = 7$



61. $7^{x} = 7^{3}$
We have a single term with the same base on both sides of the equation. Therefore, we can set the exponents equal to each other: $x = 3$.
The solution set is $\{3\}$.

62. $5^{x} = 5^{-6}$
We have a single term with the same base on both sides of the equation. Therefore, we can set the

exponents equal to each other: $x=-6$.
The solution set is $\{-6\}$.

63. $2^{-x}=16$
$2^{-x}=2^4$
$-x=4$
$x=-4$
The solution set is $\{-4\}$.

64. $3^{-x}=81$
$3^{-x}=3^4$
$-x=4$
$x=-4$
The solution set is $\{-4\}$.

65. $\left(\frac{1}{5}\right)^x=\frac{1}{25}$
$\left(\frac{1}{5}\right)^x=\frac{1}{5^2}$
$\left(\frac{1}{5}\right)^x=\left(\frac{1}{5}\right)^2$
$x=2$
The solution set is $\{2\}$.

66. $\left(\frac{1}{4}\right)^x=\frac{1}{64}$
$\left(\frac{1}{4}\right)^x=\frac{1}{4^3}$
$\left(\frac{1}{4}\right)^x=\left(\frac{1}{4}\right)^3$
$x=3$
The solution set is $\{3\}$.

67. $2^{2x-1}=4$
$2^{2x-1}=2^2$
$2x-1=2$
$2x=3$
$x=\frac{3}{2}$
The solution set is $\left\{\frac{3}{2}\right\}$.

68. $5^{x+3}=\frac{1}{5}$
$5^{x+3}=5^{-1}$
$x+3=-1$
$x=-4$
The solution set is $\{-4\}$.

69. $3^{x^3}=9^x$
$3^{x^3}=\left(3^2\right)^x$
$3^{x^3}=3^{2x}$
$x^3=2x$
$x^3-2x=0$
$x\left(x^2-2\right)=0$
$x=0$ or $x^2-2=0$
$x^2=2$
$x=\pm\sqrt{2}$
The solution set is $\left\{-\sqrt{2},0,\sqrt{2}\right\}$.

70. $4^{x^2}=2^x$
$\left(2^2\right)^{x^2}=2^x$
$2^{2x^2}=2^x$
$2x^2=x$
$2x^2-x=0$
$x(2x-1)=0$
$x=0$ or $2x-1=0$
$2x=1$
$x=\frac{1}{2}$
The solution set is $\left\{0,\frac{1}{2}\right\}$.

71. $8^{-x+14}=16^x$
$\left(2^3\right)^{-x+14}=\left(2^4\right)^x$
$2^{-3x+42}=2^{4x}$
$-3x+42=4x$
$42=7x$
$6=x$
The solution set is $\{6\}$.

72.
$$9^{-x+15} = 27^x$$
$$\left(3^2\right)^{-x+15} = \left(3^3\right)^x$$
$$3^{-2x+30} = 3^{3x}$$
$$-2x+30 = 3x$$
$$30 = 5x$$
$$6 = x$$
The solution set is $\{6\}$.

73.
$$3^{x^2-7} = 27^{2x}$$
$$3^{x^2-7} = \left(3^3\right)^{2x}$$
$$3^{x^2-7} = 3^{6x}$$
$$x^2 - 7 = 6x$$
$$x^2 - 6x - 7 = 0$$
$$(x-7)(x+1) = 0$$
$$x-7=0 \quad \text{or} \quad x+1=0$$
$$x = 7 \qquad\qquad x = -1$$
The solution set is $\{-1, 7\}$.

74.
$$5^{x^2+8} = 125^{2x}$$
$$5^{x^2+8} = \left(5^3\right)^{2x}$$
$$5^{x^2+8} = 5^{6x}$$
$$x^2 + 8 = 6x$$
$$x^2 - 6x + 8 = 0$$
$$(x-4)(x-2) = 0$$
$$x-4=0 \quad \text{or} \quad x-2=0$$
$$x = 4 \qquad\qquad x = 2$$
The solution set is $\{2, 4\}$.

75.
$$4^x \cdot 2^{x^2} = 16^2$$
$$\left(2^2\right)^x \cdot 2^{x^2} = \left(2^4\right)^2$$
$$2^{2x} \cdot 2^{x^2} = 2^8$$
$$2^{2x+x^2} = 2^8$$
$$x^2 + 2x = 8$$
$$x^2 + 2x - 8 = 0$$
$$(x+4)(x-2) = 0$$
$$x+4=0 \quad \text{or} \quad x-2=0$$
$$x = -4 \qquad\qquad x = 2$$
The solution set is $\{-4, 2\}$.

76.
$$9^{2x} \cdot 27^{x^2} = 3^{-1}$$
$$\left(3^2\right)^{2x} \cdot \left(3^3\right)^{x^2} = 3^{-1}$$
$$3^{4x} \cdot 3^{3x^2} = 3^{-1}$$
$$3^{4x+3x^2} = 3^{-1}$$
$$3x^2 + 4x = -1$$
$$3x^2 + 4x + 1 = 0$$
$$(3x+1)(x+1) = 0$$
$$3x+1=0 \quad \text{or} \quad x+1=0$$
$$3x = -1 \qquad\qquad x = -1$$
$$x = -\frac{1}{3}$$
The solution set is $\left\{-1, -\frac{1}{3}\right\}$.

77.
$$e^x = e^{3x+8}$$
$$x = 3x + 8$$
$$-2x = 8$$
$$x = -4$$
The solution set is $\{-4\}$.

78.
$$e^{3x} = e^{2-x}$$
$$3x = 2 - x$$
$$4x = 2$$
$$x = \frac{1}{2}$$
The solution set is $\left\{\frac{1}{2}\right\}$.

79.
$$e^{x^2} = e^{3x} \cdot \frac{1}{e^2}$$
$$e^{x^2} = e^{3x} \cdot e^{-2}$$
$$e^{x^2} = e^{3x-2}$$
$$x^2 = 3x - 2$$
$$x^2 - 3x + 2 = 0$$
$$(x-2)(x-1) = 0$$
$$x-2=0 \quad \text{or} \quad x-1=0$$
$$x = 2 \qquad\qquad x = 1$$
The solution set is $\{1, 2\}$.

80. $\left(e^4\right)^x \cdot e^{x^2} = e^{12}$

$$e^{4x} \cdot e^{x^2} = e^{12}$$
$$e^{4x+x^2} = e^{12}$$
$$x^2 + 4x = 12$$
$$x^2 + 4x - 12 = 0$$
$$(x+6)(x-2) = 0$$
$$x + 6 = 0 \quad \text{or} \quad x - 2 = 0$$
$$x = -6 \qquad x = 2$$

The solution set is $\{-6, 2\}$.

81. If $4^x = 7$, then $\left(4^x\right)^{-2} = 7^{-2}$

$$4^{-2x} = \frac{1}{7^2}$$
$$4^{-2x} = \frac{1}{49}$$

82. If $2^x = 3$, then $\left(2^x\right)^{-2} = 3^{-2}$

$$2^{-2x} = \frac{1}{3^2}$$
$$\left(2^2\right)^{-x} = \frac{1}{9}$$
$$4^{-x} = \frac{1}{9}$$

83. If $3^{-x} = 2$, then $\left(3^{-x}\right)^{-2} = 2^{-2}$

$$3^{2x} = \frac{1}{2^2}$$
$$3^{2x} = \frac{1}{4}$$

84. If $5^{-x} = 3$, then $\left(5^{-x}\right)^{-3} = 3^{-3}$

$$5^{3x} = \frac{1}{3^3}$$
$$5^{3x} = \frac{1}{27}$$

85. We need a function of the form $f(x) = k \cdot a^{p \cdot x}$, with $a > 0,\ a \neq 1$. The graph contains the points $\left(-1, \frac{1}{3}\right)$, $(0,1)$, $(1,3)$, and $(2,9)$. In other words, $f(-1) = \frac{1}{3}$, $f(0) = 1$, $f(1)=3$, and $f(2) = 9$.

Therefore, $f(0) = k \cdot a^{p \cdot (0)}$

$$1 = k \cdot a^0$$
$$1 = k \cdot 1$$
$$1 = k$$

and $f(1) = a^{p \cdot (1)}$

$$3 = a^p$$

Let's use $a = 3,\ p = 1$. Then $f(x) = 3^x$. Now we need to verify that this function yields the other known points on the graph. $f(-1) = 3^{-1} = \frac{1}{3}$;

$f(2) = 3^2 = 9$

So we have the function $f(x) = 3^x$.

86. We need a function of the form $f(x) = k \cdot a^{p \cdot x}$, with $a > 0,\ a \neq 1$. The graph contains the points $\left(-1, \frac{1}{5}\right)$, $(0,1)$, and $(1,5)$. In other words, $f(-1) = \frac{1}{5}$, $f(0) = 1$, and $f(1) = 5$. Therefore,

$$f(0) = k \cdot a^{p \cdot (0)}$$
$$1 = k \cdot a^0$$
$$1 = k \cdot 1$$
$$1 = k$$

and $f(1) = a^{p \cdot (1)}$

$$5 = a^p$$

Let's use $a = 5,\ p = 1$. Then $f(x) = 5^x$. Now we need to verify that this function yields the other known point on the graph.

$$f(-1) = 5^{-1} = \frac{1}{5}$$

So we have the function $f(x) = 5^x$.

87. We need a function of the form $f(x)=k\cdot a^{p\cdot x}$, with $a>0,\ a\neq 1$. The graph contains the points $\left(-1,-\frac{1}{6}\right)$, $(0,-1)$, $(1,-6)$, and $(2,-36)$. In other words, $f(-1)=-\frac{1}{6}$, $f(0)=-1$, $f(1)=-6$, and $f(2)=-36$.

Therefore, $f(0)=k\cdot a^{p\cdot(0)}$ and $f(1)=-a^{p\cdot(1)}$.

$$-1=k\cdot a^0 \qquad -6=-a^p$$
$$-1=k\cdot 1 \qquad 6=a^p$$
$$-1=k$$

Let's use $a=6,\ p=1$. Then $f(x)=-6^x$.

Now we need to verify that this function yields the other known points on the graph.

$f(-1)=-6^{-1}=-\frac{1}{6}$; $\quad f(2)=-6^2=-36$

So we have the function $f(x)=-6^x$.

88. We need a function of the form $f(x)=k\cdot a^{p\cdot x}$, with $a>0,\ a\neq 1$. The graph contains the points $\left(-1,-\frac{1}{e}\right)$, $(0,-1)$, $(1,-e)$, and $\left(2,-e^2\right)$. In other words, $f(-1)=-\frac{1}{e}$, $f(0)=-1$, $f(1)=-e$, and $f(2)=-e^2$.

Therefore, $f(0)=k\cdot a^{p\cdot(0)}$

$$-1=k\cdot a^0$$
$$-1=k\cdot 1$$
$$-1=k$$

and $f(1)=-a^{p\cdot(1)}$

$$-e=-a^p$$
$$e=a^p$$

Let's use $a=e,\ p=1$. Then $f(x)=-e^x$. Now we need to verify that this function yields the other known points on the graph.

$f(-1)=-e^{-1}=-\frac{1}{e}$

$f(2)=-e^2$

So we have the function $f(x)=-e^x$.

89. We need a function of the form $f(x)=k\cdot a^{p\cdot x}+b$, with $a>0,\ a\neq 1$ and b is the vertical shift of 2 units upward. The graph contains the points $(0,3)$, and $(1,5)$. In other words, $f(0)=1$ and $f(1)=3$. We can assume the graph has the same shape as the graph of $f(x)=k\cdot a^{p\cdot x}$. The reference (unshifted) graph would contain the points $(0,1)$, and $(1,3)$.

Therefore, $f(0)=k\cdot a^{p\cdot(0)}$ and $f(1)=a^{p\cdot(1)}$

$$1=k\cdot a^0 \qquad 3=a^p$$
$$1=k\cdot 1$$
$$1=k$$

Let's use $a=3,\ p=1$. Then $f(x)=3^x$. To shift the graph up by 2 units we would have $f(x)=3^x+2$. Now we need to verify that this function yields the other known points on the graph.

$f(0)=3^0+2=3$

$f(1)=3^1+2=5$

So we have the function $f(x)=3^x+2$.

90. We need a function of the form $f(x)=k\cdot a^{p\cdot x}+b$, with $a>0,\ a\neq 1$ and b is the vertical shift of 3 units downward. The graph contains the points $(0,-2)$, and $(-2,1)$. In other words, $f(0)=-2$ and $f(-2)=1$. We can assume the graph has the same shape as the graph of $f(x)=k\cdot a^{p\cdot x}$. The reference (unshifted) graph would contain the points $(0,1)$, and $(-2,4)$.

Therefore, $f(0)=k\cdot a^{p\cdot(0)}$ and $f(-2)=a^{p\cdot(-2)}$

$$1=k\cdot a^0 \qquad \frac{1}{4}=a^{2p}$$
$$1=k\cdot 1 \qquad \frac{1}{2}=a^p$$
$$1=k$$

Let's use $a=\frac{1}{2},\ p=1$. Then $f(x)=\frac{1}{2}^x$. To shift the graph down by 3 units we would have $f(x)=\frac{1}{2}^x-3$. Now we need to verify that this function yields the other known points on the graph.

$f(0)=\frac{1}{2}^0-3=-2$

$f(-2)=\frac{1}{2}^{-2}-3=4-3=1$

So we have the function $f(x)=\frac{1}{2}^{x}-3$.

91. a. $f(4)=2^4=16$

The point $(4,16)$ is on the graph of *f*.

b. $f(x)=\frac{1}{16}$

$2^x=\frac{1}{16}$

$2^x=\frac{1}{2^4}$

$2^x=2^{-4}$

$x=-4$

The point $\left(-4,\frac{1}{16}\right)$ is on the graph of *f*.

92. a. $f(4)=3^4=81$

The point $(4,81)$ is on the graph of *f*.

b. $f(x)=\frac{1}{9}$

$3^x=\frac{1}{9}$

$3^x=\frac{1}{3^2}$

$3^x=3^{-2}$

$x=-2$

The point $\left(-2,\frac{1}{9}\right)$ is on the graph of *f*.

93. a. $g(-1)=4^{-1}+2=\frac{1}{4}+2=\frac{9}{4}$

The point $\left(-1,\frac{9}{4}\right)$ is on the graph of *g*.

b. $g(x)=66$

$4^x+2=66$

$4^x=64$

$4^x=4^3$

$x=3$

The point $(3,66)$ is on the graph of *g*.

94. a. $g(-1)=5^{-1}-3=\frac{1}{5}-3=-\frac{14}{5}$

The point $\left(-1,-\frac{14}{15}\right)$ is on the graph of *g*.

b. $g(x)=122$

$5^x-3=122$

$5^x=125$

$5^x=5^3$

$x=3$

The point $(3,122)$ is on the graph of *g*.

95. a. $H(-6)=\left(\frac{1}{2}\right)^{-6}-4=(2)^6-4=60$

The point $(-6,60)$ is on the graph of *H*.

b. $H(x)=12$

$\left(\frac{1}{2}\right)^x-4=12$

$\left(\frac{1}{2}\right)^x=16$

$(2)^{-x}=2^4$

$-x=4$

$x=-4$

The point $(-4,12)$ is on the graph of *H*.

c. $\left(\frac{1}{2}\right)^x-4=0$

$\left(\frac{1}{2}\right)^x=4$

$\left(2^{-1}\right)^x=2^2$

$2^{-x}=2^2$

$-x=2$

$x=-2$

The zero of *H* is $x=-2$.

96. **a.** $F(-5)=\left(\frac{1}{3}\right)^{-5}-3=(3)^5-3=240$

The point $(-5,240)$ is on the graph of F.

b. $F(x)=24$

$$\left(\frac{1}{3}\right)^x-3=24$$
$$\left(\frac{1}{3}\right)^x=27$$
$$3^{-x}=3^3$$
$$-x=3$$
$$x=-3$$

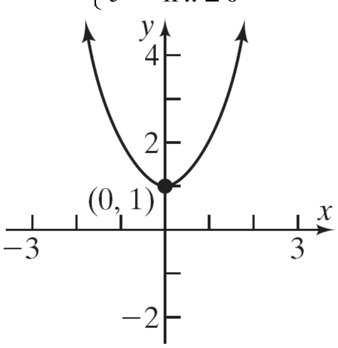
The point $(-3,24)$ is on the graph of F.

c. $\left(\frac{1}{3}\right)^x-3=0$

$$\left(\frac{1}{3}\right)^x=3$$
$$\left(3^{-1}\right)^x=3^1$$
$$3^{-x}=3^1$$
$$-x=1$$
$$x=-1$$

The zero of F is $x=-1$.

97. $f(x)=\begin{cases} e^{-x} & \text{if } x<0 \\ e^{x} & \text{if } x\ge 0 \end{cases}$

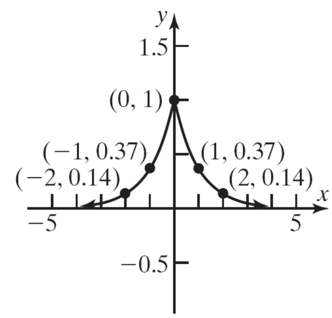


Domain: $(-\infty,\infty)$

Range: $\{y \mid y\ge 1\}$ or $[1,\infty)$

Intercept: $(0,1)$

98. $f(x)=\begin{cases} e^{x} & \text{if } x<0 \\ e^{-x} & \text{if } x\ge 0 \end{cases}$

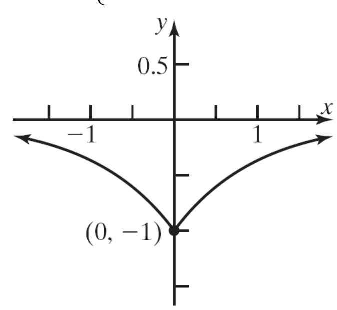


Domain: $(-\infty,\infty)$

Range: $\{y \mid 0<y\le 1\}$ or $(0,1]$

Intercept: $(0,1)$

99. $f(x)=\begin{cases} -e^{x} & \text{if } x<0 \\ -e^{-x} & \text{if } x\ge 0 \end{cases}$

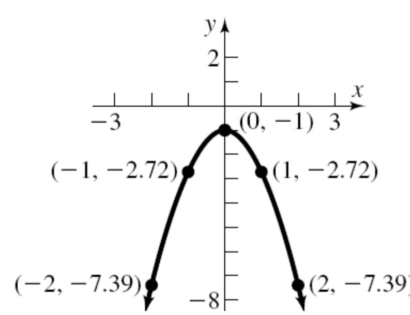


Domain: $(-\infty,\infty)$

Range: $\{y \mid -1\le y<0\}$ or $[-1,0)$

Intercept: $(0,-1)$

100. $f(x)=\begin{cases} -e^{-x} & \text{if } x<0 \\ -e^{x} & \text{if } x\ge 0 \end{cases}$



Domain: $(-\infty,\infty)$

Range: $\{y \mid y\le -1\}$ or $(-\infty,-1]$

Intercept: $(0,-1)$

101. $p(n) = 100(0.97)^n$

a. $p(10) = 100(0.97)^{10} \approx 74\%$ of light

b. $p(25) = 100(0.97)^{25} \approx 47\%$ of light

102. $p(h) = 760e^{-0.145h}$

a. $p(2) = 760e^{-0.145(2)}$
$= 760e^{-0.290}$
≈ 568.68 mm of Hg

b. $p(10) = 760e^{-0.145(10)}$
$= 760e^{-1.45}$
≈ 178.27 mm of Hg

103. $p(x) = 16,630(0.90)^x$

a. $p(3) = 16,630(0.90)^3 \approx \$12,123$

b. $p(9) = 16,630(0.90)^9 \approx \$6,443$

104. $A(n) = A_0 e^{-0.35n}$

a. $A(3) = 100e^{-0.35(3)}$
$= 100e^{-1.05}$
≈ 34.99 square millimeters

b. $A(10) = 100e^{-0.35(10)}$
$= 100e^{-3.5}$
≈ 3.02 square millimeters

105. $D(h) = 5e^{-0.4h}$

$D(1) = 5e^{-0.4(1)} = 5e^{-0.4} \approx 3.35$

After 1 hours, 3.35 milligrams will be present.

$D(6) = 5e^{-0.4(6)} = 5e^{-2.4} \approx 0.45$ milligrams

After 6 hours, 0.45 milligrams will be present.

106. $N = P\left(1 - e^{-0.15d}\right)$

$N(3) = 1000\left(1 - e^{-0.15(3)}\right)$
$= 1000\left(1 - e^{-0.45}\right) \approx 362$

After 3 days, 362 students will have heard the rumor.

107. $F(t) = 1 - e^{-0.1t}$

a. $F(10) = 1 - e^{-0.1(10)} = 1 - e^{-1} \approx 0.632$

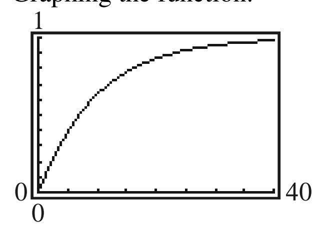
The probability that a car will arrive within 10 minutes of 12:00 PM is 0.632.

b. $F(40) = 1 - e^{-0.1(40)} = 1 - e^{-4} \approx 0.982$

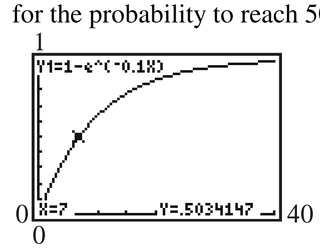
The probability that a car will arrive within 40 minutes of 12:00 PM is 0.982.

c. As $t \to \infty$, $F(t) = 1 - e^{-0.1t} \to 1 - 0 = 1$

d. Graphing the function:



e. $F(7) \approx 0.50$, so about 7 minutes are needed for the probability to reach 50%.



108. $F(t) = 1 - e^{-0.15t}$

a. $F(15) = 1 - e^{-0.15(15)} = 1 - e^{-2.25} \approx 0.895$

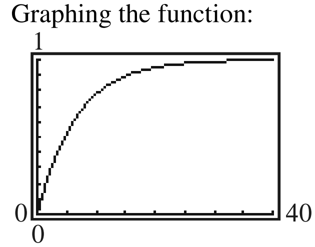
The probability that a car will arrive within 15 minutes of 5:00 PM is 0.895.

b. $F(30) = 1 - e^{-0.15(30)} = 1 - e^{-4.5} \approx 0.989$

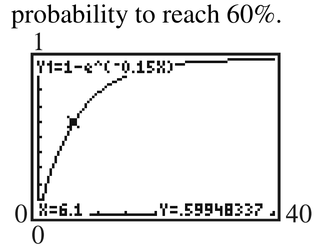
The probability that a car will arrive within 30 minutes of 5:00 PM is 0.989.

c. As $t \to \infty$, $F(t) = 1 - e^{-0.15t} \to 1 - 0 = 1$

d. Graphing the function:



e. $F(6) \approx 0.60$, so 6 minutes are needed for the probability to reach 60%.



109. $P(x)=\dfrac{20^x e^{-20}}{x!}$

a. $P(15)=\dfrac{20^{15} e^{-20}}{15!}\approx 0.0516 \text{ or } 5.16\%$

The probability that 15 cars will arrive between 5:00 PM and 6:00 PM is 5.16%.

b. $P(20)=\dfrac{20^{20} e^{-20}}{20!}\approx 0.0888 \text{ or } 8.88\%$

The probability that 20 cars will arrive between 5:00 PM and 6:00 PM is 8.88%.

110. $P(x)=\dfrac{4^x e^{-4}}{x!}$

a. $P(5)=\dfrac{4^5 e^{-4}}{5!}\approx 0.1563 \text{ or } 15.63\%$

The probability that 5 people will arrive within the next minute is 15.63%.

b. $P(8)=\dfrac{4^8 e^{-4}}{8!}\approx 0.0298 \text{ or } 2.98\%$

The probability that 8 people will arrive within the next minute is 2.98%.

111. $R=10^{\left(\frac{4221}{T+459.4}-\frac{4221}{D+459.4}+2\right)}$

a. $R=10^{\left(\frac{4221}{50+459.4}-\frac{4221}{41+459.4}+2\right)}\approx 70.95\%$

b. $R=10^{\left(\frac{4221}{68+459.4}-\frac{4221}{59+459.4}+2\right)}\approx 72.62\%$

c. $R=10^{\left(\frac{4221}{T+459.4}-\frac{4221}{T+459.4}+2\right)}=10^2=100\%$

112. $L(t)=500\left(1-e^{-0.0061t}\right)$

a.
$$\begin{aligned}L(30)&=500\left(1-e^{-0.0061(30)}\right)\\&=500\left(1-e^{-0.183}\right)\\&\approx 84\end{aligned}$$

The student will learn about 84 words after 30 minutes.

b.
$$\begin{aligned}L(60)&=500\left(1-e^{-0.0061(60)}\right)\\&=500\left(1-e^{-0.366}\right)\\&\approx 153\end{aligned}$$

The student will learn about 153 words after 60 minutes.

113. $I=\dfrac{E}{R}\left[1-e^{-\left(\frac{R}{L}\right)t}\right]$

a. $I_1=\dfrac{120}{10}\left[1-e^{-\left(\frac{10}{5}\right)0.3}\right]=12\left[1-e^{-0.6}\right]\approx 5.414$

amperes after 0.3 second

$I_1=\dfrac{120}{10}\left[1-e^{-\left(\frac{10}{5}\right)0.5}\right]=12\left[1-e^{-1}\right]\approx 7.585$

amperes after 0.5 second

$I_1=\dfrac{120}{10}\left[1-e^{-\left(\frac{10}{5}\right)1}\right]=12\left[1-e^{-2}\right]\approx 10.376$

amperes after 1 second

b. As $t\to\infty$, $e^{-\left(\frac{10}{5}\right)t}\to 0$. Therefore, as,

$t\to\infty,\ I_1=\dfrac{120}{10}\left[1-e^{-\left(\frac{10}{5}\right)t}\right]\to 12[1-0]=12$,

which means the maximum current is 12 amperes.

c. See the graph at the end of the solution.

d. $I_2=\dfrac{120}{5}\left[1-e^{-\left(\frac{5}{10}\right)0.3}\right]=24\left[1-e^{-0.15}\right]$

≈ 3.343 amperes after 0.3 second

$I_2=\dfrac{120}{5}\left[1-e^{-\left(\frac{5}{10}\right)0.5}\right]=24\left[1-e^{-0.25}\right]$

≈ 5.309 amperes after 0.5 second

$I_2=\dfrac{120}{5}\left[1-e^{-\left(\frac{5}{10}\right)1}\right]=24\left[1-e^{-0.5}\right]$

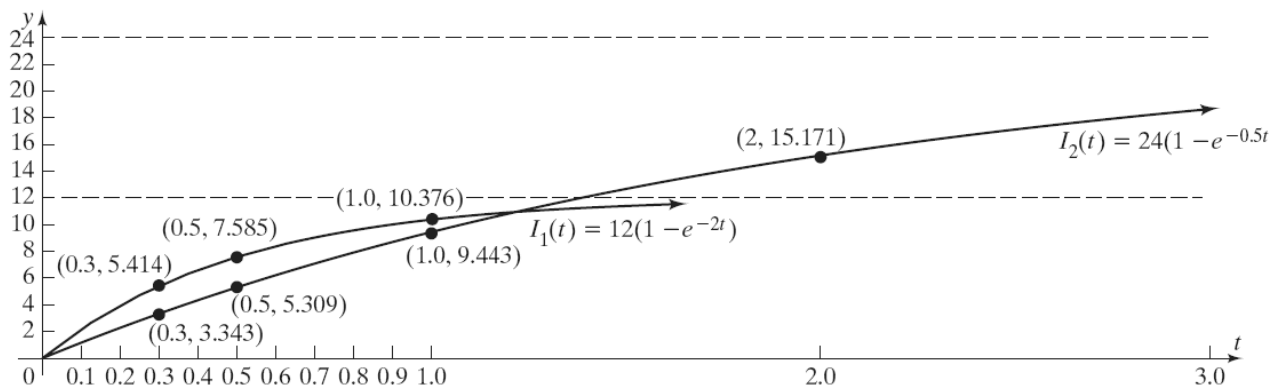
≈ 9.443 amperes after 1 second

e. As $t\to\infty$, $e^{-\left(\frac{5}{10}\right)t}\to 0$. Therefore, as,

$t\to\infty,\ I_1=\dfrac{120}{5}\left[1-e^{-\left(\frac{10}{5}\right)t}\right]\to 24[1-0]=24$,

which means the maximum current is 24 amperes.

f. See the graph that follows.



114. $I = \frac{E}{R} \cdot e^{\left(\frac{-t}{RC}\right)}$

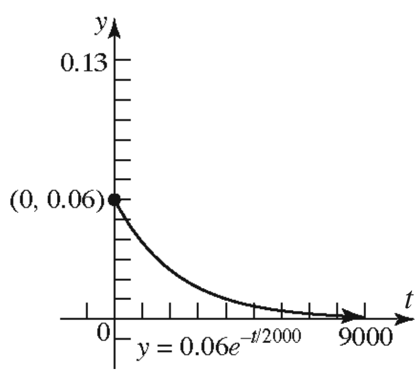
a. $I_1 = \frac{120}{2000} \cdot e^{\left(\frac{-0}{2000 \cdot 1}\right)} = \frac{120}{2000} e^0 = 0.06$
amperes initially.

$I_1 = \frac{120}{2000} \cdot e^{\left(\frac{-1000}{2000 \cdot 1}\right)} = \frac{120}{2000} e^{-1/2} \approx 0.0364$
amperes after 1000 microseconds

$I_1 = \frac{120}{2000} \cdot e^{\left(\frac{-3000}{2000 \cdot 1}\right)} = \frac{120}{2000} e^{-1.5} \approx 0.0134$
amperes after 3000 microseconds

b. The maximum current occurs at $t = 0$. Therefore, the maximum current is 0.06 amperes.

c. Graphing the function:



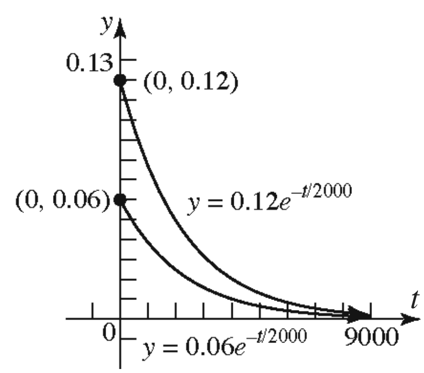
d. $I_2 = \frac{120}{1000} \cdot e^{\left(\frac{-0}{1000 \cdot 2}\right)} = \frac{120}{1000} e^0 = 0.12$
amperes initially.

$I_2 = \frac{120}{1000} \cdot e^{\left(\frac{-1000}{1000 \cdot 2}\right)} = \frac{120}{1000} e^{-1/2} \approx 0.0728$
amperes after 1000 microseconds

$I_2 = \frac{120}{1000} \cdot e^{\left(\frac{-3000}{1000 \cdot 2}\right)} = \frac{120}{1000} e^{-1.5} \approx 0.0268$
amperes after 3000 microseconds

e. The maximum current occurs at $t = 0$. Therefore, the maximum current is 0.12 amperes.

f. Graphing the functions:



115. Since the growth rate is 3 then $a=3$. So we have

$$f(x)=C\cdot 3^x \qquad \text{So} \quad f(7)=\frac{12}{3^6}\cdot 3^7$$
$$f(6)=C\cdot 3^6 \qquad\qquad =12\cdot 3$$
$$12=C\cdot 3^6 \qquad\qquad =36$$
$$\frac{12}{3^6}=C$$

So $f(7)=36$

116. $2+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\ldots+\frac{1}{n!}$

$n=4;\quad 2+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}\approx 2.7083$

$n=6;\quad 2+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\frac{1}{5!}+\frac{1}{6!}\approx 2.7181$

$n=8;\quad 2+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\frac{1}{5!}+\frac{1}{6!}+\frac{1}{7!}+\frac{1}{8!}$
≈ 2.7182788

$n=10;\quad 2+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\frac{1}{5!}+\frac{1}{6!}+\frac{1}{7!}+\frac{1}{8!}+\frac{1}{9!}+\frac{1}{10!}$
≈ 2.7182818

$e\approx 2.718281828$

117. $2+1=3$

$$2+\cfrac{1}{1+1}=2.5<e$$

$$2+\cfrac{1}{1+\cfrac{1}{2+2}}=2.8>e$$

$$2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{2}{3+3}}}=2.7<e$$

$$2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{2}{3+\cfrac{3}{4+4}}}}\approx 2.721649485>e$$

$$2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{2}{3+\cfrac{3}{4+\cfrac{4}{5+5}}}}}\approx 2.717770035<e$$

$$2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{2}{3+\cfrac{3}{4+\cfrac{4}{5+\cfrac{5}{6+6}}}}}}\approx 2.718348855>e$$

$e\approx 2.718281828$

118. $f(x)=a^x$

$$\frac{f(x+h)-f(x)}{h}=\frac{a^{x+h}-a^x}{h}=\frac{a^xa^h-a^x}{h}=\frac{a^x\left(a^h-1\right)}{h}=a^x\left(\frac{a^h-1}{h}\right)$$

119. $f(x)=a^x$

$f(A+B)=a^{A+B}=a^A\cdot a^B=f(A)\cdot f(B)$

120. $f(x)=a^x$

$f(-x)=a^{-x}=\frac{1}{a^x}=\frac{1}{f(x)}$

121. $f(x)=a^x$

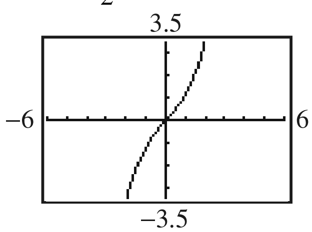
$f(\alpha x)=a^{\alpha x}=\left(a^x\right)^\alpha=\left[f(x)\right]^\alpha$

122. $\sinh x=\frac{1}{2}\left(e^x-e^{-x}\right)$

a. $f(-x) = \sinh(-x)$
$= \frac{1}{2}\left(e^{-x} - e^{x}\right)$
$= -\frac{1}{2}\left(e^{x} - e^{-x}\right)$
$= -\sinh x$
$= -f(x)$

Therefore, $f(x) = \sinh x$ is an odd function.

b. Let $Y_1 = \frac{1}{2}\left(e^{x} - e^{-x}\right)$.

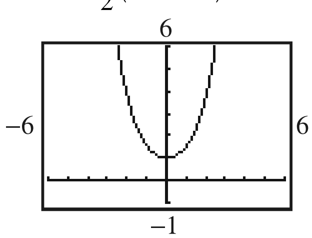


123. $\cosh x = \frac{1}{2}\left(e^{x} + e^{-x}\right)$

a. $f(-x) = \cosh(-x)$
$= \frac{1}{2}\left(e^{-x} + e^{x}\right)$
$= \frac{1}{2}\left(e^{x} + e^{-x}\right)$
$= \cosh x$
$= f(x)$

Thus, $f(x) = \cosh x$ is an even function.

b. Let $Y_1 = \frac{1}{2}\left(e^{x} + e^{-x}\right)$.



c. $(\cosh x)^2 - (\sinh x)^2$
$= \left(\frac{e^{x} + e^{-x}}{2}\right)^2 - \left(\frac{e^{x} - e^{-x}}{2}\right)^2$
$= \frac{e^{2x} + 2 + e^{-2x}}{4} - \frac{e^{2x} - 2 + e^{-2x}}{4}$
$= \frac{e^{2x} + 2 + e^{-2x} - e^{2x} + 2 - e^{-2x}}{4}$
$= \frac{4}{4}$
$= 1$

124. $f(x) = 2^{\left(2^{x}\right)} + 1$

$f(1) = 2^{\left(2^{1}\right)} + 1 = 2^2 + 1 = 4 + 1 = 5$

$f(2) = 2^{\left(2^{2}\right)} + 1 = 2^4 + 1 = 16 + 1 = 17$

$f(3) = 2^{\left(2^{3}\right)} + 1 = 2^8 + 1 = 256 + 1 = 257$

$f(4) = 2^{\left(2^{4}\right)} + 1 = 2^{16} + 1 = 65,536 + 1 = 65,537$

$f(5) = 2^{\left(2^{5}\right)} + 1 = 2^{32} + 1 = 4,294,967,296 + 1$
$= 4,294,967,297$
$= 641 \times 6,700,417$

125. Since the number of bacteria doubles every minute, half of the container is full one minute before it is full. Thus, it takes 59 minutes to fill the container.

126. Answers will vary.

127. Answers will vary.

128. Given the function $f(x) = a^{x}$, with $a > 1$,
If $x > 0$, the graph becomes steeper as a increases.
If $x < 0$, the graph becomes less steep as a increases.

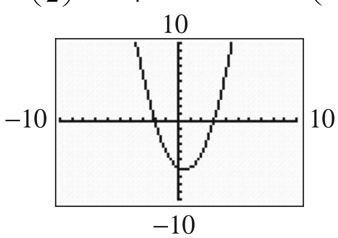
129. Using the laws of exponents, we have:
$a^{-x} = \frac{1}{a^{x}} = \left(\frac{1}{a}\right)^{x}$. So $y = a^{-x}$ and $y = \left(\frac{1}{a}\right)^{x}$ will have the same graph.

Section 5.4

1. a.
$$3x-7\le 8-2x$$
$$5x\le 15$$
$$x\le 3$$
The solution set is $\{x\mid x\le 3\}$.

b. $x^2-x-6>0$

We graph the function $f(x)=x^2-x-6$.
The intercepts are
y-intercept: $f(0)=-6$
x-intercepts: $x^2-x-6=0$
$$(x+2)(x-3)=0$$
$$x=-2,\ x=3$$
The vertex is at $x=\frac{-b}{2a}=\frac{-(-1)}{2(1)}=\frac{1}{2}$. Since $f\left(\frac{1}{2}\right)=-\frac{25}{4}$, the vertex is $\left(\frac{1}{2},-\frac{25}{4}\right)$.



The graph is above the x-axis when $x<-2$ or $x>3$. Since the inequality is strict, the solution set is $\{x\mid x<-2 \text{ or } x>3\}$ or, using interval notation, $(-\infty,-2)\cup(3,\infty)$.

2. $\frac{x-1}{x+4}>0$

$f(x)=\frac{x-1}{x+4}$

f is zero or undefined when $x=1$ or $x=-4$.

Interval	$(-\infty,-4)$	$(-4,1)$	$(1,\infty)$
Test Value	-5	0	2
Value of f	6	$-\frac{1}{4}$	$\frac{1}{6}$
Conclusion	positive	negative	positive

The solution set is $\{x\mid x<-4 \text{ or } x>1\}$ or, using interval notation, $(-\infty,-4)\cup(1,\infty)$.

3.
$$2x+3=9$$
$$2x=6$$
$$x=3$$

4. $\{x\mid x>0\}$ or $(0,\ \infty)$

5. $\left(\frac{1}{a},-1\right)$, $(1,0)$, $(a,1)$

6. 1

7. False. If $y=\log_a x$, then $x=a^y$.

8. True

9. $9=3^2$ is equivalent to $2=\log_3 9$.

10. $16=4^2$ is equivalent to $2=\log_4 16$.

11. $a^2=1.6$ is equivalent to $2=\log_a 1.6$.

12. $a^3=2.1$ is equivalent to $3=\log_a 2.1$.

13. $2^x=7.2$ is equivalent to $x=\log_2 7.2$.

14. $3^x=4.6$ is equivalent to $x=\log_3 4.6$.

15. $e^x=8$ is equivalent to $x=\ln 8$.

16. $e^{2.2}=M$ is equivalent to $2.2=\ln M$.

17. $\log_2 8=3$ is equivalent to $2^3=8$.

18. $\log_3\left(\frac{1}{9}\right)=-2$ is equivalent to $3^{-2}=\frac{1}{9}$.

19. $\log_a 3=6$ is equivalent to $a^6=3$.

20. $\log_b 4=2$ is equivalent to $b^2=4$.

21. $\log_3 2=x$ is equivalent to $3^x=2$.

22. $\log_2 6=x$ is equivalent to $2^x=6$.

23. $\ln 4=x$ is equivalent to $e^x=4$.

24. $\ln x=4$ is equivalent to $e^4=x$.

25. $\log_2 1 = 0$ since $2^0 = 1$.

26. $\log_8 8 = 1$ since $8^1 = 8$.

27. $\log_5 25 = 2$ since $5^2 = 25$.

28. $\log_3\left(\frac{1}{9}\right) = -2$ since $3^{-2} = \frac{1}{9}$.

29. $\log_{1/2} 16 = -4$ since $\left(\frac{1}{2}\right)^{-4} = 2^4 = 16$.

30. $\log_{1/3} 9 = -2$ since $\left(\frac{1}{3}\right)^{-2} = 3^2 = 9$.

31. $\log_{10}\sqrt{10} = \frac{1}{2}$ since $10^{1/2} = \sqrt{10}$.

32. $\log_5 \sqrt[3]{25} = \frac{2}{3}$ since $5^{2/3} = 25^{1/3} = \sqrt[3]{25}$.

33. $\log_{\sqrt{2}} 4 = 4$ since $\left(\sqrt{2}\right)^4 = 4$.

34. $\log_{\sqrt{3}} 9 = 4$ since $\left(\sqrt{3}\right)^4 = 9$.

35. $\ln\sqrt{e} = \frac{1}{2}$ since $e^{1/2} = \sqrt{e}$.

36. $\ln e^3 = 3$ since $e^3 = e^3$.

37. $f(x) = \ln(x-3)$ requires $x-3>0$.

$x-3>0$

$x>3$

The domain of f is $\{x \mid x>3\}$ or $(3,\infty)$.

38. $g(x) = \ln(x-1)$ requires $x-1>0$.

$x-1>0$

$x>1$

The domain of g is $\{x \mid x>1\}$ or $(1,\infty)$.

39. $F(x) = \log_2 x^2$ requires $x^2>0$.

$x^2>0$ for all $x \neq 0$.

The domain of F is $\{x \mid x \neq 0\}$.

40. $H(x) = \log_5 x^3$ requires $x^3>0$.

$x^3>0$ for all $x>0$.

The domain of H is $\{x \mid x>0\}$ or $(0,\infty)$.

41. $f(x) = 3 - 2\log_4\left[\frac{x}{2}-5\right]$ requires $\frac{x}{2}-5>0$.

$\frac{x}{2}-5>0$

$\frac{x}{2}>5$

$x>10$

The domain of *f* is $\{x \mid x>10\}$ or $(10,\infty)$.

42. $g(x) = 8 + 5\ln(2x+3)$ requires $2x+3>0$.

$2x+3>0$

$2x>-3$

$x>-\frac{3}{2}$

The domain of *g* is $\left\{x \,\middle|\, x>-\frac{3}{2}\right\}$ or $\left(-\frac{3}{2},\infty\right)$.

43. $f(x) = \ln\left(\frac{1}{x+1}\right)$ requires $\frac{1}{x+1}>0$.

$p(x) = \frac{1}{x+1}$ is undefined when $x=-1$.

Interval	$(-\infty,-1)$	$(-1,\infty)$
Test Value	-2	0
Value of *p*	-1	1
Conclusion	negative	positive

The domain of *f* is $\{x \mid x>-1\}$ or $(-1,\infty)$.

44. $g(x) = \ln\left(\frac{1}{x-5}\right)$ requires $\frac{1}{x-5}>0$.

$p(x) = \frac{1}{x-5}$ is undefined when $x=5$.

Interval	$(-\infty,5)$	$(5,\infty)$
Test Value	4	6
Value of *p*	-1	1
Conclusion	negative	positive

The domain of *g* is $\{x \mid x>5\}$ or $(5,\infty)$.

45. $g(x) = \log_5\left(\frac{x+1}{x}\right)$ requires $\frac{x+1}{x} > 0$.

$p(x) = \frac{x+1}{x}$ is zero or undefined when $x = -1$ or $x = 0$.

Interval	$(-\infty, -1)$	$(-1, 0)$	$(0, \infty)$
Test Value	-2	$-\frac{1}{2}$	1
Value of p	$\frac{1}{2}$	-1	2
Conclusion	positive	negative	positive

The domain of g is $\{x \mid x < -1 \text{ or } x > 0\}$; $(-\infty, -1) \cup (0, \infty)$.

46. $h(x) = \log_3\left(\frac{x}{x-1}\right)$ requires $\frac{x}{x-1} > 0$.

$p(x) = \frac{x}{x-1}$ is zero or undefined when $x = 0$ or $x = 1$.

Interval	$(-\infty, 0)$	$(0, 1)$	$(1, \infty)$
Test Value	-1	$-\frac{1}{2}$	2
Value of p	$\frac{1}{2}$	-1	2
Conclusion	positive	negative	positive

The domain of h is $\{x \mid x < 0 \text{ or } x > 1\}$; $(-\infty, 0) \cup (1, \infty)$.

47. $f(x) = \sqrt{\ln x}$ requires $\ln x \geq 0$ and $x > 0$

$$\ln x \geq 0$$
$$x \geq e^0$$
$$x \geq 1$$

The domain of h is $\{x \mid x \geq 1\}$ or $[1, \infty)$.

48. $g(x) = \frac{1}{\ln x}$ requires $\ln x \neq 0$ and $x > 0$

$$\ln x \neq 0$$
$$x \neq e^0$$
$$x \neq 1$$

The domain of h is $\{x \mid x > 0 \text{ and } x \neq 1\}$; $(0,1) \cup (1, \infty)$.

49. $\ln\left(\frac{5}{3}\right) \approx 0.511$

50. $\frac{\ln(5)}{3} \approx 0.536$

51. $\frac{\ln \frac{10}{3}}{0.04} \approx 30.099$

52. $\frac{\ln \frac{2}{3}}{-0.1} \approx 4.055$

53. $\frac{\ln 4 + \ln 2}{\log 4 + \log 2} \approx 2.303$

54. $\frac{\log 15 + \log 20}{\ln 15 + \ln 20} \approx 0.434$

55. $\frac{2\ln 5 + \log 50}{\log 4 - \ln 2} \approx -53.991$

56. $\frac{3\log 80 - \ln 5}{\log 5 + \ln 20} \approx 1.110$

57. If the graph of $f(x) = \log_a x$ contains the point $(2, 2)$, then $f(2) = \log_a 2 = 2$. Thus,

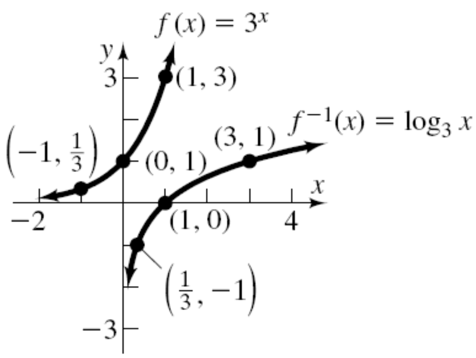
$$\log_a 2 = 2$$
$$a^2 = 2$$
$$a = \pm\sqrt{2}$$

Since the base a must be positive by definition, we have that $a = \sqrt{2}$.

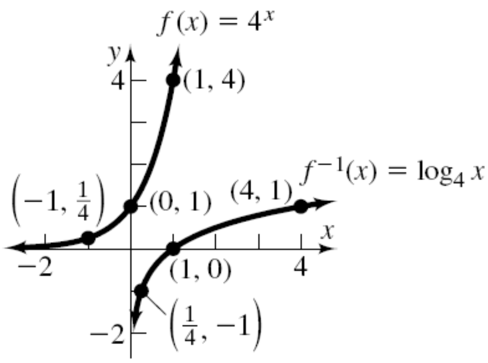
58. If the graph of $f(x) = \log_a x$ contains the point $\left(\frac{1}{2}, -4\right)$, then $f\left(\frac{1}{2}\right) = \log_a\left(\frac{1}{2}\right) = -4$. Thus,

$$\log_a\left(\frac{1}{2}\right) = -4$$
$$a^{-4} = \frac{1}{2}$$
$$\frac{1}{a^4} = \frac{1}{2}$$
$$a^4 = 2$$
$$a = 2^{1/4} \approx 1.189$$

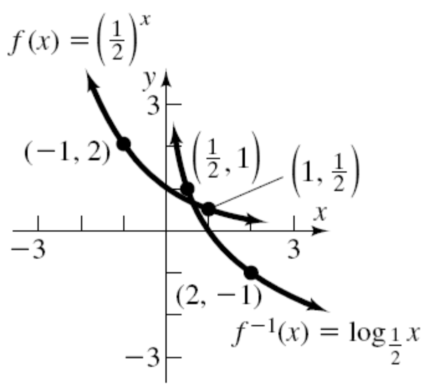
59.



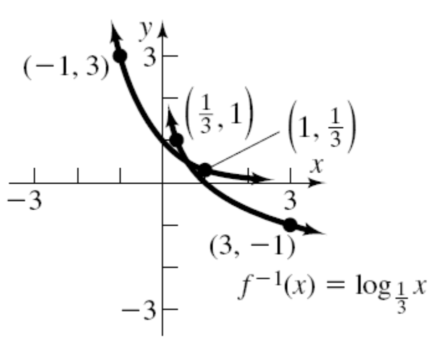
60.



61.



62.



63. B

64. F

65. D

66. H

67. A

68. C

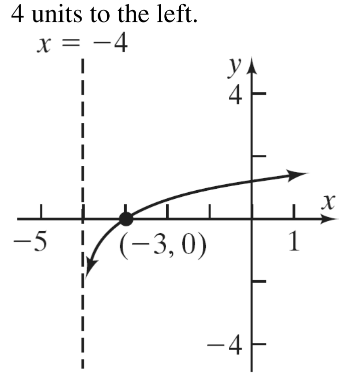
69. E

70. G

71. $f(x) = \ln(x+4)$

a. Domain: $(-4, \infty)$

b. Using the graph of $y = \ln x$, shift the graph 4 units to the left.



c. Range: $(-\infty, \infty)$
Vertical Asymptote: $x = -4$

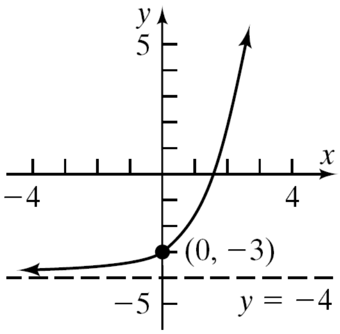
d.

$$\begin{aligned} f(x) &= \ln(x+4) \\ y &= \ln(x+4) \\ x &= \ln(y+4) \quad \text{Inverse} \\ y+4 &= e^x \\ y &= e^x - 4 \\ f^{-1}(x) &= e^x - 4 \end{aligned}$$

e. The domain of the inverse found in part (d) is all real numbers.

Since the domain of f is the range of f^{-1}, we can use the result from part (a) to say that the range of f^{-1} is $(-4, \infty)$.

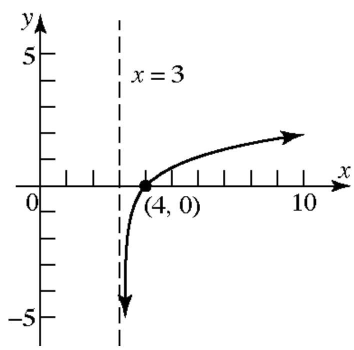
f. Shift the graph of $y = e^x$ down 4 units.



72. $f(x) = \ln(x-3)$

a. Domain: $(3, \infty)$

b. Using the graph of $y = \ln x$, shift the graph 3 units to the right.



c. Range: $(-\infty, \infty)$
Vertical Asymptote: $x = 3$

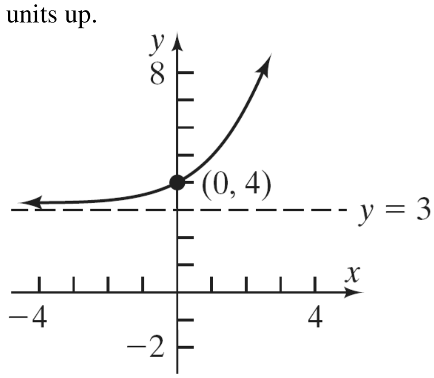
d.

$$\begin{aligned} f(x) &= \ln(x-3) \\ y &= \ln(x-3) \\ x &= \ln(y-3) \quad \text{Inverse} \\ y-3 &= e^x \\ y &= e^x + 3 \\ f^{-1}(x) &= e^x + 3 \end{aligned}$$

e. The domain of the inverse found in part (d) is all real numbers.

Since the domain of f is the range of f^{-1}, we can use the result from part (a) to say that the range of f^{-1} is $(3, \infty)$.

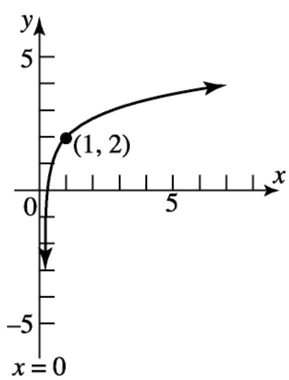
f. Using the graph of $y = e^x$, shift the graph 3 units up.



73. $f(x) = 2 + \ln x$

a. Domain: $(0, \infty)$

b. Using the graph of $y = \ln x$, shift up 2 units.



c. Range: $(-\infty, \infty)$
Vertical Asymptote: $x = 0$

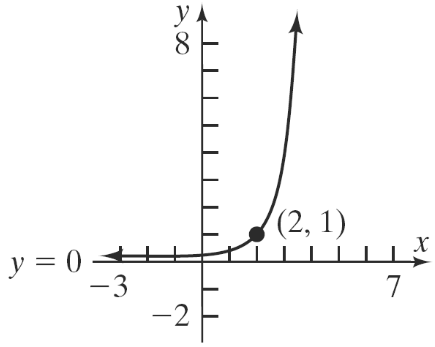
d.

$$\begin{aligned} f(x) &= 2 + \ln x \\ y &= 2 + \ln x \\ x &= 2 + \ln y \quad \text{Inverse} \\ x-2 &= \ln y \\ y &= e^{x-2} \\ f^{-1}(x) &= e^{x-2} \end{aligned}$$

e. The domain of the inverse found in part (d) is all real numbers.

Since the domain of f is the range of f^{-1}, we can use the result from part (a) to say that the range of f^{-1} is $(0, \infty)$.

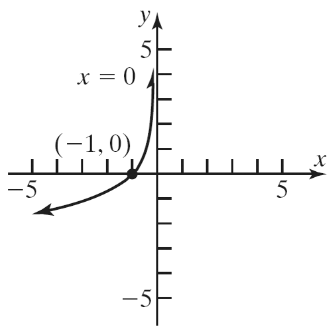
f. Using the graph of $y = e^x$, shift the graph 2 units to the right.



74. $f(x) = -\ln(-x)$

a. Domain: $(-\infty, 0)$

b. Using the graph of $y = \ln x$, reflect the graph about the y-axis, and reflect about the x-axis.



c. Range: $(-\infty, \infty)$
Vertical Asymptote: $x = 0$

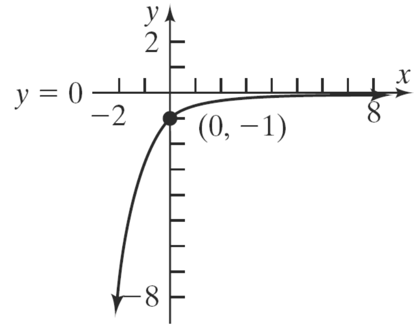
d.

$$\begin{aligned} f(x) &= -\ln(-x) \\ y &= -\ln(-x) \\ x &= -\ln(-y) \quad \text{Inverse} \\ -x &= \ln(-y) \\ -y &= e^{-x} \\ y &= -e^{-x} \\ f^{-1}(x) &= -e^{-x} \end{aligned}$$

e. The domain of the inverse found in part (d) is all real numbers.

Since the domain of f is the range of f^{-1}, we can use the result from part (a) to say that the range of f^{-1} is $(-\infty, 0)$.

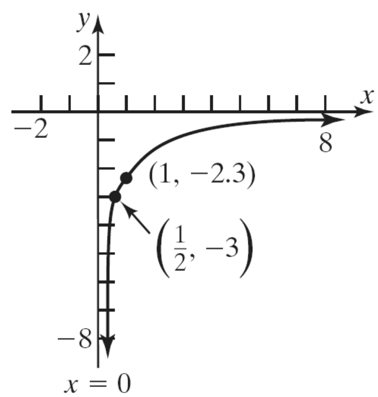
f. Using the graph of $y = e^x$, reflect the graph about the y-axis, and reflect about the x-axis.



75. $f(x) = \ln(2x) - 3$

a. Domain: $(0, \infty)$

b. Using the graph of $y = \ln x$, compress the graph horizontally by a factor of $\frac{1}{2}$, and shift down 3 units.



c. Range: $(-\infty, \infty)$
Vertical Asymptote: $x = 0$

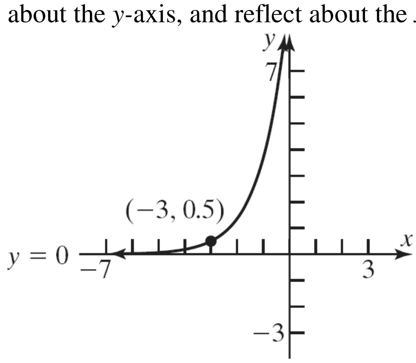
d.

$$\begin{aligned} f(x) &= \ln(2x) - 3 \\ y &= \ln(2x) - 3 \\ x &= \ln(2y) - 3 \quad \text{Inverse} \\ x + 3 &= \ln(2y) \\ 2y &= e^{x+3} \\ y &= \frac{1}{2}e^{x+3} \\ f^{-1}(x) &= \frac{1}{2}e^{x+3} \end{aligned}$$

e. The domain of the inverse found in part (d) is all real numbers.

Since the domain of f is the range of f^{-1}, we can use the result from part (a) to say that the range of f^{-1} is $(0,\infty)$.

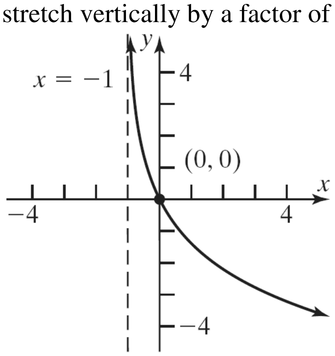
f. Using the graph of $y = e^x$, reflect the graph about the y-axis, and reflect about the x-axis.



76. $f(x) = -2\ln(x+1)$

a. Domain: $(-1, \infty)$

b. Using the graph of $y = \ln x$, shift the graph to the left 1 unit, reflect about the x-axis and stretch vertically by a factor of 2.



c. Range: $(-\infty, \infty)$
Vertical Asymptote: $x = -1$

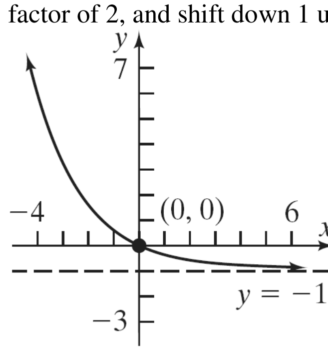
d.

$$f(x) = -2\ln(x+1)$$
$$y = -2\ln(x+1)$$
$$x = -2\ln(y+1) \quad \text{Inverse}$$
$$-\frac{x}{2} = \ln(y+1)$$
$$y+1 = e^{-x/2}$$
$$y = e^{-x/2} - 1$$
$$f^{-1}(x) = e^{-x/2} - 1$$

e. The domain of the inverse found in part (d) is all real numbers.

Since the domain of f is the range of f^{-1}, we can use the result from part (a) to say that the range of f^{-1} is $(-1,\infty)$.

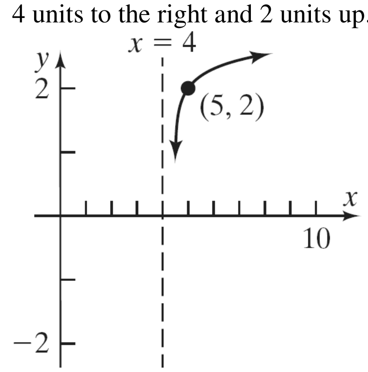
f. Using the graph of $y = e^x$, reflect the graph about the y-axis, stretch horizontally by a factor of 2, and shift down 1 unit.



77. $f(x) = \log(x-4) + 2$

a. Domain: $(4, \infty)$

b. Using the graph of $y = \log x$, shift the graph 4 units to the right and 2 units up.



c. Range: $(-\infty, \infty)$
Vertical Asymptote: $x = 4$

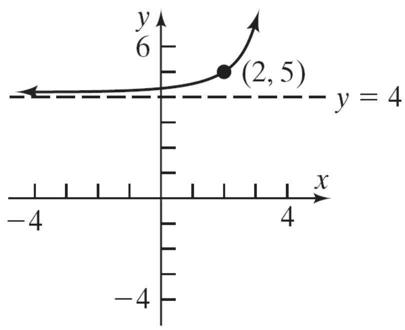
d.

$$f(x) = \log(x-4) + 2$$
$$y = \log(x-4) + 2$$
$$x = \log(y-4) + 2 \quad \text{Inverse}$$
$$x - 2 = \log(y-4)$$
$$y - 4 = 10^{x-2}$$
$$y = 10^{x-2} + 4$$
$$f^{-1}(x) = 10^{x-2} + 4$$

e. The domain of the inverse found in part (d) is all real numbers.

Since the domain of f is the range of f^{-1}, we can use the result from part (a) to say that the range of f^{-1} is $(4,\infty)$.

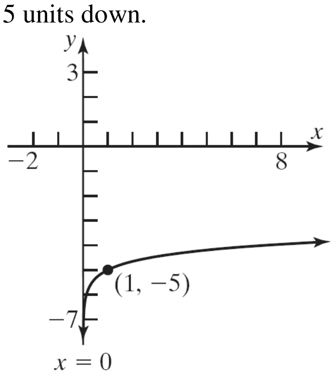
f. Using the graph of $y = 10^x$, shift the graph 2 units to the right and 4 units up.



78. $f(x) = \frac{1}{2}\log x - 5$

a. Domain: $(0, \infty)$

b. Using the graph of $y = \log x$, compress the graph vertically by a factor of $\frac{1}{2}$, and shift it 5 units down.



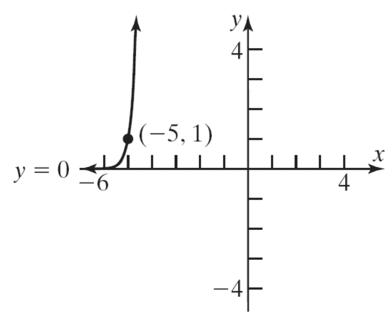
c. Range: $(-\infty, \infty)$
Vertical Asymptote: $x = 0$

d.
$$f(x) = \frac{1}{2}\log x - 5$$
$$y = \frac{1}{2}\log x - 5$$
$$x = \frac{1}{2}\log y - 5 \quad \text{Inverse}$$
$$x + 5 = \frac{1}{2}\log y$$
$$2(x+5) = \log y$$
$$y = 10^{2(x+5)}$$
$$f^{-1}(x) = 10^{2(x+5)}$$

e. The domain of the inverse found in part (d) is all real numbers.

Since the domain of f is the range of f^{-1}, we can use the result from part (a) to say that the range of f^{-1} is $(0,\infty)$.

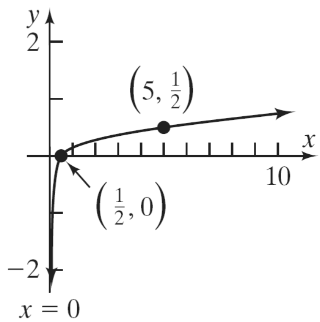
f. Using the graph of $y = 10^x$, shift the graph 5 units to the left, and compress horizontally by a factor of $\frac{1}{2}$.



79. $f(x) = \frac{1}{2}\log(2x)$

a. Domain: $(0, \infty)$

b. Using the graph of $y = \log x$, compress the graph horizontally by a factor of $\frac{1}{2}$, and compress vertically by a factor of $\frac{1}{2}$.



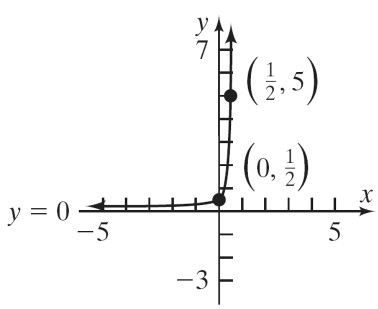
c. Range: $(-\infty, \infty)$
Vertical Asymptote: $x = 0$

d.
$$f(x) = \frac{1}{2}\log(2x)$$
$$y = \frac{1}{2}\log(2x)$$
$$x = \frac{1}{2}\log(2y) \quad \text{Inverse}$$
$$2x = \log(2y)$$
$$2y = 10^{2x}$$
$$y = \frac{1}{2}\cdot 10^{2x}$$
$$f^{-1}(x) = \frac{1}{2}\cdot 10^{2x}$$

e. The domain of the inverse found in part (d) is all real numbers.

Since the domain of f is the range of f^{-1}, we can use the result from part (a) to say that the range of f^{-1} is $(0, \infty)$.

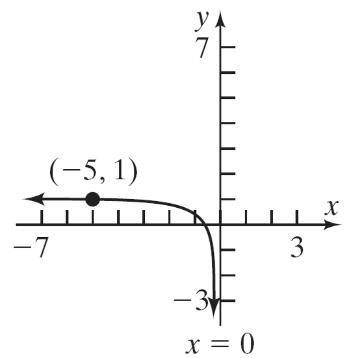
f. Using the graph of $y = 10^x$, compress the graph horizontally by a factor of $\frac{1}{2}$, and compress vertically by a factor of $\frac{1}{2}$.



80. $f(x) = \log(-2x)$

a. Domain: $(-\infty, 0)$

b. Using the graph of $y = \log x$, reflect the graph across the *y*-axis and compress horizontally by a factor of $\frac{1}{2}$.

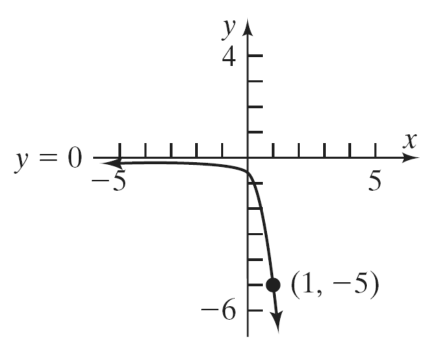


c. Range: $(-\infty, \infty)$
Vertical Asymptote: $x = 0$

d.
$$f(x) = \log(-2x)$$
$$y = \log(-2x)$$
$$x = \log(-2y) \quad \text{Inverse}$$
$$-2y = 10^x$$
$$f^{-1}(x) = -\frac{1}{2}\cdot 10^x$$

e. The domain of the inverse found in part (d) is all real numbers.
Since the domain of f is the range of f^{-1}, we can use the result from part (a) to say that the range of f^{-1} is $(-\infty, 0)$.

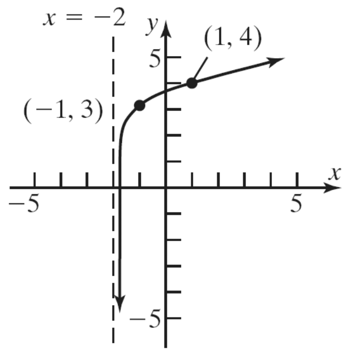
f. Using the graph of $y = 10^x$, reflect the graph across the *x*-axis and compress vertically by a factor of $\frac{1}{2}$.



81. $f(x) = 3 + \log_3(x+2)$

a. Domain: $(-2, \infty)$

b. Using the graph of $y = \log_3 x$, shift 2 units to the left, and shift up 3 units.



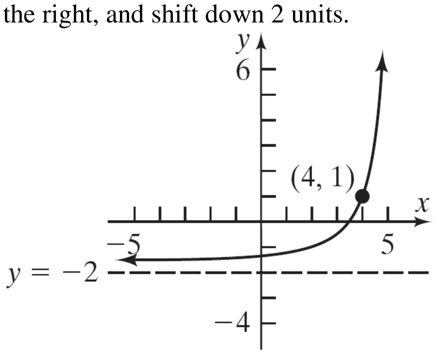
c. Range: $(-\infty, \infty)$
Vertical Asymptote: $x = -2$

d.
$$f(x) = 3 + \log_3(x+2)$$
$$y = 3 + \log_3(x+2)$$
$$x = 3 + \log_3(y+2) \quad \text{Inverse}$$
$$x - 3 = \log_3(y+2)$$
$$y + 2 = 3^{x-3}$$
$$y = 3^{x-3} - 2$$
$$f^{-1}(x) = 3^{x-3} - 2$$

e. The domain of the inverse found in part (d) is all real numbers.

Since the domain of f is the range of f^{-1}, we can use the result from part (a) to say that the range of f^{-1} is $(-2, \infty)$.

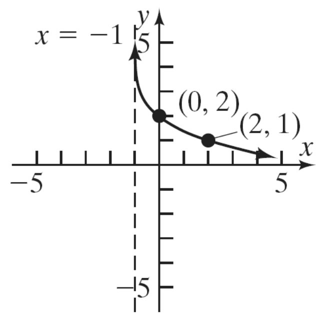
f. Using the graph of $y = 3^x$, shift 3 units to the right, and shift down 2 units.



82. $f(x) = 2 - \log_3(x+1)$

a. Domain: $(-1, \infty)$

b. Using the graph of $y = \log_3 x$, shift 1 unit to the left, reflect the graph about the x-axis, and shift 2 units up.



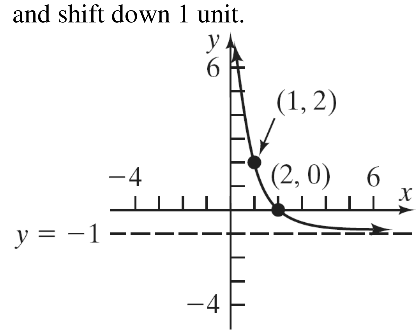
c. Range: $(-\infty, \infty)$
Vertical Asymptote: $x = -1$

d.
$$f(x) = 2 - \log_3(x+1)$$
$$y = 2 - \log_3(x+1)$$
$$x = 2 - \log_3(y+1) \quad \text{Inverse}$$
$$x - 2 = -\log_3(y+1)$$
$$2 - x = \log_3(y+1)$$
$$y + 1 = 3^{2-x}$$
$$y = 3^{2-x} - 1$$
$$f^{-1}(x) = 3^{2-x} - 1$$

e. The domain of the inverse found in part (d) is all real numbers.

Since the domain of f is the range of f^{-1}, we can use the result from part (a) to say that the range of f^{-1} is $(-1, \infty)$.

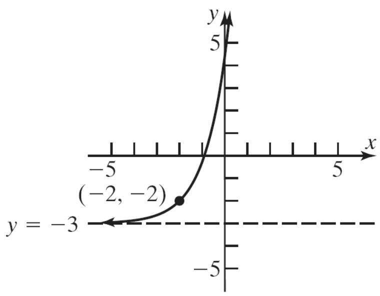
f. Using the graph of $y = 3^x$, reflect the graph about the y-axis, shift 2 units to the right, and shift down 1 unit.



83. $f(x) = e^{x+2} - 3$

a. Domain: $(-\infty, \infty)$

b. Using the graph of $y = e^x$, shift the graph two units to the left, and shift 3 units down.



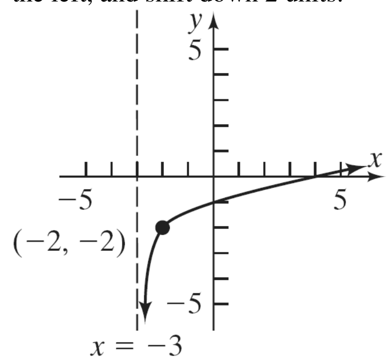
c. Range: $(-3, \infty)$
Horizontal Asymptote: $y = -3$

d.
$$
\begin{aligned}
f(x) &= e^{x+2} - 3 \\
y &= e^{x+2} - 3 \\
x &= e^{y+2} - 3 \quad \text{Inverse} \\
x + 3 &= e^{y+2} \\
y + 2 &= \ln(x+3) \\
y &= \ln(x+3) - 2 \\
f^{-1}(x) &= \ln(x+3) - 2
\end{aligned}
$$

e. For the domain of f^{-1} we need
$x + 3 > 0$
$x > -3$
So the domain of the inverse found in part (d) is $(-3, \infty)$.

Since the domain of f is the range of f^{-1}, we can use the result from part (a) to say that the range of f^{-1} is $(-\infty, \infty)$.

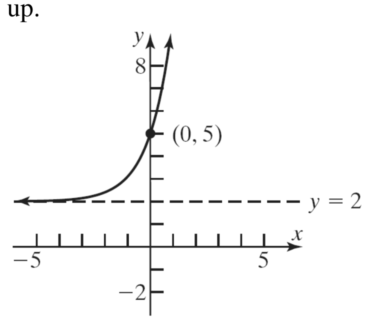
f. Using the graph of $y = \ln x$, shift 3 units to the left, and shift down 2 units.



84. $f(x) = 3e^x + 2$

a. Domain: $(-\infty, \infty)$

b. Using the graph of $y = e^x$, stretch the graph vertically by a factor of 3, and shift 2 units up.



c. Range: $(2, \infty)$
Horizontal Asymptote: $y = 2$

d.
$$
\begin{aligned}
f(x) &= 3e^x + 2 \\
y &= 3e^x + 2 \\
x &= 3e^y + 2 \quad \text{Inverse} \\
x - 2 &= 3e^y \\
\frac{x-2}{3} &= e^y \\
y &= \ln\left(\frac{x-2}{3}\right) \\
f^{-1}(x) &= \ln\left(\frac{x-2}{3}\right)
\end{aligned}
$$

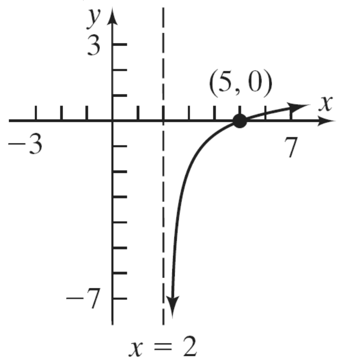
e. For the domain of f^{-1} we need

$$\frac{x-2}{3} > 0$$
$$x-2>0$$
$$x>2$$

The domain of the inverse found in part (d) is $(2,\infty)$.

Since the domain of f is the range of f^{-1}, we can use the result from part (a) to say that the range of f^{-1} is $(-\infty,\infty)$.

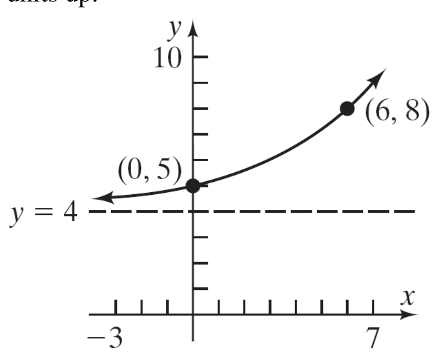
f. Using the graph of $y=\ln x$, shift 3 units to the left, and shift down 2 units.



85. $f(x)=2^{x/3}+4$

a. Domain: $(-\infty,\ \infty)$

b. Using the graph of $y=2^x$, stretch the graph horizontally by a factor of 3, and shift 4 units up.



c. Range: $(4,\infty)$

Horizontal Asymptote: $y=4$

d.

$$f(x)=2^{x/3}+4$$
$$y=2^{x/3}+4$$
$$x=2^{y/3}+4 \quad \text{Inverse}$$
$$x-4=2^{y/3}$$
$$\frac{y}{3}=\log_2(x-4)$$
$$y=3\log_2(x-4)$$
$$f^{-1}(x)=3\log_2(x-4)$$

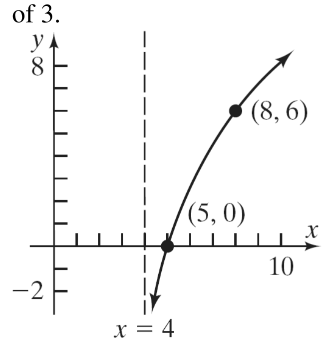
e. For the domain of f^{-1} we need

$$x-4>0$$
$$x>4$$

The domain of the inverse found in part (d) is $(4,\infty)$.

Since the domain of f is the range of f^{-1}, we can use the result from part (a) to say that the range of f^{-1} is $(-\infty,\infty)$.

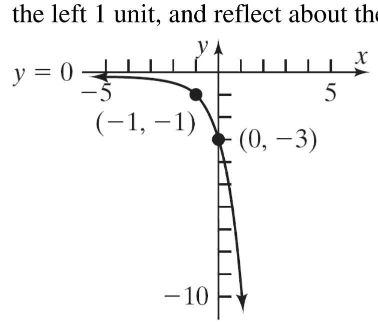
f. Using the graph of $y=\log_2 x$, shift 4 units to the right, and stretch vertically by a factor of 3.



86. $f(x)=-3^{x+1}$

a. Domain: $(-\infty,\ \infty)$

b. Using the graph of $y=3^x$, shift the graph to the left 1 unit, and reflect about the x-axis.



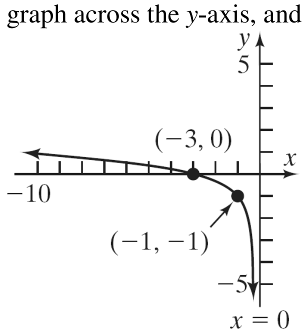
c. Range: $(-\infty, 0)$
Horizontal Asymptote: $y = 0$

d.
$$f(x) = -3^{x+1}$$
$$y = -3^{x+1}$$
$$x = -3^{y+1} \quad \text{Inverse}$$
$$-x = 3^{y+1}$$
$$y+1 = \log_3(-x)$$
$$y = \log_3(-x) - 1$$
$$f^{-1}(x) = \log_3(-x) - 1$$

e. For the domain of f^{-1} we need
$$-x > 0$$
$$x < 0$$
The domain of the inverse found in part (d) is $(-\infty, 0)$.

Since the domain of f is the range of f^{-1}, we can use the result from part (a) to say that the range of f^{-1} is $(-\infty, \infty)$.

f. Using the graph of $y = \log_3 x$, reflect the graph across the y-axis, and shift down 1 unit.



87. $\log_3 x = 2$
$$x = 3^2$$
$$x = 9$$
The solution set is $\{9\}$.

88. $\log_5 x = 3$
$$x = 5^3$$
$$x = 125$$
The solution set is $\{125\}$.

89. $\log_2(2x+1) = 3$
$$2x+1 = 2^3$$
$$2x+1 = 8$$
$$2x = 7$$
$$x = \frac{7}{2}$$
The solution set is $\left\{\frac{7}{2}\right\}$.

90. $\log_3(3x-2) = 2$
$$3x-2 = 3^2$$
$$3x-2 = 9$$
$$3x = 11$$
$$x = \frac{11}{3}$$
The solution set is $\left\{\frac{11}{3}\right\}$.

91. $\log_x 4 = 2$
$$x^2 = 4$$
$$x = 2 \quad (x \neq -2, \text{ base is positive})$$
The solution set is $\{2\}$.

92. $\log_x\left(\frac{1}{8}\right) = 3$
$$x^3 = \frac{1}{8}$$
$$x = \frac{1}{2}$$
The solution set is $\left\{\frac{1}{2}\right\}$.

93. $\ln e^x = 5$
$$e^x = e^5$$
$$x = 5$$
The solution set is $\{5\}$.

94. $\ln e^{-2x} = 8$
$$e^{-2x} = e^8$$
$$-2x = 8$$
$$x = -4$$
The solution set is $\{-4\}$.

95. $\log_4 64 = x$
$$4^x = 64$$
$$4^x = 4^3$$
$$x = 3$$
The solution set is $\{3\}$.

96. $\log_5 625 = x$
$$5^x = 625$$
$$5^x = 5^4$$
$$x = 4$$
The solution set is $\{4\}$.

97. $\log_3 243 = 2x+1$
$$3^{2x+1} = 243$$
$$3^{2x+1} = 3^5$$
$$2x+1 = 5$$
$$2x = 4$$
$$x = 2$$
The solution set is $\{2\}$.

98. $\log_6 36 = 5x+3$
$$6^{5x+3} = 36$$
$$6^{5x+3} = 6^2$$
$$5x+3 = 2$$
$$5x = -1$$
$$x = -\frac{1}{5}$$
The solution set is $\left\{-\frac{1}{5}\right\}$.

99. $e^{3x} = 10$
$$3x = \ln 10$$
$$x = \frac{\ln 10}{3}$$
The solution set is $\left\{\frac{\ln 10}{3}\right\}$.

100. $e^{-2x} = \frac{1}{3}$
$$-2x = \ln\left(\frac{1}{3}\right)$$
$$-2x = \ln\left(3^{-1}\right)$$
$$-2x = -\ln 3$$
$$2x = \ln 3$$
$$x = \frac{\ln 3}{2}$$
The solution set is $\left\{\frac{\ln 3}{2}\right\}$.

101. $e^{2x+5} = 8$
$$2x+5 = \ln 8$$
$$2x = -5+\ln 8$$
$$x = \frac{-5+\ln 8}{2}$$
The solution set is $\left\{\frac{-5+\ln 8}{2}\right\}$.

102. $e^{-2x+1} = 13$
$$-2x+1 = \ln 13$$
$$-2x = -1+\ln 13$$
$$x = \frac{-1+\ln 13}{-2} = \frac{1-\ln 13}{2}$$
The solution set is $\left\{\frac{1-\ln 13}{2}\right\}$.

103. $\log_3\left(x^2+1\right) = 2$
$$x^2+1 = 3^2$$
$$x^2+1 = 9$$
$$x^2 = 8$$
$$x = \pm\sqrt{8} = \pm 2\sqrt{2}$$
The solution set is $\left\{-2\sqrt{2},\ 2\sqrt{2}\right\}$.

104. $\log_5\left(x^2+x+4\right) = 2$
$$x^2+x+4 = 5^2$$
$$x^2+x+4 = 25$$
$$x^2+x-21 = 0$$
$$x = \frac{-1\pm\sqrt{1^2-4(1)(-21)}}{2(1)} = \frac{-1\pm\sqrt{85}}{2}$$
The solution set is $\left\{\frac{-1-\sqrt{85}}{2},\ \frac{-1+\sqrt{85}}{2}\right\}$.

105. $\log_2 8^x = -3$

$8^x = 2^{-3}$

$\left(2^3\right)^x = 2^{-3}$

$2^{3x} = 2^{-3}$

$3x = -3$

$x = -1$

The solution set is $\{-1\}$.

106. $\log_3 3^x = -1$

$3^x = 3^{-1}$

$x = -1$

The solution set is $\{-1\}$.

107. $5e^{0.2x} = 7$

$e^{0.2x} = \frac{7}{5}$

$0.2x = \ln\frac{7}{5}$

$5(0.2x) = 5\left(\ln\frac{7}{5}\right)$

$x = 5\ln\frac{7}{5}$

The solution set is $\left\{5\ln\frac{7}{5}\right\}$.

108. $8\cdot 10^{2x-7} = 3$

$10^{2x-7} = \frac{3}{8}$

$2x-7 = \log\frac{3}{8}$

$2x = 7+\log\frac{3}{8}$

$x = \frac{1}{2}\left(7+\log\frac{3}{8}\right)$

The solution set is $\left\{\frac{1}{2}\left(7+\log\frac{3}{8}\right)\right\}$.

109. $2\cdot 10^{2-x} = 5$

$10^{2-x} = \frac{5}{2}$

$2-x = \log\frac{5}{2}$

$-x = -2+\log\frac{5}{2}$

$x = 2-\log\frac{5}{2}$

The solution set is $\left\{2-\log\frac{5}{2}\right\}$.

110. $4e^{x+1} = 5$

$e^{x+1} = \frac{5}{4}$

$x+1 = \ln\frac{5}{4}$

$x = -1+\ln\frac{5}{4}$

The solution set is $\left\{-1+\ln\frac{5}{4}\right\}$.

111. **a.** $G(x) = \log_3(2x+1)-2$

We require that $2x+1$ be positive.

$2x+1>0$

$2x>-1$

$x>-\frac{1}{2}$

Domain: $\left\{x \mid x>-\frac{1}{2}\right\}$ or $\left(-\frac{1}{2},\infty\right)$

b. $G(40) = \log_3(2\cdot 40+1)-2$

$= \log_3 81-2$

$= 4-2$

$= 2$

The point $(40,2)$ is on the graph of G.

c. $G(x) = 3$

$\log_3(2x+1)-2 = 3$

$\log_3(2x+1) = 5$

$2x+1 = 3^5$

$2x+1 = 243$

$2x = 242$

$x = 121$

The point $(121,3)$ is on the graph of G.

d. $G(x)=0$

$\log_3(2x+1)-2=0$

$\log_3(2x+1)=2$

$2x+1=3^2$

$2x+1=9$

$2x=8$

$x=4$

The zero of G is $x=4$.

112. a. $F(x)=\log_2(x+1)-3$

We require that $x+1$ be positive.

$x+1>0$

$x>-1$

Domain: $\{x \mid x>-1\}$ or $(-1,\infty)$

b. $F(7)=\log_2(7+1)-3$

$=\log_2(8)-3$

$=3-3$

$=0$

The point $(7,0)$ is on the graph of F.

c. $F(x)=-1$

$\log_2(x+1)-3=-1$

$\log_2(x+1)=2$

$x+1=2^2$

$x+1=4$

$x=3$

The point $(3,-1)$ is on the graph of F.

d. $F(x)=0$

$\log_2(x+1)-3=0$

$\log_2(x+1)=3$

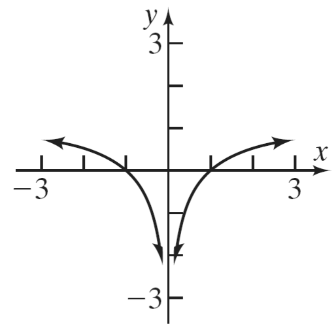
$x+1=2^3$

$x+1=8$

$x=7$

The zero of G is $x=7$.

113. $f(x)=\begin{cases}\ln(-x) & \text{if } x<0\\ \ln x & \text{if } x>0\end{cases}$

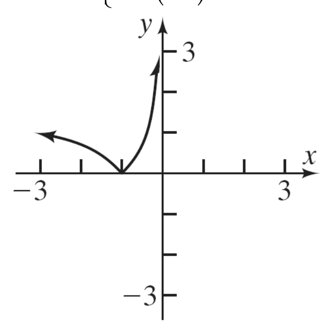


Domain: $\{x \mid x\neq 0\}$

Range: $(-\infty,\infty)$

Intercepts: $(-1,0)$, $(1,0)$

114. $f(x)=\begin{cases}\ln(-x) & \text{if } x\le -1\\ -\ln(-x) & \text{if } -1<x<0\end{cases}$

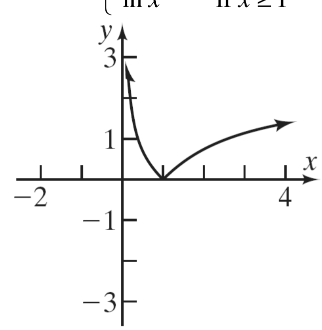


Domain: $\{x \mid x<0\}$; $(-\infty,0)$

Range: $\{y \mid y\ge 0\}$; $[0,\infty)$

Intercept: $(-1,0)$

115. $f(x)=\begin{cases}-\ln x & \text{if } 0<x<1\\ \ln x & \text{if } x\ge 1\end{cases}$

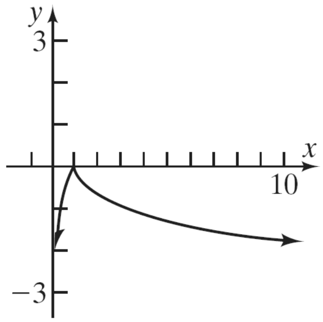


Domain: $\{x \mid x>0\}$; $(0,\infty)$

Range: $\{y \mid y\ge 0\}$; $[0,\infty)$

Intercept: $(1,0)$

116. $f(x)=\begin{cases}\ln x & \text{if } 0<x<1\\ -\ln x & \text{if } x\ge 1\end{cases}$



Domain: $\{x \mid x>0\}$; $(0,\infty)$
Range: $\{y \mid y\le 0\}$; $(-\infty,0]$
Intercept: $(1,0)$

117. $\text{pH}=-\log_{10}\left[\text{H}^+\right]$

a. $\text{pH}=-\log_{10}[0.1]=-(-1)=1$

b. $\text{pH}=-\log_{10}[0.01]=-(-2)=2$

c. $\text{pH}=-\log_{10}[0.001]=-(-3)=3$

d. As the H^+ decreases, the pH increases.

e.
$$\begin{aligned}3.5&=-\log_{10}\left[\text{H}^+\right]\\-3.5&=\log_{10}\left[\text{H}^+\right]\\\left[\text{H}^+\right]&=10^{-3.5}\\&\approx 3.16\times10^{-4}\\&=0.000316\end{aligned}$$

f.
$$\begin{aligned}7.4&=-\log_{10}\left[\text{H}^+\right]\\-7.4&=\log_{10}\left[\text{H}^+\right]\\\left[\text{H}^+\right]&=10^{-7.4}\\&\approx 3.981\times10^{-8}\\&=0.00000003981\end{aligned}$$

118.
$$\begin{aligned}H&=-\left(p_1\log p_1+p_2\log p_2+\cdots+p_n\log p_n\right)\\&=-p_1\log p_1-p_2\log p_2-\cdots-p_n\log p_n\end{aligned}$$

a.
$$\begin{aligned}H&=-0.008\log(0.008)-0.045\log(0.045)\\&\quad-0.125\log(0.125)-0.159\log(0.159)\\&\quad-0.001\log(0.001)-0.662\log(0.662)\\&\approx 0.4388\end{aligned}$$

b. $H_{\max}=\log(6)\approx 0.7782$

c. $E=\dfrac{H}{H_{\max}}=\dfrac{0.4388}{0.7782}\approx 0.5639$

d. (answers may vary)

Race	Proportion
American Indian/ Native Alaskan	0.009
Asian	0.048
Black or African American	0.126
Hispanic	0.163
White	0.724
Native Hawaiian/Pacific Islander	0.002

$$\begin{aligned}H&=-0.009\log(0.009)-0.048\log(0.048)\\&\quad-0.126\log(0.126)-0.163\log(0.163)\\&\quad-0.724\log(0.724)-0.002\log(0.002)\\&\approx 0.4304\end{aligned}$$

The United States appears to be growing more diverse.

119. $p=760e^{-0.145h}$

a.
$$\begin{aligned}320&=760e^{-0.145h}\\\frac{320}{760}&=e^{-0.145h}\\\ln\left(\frac{320}{760}\right)&=-0.145h\\h&=\frac{\ln\left(\frac{320}{760}\right)}{-0.145}\approx 5.97\end{aligned}$$

Approximately 5.97 kilometers.

b.
$$\begin{aligned}667&=760e^{-0.145h}\\\frac{667}{760}&=e^{-0.145h}\\\ln\left(\frac{667}{760}\right)&=-0.145h\\h&=\frac{\ln\left(\frac{667}{760}\right)}{-0.145}\approx 0.90\end{aligned}$$

Approximately 0.90 kilometers.

120. $A = A_0 e^{-0.35n}$

a. $50 = 100e^{-0.35n}$

$0.5 = e^{-0.35n}$

$\ln(0.5) = -0.35n$

$t = \frac{\ln(0.5)}{-0.35} \approx 1.98$

Approximately 2 days.

b. $10 = 100e^{-0.35n}$

$0.1 = e^{-0.35n}$

$\ln(0.1) = -0.35n$

$t = \frac{\ln(0.1)}{-0.35} \approx 6.58$

About 6.58 days, or 6 days and 14 hours.

121. $F(t) = 1 - e^{-0.1t}$

a. $0.5 = 1 - e^{-0.1t}$

$-0.5 = -e^{-0.1t}$

$0.5 = e^{-0.1t}$

$\ln(0.5) = -0.1t$

$t = \frac{\ln(0.5)}{-0.1} \approx 6.93$

Approximately 6.93 minutes.

b. $0.8 = 1 - e^{-0.1t}$

$-0.2 = -e^{-0.1t}$

$0.2 = e^{-0.1t}$

$\ln(0.2) = -0.1t$

$t = \frac{\ln(0.2)}{-0.1} \approx 16.09$

Approximately 16.09 minutes.

c. It is impossible for the probability to reach 100% because $e^{-0.1t}$ will never equal zero; thus, $F(t) = 1 - e^{-0.1t}$ will never equal 1.

122. $F(t) = 1 - e^{-0.15t}$

a. $0.50 = 1 - e^{-0.15t}$

$-0.5 = e^{-(R/L)t}$

$0.5 = e^{-0.15t}$

$\ln(0.5) = -0.15t$

$t = \frac{\ln(0.5)}{-0.15} \approx 4.62$

Approximately 4.62 minutes, or 4 minutes and 37 seconds.

b. $0.80 = 1 - e^{-0.15t}$

$-0.2 = -e^{-0.15t}$

$0.2 = e^{-0.15t}$

$\ln(0.2) = -0.15t$

$t = \frac{\ln(0.2)}{-0.15} \approx 10.73$

Approximately 10.73 minutes, or 10 minutes and 44 seconds.

123. $D = 5e^{-0.4h}$

$2 = 5e^{-0.4h}$

$0.4 = e^{-0.4h}$

$\ln(0.4) = -0.4h$

$h = \frac{\ln(0.4)}{-0.4} \approx 2.29$

Approximately 2.29 hours, or 2 hours and 17 minutes.

124. $N = P\left(1 - e^{-0.15d}\right)$

$450 = 1000\left(1 - e^{-0.15d}\right)$

$0.45 = 1 - e^{-0.15d}$

$-0.55 = -e^{-0.15d}$

$0.55 = e^{-0.15d}$

$\ln(0.55) = -0.15d$

$d = \frac{\ln(0.55)}{-0.15} \approx 3.99$

Approximately 4 days.

125. $I = \frac{E}{R}\left[1 - e^{-(R/L)t}\right]$

Substituting $E = 12$, $R = 10$, $L = 5$, and $I = 0.5$, we obtain:

$$0.5 = \frac{12}{10}\left[1 - e^{-(10/5)t}\right]$$
$$\frac{5}{12} = 1 - e^{-2t}$$
$$e^{-2t} = \frac{7}{12}$$
$$-2t = \ln(7/12)$$
$$t = \frac{\ln(7/12)}{-2} \approx 0.2695$$

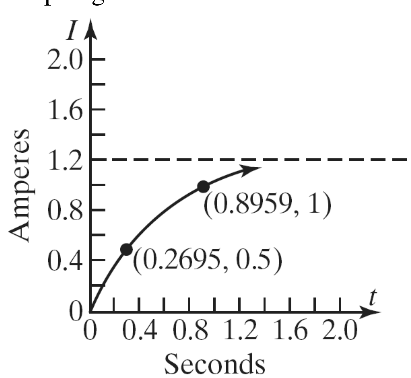
It takes approximately 0.2695 second to obtain a current of 0.5 ampere.

Substituting $E = 12$, $R = 10$, $L = 5$, and $I = 1.0$, we obtain:

$$1.0 = \frac{12}{10}\left[1 - e^{-(10/5)t}\right]$$
$$\frac{10}{12} = 1 - e^{-2t}$$
$$e^{-2t} = \frac{1}{6}$$
$$-2t = \ln(1/6)$$
$$t = \frac{\ln(1/6)}{-2} \approx 0.8959$$

It takes approximately 0.8959 second to obtain a current of 0.5 ampere.

Graphing:



126. $L(t) = A\left(1 - e^{-kt}\right)$

a.
$$20 = 200\left(1 - e^{-k(5)}\right)$$
$$0.1 = 1 - e^{-5k}$$
$$e^{-5k} = 0.9$$
$$-5k = \ln 0.9$$
$$k = -\frac{\ln 0.9}{5} \approx 0.0211$$

b.
$$L(10) = 200\left(1 - e^{-\left(-\frac{\ln 0.9}{5}\right)(10)}\right)$$
$$= 200\left(1 - e^{2\ln 0.9}\right)$$
$$= 38 \text{ words}$$

c.
$$L(15) = 200\left(1 - e^{-\left(-\frac{\ln 0.9}{5}\right)(15)}\right)$$
$$= 200\left(1 - e^{3\ln 0.9}\right)$$
$$\approx 54 \text{ words}$$

d.
$$180 = 200\left(1 - e^{-\left(-\frac{\ln 0.9}{5}\right)t}\right)$$
$$0.9 = 1 - e^{\frac{\ln 0.9}{5}t}$$
$$e^{\frac{\ln 0.9}{5}t} = 0.1$$
$$\frac{\ln 0.9}{5}t = \ln 0.1$$
$$t = \frac{\ln 0.1}{\frac{\ln 0.9}{5}} \approx 109.27 \text{ minutes}$$

127.
$$L\left(10^{-7}\right) = 10\log\left(\frac{10^{-7}}{10^{-12}}\right)$$
$$= 10\log\left(10^{5}\right)$$
$$= 10 \cdot 5$$
$$= 50 \text{ decibels}$$

128.
$$L\left(10^{-1}\right) = 10\log\left(\frac{10^{-1}}{10^{-12}}\right)$$
$$= 10\log\left(10^{11}\right)$$
$$= 10 \cdot 11$$
$$= 110 \text{ decibels}$$

129.
$$L\left(10^{-3}\right) = 10\log\left(\frac{10^{-3}}{10^{-12}}\right)$$
$$= 10\log\left(10^{9}\right)$$
$$= 10 \cdot 9$$
$$= 90 \text{ decibels}$$

130. Intensity of car:

$$70 = 10\log\left(\frac{x}{10^{-12}}\right)$$
$$7 = \log\left(\frac{x}{10^{-12}}\right)$$
$$10^7 = \frac{x}{10^{-12}}$$
$$x = 10^{-5}$$

Intensity of truck is $10 \cdot 10^{-5} = 10^{-4}$.

$$L\left(10^{-4}\right) = 10\log\left(\frac{10^{-4}}{10^{-12}}\right) = 10\log\left(10^8\right) = 10 \cdot 8 = 80 \text{ decibels}$$

131. $M(125{,}892) = \log\left(\frac{125{,}892}{10^{-3}}\right) \approx 8.1$

132. $M(50{,}119) = \log\left(\frac{50{,}119}{10^{-3}}\right) \approx 7.7$

133. $R = e^{kx}$

a.
$$1.4 = e^{k(0.03)}$$
$$1.4 = e^{0.03k}$$
$$\ln(1.4) = 0.03k$$
$$k = \frac{\ln(1.4)}{0.03} \approx 11.216$$

b. $R = e^{11.216(0.17)} = e^{1.90672} \approx 6.73$

c.
$$100 = e^{11.216x}$$
$$100 = e^{11.216x}$$
$$\ln(100) = 11.216x$$
$$x = \frac{\ln(100)}{11.216} \approx 0.41 \text{ percent}$$

d.
$$5 = e^{11.216x}$$
$$\ln 5 = 11.216x$$
$$x = \frac{\ln 5}{11.216} \approx 0.14 \text{ percent}$$

At a percent concentration of 0.14 or higher, the driver should be charged with a DUI.

e. Answers will vary.

134. No. Explanations will vary.

135. If the base of a logarithmic function equals 1, we would have the following:

$$f(x) = \log_1(x)$$
$$f^{-1}(x) = 1^x = 1 \text{ for every real number } x.$$

In other words, f^{-1} would be a constant function and, therefore, f^{-1} would not be one-to-one.

136. New = Old$\left(e^{Rt}\right)$

Age	Depriciation rate
1	$38{,}000 = 36{,}600e^{R(1)}$; $\frac{38{,}000}{36{,}600} = e^R$; $R = \ln\left(\frac{38{,}000}{36{,}600}\right) \approx 0.03754 \approx 3.8\%$
2	$38{,}000 = 32{,}400e^{R(2)}$; $\frac{38{,}000}{32{,}400} = e^{2R}$; $\ln\left(\frac{38{,}000}{32{,}400}\right) = 2R$; $R = \frac{\ln\left(\frac{38{,}000}{32{,}400}\right)}{2} \approx 0.07971 \approx 8\%$
3	$38{,}000 = 28{,}750e^{R(3)}$; $\frac{38{,}000}{28{,}750} = e^{3R}$; $\ln\left(\frac{38{,}000}{28{,}750}\right) = 3R$; $R = \frac{\ln\left(\frac{38{,}000}{28{,}750}\right)}{3} \approx 0.0930 \approx 9.3\%$
4	$38{,}000 = 25{,}400e^{R(4)}$; $\frac{38{,}000}{25{,}400} = e^{4R}$; $\ln\left(\frac{38{,}000}{25{,}400}\right) = 4R$; $R = \frac{\ln\left(\frac{38{,}000}{24{,}500}\right)}{4} \approx 0.1007 \approx 10.1\%$

Age	Depriciation rate
5	$38,000 = 21,200e^{R\,(5)}$ $\frac{38,000}{21,200} = e^{5R}$ $\ln\left(\frac{38,000}{21,200}\right) = 5R$ $R = \frac{\ln\left(\frac{38,000}{21,200}\right)}{5} \approx 0.1167 \approx 11.7\%$

Answers will vary.

Section 5.5

1. 0

2. 1

3. M

4. r

5. $\log_a M$; $\log_a N$

6. $\log_a M$; $\log_a N$

7. $r\log_a M$

8. 6

9. 7

10. False: $\ln(x+3) - \ln(2x) = \ln\left(\frac{x+3}{2x}\right)$

11. False: $\log_2\left(3x^4\right) = \log_2 3 + 4\log_2 x$

12. False: $\frac{\ln 8}{\ln 4} = \frac{3}{2}$

13. $\log_3 3^{71} = 71$

14. $\log_2 2^{-13} = -13$

15. $\ln e^{-4} = -4$

16. $\ln e^{\sqrt{2}} = \sqrt{2}$

17. $2^{\log_2 7} = 7$

18. $e^{\ln 8} = 8$

19. $\log_8 2 + \log_8 4 = \log_8(4\cdot 2) = \log_8 8 = 1$

20.
$$\begin{aligned}\log_6 9 + \log_6 4 &= \log_6(9\cdot 4)\\ &= \log_6 36\\ &= \log_6 6^2\\ &= 2\end{aligned}$$

21. $\log_6 18 - \log_6 3 = \log_6 \frac{18}{3} = \log_6 6 = 1$

22. $\log_8 16 - \log_8 2 = \log_8 \frac{16}{2} = \log_8 8 = 1$

23.
$$\begin{aligned}\log_2 6\cdot\log_6 8 &= \log_6 8^{\log_2 6}\\ &= \log_6\left(2^3\right)^{\log_2 6}\\ &= \log_6 2^{3\log_2 6}\\ &= \log_6 2^{\log_2 6^3}\\ &= \log_6 6^3\\ &= 3\end{aligned}$$

24.
$$\begin{aligned}\log_3 8\cdot\log_8 9 &= \log_8 9^{\log_3 8}\\ &= \log_8\left(3^2\right)^{\log_3 8}\\ &= \log_8 3^{2\log_3 8}\\ &= \log_8 3^{\log_3 8^2}\\ &= \log_8 8^2\\ &= 2\end{aligned}$$

25. $3^{\log_3 5 - \log_3 4} = 3^{\log_3 \frac{5}{4}} = \frac{5}{4}$

26. $5^{\log_5 6 + \log_5 7} = 5^{\log_5(6\cdot 7)} = 5^{\log_5 42} = 42$

27. $e^{\log_{e^2} 16}$

Let $a = \log_{e^2} 16$, then $\left(e^2\right)^a = 16$.

$$e^{2a} = 16$$
$$e^{2a} = 4^2$$
$$\left(e^{2a}\right)^{1/2} = \left(4^2\right)^{1/2}$$
$$e^a = 4$$
$$a = \ln 4$$

Thus, $e^{\log_{e^2} 16} = e^{\ln 4} = 4$.

28. $e^{\log_{e^2} 9}$

Let $a = \log_{e^2} 9$, then $\left(e^2\right)^a = 9.$

$$e^{2a} = 9$$
$$e^{2a} = 3^2$$
$$\left(e^{2a}\right)^{1/2} = \left(3^2\right)^{1/2}$$
$$e^a = 3$$
$$a = \ln 3$$

Thus, $e^{\log_{e^2} 9} = e^{\ln 3} = 3$.

29. $\ln 6 = \ln(2 \cdot 3) = \ln 2 + \ln 3 = a + b$

30. $\ln \frac{2}{3} = \ln 2 - \ln 3 = a - b$

31. $\ln 1.5 = \ln \frac{3}{2} = \ln 3 - \ln 2 = b - a$

32. $\ln 0.5 = \ln \frac{1}{2} = \ln 1 - \ln 2 = 0 - a = -a$

33. $\ln 8 = \ln 2^3 = 3 \cdot \ln 2 = 3a$

34. $\ln 27 = \ln 3^3 = 3 \cdot \ln 3 = 3b$

35.
$$\ln \sqrt[5]{6} = \ln 6^{1/5} = \frac{1}{5}\ln 6 = \frac{1}{5}\ln(2 \cdot 3) = \frac{1}{5}(\ln 2 + \ln 3) = \frac{1}{5}(a + b)$$

36.
$$\ln \sqrt[4]{\frac{2}{3}} = \ln\left(\frac{2}{3}\right)^{1/4} = \frac{1}{4}\ln\frac{2}{3} = \frac{1}{4}(\ln 2 - \ln 3) = \frac{1}{4}(a - b)$$

37. $\log_5(25x) = \log_5 25 + \log_5 x = 2 + \log_5 x$

38. $\log_3 \frac{x}{9} = \log_3 \frac{x}{3^2} = \log_3 x - \log_3 3^2 = \log_3 x - 2$

39. $\log_2 z^3 = 3\log_2 z$

40. $\log_7 x^5 = 5\log_7 x$

41. $\ln(ex) = \ln e + \ln x = 1 + \ln x$

42. $\ln \frac{e}{x} = \ln e - \ln x = 1 - \ln x$

43. $\ln\left(\frac{x}{e^x}\right) = \ln x - \ln e^x = \ln x - x$

44. $\ln\left(xe^x\right) = \ln x + \ln e^x = \ln x + x$

45.
$$\log_a\left(u^2v^3\right) = \log_a u^2 + \log_a v^3 = 2\log_a u + 3\log_a v$$

46. $\log_2\left(\frac{a}{b^2}\right) = \log_2 a - \log_2 b^2 = \log_2 a - 2\log_2 b$

47.
$$\ln\left(x^2\sqrt{1-x}\right) = \ln x^2 + \ln\sqrt{1-x} = \ln x^2 + \ln(1-x)^{1/2} = 2\ln x + \frac{1}{2}\ln(1-x)$$

48.
$$\ln\left(x\sqrt{1+x^2}\right) = \ln x + \ln\sqrt{1+x^2} = \ln x + \ln\left(1+x^2\right)^{1/2} = \ln x + \frac{1}{2}\ln\left(1+x^2\right)$$

49. $\log_2\left(\dfrac{x^3}{x-3}\right)=\log_2 x^3-\log_2(x-3)$
$=3\log_2 x-\log_2(x-3)$

50. $\log_5\left(\dfrac{\sqrt[3]{x^2+1}}{x^2-1}\right)$
$=\log_5\left(x^2+1\right)^{1/3}-\log_5(x^2-1)$
$=\dfrac{1}{3}\log_5\left(x^2+1\right)-\log_5\left(x^2-1\right)$
$=\dfrac{1}{3}\log_5\left(x^2+1\right)-\log_5\left((x+1)(x-1)\right)$
$=\dfrac{1}{3}\log_5\left(x^2+1\right)-\log_5\left(x+1\right)-\log_5\left(x-1\right)$

51. $\log\left[\dfrac{x(x+2)}{(x+3)^2}\right]=\log\left[x(x+2)\right]-\log(x+3)^2$
$=\log x+\log(x+2)-2\log(x+3)$

52. $\log\left[\dfrac{x^3\sqrt{x+1}}{(x-2)^2}\right]=\log\left(x^3\sqrt{x+1}\right)-\log(x-2)^2$
$=\log x^3+\log(x+1)^{1/2}-2\log(x-2)$
$=3\log x+\dfrac{1}{2}\log(x+1)-2\log(x-2)$

53. $\ln\left[\dfrac{x^2-x-2}{(x+4)^2}\right]^{1/3}$
$=\dfrac{1}{3}\ln\left[\dfrac{(x-2)(x+1)}{(x+4)^2}\right]$
$=\dfrac{1}{3}\left[\ln(x-2)(x+1)-\ln(x+4)^2\right]$
$=\dfrac{1}{3}\left[\ln(x-2)+\ln(x+1)-2\ln(x+4)\right]$
$=\dfrac{1}{3}\ln(x-2)+\dfrac{1}{3}\ln(x+1)-\dfrac{2}{3}\ln(x+4)$

54. $\ln\left[\dfrac{(x-4)^2}{x^2-1}\right]^{2/3}$
$=\dfrac{2}{3}\ln\left[\dfrac{(x-4)^2}{x^2-1}\right]$
$=\dfrac{2}{3}\left[\ln(x-4)^2-\ln\left(x^2-1\right)\right]$
$=\dfrac{2}{3}\left[2\ln(x-4)-\ln\left((x+1)(x-1)\right)\right]$
$=\dfrac{2}{3}\left[2\ln(x-4)-\ln(x+1)-\ln(x-1)\right]$
$=\dfrac{4}{3}\ln(x-4)-\dfrac{2}{3}\ln(x+1)-\dfrac{2}{3}\ln(x-1)$

55. $\ln\dfrac{5x\sqrt{1+3x}}{(x-4)^3}$
$=\ln\left(5x\sqrt{1+3x}\right)-\ln(x-4)^3$
$=\ln 5+\ln x+\ln\sqrt{1+3x}-3\ln(x-4)$
$=\ln 5+\ln x+\ln(1+3x)^{1/2}-3\ln(x-4)$
$=\ln 5+\ln x+\dfrac{1}{2}\ln(1+3x)-3\ln(x-4)$

56. $\ln\left[\dfrac{5x^2\,\sqrt[3]{1-x}}{4(x+1)^2}\right]$
$=\ln\left(5x^2\,\sqrt[3]{1-x}\right)-\ln\left(4(x+1)^2\right)$
$=\ln 5+\ln x^2+\ln(1-x)^{1/3}-\left[\ln 4+\ln(x+1)^2\right]$
$=\ln 5+2\ln x+\dfrac{1}{3}\ln(1-x)-\ln 4-2\ln(x+1)$

57. $3\log_5 u+4\log_5 v=\log_5 u^3+\log_5 v^4$
$=\log_5\left(u^3v^4\right)$

58. $2\log_3 u-\log_3 v=\log_3 u^2-\log_3 v$
$=\log_3\left(\dfrac{u^2}{v}\right)$

59. $\log_3 \sqrt{x} - \log_3 x^3 = \log_3 \left(\frac{\sqrt{x}}{x^3}\right)$
$= \log_3 \left(\frac{x^{1/2}}{x^3}\right)$
$= \log_3 x^{-5/2}$
$= \log_3 \left(\frac{1}{x^{5/2}}\right)$

60. $\log_2 \left(\frac{1}{x}\right) + \log_2 \left(\frac{1}{x^2}\right) = \log_2 \left(\frac{1}{x} \cdot \frac{1}{x^2}\right)$
$= \log_2 \left(\frac{1}{x^3}\right)$

61. $\log_4 \left(x^2 - 1\right) - 5\log_4 (x+1)$
$= \log_4 \left(x^2 - 1\right) - \log_4 (x+1)^5$
$= \log_4 \left[\frac{x^2-1}{(x+1)^5}\right]$
$= \log_4 \left[\frac{(x+1)(x-1)}{(x+1)^5}\right]$
$= \log_4 \left[\frac{x-1}{(x+1)^4}\right]$

62. $\log\left(x^2 + 3x + 2\right) - 2\log_2 (x+1)$
$= \log\left(x^2 + 3x + 2\right) - \log_2 (x+1)^2$
$= \log\left(\frac{x^2+3x+2}{(x+1)^2}\right)$
$= \log\left(\frac{(x+2)(x+1)}{(x+1)^2}\right)$
$= \log\left(\frac{x+2}{x+1}\right)$

63. $\ln\left(\frac{x}{x-1}\right) + \ln\left(\frac{x+1}{x}\right) - \ln\left(x^2 - 1\right)$
$= \ln\left[\frac{x}{x-1} \cdot \frac{x+1}{x}\right] - \ln\left(x^2-1\right)$
$= \ln\left[\frac{x+1}{x-1} \div \left(x^2-1\right)\right]$
$= \ln\left[\frac{x+1}{(x-1)\left(x^2-1\right)}\right]$
$= \ln\left[\frac{x+1}{(x-1)(x-1)(x+1)}\right]$
$= \ln\left(\frac{1}{(x-1)^2}\right)$
$= \ln(x-1)^{-2}$
$= -2\ln(x-1)$

64. $\log\left(\frac{x^2+2x-3}{x^2-4}\right) - \log\left(\frac{x^2+7x+6}{x+2}\right)$
$= \log\left[\frac{\left(\frac{x^2+2x-3}{x^2-4}\right)}{\left(\frac{x^2+7x+6}{x+2}\right)}\right]$
$= \log\left[\frac{(x+3)(x-1)}{(x-2)(x+2)} \cdot \frac{x+2}{(x+6)(x+1)}\right]$
$= \log\left[\frac{(x+3)(x-1)}{(x-2)(x+6)(x+1)}\right]$

65. $8\log_2 \sqrt{3x-2} - \log_2 \left(\frac{4}{x}\right) + \log_2 4$
$= \log_2 \left(\sqrt{3x-2}\right)^8 - \left(\log_2 4 - \log_2 x\right) + \log_2 4$
$= \log_2 (3x-2)^4 - \log_2 4 + \log_2 x + \log_2 4$
$= \log_2 (3x-2)^4 + \log_2 x$
$= \log_2 \left[x(3x-2)^4\right]$

66. $21\log_3 \sqrt[3]{x} + \log_3 \left(9x^2\right) - \log_3 9$
$= \log_3 \left(x^{1/3}\right)^{21} + \log_3 (9) + \log_3 \left(x^2\right) - \log_3 9$
$= \log_3 x^7 + \log_3 x^2$
$= \log_3 \left(x^7 \cdot x^2\right)$
$= \log_3 \left(x^9\right)$

67. $2\log_a\left(5x^3\right)-\frac{1}{2}\log_a(2x+3)$
$=\log_a\left(5x^3\right)^2-\log_a(2x+3)^{1/2}$
$=\log_a\left(25x^6\right)-\log_a\sqrt{2x+3}$
$=\log_a\left[\frac{25x^6}{\sqrt{2x+3}}\right]$

68. $\frac{1}{3}\log\left(x^3+1\right)+\frac{1}{2}\log\left(x^2+1\right)$
$=\log\left(x^3+1\right)^{1/3}+\log\left(x^2+1\right)^{1/2}$
$=\log\left[\sqrt[3]{x^3+1}\cdot\sqrt{x^2+1}\right]$

69. $2\log_2(x+1)-\log_2(x+3)-\log_2(x-1)$
$=\log_2(x+1)^2-\log_2(x+3)-\log_2(x-1)$
$=\log_2\frac{(x+1)^2}{(x+3)}-\log_2(x-1)$
$=\log_2\left[\frac{(x+1)^2}{(x+3)(x-1)}\right]$

70. $3\log_5(3x+1)-2\log_5(2x-1)-\log_5 x$
$=\log_5(3x+1)^3-\log_5(2x-1)^2-\log_5 x$
$=\log_5\frac{(3x+1)^3}{(2x-1)^2}-\log_5 x$
$=\log_5\left[\frac{(3x+1)^3}{x(2x-1)^2}\right]$

71. $\log_3 21=\frac{\log 21}{\log 3}\approx 2.771$

72. $\log_5 18=\frac{\log 18}{\log 5}\approx 1.796$

73. $\log_{1/3} 71=\frac{\log 71}{\log(1/3)}=\frac{\log 71}{-\log 3}\approx -3.880$

74. $\log_{1/2} 15=\frac{\log 15}{\log(1/2)}=\frac{\log 15}{-\log 2}\approx -3.907$

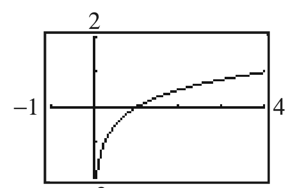
75. $\log_{\sqrt{2}} 7=\frac{\log 7}{\log\sqrt{2}}\approx 5.615$

76. $\log_{\sqrt{5}} 8=\frac{\log 8}{\log\sqrt{5}}\approx 2.584$

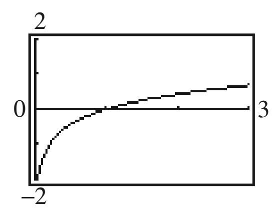
77. $\log_\pi e=\frac{\ln e}{\ln\pi}\approx 0.874$

78. $\log_\pi\sqrt{2}=\frac{\ln\sqrt{2}}{\ln\pi}\approx 0.303$

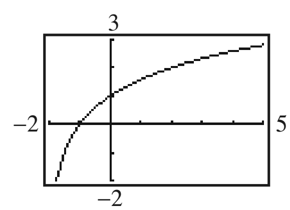
79. $y=\log_4 x=\frac{\ln x}{\ln 4}$ or $y=\frac{\log x}{\log 4}$



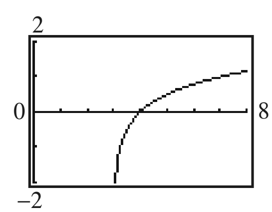
80. $y=\log_5 x=\frac{\ln x}{\ln 5}$ or $y=\frac{\log x}{\log 5}$



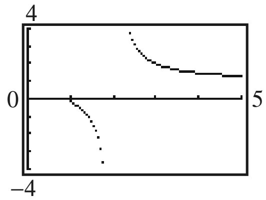
81. $y=\log_2(x+2)=\frac{\ln(x+2)}{\ln 2}$ or $y=\frac{\log(x+2)}{\log 2}$



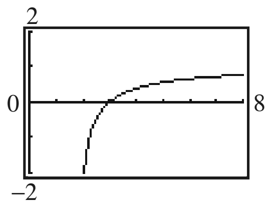
82. $y=\log_4(x-3)=\frac{\ln(x-3)}{\ln 4}$ or $y=\frac{\log(x-3)}{\log 4}$



83. $y = \log_{x-1}(x+1) = \dfrac{\ln(x+1)}{\ln(x-1)}$ or $y = \dfrac{\log(x+1)}{\log(x-1)}$



84. $y = \log_{x+2}(x-2) = \dfrac{\ln(x-2)}{\ln(x+2)}$ or $y = \dfrac{\log(x-2)}{\log(x+2)}$



85. $f(x) = \ln x$; $g(x) = e^x$; $h(x) = x^2$

a. $(f \circ g)(x) = f(g(x)) = \ln(e^x) = x$
Domain: $\{x \mid x \text{ is any real number}\}$ or $(-\infty, \infty)$

b. $(g \circ f)(x) = g(f(x)) = e^{\ln x} = x$
Domain: $\{x \mid x > 0\}$ or $(0, \infty)$
(Note: the restriction on the domain is due to the domain of $\ln x$)

c. $(f \circ g)(5) = 5$ [from part (a)]

d. $(f \circ h)(x) = f(h(x)) = \ln(x^2)$
Domain: $\{x \mid x \neq 0\}$ or $(-\infty, 0) \cup (0, \infty)$

e. $(f \circ h)(e) = \ln(e^2) = 2\ln e = 2 \cdot 1 = 2$

86. $f(x) = \log_2 x$; $g(x) = 2^x$; $h(x) = 4x$

a. $(f \circ g)(x) = f(g(x)) = \log_2(2^x) = x$
Domain: $\{x \mid x \text{ is any real number}\}$ or $(-\infty, \infty)$

b. $(g \circ f)(x) = g(f(x)) = 2^{\log_2 x} = x$
Domain: $\{x \mid x > 0\}$ or $(0, \infty)$
(Note: the restriction on the domain is due to the domain of $\log_2 x$)

c. $(f \circ g)(3) = 3$ [from part (a)]

d. $(f \circ h)(x) = f(h(x)) = \log_2(4x)$
or
$= \log_2 4 + \log_2 x = 2 + \log_2 x$
Domain: $\{x \mid x > 0\}$ or $(0, \infty)$

e. $(f \circ h)(8) = \log_2(4 \cdot 8) = \log_2 32 = 5$
or
$= 2 + \log_2 8 = 2 + 3 = 5$

87. $\ln y = \ln x + \ln C$
$\ln y = \ln(xC)$
$y = Cx$

88. $\ln y = \ln(x + C)$
$y = x + C$

89. $\ln y = \ln x + \ln(x+1) + \ln C$
$\ln y = \ln(x(x+1)C)$
$y = Cx(x+1)$

90. $\ln y = 2\ln x - \ln(x+1) + \ln C$
$\ln y = \ln\left(\dfrac{x^2 C}{x+1}\right)$
$y = \dfrac{Cx^2}{x+1}$

91. $\ln y = 3x + \ln C$
$\ln y = \ln e^{3x} + \ln C$
$\ln y = \ln(Ce^{3x})$
$y = Ce^{3x}$

92. $\ln y = -2x + \ln C$
$\ln y = \ln e^{-2x} + \ln C$
$\ln y = \ln(Ce^{-2x})$
$y = Ce^{-2x}$

93. $\ln(y-3) = -4x + \ln C$
$\ln(y-3) = \ln e^{-4x} + \ln C$
$\ln(y-3) = \ln(Ce^{-4x})$
$y - 3 = Ce^{-4x}$
$y = Ce^{-4x} + 3$

94. $\ln(y+4)=5x+\ln C$

$\ln(y+4)=\ln e^{5x}+\ln C$

$\ln(y+4)=\ln\left(Ce^{5x}\right)$

$y+4=Ce^{5x}$

$y=Ce^{5x}-4$

95. $3\ln y=\frac{1}{2}\ln(2x+1)-\frac{1}{3}\ln(x+4)+\ln C$

$\ln y^3=\ln(2x+1)^{1/2}-\ln(x+4)^{1/3}+\ln C$

$\ln y^3=\ln\left[\frac{C(2x+1)^{1/2}}{(x+4)^{1/3}}\right]$

$y^3=\frac{C(2x+1)^{1/2}}{(x+4)^{1/3}}$

$y=\left[\frac{C(2x+1)^{1/2}}{(x+4)^{1/3}}\right]^{1/3}$

$y=\frac{\sqrt[3]{C}\,(2x+1)^{1/6}}{(x+4)^{1/9}}$

96. $2\ln y=-\frac{1}{2}\ln x+\frac{1}{3}\ln\left(x^2+1\right)+\ln C$

$\ln y^2=-\ln x^{1/2}+\ln\left(x^2+1\right)^{1/3}+\ln C$

$\ln y^2=\ln\left[\frac{C\left(x^2+1\right)^{1/3}}{x^{1/2}}\right]$

$y^2=\frac{C\left(x^2+1\right)^{1/3}}{x^{1/2}}$

$y=\left[\frac{C\left(x^2+1\right)^{1/3}}{x^{1/2}}\right]^{1/2}$

$y=\frac{\sqrt{C}\left(x^2+1\right)^{1/6}}{x^{1/4}}$

97. $\log_2 3\cdot\log_3 4\cdot\log_4 5\cdot\log_5 6\cdot\log_6 7\cdot\log_7 8$

$=\frac{\log 3}{\log 2}\cdot\frac{\log 4}{\log 3}\cdot\frac{\log 5}{\log 4}\cdot\frac{\log 6}{\log 5}\cdot\frac{\log 7}{\log 6}\cdot\frac{\log 8}{\log 7}$

$=\frac{\log 8}{\log 2}=\frac{\log 2^3}{\log 2}$

$=\frac{3\log 2}{\log 2}$

$=3$

98. $\log_2 4\cdot\log_4 6\cdot\log_6 8=\frac{\log 4}{\log 2}\cdot\frac{\log 6}{\log 4}\cdot\frac{\log 8}{\log 6}$

$=\frac{\log 8}{\log 2}$

$=\frac{\log 2^3}{\log 2}$

$=\frac{3\log 2}{\log 2}$

$=3$

99. $\log_2 3\cdot\log_3 4\cdots\log_n(n+1)\cdot\log_{n+1}2$

$=\frac{\log 3}{\log 2}\cdot\frac{\log 4}{\log 3}\cdots\frac{\log(n+1)}{\log n}\cdot\frac{\log 2}{\log(n+1)}$

$=\frac{\log 2}{\log 2}$

$=1$

100. $\log_2 2\cdot\log_2 4\cdot\ldots\cdot\log_2 2^n$

$=\log_2 2\cdot\log_2 2^2\cdots\log_2 2^n$

$=1\cdot 2\cdot 3\cdots n$

$=n!$

101. $\log_a\left(x+\sqrt{x^2-1}\right)+\log_a\left(x-\sqrt{x^2-1}\right)$:

$=\log_a\left[\left(x+\sqrt{x^2-1}\right)\left(x-\sqrt{x^2-1}\right)\right]$

$=\log_a\left[x^2-\left(x^2-1\right)\right]$

$=\log_a\left[x^2-x^2+1\right]$

$=\log_a 1$

$=0$

102. $\log_a\left(\sqrt{x}+\sqrt{x-1}\right)+\log_a\left(\sqrt{x}-\sqrt{x-1}\right)$

$=\log_a\left[\left(\sqrt{x}+\sqrt{x-1}\right)\left(\sqrt{x}-\sqrt{x-1}\right)\right]$

$=\log_a\left[x-(x-1)\right]$

$=\log_a\left[x-x+1\right]$

$=\log_a 1$

$=0$

103. $2x+\ln\left(1+e^{-2x}\right)=\ln e^{2x}+\ln\left(1+e^{-2x}\right)$
$=\ln\left(e^{2x}\left(1+e^{-2x}\right)\right)$
$=\ln\left(e^{2x}+e^{0}\right)$
$=\ln\left(e^{2x}+1\right)$

104. $$\frac{f(x+h)-f(x)}{h}=\frac{\log_a(x+h)-\log_a x}{h}=\frac{\log_a\left(\frac{x+h}{x}\right)}{h}=\frac{1}{h}\cdot\log_a\left(1+\frac{h}{x}\right)=\log_a\left(1+\frac{h}{x}\right)^{\frac{1}{h}},\quad h\neq 0$$

105. $f(x)=\log_a x$ means that $x=a^{f(x)}$.
Now, raising both sides to the -1 power, we obtain $x^{-1}=\left(a^{f(x)}\right)^{-1}=\left(a^{-1}\right)^{f(x)}=\left(\frac{1}{a}\right)^{f(x)}$.
$x^{-1}=\left(\frac{1}{a}\right)^{f(x)}$ means that $\log_{1/a}x^{-1}=f(x)$.
Thus, $\log_{1/a}x^{-1}=f(x)$
$-\log_{1/a}x=f(x)$
$-f(x)=\log_{1/a}x$

106. $f(AB)=\log_a(AB)$
$=\log_a A+\log_a B$
$=f(A)+f(B)$

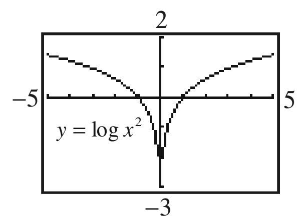
107. $f(x)=\log_a x$
$f\left(\frac{1}{x}\right)=\log_a\left(\frac{1}{x}\right)$
$=\log_a 1-\log_a x$
$=-\log_a x$
$=-f(x)$

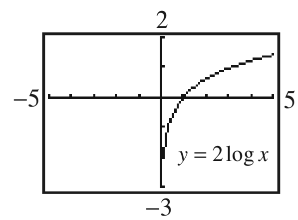
108. $f\left(x\right)=\log_a x$
$f\left(x^{\alpha}\right)=\log_a x^{\alpha}=\alpha\log_a x=\alpha f(x)$

109. If $A=\log_a M$ and $B=\log_a N$, then $a^A=M$ and $a^B=N$.
$$\log_a\left(\frac{M}{N}\right)=\log_a\left(\frac{a^A}{a^B}\right)=\log_a a^{A-B}=A-B=\log_a M-\log_a N$$

110. $\log_a\left(\frac{1}{N}\right)=\log_a N^{-1}$
$=-1\cdot\log_a N$
$=-\log_a N,\quad a\neq 1$

111. $Y_1=\log x^2$ $\qquad$ $Y_2=2\log x$





The domain of $Y_1=\log_a x^2$ is $\{x \mid x\neq 0\}$. The domain of $Y_2=2\log_a x$ is $\{x \mid x>0\}$. These two domains are different because the logarithm property $\log_a x^n=n\cdot\log_a x$ holds only when $\log_a x$ exists.

112. Answers may vary. One possibility follows:
Let $a=2$, $x=8$, and $r=3$. Then $\left(\log_a x\right)^r=\left(\log_2 8\right)^3=3^3=27$. But $r\log_a x=3\log_2 8=3\cdot 3=9$. Thus, $\left(\log_2 8\right)^3\neq 3\log_2 8$ and, in general, $\left(\log_a x\right)^r\neq r\log_a x$.

113. Answers may vary. One possibility follows:
Let $x=4$ and $y=4$. Then $\log_2(x+y)=\log_2(4+4)=\log_2 8=3$. But $\log_2 x+\log_2 y=\log_2 4+\log_2 4=2+2=4$. Thus, $\log_2(4+4)\neq\log_2 4+\log_2 4$ and, in general, $\log_2(x+y)\neq\log_2 x+\log_2 y$.

114. No. $\log_3(-5)$ does not exist. The argument of a logarithm must be nonnegative.

Section 5.6

1. $x^2 - 7x - 30 = 0$

$(x+3)(x-10) = 0$

$x+3=0$ or $x-10=0$

$x=-3$ or $x=10$

The solution set is $\{-3, 10\}$.

2. Let $u = x+3$. Then

$(x+3)^2 - 4(x+3) + 3 = 0$

$u^2 - 4u + 3 = 0$

$(u-1)(u-3) = 0$

$u-1=0$ or $u-3=0$

$u=1$ or $u=3$

Back substituting $u = x+3$, we obtain

$x+3=1$ or $x+3=3$

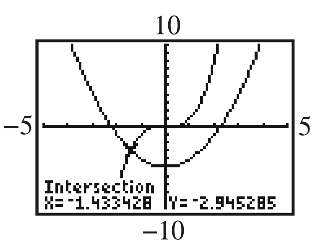
$x=-2$ or $x=0$

The solution set is $\{-2, 0\}$.

3. $x^3 = x^2 - 5$

Using INTERSECT to solve:

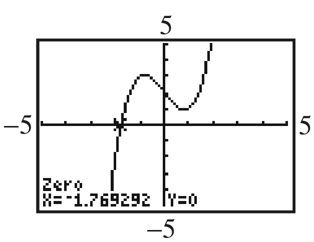
$y_1 = x^3$; $y_2 = x^2 - 5$



Thus, $x \approx -1.43$, so the solution set is $\{-1.43\}$.

4. $x^3 - 2x + 2 = 0$

Using ZERO to solve: $y_1 = x^3 - 2x + 2$



Thus, $x \approx -1.77$, so the solution set is $\{-1.77\}$.

5. $\log_4 x = 2$

$x = 4^2$

$x = 16$

The solution set is $\{16\}$.

6. $\log\ (x+6) = 1$

$x+6 = 10^1$

$x+6 = 10$

$x = 4$

The solution set is $\{4\}$.

7. $\log_2(5x) = 4$

$5x = 2^4$

$5x = 16$

$x = \frac{16}{5}$

The solution set is $\left\{\frac{16}{5}\right\}$.

8. $\log_3(3x-1) = 2$

$3x-1 = 3^2$

$3x-1 = 9$

$3x = 10$

$x = \frac{10}{3}$

The solution set is $\left\{\frac{10}{3}\right\}$.

9. $\log_4(x+2) = \log_4 8$

$x+2 = 8$

$x = 6$

The solution set is $\{6\}$.

10. $\log_5(2x+3) = \log_5 3$

$2x+3 = 3$

$2x = 0$

$x = 0$

The solution set is $\{0\}$.

11. $\frac{1}{2}\log_3 x = 2\log_3 2$

$\log_3 x^{1/2} = \log_3 2^2$

$x^{1/2} = 4$

$x = 16$

The solution set is $\{16\}$.

12. $-2\log_4 x = \log_4 9$
$$\log_4 x^{-2} = \log_4 9$$
$$x^{-2} = 9$$
$$\frac{1}{x^2} = 9$$
$$x^2 = \frac{1}{9}$$
$$x = \pm\frac{1}{3}$$
Since $\log_4\left(-\frac{1}{3}\right)$ is undefined, the solution set is $\left\{\frac{1}{3}\right\}$.

13. $3\log_2 x = -\log_2 27$
$$\log_2 x^3 = \log_2 27^{-1}$$
$$x^3 = 27^{-1}$$
$$x^3 = \frac{1}{27}$$
$$x = \frac{1}{3}$$
The solution set is $\left\{\frac{1}{3}\right\}$.

14. $2\log_5 x = 3\log_5 4$
$$\log_5 x^2 = \log_5 4^3$$
$$x^2 = 64$$
$$x = \pm 8$$
Since $\log_5(-8)$ is undefined, the solution set is $\{8\}$.

15. $3\log_2(x-1) + \log_2 4 = 5$
$$\log_2(x-1)^3 + \log_2 4 = 5$$
$$\log_2\left(4(x-1)^3\right) = 5$$
$$4(x-1)^3 = 2^5$$
$$(x-1)^3 = \frac{32}{4}$$
$$(x-1)^3 = 8$$
$$x-1 = 2$$
$$x = 3$$
The solution set is $\{3\}$.

16. $2\log_3(x+4) - \log_3 9 = 2$
$$\log_3(x+4)^2 - \log_3 3^2 = 2$$
$$\log_3(x+4)^2 - 2 = 2$$
$$\log_3(x+4)^2 = 4$$
$$(x+4)^2 = 3^4$$
$$(x+4)^2 = 81$$
$$x+4 = \pm 9$$
$$x = -4 \pm 9$$
$$x = 5 \text{ or } x = -13$$
Since $\log_3(-13+4) = \log_3(-9)$ is undefined, the solution set is $\{5\}$.

17. $\log x + \log(x+15) = 2$
$$\log\left(x(x+15)\right) = 2$$
$$x(x+15) = 10^2$$
$$x^2 + 15x - 100 = 0$$
$$(x+20)(x-5) = 0$$
$$x = -20 \text{ or } x = 5$$
Since $\log(-20)$ is undefined, the solution set is $\{5\}$.

18. $\log x + \log(x-21) = 2$
$$\log\left(x(x-21)\right) = 2$$
$$x(x-21) = 10^2$$
$$x^2 - 21x - 100 = 0$$
$$(x+4)(x-25) = 0$$
$$x = -4 \text{ or } x = 25$$
Since $\log(-4)$ is undefined, the solution set is $\{25\}$.

19. $\log(2x+1) = 1 + \log(x-2)$
$$\log(2x+1) - \log(x-2) = 1$$
$$\log\left(\frac{2x+1}{x-2}\right) = 1$$
$$\frac{2x+1}{x-2} = 10^1$$
$$2x+1 = 10(x-2)$$
$$2x+1 = 10x-20$$
$$-8x = -21$$
$$x = \frac{-21}{-8} = \frac{21}{8}$$
The solution set is $\left\{\frac{21}{8}\right\}$.

20. $\log(2x) - \log(x-3) = 1$

$$\log\left(\frac{2x}{x-3}\right) = 1$$

$$\frac{2x}{x-3} = 10^1$$

$$2x = 10(x-3)$$

$$2x = 10x - 30$$

$$-8x = -30$$

$$x = \frac{-30}{-8} = \frac{15}{4}$$

The solution set is $\left\{\frac{15}{4}\right\}$.

21. $\log_2(x+7) + \log_2(x+8) = 1$

$$\log_2[(x+7)(x+8)] = 1$$

$$(x+7)(x+8) = 2^1$$

$$x^2 + 8x + 7x + 56 = 2$$

$$x^2 + 15x + 54 = 0$$

$$(x+9)(x+6) = 0$$

$$x = -9 \text{ or } x = -6$$

Since $\log_2(-9+7) = \log_2(-2)$ is undefined, the solution set is $\{-6\}$.

22. $\log_6(x+4) + \log_6(x+3) = 1$

$$\log_6[(x+4)(x+3)] = 1$$

$$(x+4)(x+3) = 6^1$$

$$x^2 + 3x + 4x + 12 = 6$$

$$x^2 + 7x + 6 = 0$$

$$(x+6)(x+1) = 0$$

$$x = -6 \text{ or } x = -1$$

Since $\log_6(-6+4) = \log_6(-2)$ is undefined, the solution set is $\{-1\}$.

23. $\log_8(x+6) = 1 - \log_8(x+4)$

$$\log_8(x+6) + \log_8(x+4) = 1$$

$$\log_8[(x+6)(x+4)] = 1$$

$$(x+6)(x+4) = 8^1$$

$$x^2 + 4x + 6x + 24 = 8$$

$$x^2 + 10x + 16 = 0$$

$$(x+8)(x+2) = 0$$

$$x = -8 \text{ or } x = -2$$

Since $\log_8(-8+6) = \log_8(-2)$ is undefined, the solution set is $\{-2\}$.

24. $\log_5(x+3) = 1 - \log_5(x-1)$

$$\log_5(x+3) + \log_5(x-1) = 1$$

$$\log_5[(x+3)(x-1)] = 1$$

$$(x+3)(x-1) = 5^1$$

$$x^2 - x + 3x - 3 = 5$$

$$x^2 + 2x - 8 = 0$$

$$(x+4)(x-2) = 0$$

$$x = -4 \text{ or } x = 2$$

Since $\log_5(-4+3) = \log_5(-1)$ is undefined, the solution set is $\{2\}$.

25. $\ln x + \ln(x+2) = 4$

$$\ln(x(x+2)) = 4$$

$$x(x+2) = e^4$$

$$x^2 + 2x - e^4 = 0$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-e^4)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{4 + 4e^4}}{2}$$

$$= \frac{-2 \pm 2\sqrt{1+e^4}}{2}$$

$$= -1 \pm \sqrt{1+e^4}$$

$$x = -1 - \sqrt{1+e^4} \text{ or } x = -1 + \sqrt{1+e^4}$$

$$\approx -8.456 \qquad \approx 6.456$$

Since $\ln(-8.456)$ is undefined, the solution set is $\left\{-1 + \sqrt{1+e^4}\right\} \approx \{6.456\}$.

26. $\ln(x+1) - \ln x = 2$

$$\ln\left(\frac{x+1}{x}\right) = 2$$

$$\frac{x+1}{x} = e^2$$

$$x + 1 = e^2 x$$

$$e^2 x - x = 1$$

$$x\left(e^2 - 1\right) = 1$$

$$x = \frac{1}{e^2 - 1} \approx 0.157$$

The solution set is $\left\{\frac{1}{e^2-1}\right\} \approx \{0.157\}$.

27. $\log_3(x+1)+\log_3(x+4)=2$

$$\log_3\left[(x+1)(x+4)\right]=2$$
$$(x+1)(x+4)=3^2$$
$$x^2+4x+x+4=9$$
$$x^2+5x-5=0$$
$$x=\frac{-5\pm\sqrt{5^2-4(1)(-5)}}{2(1)}$$
$$=\frac{-5\pm\sqrt{45}}{2}$$
$$=\frac{-5\pm3\sqrt{5}}{2}$$
$$x=\frac{-5-3\sqrt{5}}{2} \text{ or } x=\frac{-5+3\sqrt{5}}{2}$$
$$\approx -5.854 \qquad \approx 0.854$$

Since $\log_3(-8.854+1)=\log_3(-7.854)$ is undefined, the solution set is $\left\{\frac{-5+3\sqrt{5}}{2}\right\}\approx\{0.854\}$.

28. $\log_2(x+1)+\log_2(x+7)=3$

$$\log_2\left[(x+1)(x+7)\right]=3$$
$$(x+1)(x+7)=2^3$$
$$x^2+7x+x+7=8$$
$$x^2+8x-1=0$$
$$x=\frac{-8\pm\sqrt{8^2-4(1)(-1)}}{2(1)}$$
$$=\frac{-8\pm\sqrt{68}}{2}$$
$$=\frac{-8\pm2\sqrt{17}}{2}$$
$$=-4\pm\sqrt{17}$$
$$x=-4-\sqrt{17} \text{ or } x=-4+\sqrt{17}$$
$$\approx -8.123 \qquad \approx 0.123$$

Since $\log_2(-8.123+1)=\log_2(-7.123)$ is undefined, the solution set is $\left\{-4+\sqrt{17}\right\}\approx\{0.123\}$.

29. $\log_{1/3}(x^2+x)-\log_{1/3}(x^2-x)=-1$

$$\log_{1/3}\left(\frac{x^2+x}{x^2-x}\right)=-1$$
$$\frac{x^2+x}{x^2-x}=\left(\frac{1}{3}\right)^{-1}$$
$$\frac{x^2+x}{x^2-x}=3$$
$$x^2+x=3\left(x^2-x\right)$$
$$x^2+x=3x^2-3x$$
$$-2x^2+4x=0$$
$$-2x(x-2)=0$$
$$-2x=0 \text{ or } x-2=0$$
$$x=0 \text{ or } x=2$$

Since each of the original logarithms are not defined for $x=0$, but are defined for $x=2$, the solution set is $\{2\}$.

30. $\log_4(x^2-9)-\log_4(x+3)=3$

$$\log_4\left(\frac{x^2-9}{x+3}\right)=3$$
$$\frac{(x-3)(x+3)}{x+3}=4^3$$
$$x-3=64$$
$$x=67$$

Since each of the original logarithms is defined for $x=67$, the solution set is $\{67\}$.

31. $\log_a(x-1)-\log_a(x+6)=\log_a(x-2)-\log_a(x+3)$

$$\log_a\left(\frac{x-1}{x+6}\right)=\log_a\left(\frac{x-2}{x+3}\right)$$
$$\frac{x-1}{x+6}=\frac{x-2}{x+3}$$
$$(x-1)(x+3)=(x-2)(x+6)$$
$$x^2+2x-3=x^2+4x-12$$
$$2x-3=4x-12$$
$$9=2x$$
$$x=\frac{9}{2}$$

Since each of the original logarithms is defined for $x=\frac{9}{2}$, the solution set is $\left\{\frac{9}{2}\right\}$.

32. $\log_a x + \log_a(x-2) = \log_a(x+4)$

$$\log_a\left(x(x-2)\right) = \log_a(x+4)$$
$$x(x-2) = x+4$$
$$x^2 - 2x = x+4$$
$$x^2 - 3x - 4 = 0$$
$$(x-4)(x+1) = 0$$
$$x = 4 \text{ or } x = -1$$

Since $\log_a(-1)$ is undefined, the solution set is $\{4\}$.

33. $2^{x-5} = 8$

$$2^{x-5} = 2^3$$
$$x - 5 = 3$$
$$x = 8$$

The solution set is $\{8\}$.

34. $5^{-x} = 25$

$$5^{-x} = 5^2$$
$$-x = 2$$
$$x = -2$$

The solution set is $\{-2\}$.

35. $2^x = 10$

$$x = \log_2 10 = \frac{\ln 10}{\ln 2} \approx 3.322$$

The solution set is $\{\log_2 10\} = \left\{\frac{\ln 10}{\ln 2}\right\} \approx \{3.322\}$.

36. $3^x = 14$

$$x = \log_3 14 = \frac{\ln 14}{\ln 3} \approx 2.402$$

The solution set is $\{\log_3 14\} = \left\{\frac{\ln 14}{\ln 3}\right\} \approx \{2.402\}$.

37. $8^{-x} = 1.2$

$$-x = \log_8 1.2$$
$$x = -\log_8 1.2 = -\frac{\log(1.2)}{\log 8} \approx -0.088$$

The solution set is $\{-\log_8 1.2\} = \left\{\frac{\log(1.2)}{-\log 8}\right\} \approx \{-0.088\}$.

38. $2^{-x} = 1.5$

$$-x = \log_2 1.5$$
$$x = -\log_2 1.5 = -\frac{\log 1.5}{\log 2} \approx -0.585$$

The solution set is $\{-\log_2 1.5\} = \left\{-\frac{\ln 1.5}{\ln 2}\right\} \approx \{-0.585\}$.

39. $5\left(2^{3x}\right) = 8$

$$2^{3x} = \frac{8}{5}$$
$$3x = \log_2\left(\frac{8}{5}\right)$$
$$x = \frac{1}{3}\log_2\left(\frac{8}{5}\right) = \frac{\ln(8/5)}{3\ln 2} \approx 0.226$$

The solution set is $\left\{\frac{1}{3}\log_2\left(\frac{8}{5}\right)\right\} = \left\{\frac{\ln(8/5)}{3\ln 2}\right\} \approx \{0.226\}$.

40. $0.3\left(4^{0.2x}\right) = 0.2$

$$4^{0.2x} = \frac{2}{3}$$
$$0.2x = \log_4\left(\frac{2}{3}\right)$$
$$x = \frac{\log_4(2/3)}{0.2} = \frac{\ln(2/3)}{0.2\ln 4} \approx -1.462$$

The solution set is $\left\{\frac{\log_4(2/3)}{0.2}\right\} = \left\{\frac{\ln(2/3)}{0.2\ln 4}\right\} \approx \{-1.462\}$.

41. $3^{1-2x} = 4^x$

$$\ln\left(3^{1-2x}\right) = \ln\left(4^x\right)$$
$$(1-2x)\ln 3 = x\ln 4$$
$$\ln 3 - 2x\ln 3 = x\ln 4$$
$$\ln 3 = 2x\ln 3 + x\ln 4$$
$$\ln 3 = x(2\ln 3 + \ln 4)$$
$$x = \frac{\ln 3}{2\ln 3 + \ln 4} \approx 0.307$$

The solution set is $\left\{\frac{\ln 3}{2\ln 3 + \ln 4}\right\} \approx \{0.307\}$.

42.
$$
\begin{aligned}
2^{x+1} &= 5^{1-2x}\\
\ln\left(2^{x+1}\right) &= \ln\left(5^{1-2x}\right)\\
(x+1)\ln 2 &= (1-2x)\ln 5\\
x\ln 2+\ln 2 &= \ln 5-2x\ln 5\\
x\ln 2+2x\ln 5 &= \ln 5-\ln 2\\
x(\ln 2+2\ln 5) &= \ln 5-\ln 2\\
x\left(\ln 2+\ln 25\right) &= \ln\tfrac{5}{2}\\
x\left(\ln 50\right) &= \ln\tfrac{5}{2}\\
x &= \frac{\ln\frac{5}{2}}{\ln 50}\approx 0.234
\end{aligned}
$$

The solution set is $\left\{\frac{\ln\frac{5}{2}}{\ln 50}\right\}\approx\{0.234\}$.

43.
$$
\begin{aligned}
\left(\frac{3}{5}\right)^x &= 7^{1-x}\\
\ln\left(\frac{3}{5}\right)^x &= \ln\left(7^{1-x}\right)\\
x\ln(3/5) &= (1-x)\ln 7\\
x\ln(3/5) &= \ln 7-x\ln 7\\
x\ln(3/5)+x\ln 7 &= \ln 7\\
x\left(\ln(3/5)+\ln 7\right) &= \ln 7\\
x &= \frac{\ln 7}{\ln(3/5)+\ln 7}\approx 1.356
\end{aligned}
$$

The solution set is $\left\{\frac{\ln 7}{\ln(3/5)+\ln 7}\right\}\approx\{1.356\}$.

44.
$$
\begin{aligned}
\left(\frac{4}{3}\right)^{1-x} &= 5^x\\
\ln\left(\frac{4}{3}\right)^{1-x} &= \ln\left(5^x\right)\\
(1-x)\ln(4/3) &= x\ln 5\\
\ln(4/3)-x\ln(4/3) &= x\ln 5\\
\ln(4/3) &= x\ln 5+x\ln(4/3)\\
\ln(4/3) &= x\left(\ln 5+\ln(4/3)\right)\\
\ln(4/3) &= x\left(\ln\tfrac{20}{3}\right)\\
x &= \frac{\ln\frac{4}{3}}{\ln\frac{20}{3}}\approx 0.152
\end{aligned}
$$

The solution set is $\left\{\frac{\ln\frac{4}{3}}{\ln\frac{20}{3}}\right\}\approx\{0.152\}$.

45.
$$
\begin{aligned}
1.2^x &= (0.5)^{-x}\\
\ln 1.2^x &= \ln(0.5)^{-x}\\
x\ln(1.2) &= -x\ln(0.5)\\
x\ln(1.2)+x\ln(0.5) &= 0\\
x\left(\ln(1.2)+\ln(0.5)\right) &= 0\\
x &= 0
\end{aligned}
$$

The solution set is $\{0\}$.

46.
$$
\begin{aligned}
0.3^{1+x} &= 1.7^{2x-1}\\
\ln\left(0.3^{1+x}\right) &= \ln\left(1.7^{2x-1}\right)\\
(1+x)\ln(0.3) &= (2x-1)\ln(1.7)\\
\ln(0.3)+x\ln(0.3) &= 2x\ln(1.7)-\ln(1.7)\\
x\ln(0.3)-2x\ln(1.7) &= -\ln(1.7)-\ln(0.3)\\
x\left(\ln(0.3)-2\ln(1.7)\right) &= -\ln(1.7)-\ln(0.3)\\
x &= \frac{-\ln(1.7)-\ln(0.3)}{\ln(0.3)-2\ln(1.7)}=\frac{\ln 0.51}{\ln(2.89/3)}\approx -0.297
\end{aligned}
$$

The solution set is $\left\{\frac{\ln 0.51}{\ln(2.89/3)}\right\}\approx\{-0.297\}$.

47.
$$
\begin{aligned}
\pi^{1-x} &= e^x\\
\ln\pi^{1-x} &= \ln e^x\\
(1-x)\ln\pi &= x\\
\ln\pi-x\ln\pi &= x\\
\ln\pi &= x+x\ln\pi\\
\ln\pi &= x(1+\ln\pi)\\
x &= \frac{\ln\pi}{1+\ln\pi}\approx 0.534
\end{aligned}
$$

The solution set is $\left\{\frac{\ln\pi}{1+\ln\pi}\right\}\approx\{0.534\}$.

48.
$$
\begin{aligned}
e^{x+3} &= \pi^x\\
\ln e^{x+3} &= \ln\pi^x\\
x+3 &= x\ln\pi\\
3 &= x\ln\pi-x\\
3 &= x(\ln\pi-1)\\
x &= \frac{3}{\ln\pi-1}\approx 20.728
\end{aligned}
$$

The solution set is $\left\{\frac{3}{\ln\pi-1}\right\}\approx\{20.728\}$.

49. $2^{2x}+2^x-12=0$

$\left(2^x\right)^2+2^x-12=0$

$\left(2^x-3\right)\left(2^x+4\right)=0$

$2^x-3=0$ or $2^x+4=0$

$2^x=3$ or $2^x=-4$

$\ln\left(2^x\right)=\ln 3$ No solution

$x\ln 2=\ln 3$

$x=\frac{\ln 3}{\ln 2}\approx 1.585$

The solution set is $\left\{\frac{\ln 3}{\ln 2}\right\}\approx\{1.585\}$.

50. $3^{2x}+3^x-2=0$

$\left(3^x\right)^2+3^x-2=0$

$\left(3^x-1\right)\left(3^x+2\right)=0$

$3^x-1=0$ or $3^x+2=0$

$3^x=1$ or $3^x=-2$

$x=0$ No solution

The solution set is $\{0\}$.

51. $3^{2x}+3^{x+1}-4=0$

$\left(3^x\right)^2+3\cdot 3^x-4=0$

$\left(3^x-1\right)\left(3^x+4\right)=0$

$3^x-1=0$ or $3^x+4=0$

$3^x=1$ or $3^x=-4$

$x=0$ No solution

The solution set is $\{0\}$.

52. $2^{2x}+2^{x+2}-12=0$

$\left(2^x\right)^2+2^2\cdot 2^x-12=0$

$\left(2^x-2\right)\left(2^x+6\right)=0$

$2^x-2=0$ or $2^x+6=0$

$2^x=2$ or $2^x=-6$

$x=1$ No solution

The solution set is $\{1\}$.

53. $16^x+4^{x+1}-3=0$

$\left(4^2\right)^x+4\cdot 4^x-3=0$

$\left(4^x\right)^2+4\cdot 4^x-3=0$

Let $u=4^x$.

$u^2+4u-3=0$

$a=1, b=4, c=-3$

$u=\frac{-4\pm\sqrt{4^2-4(1)(-3)}}{2(1)}=\frac{-4\pm\sqrt{28}}{2}$

$=\frac{-4\pm 2\sqrt{7}}{2}=-2\pm\sqrt{7}$

Therefore, we get

~~$4^x=-2-\sqrt{7}$~~ or $4^x=-2+\sqrt{7}$

$x=\log_4\left(-2+\sqrt{7}\right)$

(we ignore the first solution since 4^x is never negative)

The solution set is $\left\{\log_4\left(-2+\sqrt{7}\right)\right\}\approx\{-0.315\}$.

54. $9^x-3^{x+1}+1=0$

$\left(3^2\right)^x-3\cdot 3^x+1=0$

$\left(3^x\right)^2-3\cdot 3^x+1=0$

Let $u=3^x$.

$u^2-3u+1=0$

$a=1, b=-3, c=1$

$u=\frac{3\pm\sqrt{(-3)^2-4(1)(1)}}{2(1)}=\frac{3\pm\sqrt{5}}{2}$

Therefore, we get

$3^x=\frac{3\pm\sqrt{5}}{2}$

$x=\log_3\left(\frac{3\pm\sqrt{5}}{2}\right)$

The solution set is

$\left\{\log_3\left(\frac{3-\sqrt{5}}{2}\right),\log_3\left(\frac{3+\sqrt{5}}{2}\right)\right\}$

$\approx\{-0.876, 0.876\}$.

55. $25^x - 8\cdot 5^x = -16$

$\left(5^2\right)^x - 8\cdot 5^x = -16$

$\left(5^x\right)^2 - 8\cdot 5^x = -16$

Let $u = 5^x$.

$u^2 - 8u = -16$

$u^2 - 8u + 16 = 0$

$(u-4)^2 = 0$

$u = 4$

Therefore, we get

$5^x = 4$

$x = \log_5 4$

The solution set is $\{\log_5 4\} \approx \{0.861\}$.

56. $36^x - 6\cdot 6^x = -9$

$\left(6^2\right)^x - 6\cdot 6^x + 9 = 0$

$\left(6^x\right)^2 - 6\cdot 6^x + 9 = 0$

$\left(6^x - 3\right)^2 = 0$

$6^x = 3$

$x = \log_6 3$

The solution set is $\{\log_6 3\} \approx \{0.613\}$.

57. $3\cdot 4^x + 4\cdot 2^x + 8 = 0$

$3\cdot\left(2^2\right)^x + 4\cdot 2^x + 8 = 0$

$3\cdot\left(2^x\right)^2 + 4\cdot 2^x + 8 = 0$

Let $u = 2^x$.

$3u^2 + 4u + 8 = 0$

$a = 3, b = 4, c = 8$

$$u = \frac{-4 \pm \sqrt{4^2 - 4(3)(8)}}{2(3)}$$

$$= \frac{-4 \pm \sqrt{-80}}{6} = \text{not real}$$

The equation has no real solution.

58. $2\cdot 49^x + 11\cdot 7^x + 5 = 0$

$2\cdot\left(7^2\right)^x + 11\cdot 7^x + 5 = 0$

$2\cdot\left(7^x\right)^2 + 11\cdot 7^x + 5 = 0$

Let $u = 7^x$.

$2u^2 + 11u + 5 = 0$

$(2u+1)(u+5) = 0$

$2u + 1 = 0$ or $u + 5 = 0$

$2u = -1$ $\quad u = -5$

$u = -\frac{1}{2}$

Therefore, we get

$7^x = -\frac{1}{2}$ or $7^x = -5$

Since $7^x > 0$ for all x, the equation has no real solution.

59. $4^x - 10\cdot 4^{-x} = 3$

Multiply both sides of the equation by 4^x.

$\left(4^x\right)^2 - 10\cdot 4^{-x}\cdot 4^x = 3\cdot 4^x$

$\left(4^x\right)^2 - 10 = 3\cdot 4^x$

$\left(4^x\right)^2 - 3\cdot 4^x - 10 = 0$

$\left(4^x - 5\right)\left(4^x + 2\right) = 0$

$4^x - 5 = 0$ or $4^x + 2 = 0$

$4^x = 5$ $\quad$ ~~$4^x = -2$~~

$x = \log_4 5$

The solution set is $\{\log_4 5\} \approx \{1.161\}$.

60. $3^x - 14\cdot 3^{-x} = 5$

Multiply both sides of the equation by 3^x.

$\left(3^x\right)^2 - 14\cdot 3^{-x}\cdot 3^x = 5\cdot 3^x$

$\left(3^x\right)^2 - 14 = 5\cdot 3^x$

$\left(3^x\right)^2 - 5\cdot 3^x - 14 = 0$

$\left(3^x - 7\right)\left(3^x + 2\right) = 0$

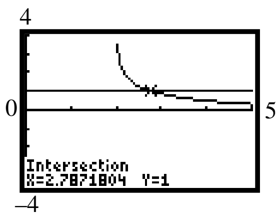
$3^x - 7 = 0$ or $3^x + 2 = 0$

$3^x = 7$ $\quad$ ~~$3^x = -2$~~

$x = \log_3 7$

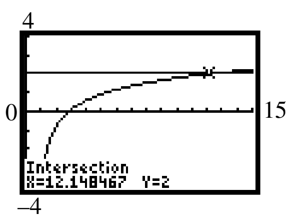
The solution set is $\{\log_3 7\} \approx \{1.771\}$.

61. $\log_5(x+1) - \log_4(x-2) = 1$
Using INTERSECT to solve:
$y_1 = \ln(x+1)/\ln(5) - \ln(x-2)/\ln(4)$
$y_2 = 1$



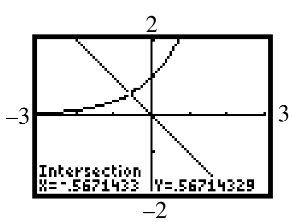
Thus, $x \approx 2.79$, so the solution set is $\{2.79\}$.

62. $\log_2(x-1) - \log_6(x+2) = 2$
Using INTERSECT to solve:
$y_1 = \ln(x-1)/\ln(2) - \ln(x+2)/\ln(6)$
$y_2 = 2$



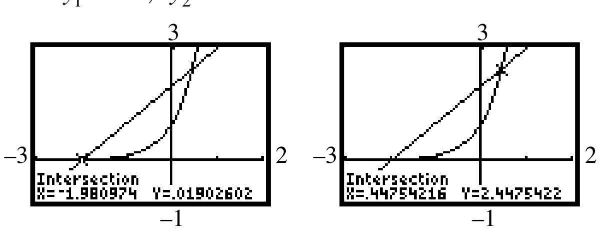
Thus, $x \approx 12.15$, so the solution set is $\{12.15\}$.

63. $e^x = -x$
Using INTERSECT to solve: $y_1 = e^x$; $y_2 = -x$



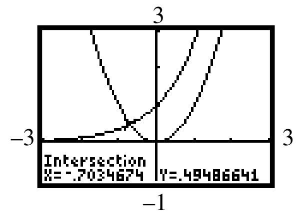
Thus, $x \approx -0.57$, so the solution set is $\{-0.57\}$.

64. $e^{2x} = x+2$
Using INTERSECT to solve:
$y_1 = e^{2x}$; $y_2 = x+2$



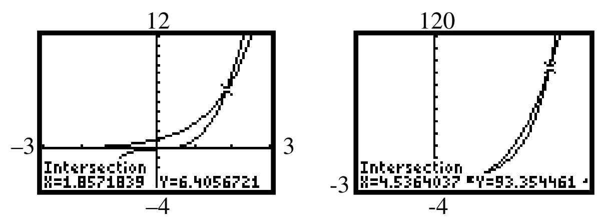
Thus, $x \approx -1.98$ or $x \approx 0.45$, so the solution set is $\{-1.98, 0.45\}$.

65. $e^x = x^2$
Using INTERSECT to solve:
$y_1 = e^x$; $y_2 = x^2$



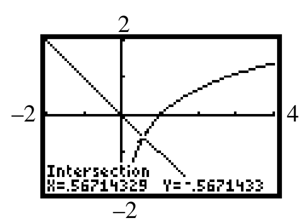
Thus, $x \approx -0.70$, so the solution set is $\{-0.70\}$.

66. $e^x = x^3$
Using INTERSECT to solve:
$y_1 = e^x$; $y_2 = x^3$



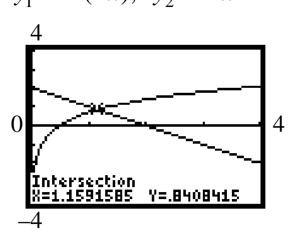
Thus, $x \approx 1.86$ or $x \approx 4.54$, so the solution set is $\{1.86, 4.54\}$.

67. $\ln x = -x$
Using INTERSECT to solve:
$y_1 = \ln x$; $y_2 = -x$



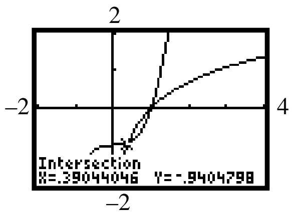
Thus, $x \approx 0.57$, so the solution set is $\{0.57\}$.

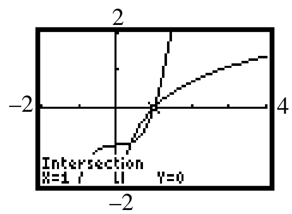
68. $\ln(2x) = -x+2$
Using INTERSECT to solve:
$y_1 = \ln(2x)$; $y_2 = -x+2$



Thus, $x \approx 1.16$, so the solution set is $\{1.16\}$.

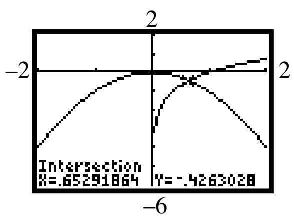
69. $\ln x = x^3 - 1$
Using INTERSECT to solve:
$y_1 = \ln x;\ \ y_2 = x^3 - 1$





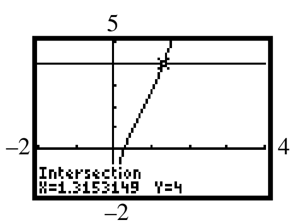
Thus, $x \approx 0.39$ or $x = 1$, so the solution set is $\{0.39,\ 1\}$.

70. $\ln x = -x^2$
Using INTERSECT to solve:
$y_1 = \ln x;\ \ y_2 = -x^2$



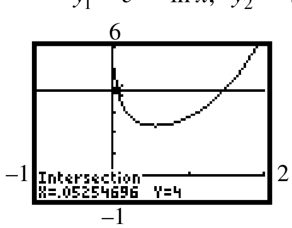
Thus, $x \approx 0.65$, so the solution set is $\{0.65\}$.

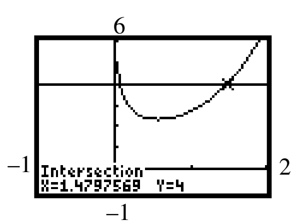
71. $e^x + \ln x = 4$
Using INTERSECT to solve:
$y_1 = e^x + \ln x;\ \ y_2 = 4$



Thus, $x \approx 1.32$, so the solution set is $\{1.32\}$.

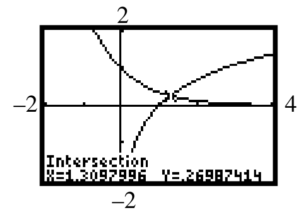
72. $e^x - \ln x = 4$
Using INTERSECT to solve:
$y_1 = e^x - \ln x;\ \ y_2 = 4$





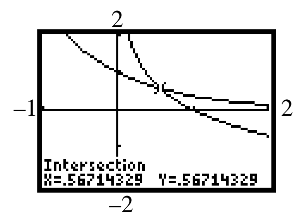
Thus, $x \approx 0.05$ or $x \approx 1.48$, so the solution set is $\{0.05,\ 1.48\}$.

73. $e^{-x} = \ln x$
Using INTERSECT to solve:
$y_1 = e^{-x};\ \ y_2 = \ln x$



Thus, $x \approx 1.31$, so the solution set is $\{1.31\}$.

74. $e^{-x} = -\ln x$
Using INTERSECT to solve:
$y_1 = e^{-x};\ \ y_2 = -\ln x$



Thus, $x \approx 0.57$, so the solution set is $\{0.57\}$

75.
$$\log_2(x+1) - \log_4 x = 1$$
$$\log_2(x+1) - \frac{\log_2 x}{\log_2 4} = 1$$
$$\log_2(x+1) - \frac{\log_2 x}{2} = 1$$
$$2\log_2(x+1) - \log_2 x = 2$$
$$\log_2(x+1)^2 - \log_2 x = 2$$
$$\log_2\left(\frac{(x+1)^2}{x}\right) = 2$$
$$\frac{(x+1)^2}{x} = 2^2$$
$$x^2 + 2x + 1 = 4x$$
$$x^2 - 2x + 1 = 0$$
$$(x-1)^2 = 0$$
$$x - 1 = 0$$
$$x = 1$$
Since each of the original logarithms is defined for $x = 1$, the solution set is $\{1\}$.

76. $\log_2(3x+2)-\log_4 x=3$

$$\log_2(3x+2)-\frac{\log_2 x}{\log_2 4}=3$$
$$\log_2(3x+2)-\frac{\log_2 x}{2}=3$$
$$2\log_2(3x+2)-\log_2 x=6$$
$$\log_2(3x+2)^2-\log_2 x=6$$
$$\log_2\left(\frac{(3x+2)^2}{x}\right)=6$$
$$\frac{(3x+2)^2}{x}=2^6$$
$$9x^2+12x+4=64x$$
$$9x^2-52x+4=0$$
$$x=\frac{52\pm\sqrt{(-52)^2-4(9)(4)}}{2(9)}$$
$$=\frac{52\pm\sqrt{2560}}{18}$$
$$=\frac{52\pm16\sqrt{10}}{18}$$
$$=\frac{26\pm8\sqrt{10}}{9}$$
$$\approx 5.700 \text{ or } 0.078$$

Since each of the original logarithms is defined for $x=0.078$ and $x=5.700$, the solution set is $\left\{\frac{26-8\sqrt{10}}{9},\ \frac{26+8\sqrt{10}}{9}\right\}\approx\{0.078,\ 5.700\}$.

77. $\log_{16} x+\log_4 x+\log_2 x=7$

$$\frac{\log_2 x}{\log_2 16}+\frac{\log_2 x}{\log_2 4}+\log_2 x=7$$
$$\frac{\log_2 x}{4}+\frac{\log_2 x}{2}+\log_2 x=7$$
$$\log_2 x+2\log_2 x+4\log_2 x=28$$
$$7\log_2 x=28$$
$$\log_2 x=4$$
$$x=2^4=16$$

Since each of the original logarithms is defined for $x=16$, the solution set is $\{16\}$.

78. $\log_9 x+3\log_3 x=14$

$$\frac{\log_3 x}{\log_3 9}+3\log_3 x=14$$
$$\frac{\log_3 x}{2}+3\log_3 x=14$$
$$\frac{7}{2}\log_3 x=14$$
$$\log_3 x=4$$
$$x=3^4=81$$

Since each of the original logarithms is defined for $x=81$, the solution set is $\{81\}$.

79. $\left(\sqrt[3]{2}\right)^{2-x}=2^{x^2}$

$$\left(2^{1/3}\right)^{2-x}=2^{x^2}$$
$$2^{\frac{1}{3}(2-x)}=2^{x^2}$$
$$\frac{1}{3}(2-x)=x^2$$
$$2-x=3x^2$$
$$3x^2+x-2=0$$
$$(3x-2)(x+1)=0$$
$$x=\frac{2}{3} \text{ or } x=-1$$

The solution set is $\left\{-1,\frac{2}{3}\right\}$.

80. $\log_2 x^{\log_2 x}=4$

$$\log_2 x\cdot\log_2 x=4$$
$$(\log_2 x)^2=4$$
$$\log_2 x=-2 \text{ or } \log_2 x=2$$
$$x=2^{-2} \text{ or } x=2^2$$
$$x=\frac{1}{4} \text{ or } x=4$$

Since each of the original logarithms is defined for both $x=\frac{1}{4}$ and $x=4$, the solution set is $\left\{\frac{1}{4},4\right\}$.

81.
$$\begin{aligned}\frac{e^x+e^{-x}}{2}&=1\\ e^x+e^{-x}&=2\\ e^x(e^x+e^{-x})&=2e^x\\ e^{2x}+1&=2e^x\\ (e^x)^2-2e^x+1&=0\\ (e^x-1)^2&=0\\ e^x-1&=0\\ e^x&=1\\ x&=0\end{aligned}$$
The solution set is $\{0\}$.

82.
$$\begin{aligned}\frac{e^x+e^{-x}}{2}&=3\\ e^x+e^{-x}&=6\\ e^x(e^x+e^{-x})&=6e^x\\ e^{2x}+1&=6e^x\\ (e^x)^2-6e^x+1&=0\\ e^x&=\frac{6\pm\sqrt{(-6)^2-4(1)(1)}}{2(1)}\\ &=\frac{6\pm\sqrt{32}}{2}=\frac{6\pm4\sqrt{2}}{2}=3\pm2\sqrt{2}\end{aligned}$$
$x=\ln\left(3-2\sqrt{2}\right)$ or $x=\ln\left(3+2\sqrt{2}\right)$

$x\approx-1.763$ or $x\approx1.763$

The solution set is $\left\{\ln\left(3-2\sqrt{2}\right),\ \ln\left(3+2\sqrt{2}\right)\right\}\approx\{-1.763,1.763\}$.

83.
$$\begin{aligned}\frac{e^x-e^{-x}}{2}&=2\\ e^x-e^{-x}&=4\\ e^x(e^x-e^{-x})&=4e^x\\ e^{2x}-1&=4e^x\\ (e^x)^2-4e^x-1&=0\\ e^x&=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-1)}}{2(1)}\\ &=\frac{4\pm\sqrt{20}}{2}=\frac{4\pm2\sqrt{5}}{2}=2\pm\sqrt{5}\end{aligned}$$
$x=\ln\left(2-\sqrt{5}\right)$ or $x=\ln\left(2+\sqrt{5}\right)$

$x\approx\ln(-0.236)$ or $x\approx1.444$

Since $\ln(-0.236)$ is undefined, the solution set is $\left\{\ln\left(2+\sqrt{5}\right)\right\}\approx\{1.444\}$.

84.
$$\begin{aligned}\frac{e^x-e^{-x}}{2}&=-2\\ e^x-e^{-x}&=-4\\ e^x\left(e^x-e^{-x}\right)&=-4e^x\\ e^{2x}-1&=-4e^x\\ (e^x)^2+4e^x-1&=0\\ e^x&=\frac{-4\pm\sqrt{4^2-4(1)(-1)}}{2(1)}\\ &=\frac{-4\pm\sqrt{20}}{2}\\ &=\frac{-4\pm2\sqrt{5}}{2}=-2\pm\sqrt{5}\end{aligned}$$
$x=\ln\left(-2-\sqrt{5}\right)$ or $x=\ln\left(-2+\sqrt{5}\right)$

$x\approx\ln(-4.236)$ or $x\approx-1.444$

Since $\ln(-4.236)$ is undefined, the solution set is $\left\{\ln\left(-2+\sqrt{5}\right)\right\}\approx\{-1.444\}$.

85.
$$\begin{aligned}\log_5 x+\log_3 x&=1\\ \frac{\ln x}{\ln 5}+\frac{\ln x}{\ln 3}&=1\\ (\ln x)\left(\frac{1}{\ln 5}+\frac{1}{\ln 3}\right)&=1\\ \ln x&=\frac{1}{\frac{1}{\ln 5}+\frac{1}{\ln 3}}\\ \ln x&=\frac{(\ln 5)(\ln 3)}{\ln 3+\ln 5}\\ \ln x&=\frac{(\ln 5)(\ln 3)}{\ln 15}\\ x&=e^{\left(\frac{(\ln 5)(\ln 3)}{\ln 15}\right)}\approx1.921\end{aligned}$$
The solution set is $\left\{e^{\left(\frac{(\ln 5)(\ln 3)}{\ln 15}\right)}\right\}\approx\{1.921\}$.

86.
$$\log_2 x + \log_6 x = 3$$
$$\frac{\ln x}{\ln 2} + \frac{\ln x}{\ln 6} = 3$$
$$(\ln x)\left(\frac{1}{\ln 2} + \frac{1}{\ln 6}\right) = 3$$
$$\ln x = \frac{3}{\frac{1}{\ln 2} + \frac{1}{\ln 6}}$$
$$\ln x = \frac{3(\ln 2)(\ln 6)}{\ln 6 + \ln 2}$$
$$\ln x = \frac{3(\ln 2)(\ln 6)}{\ln 12}$$
$$x = e^{\left(\frac{3(\ln 2)(\ln 6)}{\ln 12}\right)} \approx 4.479$$

The solution set is $\left\{e^{\left(\frac{3(\ln 2)(\ln 6)}{\ln 12}\right)}\right\} \approx \{4.479\}$.

87. a.
$$f(x) = 3$$
$$\log_2(x+3) = 3$$
$$x + 3 = 2^3$$
$$x + 3 = 8$$
$$x = 5$$

The solution set is $\{5\}$. The point $(5, 3)$ is on the graph of *f*.

b.
$$g(x) = 4$$
$$\log_2(3x+1) = 4$$
$$3x + 1 = 2^4$$
$$3x + 1 = 16$$
$$3x = 15$$
$$x = 5$$

The solution set is $\{5\}$. The point $(5, 4)$ is on the graph of *g*.

c.
$$f(x) = g(x)$$
$$\log_2(x+3) = \log_2(3x+1)$$
$$x + 3 = 3x + 1$$
$$2 = 2x$$
$$1 = x$$

The solution set is $\{1\}$, so the graphs intersect when $x = 1$. That is, at the point $(1, 2)$.

d.
$$(f+g)(x) = 7$$
$$\log_2(x+3) + \log_2(3x+1) = 7$$
$$\log_2[(x+3)(3x+1)] = 7$$
$$(x+3)(3x+1) = 2^7$$
$$3x^2 + 10x + 3 = 128$$
$$3x^2 + 10x - 125 = 0$$
$$(3x+25)(x-5) = 0$$

~~$3x + 25 = 0$, $3x = -25$, $x = -\frac{25}{3}$~~ or $x - 5 = 0$, $x = 5$

The solution set is $\{5\}$.

e.
$$(f-g)(x) = 2$$
$$\log_2(x+3) - \log_2(3x+1) = 2$$
$$\log_2 \frac{x+3}{3x+1} = 2$$
$$\frac{x+3}{3x+1} = 2^2$$
$$x + 3 = 4(3x+1)$$
$$x + 3 = 12x + 4$$
$$-1 = 11x$$
$$-\frac{1}{11} = x$$

The solution set is $\left\{-\frac{1}{11}\right\}$.

88. a.
$$f(x) = 2$$
$$\log_3(x+5) = 2$$
$$x + 5 = 3^2$$
$$x + 5 = 9$$
$$x = 4$$

The solution set is $\{4\}$. The point $(4, 2)$ is on the graph of *f*.

b.
$$g(x) = 3$$
$$\log_3(x-1) = 3$$
$$x - 1 = 3^3$$
$$x - 1 = 27$$
$$x = 28$$

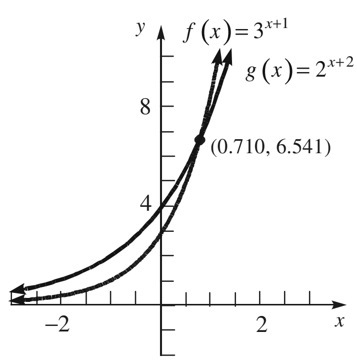
The solution set is $\{28\}$. The point $(28, 3)$ is on the graph of *g*.

c.
$$f(x) = g(x)$$
$$\log_3(x+5) = \log_3(x-1)$$
$$x+5 = x-1$$
$$5 = -1 \quad \text{False}$$
This is a contradiction, so the equation has no solution. The graphs do not intersect.

d.
$$(f+g)(x) = 3$$
$$\log_3(x+5) + \log_3(x-1) = 3$$
$$\log_3[(x+5)(x-1)] = 3$$
$$(x+5)(x-1) = 3^3$$
$$x^2+4x-5 = 27$$
$$x^2+4x-32 = 0$$
$$(x+8)(x-4) = 0$$
$$\cancel{x+8=0} \quad \text{or} \quad x-4=0$$
$$\cancel{x=-8} \qquad x=4$$
The solution set is $\{4\}$.

e.
$$(f-g)(x) = 2$$
$$\log_3(x+5) - \log_3(x-1) = 2$$
$$\log_3 \frac{x+5}{x-1} = 2$$
$$\frac{x+5}{x-1} = 3^2$$
$$x+5 = 9(x-1)$$
$$x+5 = 9x-9$$
$$14 = 8x$$
$$\frac{7}{4} = x$$
The solution set is $\left\{\frac{7}{4}\right\}$.

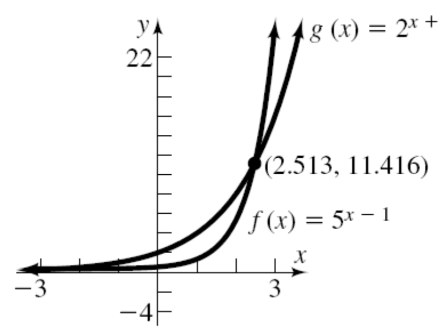
89. a.



b.
$$f(x) = g(x)$$
$$3^{x+1} = 2^{x+2}$$
$$\ln(3^{x+1}) = \ln(2^{x+2})$$
$$(x+1)\ln 3 = (x+2)\ln 2$$
$$x\ln 3 + \ln 3 = x\ln 2 + 2\ln 2$$
$$x\ln 3 - x\ln 2 = 2\ln 2 - \ln 3$$
$$x(\ln 3 - \ln 2) = 2\ln 2 - \ln 3$$
$$x = \frac{2\ln 2 - \ln 3}{\ln 3 - \ln 2} \approx 0.710$$
$$f\left(\frac{2\ln 2 - \ln 3}{\ln 3 - \ln 2}\right) \approx 6.541$$
The intersection point is roughly $(0.710, 6.541)$.

c. Based on the graph, $f(x) > g(x)$ for $x > 0.710$. The solution set is $\{x \mid x > 0.710\}$ or $(0.710, \infty)$.

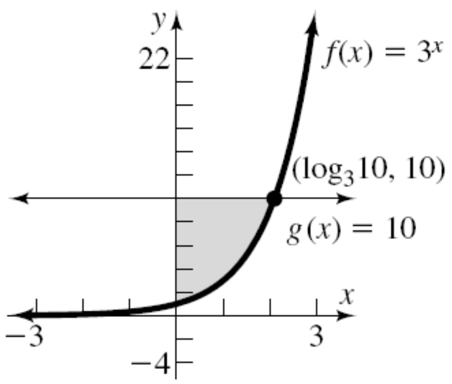
90. a.



b.
$$f(x) = g(x)$$
$$5^{x-1} = 2^{x+1}$$
$$\ln(5^{x-1}) = \ln(2^{x+1})$$
$$(x-1)\ln 5 = (x+1)\ln 2$$
$$x\ln 5 - \ln 5 = x\ln 2 + \ln 2$$
$$x\ln 5 - x\ln 2 = \ln 5 + \ln 2$$
$$x(\ln 5 - \ln 2) = \ln 5 + \ln 2$$
$$x = \frac{\ln 5 + \ln 2}{\ln 5 - \ln 2} \approx 2.513$$
$$f\left(\frac{\ln 5 + \ln 2}{\ln 5 - \ln 2}\right) \approx 11.416$$
The intersection point is roughly $(2.513, 11.416)$.

c. Based on the graph, $f(x) > g(x)$ for $x > 2.513$. The solution set is $\{x \mid x > 2.513\}$ or $(2.513, \infty)$.

91. a., b.

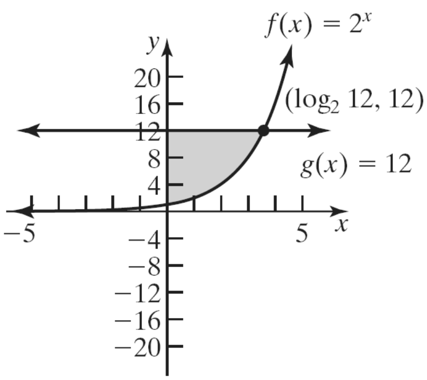


c. $f(x) = g(x)$

$$3^x = 10$$
$$x = \log_3 10$$

The intersection point is $(\log_3 10, 10)$.

92. a., b.

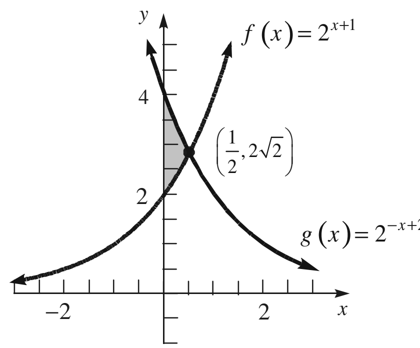


c. $f(x) = g(x)$

$$2^x = 12$$
$$x = \log_2 12$$

The intersection point is $(\log_2 12, 12)$.

93. a., b.

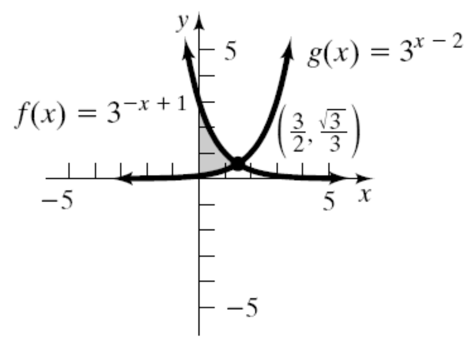


c. $f(x) = g(x)$

$$2^{x+1} = 2^{-x+2}$$
$$x + 1 = -x + 2$$
$$2x = 1$$
$$x = \frac{1}{2}$$
$$f\left(\frac{1}{2}\right) = 2^{1/2+1} = 2^{3/2} = 2\sqrt{2}$$

The intersection point is $\left(\frac{1}{2}, 2\sqrt{2}\right)$.

94. a., b.



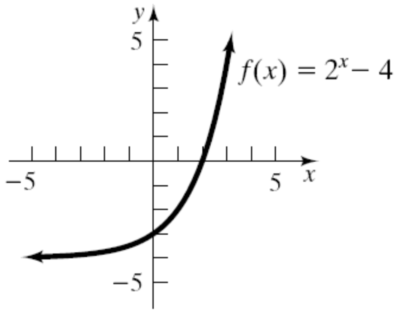
c. $f(x) = g(x)$

$$3^{-x+1} = 3^{x-2}$$
$$-x + 1 = x - 2$$
$$-2x = -3$$
$$x = \frac{3}{2}$$
$$f\left(\frac{3}{2}\right) = 3^{-3/2+1} = 3^{-1/2} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

The intersection point is $\left(\frac{3}{2}, \frac{\sqrt{3}}{3}\right)$.

95. a. $f(x) = 2^x - 4$

Using the graph of $y = 2^x$, shift the graph down 4 units.



b. $f(x) = 0$

$2^x - 4 = 0$

$2^x = 4$

$2^x = 2^2$

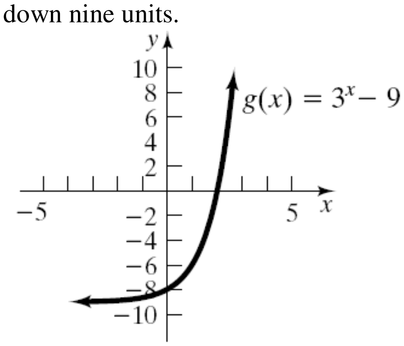
$x = 2$

The zero of *f* is $x = 2$.

c. Based on the graph, $f(x) < 0$ when $x < 2$. The solution set is $\{x \mid x < 2\}$ or $(-\infty, 2)$.

96. a. $g(x) = 3^x - 9$

Using the graph of $y = 3^x$, shift the graph down nine units.



b. $g(x) = 0$

$3^x - 9 = 0$

$3^x = 9$

$3^x = 3^2$

$x = 2$

The zero of *g* is $x = 2$.

c. Based on the graph, $g(x) > 0$ when $x > 2$. The solution set is $\{x \mid x > 2\}$ or $(2, \infty)$.

97. a.

$$309(1.009)^{t-2010} = 419$$
$$(1.009)^{t-2010} = \frac{419}{309}$$
$$\ln(1.009)^{t-2010} = \ln\left(\frac{419}{309}\right)$$
$$(t-2010)\ln(1.009) = \ln\left(\frac{419}{309}\right)$$
$$t - 2010 = \frac{\ln(419/309)}{\ln(1.009)}$$
$$t = \frac{\ln(419/309)}{\ln(1.009)} + 2010$$
$$\approx 2044$$

According to the model, the population of the U.S. will reach 419 million people around the beginning of the year 2044.

b.

$$309(1.009)^{t-2010} = 488$$
$$(1.009)^{t-2010} = \frac{488}{309}$$
$$\ln(1.009)^{t-2010} = \ln\left(\frac{488}{309}\right)$$
$$(t-2010)\ln(1.009) = \ln\left(\frac{488}{309}\right)$$
$$t - 2010 = \frac{\ln(488/309)}{\ln(1.009)}$$
$$t = \frac{\ln(488/309)}{\ln(1.009)} + 2010$$
$$\approx 2061$$

According to the model, the population of the U.S. will reach 488 million people in the beginning of the year 2061.

98. a.

$$6.91(1.0114)^{t-2011} = 9.6$$
$$(1.0114)^{t-2011} = \frac{9.6}{6.91}$$
$$\ln(1.0114)^{t-2011} = \ln\left(\frac{9.6}{6.91}\right)$$
$$(t-2011)\ln(1.0114) = \ln(9.6/6.91)$$
$$t - 2011 = \frac{\ln(9.6/6.91)}{\ln(1.0114)}$$
$$t = \frac{\ln(9.6/6.91)}{\ln(1.0114)} + 2011$$
$$\approx 2040$$

According to the model, the population of the world will reach 6.6 billion people at the beginning of the year 2040.

b.
$$\begin{aligned} 6.91(1.0114)^{t-2011} &= 12 \\ (1.0114)^{t-2011} &= \frac{12}{6.91} \\ \ln(1.0114)^{t-2011} &= \ln\left(\frac{12}{6.91}\right) \\ (t-2011)\ln(1.0114) &= \ln(12/6.91) \\ t-2011 &= \frac{\ln(12/6.91)}{\ln(1.0114)} \\ t &= \frac{\ln(12/6.91)}{\ln(1.0114)} + 2011 \\ &\approx 2060 \end{aligned}$$

According to the model, the population of the world will reach 12 billion people at the beginning of the year 2060.

99. a.
$$\begin{aligned} 16{,}500(0.82)^t &= 9{,}000 \\ (0.82)^t &= \frac{9{,}000}{16{,}500} \\ \log(0.82)^t &= \log\left(\frac{9{,}000}{16{,}500}\right) \\ t\log(0.82) &= \log\left(\frac{9{,}000}{16{,}500}\right) \\ t &= \frac{\log(9{,}000/16{,}500)}{\log(0.82)} \\ &\approx 3.05 \end{aligned}$$

According to the model, the car will be worth \$9,000 after about 3 years.

b.
$$\begin{aligned} 16{,}500(0.82)^t &= 4{,}000 \\ (0.82)^t &= \frac{4{,}000}{16{,}500} \\ \log(0.82)^t &= \log\left(\frac{4{,}000}{16{,}500}\right) \\ t\log(0.82) &= \log\left(\frac{4{,}000}{16{,}500}\right) \\ t &= \frac{\log(4{,}000/16{,}500)}{\log(0.82)} \\ &\approx 7.14 \end{aligned}$$

According to the model, the car will be worth \$4,000 after about 7.1 years.

c.
$$\begin{aligned} 16{,}500(0.82)^t &= 2{,}000 \\ (0.82)^t &= \frac{2{,}000}{16{,}500} \\ \log(0.82)^t &= \log\left(\frac{2{,}000}{16{,}500}\right) \\ t\log(0.82) &= \log\left(\frac{2{,}000}{16{,}500}\right) \\ t &= \frac{\log(2{,}000/16{,}500)}{\log(0.82)} \\ &\approx 10.63 \end{aligned}$$

According to the model, the car will be worth \$2,000 after about 10.6 years.

100. a.
$$\begin{aligned} 16{,}775(0.905)^t &= 15{,}000 \\ (0.905)^t &= \frac{15{,}000}{16{,}775} \\ \log(0.905)^t &= \log\left(\frac{15{,}000}{16{,}775}\right) \\ t\log(0.905) &= \log\left(\frac{15{,}000}{16{,}775}\right) \\ t &= \frac{\log(15{,}000/16{,}775)}{\log(0.905)} \approx 1.1 \end{aligned}$$

According to the model, the car will be worth \$15,000 after about 1.1 years.

b.
$$\begin{aligned} 16{,}775(0.905)^t &= 8{,}000 \\ (0.905)^t &= \frac{8{,}000}{16{,}775} \\ \log(0.905)^t &= \log\left(\frac{8{,}000}{16{,}775}\right) \\ t\log(0.905) &= \log\left(\frac{8{,}000}{16{,}775}\right) \\ t &= \frac{\log(8{,}000/16{,}775)}{\log(0.905)} \approx 7.4 \end{aligned}$$

According to the model, the car will be worth \$8,000 after about 7.4 years.

c. $16,775(0.905)^t = 4,000$

$$(0.905)^t = \frac{4,000}{16,775}$$

$$\log(0.905)^t = \log\left(\frac{4,000}{16,775}\right)$$

$$t\log(0.905) = \log\left(\frac{4,000}{16,775}\right)$$

$$t = \frac{\log(4,000/16,775)}{\log(0.905)} \approx 14.4$$

According to the model, the car will be worth \$4,000 after about 14.4 years.

101. Solution A: change to exponential expression; square root method; meaning of $\pm$; solve.

Solution B: $\log_a M^r = r\log_a M$; divide by 2; change to exponential expression; solve.

The power rule $\log_a M^r = r\log_a M$ only applies when $M > 0$. In this equation, $M = x-1$. Now, $x = -2$ causes $M = -2-1 = -3$. Thus, if we use the power rule, we lose the valid solution $x = -2$.

Section 5.7

1. $P = \$500,\ r = 0.06,\ t = 6 \text{ months} = 0.5 \text{ year}$
$I = Prt = (500)(0.06)(0.5) = \$15.00$

2. $P = \$5000, t = 9 \text{ months} = 0.75 \text{ year},\ I = \$500$
$500 = 5000r(0.75)$

$$r = \frac{500}{(5000)(0.75)} = \frac{2}{15} = \frac{2}{15}\cdot 100\% = \frac{40}{3}\% = 13\frac{1}{3}\%$$

The per annum interest rate was $13\frac{1}{3}\%$.

3. principal

4. I; Prt; simple interest

5. 4

6. effective rate of interest

7. $P = \$100,\ r = 0.04,\ n = 4,\ t = 2$

$$A = P\left(1+\frac{r}{n}\right)^{nt} = 100\left(1+\frac{0.04}{4}\right)^{(4)(2)} \approx \$108.29$$

8. $P = \$50,\ r = 0.06,\ n = 12,\ t = 3$

$$A = P\left(1+\frac{r}{n}\right)^{nt} = 50\left(1+\frac{0.06}{12}\right)^{(12)(3)} \approx \$59.83$$

9. $P = \$500,\ r = 0.08,\ n = 4,\ t = 2.5$

$$A = P\left(1+\frac{r}{n}\right)^{nt} = 500\left(1+\frac{0.08}{4}\right)^{(4)(2.5)} \approx \$609.50$$

10. $P = \$300,\ r = 0.12,\ n = 12,\ t = 1.5$

$$A = P\left(1+\frac{r}{n}\right)^{nt} = 300\left(1+\frac{0.12}{12}\right)^{(12)(1.5)} \approx \$358.84$$

11. $P = \$600,\ r = 0.05,\ n = 365,\ t = 3$

$$A = P\left(1+\frac{r}{n}\right)^{nt} = 600\left(1+\frac{0.05}{365}\right)^{(365)(3)} \approx \$697.09$$

12. $P = \$700,\ r = 0.06,\ n = 365,\ t = 2$

$$A = P\left(1+\frac{r}{n}\right)^{nt} = 700\left(1+\frac{0.06}{365}\right)^{(365)(2)} \approx \$789.24$$

13. $P = \$1000,\ r = 0.11,\ t = 2$
$A = Pe^{rt} = 1000e^{(0.11)(2)} \approx \$1246.08$

14. $P = \$400,\ r = 0.07,\ t = 3$
$A = Pe^{rt} = 400e^{(0.07)(3)} \approx \$493.47$

15. $A = \$100,\ r = 0.06,\ n = 12,\ t = 2$

$$P = A\left(1+\frac{r}{n}\right)^{-nt} = 100\left(1+\frac{0.06}{12}\right)^{(-12)(2)} \approx \$88.72$$

16. $A = \$75,\ r = 0.08,\ n = 4,\ t = 3$

$$P = A\left(1+\frac{r}{n}\right)^{-nt} = 75\left(1+\frac{0.08}{4}\right)^{(-4)(3)} \approx \$59.14$$

17. $A = \$1000,\ r = 0.06,\ n = 365,\ t = 2.5$

$$P = A\left(1+\frac{r}{n}\right)^{-nt} = 1000\left(1+\frac{0.06}{365}\right)^{(-365)(2.5)} \approx \$860.72$$

18. $A = \$800,\ r = 0.07,\ n = 12,\ t = 3.5$

$$P = A\left(1+\frac{r}{n}\right)^{-nt}$$
$$= 800\left(1+\frac{0.07}{12}\right)^{(-12)(3.5)} \approx \$626.61$$

19. $A = \$600,\ r = 0.04,\ n = 4,\ t = 2$

$$P = A\left(1+\frac{r}{n}\right)^{-nt} = 600\left(1+\frac{0.04}{4}\right)^{(-4)(2)} \approx \$554.09$$

20. $A = \$300,\ r = 0.03,\ n = 365,\ t = 4$

$$P = A\left(1+\frac{r}{n}\right)^{-nt}$$
$$= 300\left(1+\frac{0.03}{365}\right)^{(-365)(4)} \approx \$266.08$$

21. $A = \$80,\ r = 0.09,\ t = 3.25$

$$P = Ae^{-rt} = 80e^{(-0.09)(3.25)} \approx \$59.71$$

22. $A = \$800,\ r = 0.08,\ t = 2.5$

$$P = Ae^{-rt} = 800e^{(-0.08)(2.5)} \approx \$654.98$$

23. Suppose P dollars are invested for 1 year at 5%.

Compounded quarterly yields:

$$A = P\left(1+\frac{0.05}{4}\right)^{(4)(1)} \approx 1.05095P.$$

The interest earned is

$I = 1.05095P - P = 0.05095P$

Thus, $\quad I = Prt$

$$0.05095P = P \cdot r \cdot 1$$
$$0.05095 = r$$

The effective interest rate is 5.095%.

24. Suppose P dollars are invested for 1 year at 6%.

Compounded monthly yields:

$$A = P\left(1+\frac{0.06}{12}\right)^{(12)(1)} \approx 1.06168P.$$

The interest earned is

$I = 1.06168P - P = 0.06168P$

Thus, $\quad I = Prt$

$$0.06168P = P \cdot r \cdot 1$$
$$0.06168 = r$$

The effective interest rate is 6.168%.

25. Suppose P dollars are invested for 1 year at 5%.

Compounded continuously yields:

$A = Pe^{(0.05)(1)} \approx 1.05127P$

The interest earned is

$I = 1.05127P - P = 0.05127P$

Thus, $\quad I = Prt$

$$0.05127P = P \cdot r \cdot 1$$
$$.05127 = r$$

The effective interest rate is 5.127%.

26. Suppose P dollars are invested for 1 year at 6%.

Compounded continuously yields:

$A = Pe^{(0.06)(1)} \approx 1.06184P$

The interest earned is

$I = 1.06184P - P = 0.06184P$

Thus, $\quad I = Prt$

$$0.06184P = P \cdot r \cdot 1$$
$$0.06184 = r$$

The effective interest rate is 6.184%.

27. 6% compounded quarterly:

$$A = 10{,}000\left(1+\frac{0.06}{4}\right)^{(4)(1)} = \$10{,}613.64$$

$6\frac{1}{4}\%$ compounded annually:

$$A = 10{,}000(1+0.0625)^{1} = \$10{,}625$$

$6\frac{1}{4}\%$ compounded annually is the better deal.

28. 9% compounded quarterly:

$$A = 10{,}000\left(1+\frac{0.09}{4}\right)^{(4)(1)} \approx \$10{,}930.83$$

$9\frac{1}{4}\%$ compounded annually:

$$A = 10{,}000(1+0.0925)^{1} = \$10{,}925$$

9% compounded quarterly is the better deal.

29. 9% compounded monthly:

$$A = 10{,}000\left(1+\frac{0.09}{12}\right)^{(12)(1)} = \$10{,}938.07$$

8.8% compounded daily:

$$A = 10{,}000\left(1+\frac{0.088}{365}\right)^{365} = \$10{,}919.77$$

9% compounded monthly is the better deal.

30. 8% compounded semiannually:

$$A = 10{,}000\left(1+\frac{0.08}{2}\right)^{(2)(1)} = \$10{,}816$$

7.9% compounded daily:

$$A = 10{,}000\left(1+\frac{0.079}{365}\right)^{365} = \$10{,}821.95$$

7.9% compounded daily is the better deal.

31. $2P = P\left(1+\frac{r}{1}\right)^{3(1)}$

$2P = P\left(1+r\right)^3$

$2 = (1+r)^3$

$\sqrt[3]{2} = 1+r$

$r = \sqrt[3]{2} - 1 \approx 0.25992$

The required rate is 25.992%.

32. $2P = P\left(1+\frac{r}{1}\right)^{6(1)}$

$2P = P\left(1+r\right)^6$

$2 = (1+r)^6$

$\sqrt[6]{2} = 1+r$

$r = \sqrt[6]{2} - 1 \approx 0.12246$

The required rate is 12.246%.

33. $3P = P\left(1+\frac{r}{1}\right)^{5(1)}$

$3P = P\left(1+r\right)^5$

$3 = (1+r)^5$

$\sqrt[5]{3} = 1+r$

$r = \sqrt[5]{3} - 1 \approx 0.24573$

The required rate is 24.573%.

34. $3P = P\left(1+\frac{r}{1}\right)^{10(1)}$

$3P = P\left(1+r\right)^{10}$

$3 = (1+r)^{10}$

$\sqrt[10]{3} = 1+r$

$r = \sqrt[10]{3} - 1 \approx 0.11612$

The required rate is 11.612%.

35. a. $2P = P\left(1+\frac{0.08}{12}\right)^{12t}$

$2 = \left(1+\frac{0.08}{12}\right)^{12t}$

$\ln 2 = \ln\left(1+\frac{0.08}{12}\right)^{12t}$

$\ln 2 = 12t \ln\left(1+\frac{0.08}{12}\right)$

$t = \dfrac{\ln 2}{12\ln\left(1+\frac{0.08}{12}\right)} \approx 8.69$

It will take about 8.69 years to double.

b. $2P = Pe^{0.08t}$

$2 = e^{0.08t}$

$\ln 2 = 0.08t$

$t = \dfrac{\ln 2}{0.08} \approx 8.66$

It will take about 8.66 years to double.

36. a. $3P = P\left(1+\frac{0.06}{12}\right)^{12t}$

$3 = \left(1.005\right)^{12t}$

$\ln 3 = \ln\left(1.005\right)^{12t}$

$\ln 3 = 12t\ln\left(1.005\right)$

$t = \dfrac{\ln 3}{12\ln\left(1.005\right)} \approx 18.36$

It will take about 18.36 years to triple.

b. $3P = Pe^{0.06t}$

$3 = e^{0.06t}$

$\ln 3 = 0.06t$

$t = \dfrac{\ln 3}{0.06} \approx 18.31$

It will take about 18.31 years to triple.

37. Since the effective interest rate is 7%, we have:

$I = Prt$

$I = P \cdot 0.07 \cdot 1$

$I = 0.07P$

Thus, the amount in the account is

$A = P + 0.07P = 1.07P$

Let x be the required interest rate. Then,

$$1.07P = P\left(1+\frac{r}{4}\right)^{(4)(1)}$$
$$1.07 = \left(1+\frac{r}{4}\right)^{4}$$
$$\sqrt[4]{1.07} = 1+\frac{r}{4}$$
$$\sqrt[4]{1.07} - 1 = \frac{r}{4}$$
$$r = 4\left(\sqrt[4]{1.07}-1\right) \approx 0.06823$$

Thus, an interest rate of 6.823% compounded quarterly has an effective interest rate of 7%.

38. Since the effective interest rate is 6%, we have:

$$I = Prt$$
$$I = P \cdot 0.06 \cdot 1$$
$$I = 0.06P$$

Thus, the amount in the account is

$$A = P + 0.06P = 1.06P$$

Let x be the required interest rate. Then,

$$1.06P = Pe^{(r)(1)}$$
$$1.06 = e^{r}$$
$$r = \ln(1.06) \approx 0.05827$$

Thus, an interest rate of 5.827% compounded continuously has an effective interest rate of 6%.

39. $150 = 100\left(1+\frac{0.08}{12}\right)^{12t}$

$$1.5 \approx (1.006667)^{12t}$$
$$\ln 1.5 \approx 12t \ln(1.006667)$$
$$t \approx \frac{\ln 1.5}{12\ln(1.006667)} \approx 5.09$$

Compounded monthly, it will take about 5.09 years (or 61.02 months).

$$150 = 100e^{0.08t}$$
$$1.5 = e^{0.08t}$$
$$\ln 1.5 = 0.08t$$
$$t = \frac{\ln 1.5}{0.08} \approx 5.07$$

Compounded continuously, it will take about 5.07 years (or 60.82 months).

40. $175 = 100\left(1+\frac{0.10}{12}\right)^{12t}$

$$1.75 \approx (1.008333)^{12t}$$
$$\ln 1.75 \approx 12t \ln(1.008333)$$
$$t \approx \frac{\ln 1.75}{12\ln(1.008333)} \approx 5.62$$

Compounded monthly, it will take about 5.62 years (or 67.43 months).

$$175 = 100e^{0.10t}$$
$$1.75 = e^{0.10t}$$
$$\ln 1.75 = 0.10t$$
$$t = \frac{\ln 1.75}{0.10} \approx 5.60$$

Compounded continuously, it will take about 5.60 years (or 67.15 months).

41. $25,000 = 10,000e^{0.06t}$

$$2.5 = e^{0.06t}$$
$$\ln 2.5 = 0.06t$$
$$t = \frac{\ln 2.5}{0.06} \approx 15.27$$

It will take about 15.27 years (or 15 years, 3 months).

42. $80,000 = 25,000e^{0.07t}$

$$3.2 = e^{0.07t}$$
$$\ln 3.2 = 0.07t$$
$$t = \frac{\ln 3.2}{0.07} \approx 16.62$$

It will take about 16.62 years (or 16 years, 7 months).

43. $A = 90,000(1+0.03)^5 = \$104,335$

The house will cost $104,335 in five years.

44. $A = 200(1+0.0125)^6 \approx \$215.48$

Her bill will be $215.48 after 6 months.

45. $P = 15,000e^{(-0.05)(3)} \approx \$12,910.62$

Jerome should ask for $12,910.62.

46. $P = 3,000\left(1+\frac{0.03}{12}\right)^{(-12)(0.5)} \approx \$2955.39$

John should save $2955.39.

47. $A = 15(1+0.15)^5 = 15(1.15)^5 \approx \$30.17$ per share for a total of about \$3017.

48. $850,000 = 650,000(1+r)^3$

$$\frac{85}{65} = (1+r)^3$$

$$\sqrt[3]{\frac{85}{65}} = 1+r$$

$$r \approx \sqrt[3]{1.3077} - 1 \approx 0.0935$$

The annual return is approximately 9.35%.

49. 5.6% compounded continuously:

$A = 1000e^{(0.056)(1)} = \$1057.60$

Jim will not have enough money to buy the computer.

5.9% compounded monthly:

$$A = 1000\left(1+\frac{0.059}{12}\right)^{12} = \$1060.62$$

The second bank offers the better deal.

50. 6.8% compounded continuously for 3 months:
Amount on April 1:

$A = 1000e^{(0.068)(0.25)} = \$1017.15$

5.25% compounded monthly for 1 month:
Amount on May 1

$$A = 1017.15\left(1+\frac{0.0525}{12}\right)^{(12)(1/12)} = \$1021.60$$

51. Will: 9% compounded semiannually:

$$A = 2000\left(1+\frac{0.09}{2}\right)^{(2)(20)} = \$11,632.73$$

Henry: 8.5% compounded continuously:

$A = 2000e^{(0.085)(20)} = \$10,947.89$

Will has more money after 20 years.

52. Value of \$1000 compounded continuously at 10% for 3 years:

$A = 1000e^{(0.10)(3)} = \$1349.86$

April will have more money if she takes the \$1000 now and invests it.

53. **a.** Let x = the year, then the average annual cost C of a 4-year private college is by the function $C(x) = 25,143(1.059)^{x-2008}$.

$$\begin{aligned} C(2028) &= 25,143(1.059)^{2028-2008} \\ &= 25,143(1.059)^{20} \\ &\approx 79,129 \end{aligned}$$

In 2028, the average annual cost at a 4-year private college will be about \$79,129.

b.

$$\begin{aligned} A &= Pe^{rt} \\ 79,129 &= Pe^{0.04(18)} \\ P &= \frac{79,129}{e^{0.04(18)}} \approx \$38,516 \end{aligned}$$

An investment of \$38,516 in 2010 would pay for the cost of college at a 4-year private college in 2028.

54. $P = 100,000;\ t = 5$

a. Simple interest at 12% per annum:

$A = 100,000 + 100,000(0.12)(5) = \$160,000$

$I = \$160,000 - \$100,000 = \$60,000$

b. 11.5% compounded monthly:

$$A = 100,000\left(1+\frac{0.115}{12}\right)^{(12)(5)} \approx \$177,227$$

$I = \$177,227 - \$100,000 = \$77,227$

c. 11.25% compounded continuously:

$A = 100,000e^{(0.1125)(5)} \approx \$175,505$

$I = \$175,505 - \$100,000 = \$75,505$

Thus, simple interest at 12% is the best option since it results in the least interest.

55. $A = P\left(1+\dfrac{r}{n}\right)^{nt}$

$$A = 787\left(1+\frac{0.013}{2}\right)^{2(20)} = 787(1.0065)^{40} \approx 1019$$

The government would have to pay back approximately \$1019 billion in 2029. The amount of interest would be 1019 – 787 = \$232 billion.

56. From 2011 to 2020 would be 9 years, so t = 9.
The federal debt (in millions) would be:

$F = 14000000(1+0.078)^t = 14000000(1.078)^t$.

For t = 9:

$F = 14000000(1.078)^9 = 27523070.5483$.

The U.S. population (in millions) would be:

$P = 311(1+0.009)^t = 311(1.009)^t$. For t = 9:

$P = 311(1.009)^9 = 337.1171798$.

The per capita debt in 2020 will be
$\dfrac{27523070.5483}{337.1171798} = \$81,642$.

57. $P = 1000,\ r = 0.03,\ n = 2$
$A = 1000(1-0.03)^2 = \$940.90$

58. $P = 1000,\ r = 0.02,\ n = 3$
$A = 1000(1-0.02)^3 \approx \$941.19$

59. $P = 1000,\ A = 950,\ n = 2$
$$950 = 1000(1-r)^2$$
$$0.95 = (1-r)^2$$
$$\pm\sqrt{0.95} = 1-r$$
$$r = 1 \pm \sqrt{0.95}$$
$r \approx 0.0253$ or $r \approx 1.9747$
Disregard $r \approx 1.9747$. The inflation rate was 2.53%.

60. $P = 1000,\ A = 930,\ n = 2$
$$930 = 1000(1-r)^2$$
$$0.93 = (1-r)^2$$
$$\pm\sqrt{0.93} = 1-r$$
$$r = 1 \pm \sqrt{0.93}$$
$r \approx 0.0356$ or $r \approx 1.9644$
Disregard $r \approx 1.9644$. The inflation rate was 3.56%.

61. $r = 0.02$
$$\frac{1}{2}P = P(1-0.02)^t$$
$$0.5P = P(0.98)^t$$
$$0.5 = (0.98)^t$$
$$t = \log_{0.98}(0.5)$$
$$= \frac{\ln 0.5}{\ln 0.98} \approx 34.31$$
The purchasing power will be half in 34.31 years.

62. $r = 0.04$
$$\frac{1}{2}P = P(1-0.04)^t$$
$$0.5P = P(0.96)^t$$
$$0.5 = (0.96)^t$$
$$t = \log_{0.96}(0.5)$$
$$= \frac{\ln 0.5}{\ln 0.96} \approx 16.98$$
The purchasing power will be half in 16.98 years.

63. **a.** $A = \$10,000,\ r = 0.10,\ n = 12,\ t = 20$
$$P = 10,000\left(1+\frac{0.10}{12}\right)^{(-12)(20)} \approx \$1364.62$$

b. $A = \$10,000,\ r = 0.10,\ t = 20$
$$P = 10,000e^{(-0.10)(20)} \approx \$1353.35$$

64. $A = \$40,000,\ r = 0.08,\ n = 1,\ t = 17$
$$P = 40,000\left(1+\frac{0.08}{1}\right)^{-17} \approx \$10,810.76$$

65. $A = \$10,000,\ r = 0.08,\ n = 1,\ t = 10$
$$P = 10,000\left(1+\frac{0.08}{1}\right)^{(-1)(10)} \approx \$4631.93$$

66. $A = \$25,000,\ P = 12,485.52,\ n = 1,\ t = 8$
$$25,000 = 12,485.52\left(1+r^8\right)$$
$$\frac{25,000}{12,485.52} = (1+r)^8$$
$$\sqrt[8]{\frac{25,000}{12,485.52}} = 1+r$$
$$r = \sqrt[8]{\frac{25,000}{12,485.52}} - 1$$
$$r \approx 0.090665741$$
The annual rate of return is about 9.07%.

67. **a.**
$$t = \frac{\ln 2}{1\cdot\ln\left(1+\frac{0.12}{1}\right)}$$
$$= \frac{\ln 2}{\ln(1.12)} \approx 6.12 \text{ years}$$

b. $$t = \frac{\ln 3}{4 \cdot \ln\left(1+\frac{0.06}{4}\right)} = \frac{\ln 3}{4\ln(1.015)} \approx 18.45 \text{ years}$$

c. $$mP = P\left(1+\frac{r}{n}\right)^{nt}$$
$$m = \left(1+\frac{r}{n}\right)^{nt}$$
$$\ln m = nt \cdot \ln\left(1+\frac{r}{n}\right)$$
$$t = \frac{\ln m}{n \cdot \ln\left(1+\frac{r}{n}\right)}$$

68. a. $$t = \frac{\ln 8000 - \ln 1000}{0.10} \approx 20.79 \text{ years}$$

b. $$35 = \frac{\ln 30{,}000 - \ln 2000}{r}$$
$$r = \frac{\ln 30{,}000 - \ln 2000}{35} \approx 0.0774$$
$$r \approx 7.74\%$$

c. $$A = Pe^{rt}$$
$$\frac{A}{P} = e^{rt}$$
$$\ln\left(\frac{A}{P}\right) = rt$$
$$\ln A - \ln P = rt$$
$$t = \frac{\ln A - \ln P}{r}$$

69.a. $CPI_0 = 163.0$, $CPI = 215.3$, $n = 2008 - 1998 = 10$
$$215.3 = 163.0\left(1+\frac{r}{100}\right)^{10}$$
$$\frac{215.3}{163.0} = \left(1+\frac{r}{100}\right)^{10}$$
$$1+\frac{r}{100} = \sqrt[10]{\frac{215.3}{163.0}}$$
$$\frac{r}{100} = \sqrt[10]{\frac{215.3}{163.0}} - 1$$
$$r = 100\left(\sqrt[10]{\frac{215.3}{163.0}} - 1\right) \approx 2.82\%$$

b. $CPI_0 = 163.0$, $CPI = 300$, $r = 2.82$
$$300 = 215.3\left(1+\frac{2.82}{100}\right)^{n}$$
$$\frac{300}{215.3} = \left(1+\frac{2.82}{100}\right)^{n}$$
$$\ln\left(\frac{300}{215.3}\right) = \ln\left(1+\frac{2.82}{100}\right)^{n}$$
$$\ln\left(\frac{300}{215.3}\right) = n\ln\left(1+\frac{2.82}{100}\right)$$
$$n = \frac{\ln\left(\frac{300}{215.3}\right)}{\ln\left(1+\frac{2.82}{100}\right)} \approx 12.0 \text{ years}$$

The CPI will reach 300 about 12 years after 2008, or in the year 2020.

70. $CPI_0 = 234.2$, $r = 2.8\%$, $n = 5$
$$CPI = 234.2\left(1+\frac{2.8}{100}\right)^{5} \approx 268.9$$

In 5 years, the CPI index will be about 268.9.

71. $r = 3.1\%$
$$2 \cdot CPI_0 = CPI_0\left(1+\frac{3.1}{100}\right)^{n}$$
$$2 = (1.031)^{n}$$
$$n = \log_{1.031} 2 = \frac{\ln 2}{\ln 1.031} \approx 22.7$$

It will take about 22.7 years for the CPI index to double.

72. $CPI_0 = 100,\ CPI = 456.5,\ r = 5.57$

$$456.5 = 100\left(1 + \frac{5.57}{100}\right)^n$$
$$456.5 = 100(1.0557)^n$$
$$4.565 = (1.0557)^n$$
$$n = \log_{1.0558}(4.565)$$
$$= \frac{\ln 4.565}{\ln 1.0558} \approx 28.0 \text{ years}$$

The yeas that was used as the base period for the CPI was about 28 years before 1995, or the year 1967.

73. Answers will vary.

74. Answers will vary.

75. Answers will vary.

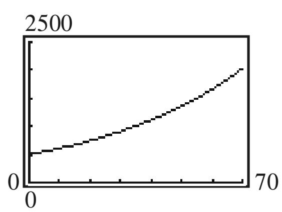
Section 5.8

1. $P(t) = 500e^{0.02t}$

a. $P(0) = 500e^{(0.02)\cdot(0)} = 500$ insects

b. growth rate = 2 %

c.



d. $P(10) = 500e^{(0.02)\cdot(10)} \approx 611$ insects

e. Find t when $P = 800$:

$$800 = 500e^{0.02t}$$
$$1.6 = e^{0.02t}$$
$$\ln 1.6 = 0.02t$$
$$t = \frac{\ln 1.6}{0.02} \approx 23.5 \text{ days}$$

f. Find t when $P = 1000$:

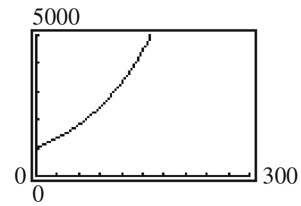
$$1000 = 500e^{0.02t}$$
$$2 = e^{0.02t}$$
$$\ln 2 = 0.02t$$
$$t = \frac{\ln 2}{0.02} \approx 34.7 \text{ days}$$

2. $N(t) = 1000e^{0.01t}$

a. $N(0) = 1000e^{(0.01)\cdot(0)} = 1000$ bacteria

b. growth rate = 1 %

c.



d. $N(4) = 1000e^{(0.01)\cdot(4)} \approx 1041$ bacteria

e. Find t when $N = 1700$:

$$1700 = 1000e^{0.01t}$$
$$1.7 = e^{0.01t}$$
$$\ln 1.7 = 0.01t$$
$$t = \frac{\ln 1.7}{0.01} \approx 53.1 \text{ hours}$$

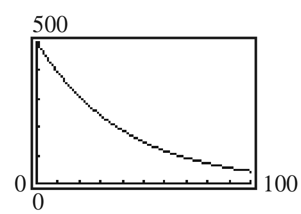
f. Find t when $N = 2000$:

$$2000 = 1000e^{0.01t}$$
$$2 = e^{0.01t}$$
$$\ln 2 = 0.01t$$
$$t = \frac{\ln 2}{0.01} \approx 69.3 \text{ hours}$$

3. $A(t) = A_0 e^{-0.0244t} = 500e^{-0.0244t}$

a. decay rate = -2.44%

b.



c. $A(10) = 500e^{(-0.0244)(10)} \approx 391.7$ grams

d. Find t when $A = 400$:

$$400 = 500e^{-0.0244t}$$
$$0.8 = e^{-0.0244t}$$
$$\ln 0.8 = -0.0244t$$
$$t = \frac{\ln 0.8}{-0.0244} \approx 9.1 \text{ years}$$

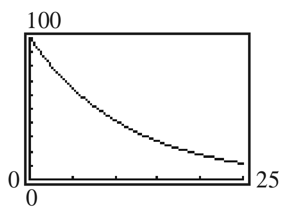
e. Find t when $A = 250$:

$$250 = 500e^{-0.0244t}$$
$$0.5 = e^{-0.0244t}$$
$$\ln 0.5 = -0.0244t$$
$$t = \frac{\ln 0.5}{-0.0244} \approx 28.4 \text{ years}$$

4. $A(t) = A_0 e^{-0.087t} = 100e^{-0.087t}$

a. decay rate $= -8.7\%$

b.



c. $A(9) = 100e^{(-0.087)(9)} \approx 45.7$ grams

d. Find t when $A = 70$:

$$70 = 100e^{-0.087t}$$
$$0.7 = e^{-0.087t}$$
$$\ln 0.7 = -0.087t$$
$$t = \frac{\ln 0.7}{-0.087} \approx 4.1 \text{ days}$$

e. Find t when $A = 50$:

$$50 = 100e^{-0.087t}$$
$$0.5 = e^{-0.087t}$$
$$\ln 0.5 = -0.087t$$
$$t = \frac{\ln 0.5}{-0.087} \approx 7.97 \text{ days}$$

5. a. $N(t) = N_0 e^{kt}$

b. If $N(t) = 1800$, $N_0 = 1000$, and $t = 1$, then

$$1800 = 1000e^{k(1)}$$
$$1.8 = e^k$$
$$k = \ln 1.8$$

If $t = 3$, then $N(3) = 1000e^{(\ln 1.8)(3)} = 5832$ mosquitoes.

c. Find t when $N(t) = 10{,}000$:

$$10{,}000 = 1000e^{(\ln 1.8)t}$$
$$10 = e^{(\ln 1.8)t}$$
$$\ln 10 = (\ln 1.8)t$$
$$t = \frac{\ln 10}{\ln 1.8} \approx 3.9 \text{ days}$$

6. a. $N(t) = N_0 e^{kt}$

b. If $N(t) = 800$, $N_0 = 500$, and $t = 1$, then

$$800 = 500e^{k(1)}$$
$$1.6 = e^k$$
$$k = \ln 1.6$$

If $t = 5$, then $N(5) = 500e^{(\ln 1.6)(5)} \approx 5243$ bacteria

c. Find t when $N(t) = 20{,}000$:

$$20{,}000 = 500e^{(\ln 1.6)t}$$
$$40 = e^{(\ln 1.6)t}$$
$$\ln 40 = (\ln 1.6)t$$
$$t = \frac{\ln 40}{\ln 1.6} \approx 7.85 \text{ hours}$$

7. a. $N(t) = N_0 e^{kt}$

b. Note that 18 months = 1.5 years, so $t = 1.5$.

$$2N_0 = N_0 e^{k(1.5)}$$
$$2 = e^{1.5k}$$
$$\ln 2 = 1.5k$$
$$k = \frac{\ln 2}{1.5}$$

If $N_0 = 10{,}000$ and $t = 2$, then

$$P(2) = 10{,}000e^{\left(\frac{\ln 2}{1.5}\right)(2)} \approx 25{,}198$$

The population 2 years from now will be 25,198.

8. a. $N(t) = N_0 e^{kt}, \ k < 0$

b. If $N(t) = 800{,}000$, $N_0 = 900{,}000$, and $t = 2007 - 2005 = 2$, then

$$800{,}000 = 900{,}000 e^{k(2)}$$
$$\frac{8}{9} = e^{2k}$$
$$\ln\left(\frac{8}{9}\right) = 2k$$
$$k = \frac{\ln(8/9)}{2}$$

If $t = 2009 - 2005 = 4$, then

$$P(4) = 900{,}000 e^{\left(\frac{\ln(8/9)}{2}\right)(4)} \approx 711{,}111$$

The population in 2009 will be 711,111.

9. Use $A = A_0 e^{kt}$ and solve for k:

$$0.5A_0 = A_0 e^{k(1690)}$$
$$0.5 = e^{1690k}$$
$$\ln 0.5 = 1690k$$
$$k = \frac{\ln 0.5}{1690}$$

When $A_0 = 10$ and $t = 50$:

$$A = 10 e^{\left(\frac{\ln 0.5}{1690}\right)(50)} \approx 9.797 \text{ grams}$$

10. Use $A = A_0 e^{kt}$ and solve for k:

$$0.5A_0 = A_0 e^{k\left(1.3\times 10^9\right)}$$
$$0.5 = e^{\left(1.3\times 10^9\right)k}$$
$$\ln 0.5 = 1.3\times 10^9 k$$
$$k = \frac{\ln 0.5}{1.3\times 10^9}$$

When $A_0 = 10$ and $t = 100$:

$$A = 10 e^{\left(\frac{\ln 0.5}{1.3\times 10^9}\right)(100)} \approx 9.999999467 \text{ grams}$$

When $A_0 = 10$ and $t = 1000$:

$$A = 10 e^{\left(\frac{\ln 0.5}{1.3\times 10^9}\right)(1000)} \approx 9.999994668 \text{ grams}$$

11. a. Use $A = A_0 e^{kt}$ and solve for k:
half-life $= 5700$ years

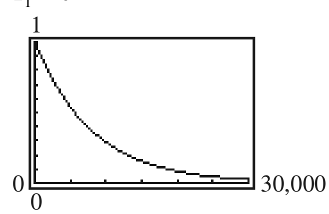
$$0.5A_0 = A_0 e^{k(5700)}$$
$$0.5 = e^{5700k}$$
$$\ln 0.5 = 5700k$$
$$k = \frac{\ln 0.5}{5700}$$

Solve for t when $A = 0.3A_0$:

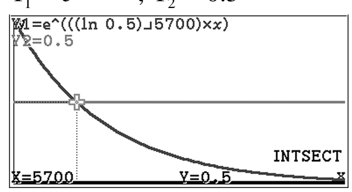
$$0.3A_0 = A_0 e^{\left(\frac{\ln 0.5}{5700}\right)t}$$
$$0.3 = e^{\left(\frac{\ln 0.5}{5700}\right)t}$$
$$\ln 0.3 = \left(\frac{\ln 0.5}{5700}\right)t$$
$$t = \frac{\ln 0.3}{\left(\frac{\ln 0.5}{5700}\right)} \approx 9901$$

The tree died approximately 9901 years ago.

b. $Y_1 = e^{\left(\frac{\ln 0.5}{5700}\right)t}$

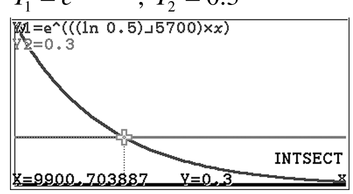


c. $Y_1 = e^{\left(\frac{\ln 0.5}{5700}\right)t}$; $Y_2 = 0.5$



Thus, 5700 years will elapse until half of the carbon 14 remains.

d. $Y_1 = e^{\left(\frac{\ln 0.5}{5700}\right)t}$; $Y_2 = 0.3$



This verifies that the tree died approximately 9901 years ago.

12. a. Use $A = A_0 e^{kt}$ and solve for k:
half-life $= 5700$ years

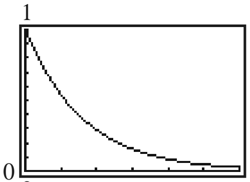
$$0.5A_0 = A_0 e^{k(5700)}$$
$$0.5A_0 = A_0 e^{k(5700)}$$
$$0.5 = e^{5700k}$$
$$\ln 0.5 = 5700k$$
$$k = \frac{\ln 0.5}{5700}$$

Solve for t when $A = 0.7A_0$:

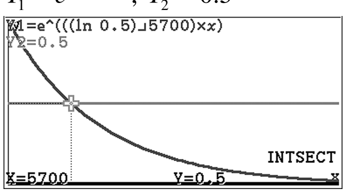
$$0.7A_0 = A_0 e^{\left(\frac{\ln 0.5}{5700}\right)t}$$
$$0.7 = e^{\left(\frac{\ln 0.5}{5700}\right)t}$$
$$\ln 0.7 = \left(\frac{\ln 0.5}{5700}\right)t$$
$$t = \frac{\ln 0.7}{\frac{\ln 0.5}{5700}} \approx 2933$$

The fossil is about 2933 years old.

b. $Y_1 = e^{\left(\frac{\ln 0.5}{5700}\right)t}$

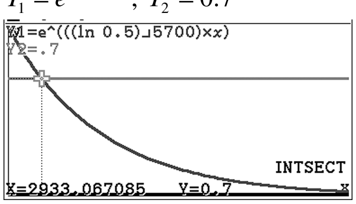


c. $Y_1 = e^{\left(\frac{\ln 0.5}{5700}\right)t}$; $Y_2 = 0.5$



Thus, 5700 years will elapse until half of the carbon 14 remains.

d. $Y_1 = e^{\left(\frac{\ln 0.5}{5700}\right)t}$; $Y_2 = 0.7$



This verifies that the fossil is approximately 2933 years ago.

13. a. Using $u = T + (u_0 - T)e^{kt}$ with $t = 5$, $T = 70$, $u_0 = 450$, and $u = 300$:

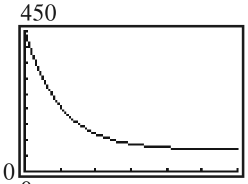
$$300 = 70 + (450 - 70)e^{k(5)}$$
$$230 = 380e^{5k}$$
$$\frac{230}{380} = e^{5k}$$
$$\ln\left(\frac{23}{38}\right) = 5k$$
$$k = \frac{\ln\left(\frac{23}{38}\right)}{5} \approx -0.1004$$

$T = 70$, $u_0 = 450$, $u = 135$:

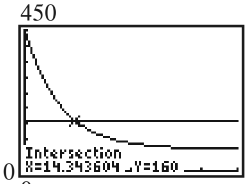
$$135 = 70 + (450 - 70)e^{\frac{\ln(23/38)}{5}t}$$
$$65 = 380e^{\frac{\ln(23/38)}{5}t}$$
$$\frac{65}{380} = e^{\frac{\ln(23/38)}{5}t}$$
$$\ln\left(\frac{65}{380}\right) = \frac{\ln(23/38)}{5}t$$
$$t = \frac{\ln(65/380)}{\left(\frac{\ln(23/38)}{5}\right)} \approx 18 \text{ minutes}$$

The pizza will be cool enough to eat at about 5:18 PM.

b. $Y_1 = 70 + (450 - 70)e^{\frac{\ln(23/38)}{5}x}$



c. $Y_1 = 70 + (450 - 70)e^{\frac{\ln(23/38)}{5}x}$; $Y_2 = 160$



The pizza will be 160˚F after about 14.3 minutes.

d. As time passes, the temperature gets closer to 70˚F.

14. a. Using $u = T + (u_0 - T)e^{kt}$ with $t = 2$, $T = 38$, $u_0 = 72$, and $u = 60$:

$$60 = 38 + (72 - 38)e^{k(2)}$$
$$22 = 34e^{2k}$$
$$\frac{22}{34} = e^{2k}$$
$$\ln\left(\frac{22}{34}\right) = 2k$$
$$k = \frac{\ln(22/34)}{2}$$

$T = 38$, $u_0 = 72$, $t = 7$

$$u = 38 + (72 - 38)e^{\left(\frac{\ln(22/34)}{2}\right)(7)}$$
$$u = 38 + 34e^{\left(\frac{\ln(22/34)}{2}\right)(7)} \approx 45.41^\circ\text{F}$$

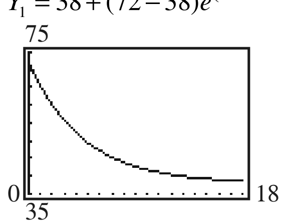
After 7 minutes the thermometer will read about 45.41˚F.

b. Find t when $u = 39^\circ\text{F}$

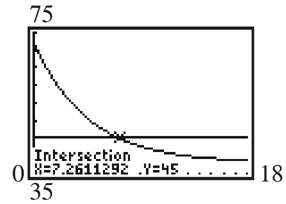
$$39 = 38 + (72 - 38)e^{\left(\frac{\ln(22/34)}{2}\right)t}$$
$$1 = 34e^{\left(\frac{\ln(22/34)}{2}\right)t}$$
$$\frac{1}{34} = e^{\left(\frac{\ln(22/34)}{2}\right)t}$$
$$\ln\left(\frac{1}{34}\right) = \left(\frac{\ln(22/34)}{2}\right)t$$
$$t = \frac{\ln(1/34)}{\left(\frac{\ln(22/34)}{2}\right)} \approx 16.2$$

The thermometer will read 39 degrees after about 16.2 minutes.

c. $Y_1 = 38 + (72 - 38)e^{\left(\frac{\ln(22/34)}{2}\right)x}$



d. $Y_1 = 38 + (72 - 38)e^{\left(\frac{\ln(22/34)}{2}\right)x}$; $Y_2 = 45$



The thermometer will read $45^\circ F$ after about 7.26 minutes.

e. As time passes, the temperature gets closer to 38˚F.

15. a. Using $u = T + (u_0 - T)e^{kt}$ with $t = 3$, $T = 35$, $u_0 = 8$, and $u = 15$:

$$15 = 35 + (8 - 35)e^{k(3)}$$
$$-20 = -27e^{3k}$$
$$\frac{20}{27} = e^{3k}$$
$$\ln\left(\frac{20}{27}\right) = 3k$$
$$k = \frac{\ln(20/27)}{3}$$

At $t = 5$: $u = 35 + (8 - 35)e^{\left(\frac{\ln(20/27)}{3}\right)(5)} \approx 18.63^\circ\text{C}$

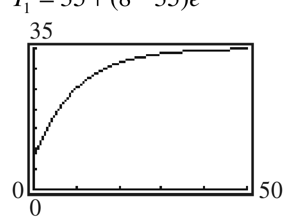
After 5 minutes, the thermometer will read approximately 18.63°C.

At $t = 10$:

$$u = 35 + (8 - 35)e^{\left(\frac{\ln(20/27)}{3}\right)(10)} \approx 25.07^\circ\text{C}$$

After 10 minutes, the thermometer will read approximately 25.07°C

b. $Y_1 = 35 + (8 - 35)e^{\left(\frac{\ln(20/27)}{3}\right)x}$



16. Using $u = T + (u_0 - T)e^{kt}$ with $t = 10$, $T = 70$, $u_0 = 28$, and $u = 35$:

$$35 = 70 + (28 - 70)e^{k(10)}$$
$$-35 = -42e^{10k}$$
$$\frac{35}{42} = e^{10k}$$
$$\ln\left(\frac{35}{42}\right) = 10k$$
$$k = \frac{\ln(35/42)}{10}$$

At $t = 30$:

$$u = 70 + (28 - 70)e^{\left(\frac{\ln(35/42)}{10}\right)(30)} \approx 45.69°\text{F}$$

After 30 minutes, the temperature of the stein will be approximately $45.69°F$.

Find the value of t so that the $u = 45°$F:

$$45 = 70 + (28 - 70)e^{\left(\frac{\ln(35/42)}{10}\right)t}$$
$$-25 = -42e^{\left(\frac{\ln(35/42)}{10}\right)t}$$
$$\frac{25}{42} = e^{\left(\frac{\ln(35/42)}{10}\right)t}$$
$$\ln\left(\frac{25}{42}\right) = \left(\frac{\ln(35/42)}{10}\right)t$$
$$t = \frac{10}{\ln(35/42)} \cdot \ln\left(\frac{25}{42}\right) \approx 28.46$$

The temperature of the stein will be $45°F$ after about 28.46 minutes.

17. Use $A = A_0 e^{kt}$ and solve for k:

$$2.2 = 2.5e^{k(24)}$$
$$0.88 = e^{24k}$$
$$\ln 0.88 = 24k$$
$$k = \frac{\ln 0.88}{24}$$

When $A_0 = 2.5$ and $t = 72$:

$$A = 2.5e^{\left(\frac{\ln 0.88}{24}\right)(72)} \approx 1.70$$

After 3 days (72 hours), the amount of free chlorine will be 1.70 parts per million.

Find t when $A = 1$:

$$1 = 2.5e^{\left(\frac{\ln 0.88}{24}\right)t}$$
$$0.4 = e^{\left(\frac{\ln 0.88}{24}\right)t}$$
$$\ln 0.4 = \left(\frac{\ln 0.88}{24}\right)t$$
$$t = \frac{24}{\ln 0.88} \cdot \ln 0.4 \approx 172$$

Ben will have to shock his pool again after 172 hours (or 7.17 days) when the level of free chlorine reaches 1.0 parts per million.

18. Use $A = A_0 e^{kt}$ and solve for k:

$$0.15 = 0.25e^{k(17)}$$
$$0.6 = e^{17k}$$
$$\ln 0.6 = 17k$$
$$k = \frac{\ln 0.6}{17}$$

When $A_0 = 0.25$ and $t = 30$:

$$A = 0.25e^{\left(\frac{\ln 0.6}{17}\right)(30)} \approx 0.10$$

After 30 minutes, approximately 0.10 M of dinitrogen pentoxide will remain.

Find t when $A = 0.01$:

$$0.01 = 0.25e^{\left(\frac{\ln 0.6}{17}\right)t}$$
$$0.04 = e^{\left(\frac{\ln 0.6}{17}\right)t}$$
$$\ln 0.04 = \left(\frac{\ln 0.6}{17}\right)t$$
$$t = \frac{17}{\ln 0.6} \cdot \ln 0.04 \approx 107$$

It will take approximately 107 minutes until 0.01 M of dinitrogen pentoxide remains.

19. Use $A = A_0 e^{kt}$ and solve for *k*:

$$0.36 = 0.40e^{k(30)}$$
$$0.9 = e^{30k}$$
$$\ln 0.9 = 30k$$
$$k = \frac{\ln 0.9}{30}$$

Note that 2 hours = 120 minutes.
When $A_0 = 0.40$ and $t = 120$:

$$A = 0.40e^{\left(\frac{\ln 0.9}{30}\right)(120)} \approx 0.26$$

After 2 hours, approximately 0.26 M of sucrose will remain.

Find t when $A = 0.10$:

$$0.10 = 0.40e^{\left(\frac{\ln 0.9}{30}\right)t}$$
$$0.25 = e^{\left(\frac{\ln 0.9}{30}\right)t}$$
$$\ln 0.25 = \left(\frac{\ln 0.9}{30}\right)t$$
$$t = \frac{30}{\ln 0.9} \cdot \ln 0.25 \approx 395$$

It will take approximately 395 minutes (or 6.58 hours) until 0.10 M of sucrose remains.

20. Use $A = A_0 e^{kt}$ and solve for k:

$$15 = 25e^{k(10)}$$
$$0.6 = e^{10k}$$
$$\ln 0.6 = 10k$$
$$k = \frac{\ln 0.6}{10}$$

When $A_0 = 25$ and $t = 24$:

$$A = 25e^{\left(\frac{\ln 0.6}{10}\right)(24)} \approx 7.34$$

There will be about 7.34 kilograms of salt left after 1 day.

Find t when $A = 0.5A_0$:

$$0.5 = 25e^{\left(\frac{\ln 0.6}{10}\right)t}$$
$$0.02 = e^{\left(\frac{\ln 0.6}{10}\right)t}$$
$$\ln 0.02 = \left(\frac{\ln 0.6}{10}\right)t$$
$$t = \frac{10}{\ln 0.6} \cdot \ln 0.02 \approx 76.6$$

It will take about 76.6 hours (about 3.19 days) until ½ kilogram of salt is left.

21. Use $A = A_0 e^{kt}$ and solve for k:

$$0.5A_0 = A_0 e^{k(8)}$$
$$0.5 = e^{8k}$$
$$\ln 0.5 = 8k$$
$$k = \frac{\ln 0.5}{8}$$

Find *t* when $A = 0.1A_0$:

$$0.1A_0 = A_0 e^{\left(\frac{\ln 0.5}{8}\right)t}$$
$$0.1 = e^{\left(\frac{\ln 0.5}{8}\right)t}$$
$$\ln 0.1 = \left(\frac{\ln 0.5}{8}\right)t$$
$$t = \frac{8}{\ln 0.5} \cdot \ln 0.1 \approx 26.6$$

The farmers need to wait about 26.6 days before using the hay.

22. Using $u = T + (u_0 - T)e^{kt}$ with $t = 2$, $T = 325$, $u_0 = 75$, and $u = 100$:

$$100 = 325 + (75 - 325)e^{k(2)}$$
$$-225 = -250e^{2k}$$
$$0.9 = e^{2k}$$
$$2k = \ln 0.9$$
$$k = \frac{\ln 0.9}{2}$$

Find the value of *t* so that *u* = 175˚F:

$$175 = 325 + (75 - 325)e^{\left(\frac{\ln 0.9}{2}\right)t}$$
$$-150 = -250e^{\left(\frac{\ln 0.9}{2}\right)t}$$
$$0.6 = e^{\left(\frac{\ln 0.9}{2}\right)t}$$
$$\ln 0.6 = \left(\frac{\ln 0.9}{2}\right)t$$
$$t = \frac{2}{\ln 0.9} \cdot \ln 0.6 \approx 9.7$$

The hotel may serve their guests about 9.7 hours after noon or at about 9:42 PM.

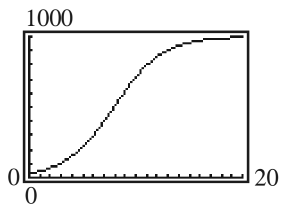
23. a. As $t \to \infty$, $e^{-0.439t} \to 0$. Thus, $P(t) \to 1000$. The carrying capacity is 1000 grams of bacteria.

b. Growth rate = 0.439 = 43.9%.

c. $P(0) = \dfrac{1000}{1+32.33e^{-0.439(0)}} = \dfrac{1000}{33.33} = 30$

The initial population was 30 grams of bacteria.

d. $Y_1 = \dfrac{1000}{1+32.33e^{-0.439x}}$

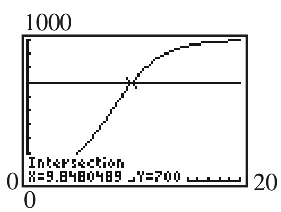


e. $P(9) = \dfrac{1000}{1+32.33e^{-0.439(9)}} \approx 616.6$

After 9 hours, the population of bacteria will be about 616.8 grams.

f. We need to find t such that $P = 700$:

$Y_1 = \dfrac{1000}{1+32.33e^{-0.439x}}$; $Y_2 = 700$

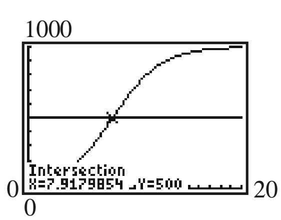


Thus, $t \approx 9.85$. The population of bacteria will be 700 grams after about 9.85 hours.

g. We need to find t such that

$P = \dfrac{1}{2}(1000) = 500$:

$Y_1 = \dfrac{1000}{1+32.33e^{-0.439x}}$; $Y_2 = 500$

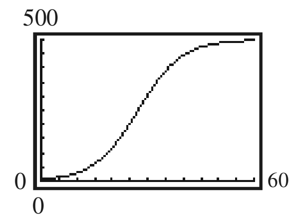


Thus, $t \approx 7.9$. The population of bacteria will reach one-half of is carrying capacity after about 7.9 hours.

24. a. As $t \to \infty$, $e^{-0.162t} \to 0$. Thus, $P(t) \to 500$. The carrying capacity is 500 bald eagles.

b. Growth rate = 0.162 = 16.2%.

c. $Y_1 = \dfrac{500}{1+82.33e^{-0.162x}}$

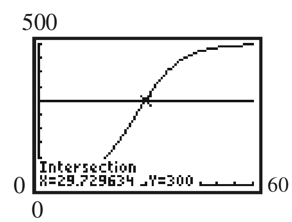


d. $P(3) = \dfrac{500}{1+82.33e^{-0.162(3)}} \approx 9.68$

After 3 years, the population is almost 10 bald eagles.

e. We need to find t such that $P = 300$:

$Y_1 = \dfrac{500}{1+82.33e^{-0.162x}}$; $Y_2 = 300$

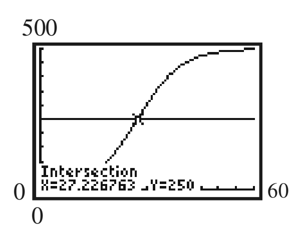


Thus, $t \approx 29.7$. The bald eagle population will be 300 in approximately 29.7 years.

f. We need to find t such that

$P = \dfrac{1}{2}(500) = 250$:

$Y_1 = \dfrac{500}{1+82.33e^{-0.162x}}$; $Y_2 = 250$



Thus, $t \approx 27.2$. The bald eagle population will reach one-half of its carrying capacity after about 27.2 years.

25. a. $y = \dfrac{6}{1+e^{-(5.085-0.1156(100))}} \approx 0.00923$

At $100°$F, the predicted number of eroded or leaky primary O-rings will be about 0.

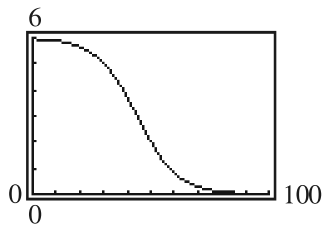
b. $y = \dfrac{6}{1+e^{-(5.085-0.1156(60))}} \approx 0.81$

At $60°\text{F}$, the predicted number of eroded or leaky primary O-rings will be about 1.

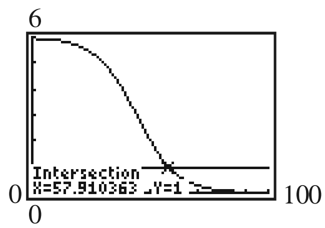
c. $y = \dfrac{6}{1+e^{-(5.085-0.1156(30))}} \approx 5.01$

At $30°\text{F}$, the predicted number of eroded or leaky primary O-rings will be about 5.

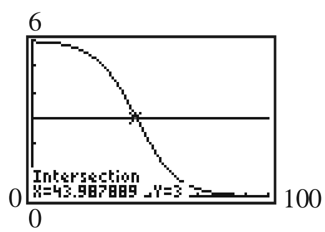
d. $Y_1 = \dfrac{6}{1+e^{-(5.085-0.1156x)}}$



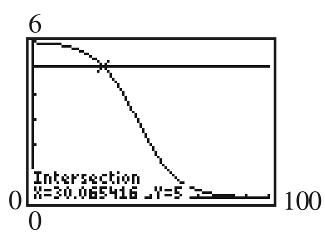
Use INTERSECT with $Y_2 = 1,\ 3,\ \text{and}\ 5$:



The predicted number of eroded or leaky O-rings is 1 when the temperature is about $57.91°\text{F}$.



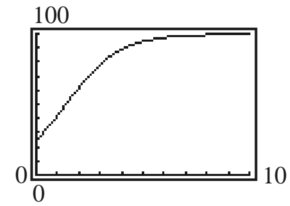
The predicted number of eroded or leaky O-rings is 3 when the temperature is about $43.99°\text{F}$.



The predicted number of eroded or leaky O-rings is 5 when the temperature is about $30.07°\text{F}$.

26. a. Growth rate = 0.799 = 79.9%.

b. $Y_1 = \dfrac{99.744}{1+3.014e^{-0.799x}}$



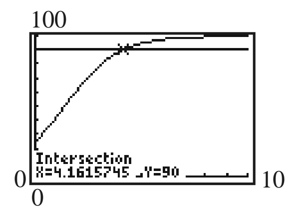
c. $t = 1990 - 1984 = 6$

$P(6) = \dfrac{99.744}{1+3.014e^{-0.799(6)}} \approx 97.3$

In 1990, about 97.3% of companies reported using Microsoft Word.

d. We need to find *t* such that *P* = 90

$Y_1 = \dfrac{99.744}{1+3.014e^{-0.799x}}$; $Y_2 = 90$



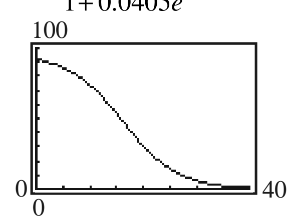
Thus, $t \approx 4.2$. Now, $1984 + 4.2 = 1988.2$. The percentage of Microsoft Word users reached 90% early in 1988.

e. Answers will vary. The maximum possible percentage of Microsoft Word users is 99.744%.

27. a. $P(0) = \dfrac{95.4993}{1+0.0405e^{0.1968(0)}} = \dfrac{95.4993}{1.0405} \approx 91.8$

In 1984, about 91.8% of households did not own a personal computer.

b. $Y_1 = \dfrac{95.4993}{1+0.0405e^{0.1968x}}$



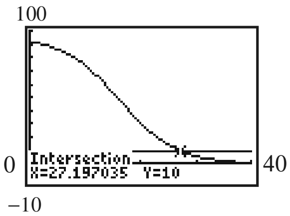
c. $t = 1995 - 1984 = 11$

$P(11) = \dfrac{95.4993}{1+0.0405e^{0.1968(11)}} \approx 70.6$

In 1995, about 70.6% of households did not own a personal computer.

d. We need to find t such that $P = 10$

$$Y_1 = \frac{95.4993}{1+0.0405e^{0.1968x}};\ Y_2 = 10$$

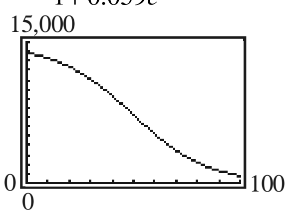


Thus, $t \approx 27.2$. Now, $1984 + 27.2 = 2011.2$. The percentage of households that do not own a personal computer will reach 10% during 2011.

28. a. $W(0) = \dfrac{14,656.248}{1+0.059e^{0.057(0)}} \approx 13,839.70$

In 1910, there were about 13,840 farm workers.

b. $Y_1 = \dfrac{14,656.248}{1+0.059e^{0.057x}}$



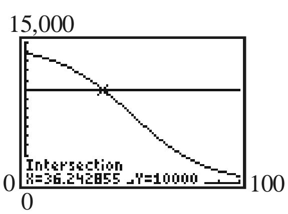
c. $t = 2010 - 1910 = 100$

$$W(100) = \frac{14,656.248}{1+0.059e^{0.057(100)}} \approx 786.56$$

In 2010, there were about 787.56 farm workers.

d. We need to find t such that $W = 10,000$.

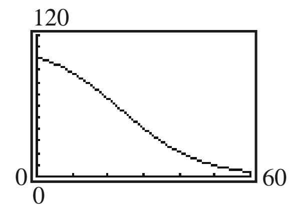
$$Y_1 = \frac{14,656.248}{1+0.059e^{0.057x}};\ Y_2 = 10,000$$



Thus, $t \approx 36.2$. Now, $1910 + 36.2 = 1946.2$. There were 10,000 farm workers in 1946.

e. As $t \to \infty$, $1+0.059e^{0.057t} \to \infty$. Thus, $W(t) \to 0$. No, it is not reasonable to use this model to predict the number of farm workers in 2060 because the number of farm workers left in the United States would be approaching 0.

29. a. $Y_1 = \dfrac{113.3198}{1+0.115e^{0.0912x}}$

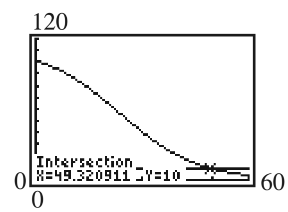


b. $P(15) = \dfrac{113.3198}{1+0.115e^{0.0912(15)}} \approx 78$

In a room of 15 people, the probability that no two people share the same birthday is about 78% or 0.78.

c. We need to find n such that $P = 10$.

$$Y_1 = \frac{113.3198}{1+0.115e^{0.0912x}};\ Y_2 = 10$$

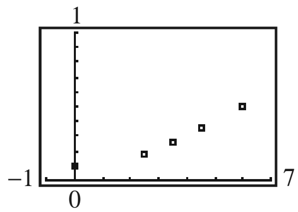


Thus, $t \approx 49.3$. The probability falls below 10% when 50 people are in the room.

d. As $n \to \infty$, $1+0.115e^{0.0912n} \to \infty$. Thus, $P(n) \to 0$. This means that as the number of people in the room increases, the more likely it will be that two will share the same birthday.

Section 5.9

1. a.



b. Using EXPonential REGression on the data yields: $y = 0.0903(1.3384)^x$

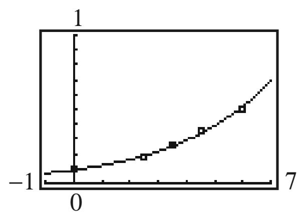
c. $y = 0.0903(1.3384)^x$

$= 0.0903\left(e^{\ln(1.3384)}\right)^x$

$= 0.0903e^{\ln(1.3384)x}$

$N(t) = 0.0903e^{0.2915t}$

d. $Y_1 = 0.0903e^{0.2915x}$



e. $N(7) = 0.0903e^{(0.2915)\cdot 7} \approx 0.69$ bacteria

f. We need to find t when $N = 0.75$:

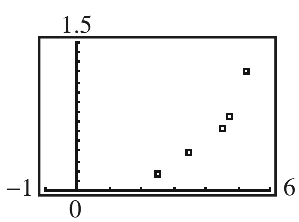
$$0.0903e^{(0.2915)\cdot t} = 0.75$$

$$e^{(0.2915)\cdot t} = \frac{0.75}{0.0903}$$

$$0.2915t = \ln\left(\frac{0.75}{0.0903}\right)$$

$$t \approx \frac{\ln\left(\frac{0.75}{0.0903}\right)}{0.2915} \approx 7.26 \text{ hours}$$

2. a.



b. Using EXPonential REGression on the data yields: $y = 0.0339(1.9474)^x$

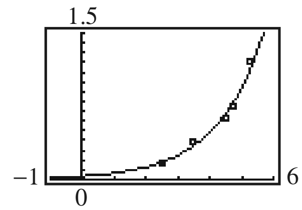
c. $y = 0.0339(1.9474)^x$

$= 0.0339\left(e^{\ln(1.9474)}\right)^x$

$= 0.0339e^{\ln(1.9474)x}$

$N(t) = 0.0339e^{(0.6665)t}$

d. $Y_1 = 0.0339e^{(0.6665)x}$



e. $N(6) = 0.0339e^{(0.6665)\cdot 6} \approx 1.85$ bacteria

f. We need to find t when $N = 2.1$:

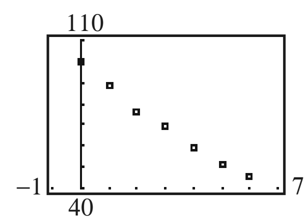
$$0.0339e^{(0.6665)t} = 2.1$$

$$e^{(0.6665)t} = \frac{2.1}{0.0339}$$

$$0.6665t = \ln\left(\frac{2.1}{0.0339}\right)$$

$$t \approx \frac{\ln\left(\frac{2.1}{0.0339}\right)}{0.6665} \approx 6.19 \text{ hours}$$

3. a.



b. Using EXPonential REGression on the data yields: $y = 100.3263(0.8769)^x$

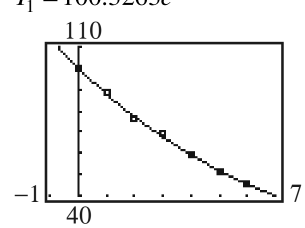
c. $y = 100.3263(0.8769)^x$

$= 100.3263\left(e^{\ln(0.8769)}\right)^x$

$= 100.3263e^{\ln(0.8769)x}$

$A(t) = 100.3263e^{(-0.1314)t}$

d. $Y_1 = 100.3263e^{(-0.1314)x}$



e. We need to find t when $A(t)=0.5\cdot A_0$

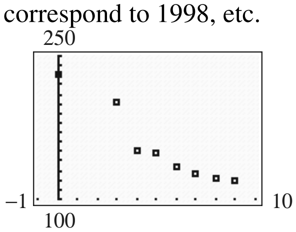
$$100.3263e^{(-0.1314)t}=(0.5)(100.3263)$$
$$e^{(-0.1314)t}=0.5$$
$$-0.1314t=\ln 0.5$$
$$t=\frac{\ln 0.5}{-0.1314}\approx 5.3 \text{ weeks}$$

f. $A(50)=100.3263e^{(-0.1314)\cdot 50}\approx 0.14$ grams

g. We need to find t when $A(t)=20$.

$$100.3263e^{(-0.1314)t}=20$$
$$e^{(-0.1314)t}=\frac{20}{100.3263}$$
$$-0.1314t=\ln\left(\frac{20}{100.3263}\right)$$
$$t=\frac{\ln\left(\frac{20}{100.3263}\right)}{-0.1314}\approx 12.3 \text{ weeks}$$

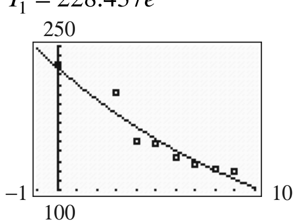
4. a. Let $x=0$ correspond to 1995, $x=3$ correspond to 1998, etc.



b. Using EXPonential REGression on the data yields: $y=228.4370(0.92301)^x$

c.
$$y=228.4370\left(e^{\ln(0.92301)}\right)^x$$
$$=228.4370e^{\ln(0.92301)x}$$
$$A(t)=228.437e^{(-0.08012)t}$$

d. $Y_1=228.437e^{(-0.08012)t}$



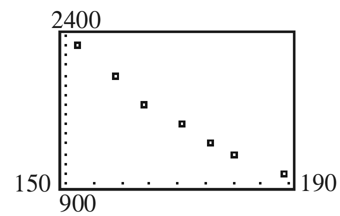
e. Note that 2010 is represented by $t=15$.
$A(15)=228.437e^{(-0.08012)\cdot 15}\approx 68.7$ billion cigarettes.

f. We need to find t when $A(t)=50$.

$$228.437e^{(-0.08012)t}=50$$
$$e^{(-0.08012)t}=\frac{50}{228.437}$$
$$-0.08012t=\ln\left(\frac{50}{228.437}\right)$$
$$t=\frac{\ln\left(\frac{50}{228.437}\right)}{-0.08012}\approx 19 \text{ years}$$

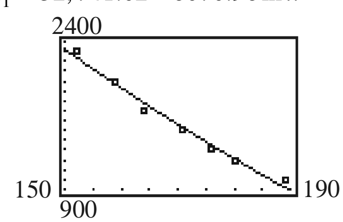
Now 1995 + 19 = 2014. The number of cigarettes exported from the U.S. will decrease to 50 billion in the year 2014.

5. a.



b. Using LnREGression on the data yields: $y=32,741.02-6070.96\ln x$

c. $Y_1=32,741.02-6070.96\ln x$

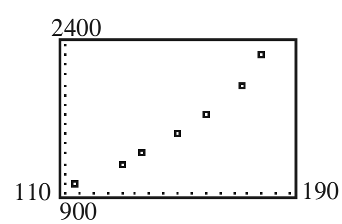


d. We need to find x when $y=1650$:

$$1650=32,741.02-6070.96\ln x$$
$$-31,091.02=-6070.96\ln x$$
$$\frac{-31,091.02}{-6070.96}=\ln x$$
$$5.1213\approx \ln x$$
$$e^{5.1213}\approx x$$
$$x\approx 168$$

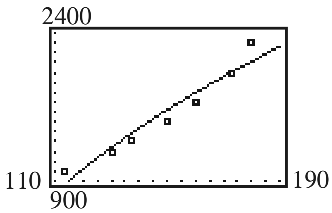
If the price were \$1650, then approximately 168 computers would be demanded.

6. a.



b. Using LnREGression on the data yields: $y = -11{,}850.72 + 2688.50 \ln x$

c. $Y_1 = -11{,}850.72 + 2688.50 \ln x$

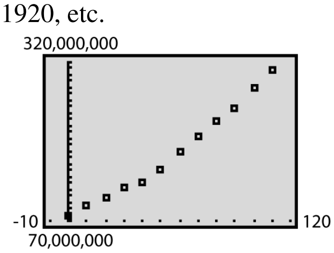


d. Find x when $y = 1650$:

$$1650 = -11{,}850.72 + 2688.50 \ln x$$
$$13{,}500.72 = 2688.50 \ln x$$
$$\frac{13{,}500.72}{2688.50} = \ln x$$
$$5.0216 \approx \ln x$$
$$e^{5.0216} \approx x$$
$$x \approx 152$$

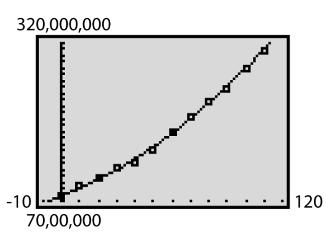
If the price were $1650, then approximately 152 computers would be supplied.

7. a. Let $x = 0$ correspond to 1900, $x = 10$ correspond to 1910, $x = 20$ correspond to 1920, etc.



b. Using LOGISTIC REGression on the data yields: $y = \dfrac{762{,}176{,}844.4}{1 + 8.7428e^{-0.0162x}}$

c. $Y_1 = \dfrac{762{,}176{,}844.4}{1 + 8.7428e^{-0.0162x}}$



d. As $x \to \infty$, $8.7428e^{-0.0162x} \to 0$, which means $1 + 8.7428e^{-0.0162x} \to 1$, so

$$y = \frac{762{,}176{,}844.4}{1 + 8.7428e^{-0.0162x}} \to 762{,}176{,}844.4$$

Therefore, the carrying capacity of the United States is approximately 762,176,844 people.

e. The year 2012 corresponds to $x = 112$, so

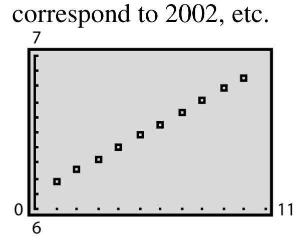
$$y = \frac{762{,}176{,}844.4}{1 + 8.7428e^{-0.0162(112)}}$$
$$\approx 314{,}362{,}768 \text{ people}$$

f. Find x when $y = 350{,}000{,}000$

$$\frac{762{,}176{,}844.4}{1 + 8.7428e^{-0.0162x}} = 350{,}000{,}000$$
$$762{,}176{,}844.4 = 350{,}000{,}000\left(1 + 8.7428e^{-0.0162x}\right)$$
$$\frac{762{,}176{,}844.4}{350{,}000{,}000} = 1 + 8.7428e^{-0.0162x}$$
$$\frac{762{,}176{,}844.4}{350{,}000{,}000} - 1 = 8.7428e^{-0.0162x}$$
$$1.17765 \approx 8.7428e^{-0.0162x}$$
$$\frac{1.17765}{8.7428} \approx e^{-0.0162x}$$
$$\ln\left(\frac{1.17765}{8.7428}\right) \approx -0.0162x$$
$$\frac{\ln\left(\frac{1.17765}{8.7428}\right)}{-0.0162} \approx x$$
$$x \approx 123.75$$

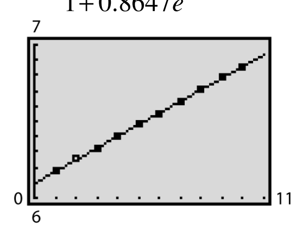
Therefore, the United States population will be 350,000,000 in the year 2023.

8. a. Let $x = 1$ correspond to 2001, $x = 2$ correspond to 2002, etc.



b. Using LOGISTIC REGression on the data yields: $y = \dfrac{11.35935}{1 + 0.8647e^{-0.0275x}}$

c. $Y_1 = \dfrac{11.35935}{1 + 0.8647e^{-0.0275x}}$:



d. As $x \to \infty$, $0.8647e^{-0.0275x} \to 0$, which means $1+0.8647e^{-0.0275x} \to 1$, so

$$y = \frac{11.35935}{1+0.8647e^{-0.0275x}} \to 11.35935$$

Therefore, the carrying capacity of the world is approximately 11.359 billion people.

e. The year 2015 corresponds to $x = 15$, so

$$y = \frac{11.35935}{1+0.8647e^{-0.0275(15)}} = 7.23\,.$$

In 2015, the population of the world was approximately 7.23 billion people.

f. We need to find x when $y = 7$:

$$\frac{11.35935}{1+0.8647e^{-0.0275x}} = 10$$

$$11.35935 = 10\left(1+0.8647e^{-0.0275x}\right)$$

$$11.35935 = 10 + 8.647e^{-0.0275x}$$

$$11.35935 - 10 = 8.647e^{-0.0275x}$$

$$10.35935 \approx 8.647e^{-0.0275x}$$

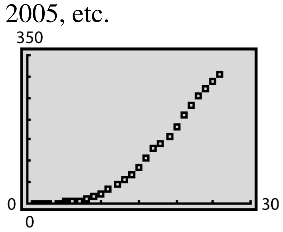
$$\frac{1.35935}{8.647} \approx e^{-0.0275x}$$

$$\ln\left(\frac{1.35935}{8.647}\right) \approx -0.0275x$$

$$x \approx \frac{\ln\left(\frac{1.35935}{8.647}\right)}{-0.0275} \approx 67.2$$

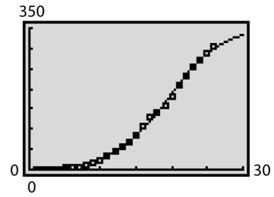
Therefore, the world population will be 10 billion in approximately the year 2067.

9. a. Let $x = 1$ correspond to 1985, $x = 10$ correspond to 1995, $x = 20$ correspond to 2005, etc.



b. Using LOGISTIC REGression on the data yields: $y = \dfrac{353.7175}{1+171.1548e^{-0.2636x}}$

c. $Y_1 = \dfrac{353.7175}{1+171.1548e^{-0.2636x}}$



d. As $x \to \infty$, $171.1548e^{-0.2636x} \to 0$, which means $1+171.1548e^{-0.2636x} \to 1$, so

$$y = \frac{353.7175}{1+171.1548e^{-0.2636x}} \to 353.7175$$

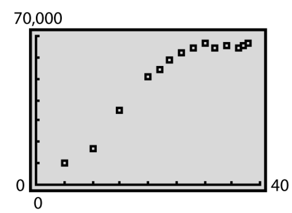
Therefore, the maximum number of cell phone subscribers in the U.S. is about 353.72 million.

e. The year 2015 corresponds to $x = 31$, so

$$y = \frac{353.7175}{1+171.1548e^{-0.2636(31)}} \approx 337.4\,.$$

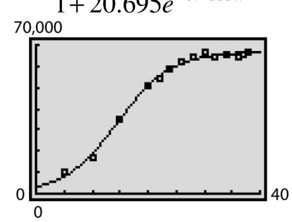
In 2015, cell phones will have approximately 337.4 million subscribers in the U.S.

10. a.



b. Let $x = 5$ correspond to 1975, $x = 10$ correspond to 1980, $x = 15$ correspond to 1985, etc. Using LOGISTIC REGression on the data yields: $y = \dfrac{66893.146}{1+20.695e^{-0.2085x}}$

c. $Y_1 = \dfrac{66893.146}{1+20.695e^{-0.2085x}}$



d. As $x \to \infty$, $20.695e^{-0.2085x} \to 0$, which means $1+20.695e^{-0.2085x} \to 1$, so

$$y = \frac{66893.146}{1+20.695e^{-0.2085x}} \to 66893.146$$

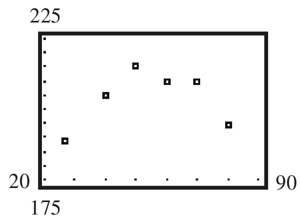
Therefore, the maximum number of cable subscribers in the U.S. is about 668,93,146.

e. The year 2015 corresponds to $x = 45$, so

$$y = \frac{66893.146}{1+20.695e^{-0.2085(45)}} \approx 66776.808 .$$

For 2015, the function predicts that the number of cable TV subscribers will be approximately 66,776,800 subscribers.

11. a.

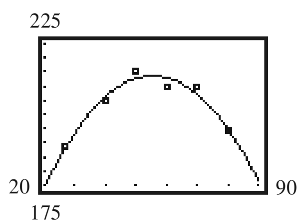


b. Based on the "upside down U-shape" of the graph, a quadratic model with $a < 0$ would best describe the data.

c. Using QUADratic REGression, the quadratic model is

$y = -0.0311x^2 + 3.4444x + 118.2493$.

d.

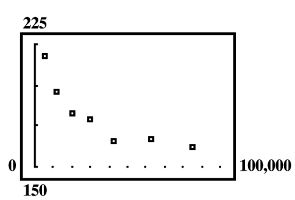


e. $y = -0.0311(35)^2 + 3.4444(35) + 118.2493$

≈ 201

The model predicts a total cholesterol of 201 for a 35-year-old male.

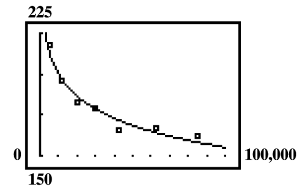
12. a.



b. Based on the graph, a logarithmic model would best describe the data.

c. Using Logarithmic REGression, the model is $y = 470.367 - 29.961\ln x$.

d.

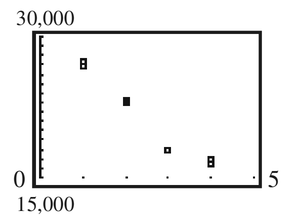


e. $y = 470.367 - 29.961\ln(55000)$

≈ 143.3

The model predicts a rate of 143.3 crimes per 1000 population.

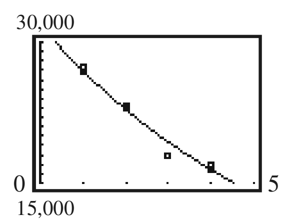
13. a.



b. An exponential model would fit best because depreciation of a car is described by exponential models in the theory of finance.

c. Using EXPonential REGression, the model is $y = 31808.51(0.8474)^x$.

d.



e. $y = 31808.51(0.8474)^5$

$\approx \$13,899$

The model predicts the asking price of a 5-year-old Chevrolet Impala SS to be $13,899.

Chapter 5 Review Exercises

1. a. $(g \circ f)(-8) = g(f(-8)) = g(2) = -4$

b. $(f \circ g)(-8) = f(g(-8)) = f(-2) = 1$

c. $(g \circ g)(7) = g(g(7)) = g(0) = -6$

d. $(g \circ f)(-5) = g(f(-5)) = g(4) = -6$

2. $f(x)=3x-5 \qquad g(x)=1-2x^2$

a. $$\begin{aligned}(f\circ g)(2)&=f(g(2))\\&=f\left(1-2(2)^2\right)\\&=f(-7)\\&=3(-7)-5\\&=-26\end{aligned}$$

b. $$\begin{aligned}(g\circ f)(-2)&=g(f(-2))\\&=g\left(3(-2)-5\right)\\&=g(-11)\\&=1-2(-11)^2\\&=-241\end{aligned}$$

c. $$\begin{aligned}(f\circ f)(4)&=f(f(4))\\&=f\left(3(4)-5\right)\\&=f(7)\\&=3(7)-5\\&=16\end{aligned}$$

d. $$\begin{aligned}(g\circ g)(-1)&=g(g(-1))\\&=g\left(1-2(-1)^2\right)\\&=g(-1)\\&=1-2(-1)^2\\&=-1\end{aligned}$$

3. $f(x)=\sqrt{x+2} \qquad g(x)=2x^2+1$

a. $$\begin{aligned}(f\circ g)(2)&=f(g(2))\\&=f\left(2(2)^2+1\right)\\&=f(9)\\&=\sqrt{9+2}\\&=\sqrt{11}\end{aligned}$$

b. $$\begin{aligned}(g\circ f)(-2)&=g(f(-2))\\&=g\left(\sqrt{-2+2}\right)\\&=g(0)\\&=2(0)^2+1\\&=1\end{aligned}$$

c. $$\begin{aligned}(f\circ f)(4)&=f(f(4))\\&=f\left(\sqrt{4+2}\right)\\&=f\left(\sqrt{6}\right)\\&=\sqrt{\sqrt{6}+2}\end{aligned}$$

d. $$\begin{aligned}(g\circ g)(-1)&=g(g(-1))\\&=g\left(2(-1)^2+1\right)\\&=g(3)\\&=2(3)^2+1\\&=19\end{aligned}$$

4. $f(x)=e^x \qquad g(x)=3x-2$

a. $$\begin{aligned}(f\circ g)(2)&=f(g(2))\\&=f\left(3(2)-2\right)\\&=f(4)\\&=e^4\end{aligned}$$

b. $$\begin{aligned}(g\circ f)(-2)&=g(f(-2))\\&=g\left(e^{-2}\right)\\&=3e^{-2}-2\\&=\frac{3}{e^2}-2\end{aligned}$$

c. $$\begin{aligned}(f\circ f)(4)&=f(f(4))\\&=f\left(e^4\right)\\&=e^{e^4}\end{aligned}$$

d. $$\begin{aligned}(g\circ g)(-1)&=g(g(-1))\\&=g\left(3(-1)-2\right)\\&=g(-5)\\&=3(-5)-2\\&=-17\end{aligned}$$

5. $f(x)=2-x \qquad g(x)=3x+1$

The domain of f is $\{x \mid x \text{ is any real number}\}$.

The domain of g is $\{x \mid x \text{ is any real number}\}$.

$$\begin{aligned}(f\circ g)(x)&=f(g(x))\\&=f(3x+1)\\&=2-(3x+1)\\&=2-3x-1\\&=1-3x\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

$$\begin{aligned}(g\circ f)(x)&=g(f(x))\\&=g(2-x)\\&=3(2-x)+1\\&=6-3x+1\\&=7-3x\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

$$\begin{aligned}(f\circ f)(x)&=f(f(x))\\&=f(2-x)\\&=2-(2-x)\\&=2-2+x\\&=x\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

$$\begin{aligned}(g\circ g)(x)&=g(g(x))\\&=g(3x+1)\\&=3(3x+1)+1\\&=9x+3+1\\&=9x+4\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

6. $f(x)=\sqrt{3x}\qquad g(x)=1+x+x^2$

The domain of f is $\{x \mid x\ge 0\}$.

The domain of g is $\{x \mid x \text{ is any real number}\}$.

$$\begin{aligned}(f\circ g)(x)&=f(g(x))\\&=f\left(1+x+x^2\right)\\&=\sqrt{3\left(1+x+x^2\right)}\\&=\sqrt{3+3x+3x^2}\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

$$\begin{aligned}(g\circ f)(x)&=g(f(x))\\&=g\left(\sqrt{3x}\right)\\&=1+\sqrt{3x}+\left(\sqrt{3x}\right)^2\\&=1+\sqrt{3x}+3x\end{aligned}$$

Domain: $\{x \mid x\ge 0\}$.

$$(f\circ f)(x)=f(f(x))=f\left(\sqrt{3x}\right)=\sqrt{3\sqrt{3x}}$$

Domain: $\{x \mid x\ge 0\}$.

$$\begin{aligned}(g\circ g)(x)&=g(g(x))\\&=g\left(1+x+x^2\right)\\&=1+\left(1+x+x^2\right)+\left(1+x+x^2\right)^2\\&=1+1+x+x^2+1+2x+3x^2+2x^3+x^4\\&=3+3x+4x^2+2x^3+x^4\end{aligned}$$

Domain: $\{x \mid x \text{ is any real number}\}$.

7. $f(x)=\dfrac{x+1}{x-1}\qquad g(x)=\dfrac{1}{x}$

The domain of f is $\{x \mid x\ne 1\}$.

The domain of g is $\{x \mid x\ne 0\}$.

$$\begin{aligned}(f\circ g)(x)&=f(g(x))\\&=f\left(\frac{1}{x}\right)=\frac{\frac{1}{x}+1}{\frac{1}{x}-1}\\&=\frac{\left(\frac{1}{x}+1\right)x}{\left(\frac{1}{x}-1\right)x}=\frac{1+x}{1-x}\end{aligned}$$

Domain $\{x \mid x\ne 0,\ x\ne 1\}$.

$$\begin{aligned}(g\circ f)(x)&=g(f(x))\\&=g\left(\frac{x+1}{x-1}\right)=\frac{1}{\left(\frac{x+1}{x-1}\right)}=\frac{x-1}{x+1}\end{aligned}$$

Domain $\{x \mid x\ne -1,\ x\ne 1\}$

$$\begin{aligned}(f\circ f)(x)&=f(f(x))\\&=f\left(\frac{x+1}{x-1}\right)=\frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1}\\&=\frac{\left(\frac{x+1}{x-1}+1\right)(x-1)}{\left(\frac{x+1}{x-1}-1\right)(x-1)}\\&=\frac{x+1+x-1}{x+1-(x-1)}=\frac{2x}{2}=x\end{aligned}$$

Domain $\{x \mid x\ne 1\}$.

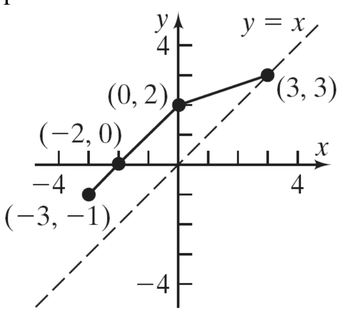
$$(g\circ g)(x)=g(g(x))=g\left(\frac{1}{x}\right)=\frac{1}{\left(\frac{1}{x}\right)}=x$$

Domain $\{x \mid x\ne 0\}$.

8. **a.** The function is one-to-one because there are no two distinct inputs that correspond to the same output.

b. The inverse is $\{(2,1),(5,3),(8,5),(10,6)\}$.

9. The function f is one-to-one because every horizontal line intersects the graph at exactly one point.



10.
$$f(x)=\frac{2x+3}{5x-2}$$
$$y=\frac{2x+3}{5x-2}$$
$$x=\frac{2y+3}{5y-2}\quad \text{Inverse}$$
$$x(5y-2)=2y+3$$
$$5xy-2x=2y+3$$
$$5xy-2y=2x+3$$
$$y(5x-2)=2x+3$$
$$y=\frac{2x+3}{5x-2}$$
$$f^{-1}(x)=\frac{2x+3}{5x-2}$$

Domain of f = Range of f^{-1}

= All real numbers except $\frac{2}{5}$.

Range of f = Domain of f^{-1}

= All real numbers except $\frac{2}{5}$.

11.
$$f(x)=\frac{1}{x-1}$$
$$y=\frac{1}{x-1}$$
$$x=\frac{1}{y-1}\quad \text{Inverse}$$
$$x(y-1)=1$$
$$xy-x=1$$
$$xy=x+1$$
$$y=\frac{x+1}{x}$$
$$f^{-1}(x)=\frac{x+1}{x}$$

Domain of f = Range of f^{-1}

= All real numbers except 1

Range of f = Domain of f^{-1}

= All real numbers except 0

12.
$$f(x)=\sqrt{x-2}$$
$$y=\sqrt{x-2}$$
$$x=\sqrt{y-2}\quad \text{Inverse}$$
$$x^2=y-2\quad x\ge 0$$
$$y=x^2+2\quad x\ge 0$$
$$f^{-1}(x)=x^2+2\quad x\ge 0$$

Domain of f = Range of $f^{-1}=\{x\,|\,x\ge 2\}$ or $[2,\infty)$

Range of f = Domain of $f^{-1}=\{x\,|\,x\ge 0\}$ or $[0,\infty)$

13.
$$f(x)=x^{1/3}+1$$
$$y=x^{1/3}+1$$
$$x=y^{1/3}+1\quad \text{Inverse}$$
$$y^{1/3}=x-1$$
$$y=(x-1)^3$$
$$f^{-1}(x)=(x-1)^3$$

Domain of f = Range of f^{-1}

= All real numbers or $(-\infty,\infty)$

Range of f = Domain of f^{-1}

= All real numbers or $(-\infty,\infty)$

14. **a.** $f(4)=3^4=81$

b. $g(9)=\log_3(9)=\log_3(3^2)=2$

c. $f(-2)=3^{-2}=\frac{1}{9}$

d. $g\left(\frac{1}{27}\right)=\log_3\left(\frac{1}{27}\right)=\log_3\left(3^{-3}\right)=-3$

15. $5^2=z$ is equivalent to $2=\log_5 z$

16. $\log_5 u=13$ is equivalent to $5^{13}=u$

17. $f(x) = \log(3x-2)$ requires:

$3x-2>0$

$x>\frac{2}{3}$

Domain: $\left\{x \middle| x>\frac{2}{3}\right\}$ or $\left(\frac{2}{3},\infty\right)$

18. $H(x) = \log_2\left(x^2-3x+2\right)$ requires

$p(x)=x^2-3x+2>0$

$(x-2)(x-1)>0$

$x=2$ and $x=1$ are the zeros of p.

Interval	$(-\infty,1)$	$(1,2)$	$(2,\infty)$
Test Value	0	$\frac{3}{2}$	3
Value of p	2	$-\frac{1}{4}$	2
Conclusion	positive	negative	positive

Thus, the domain of $H(x)=\log_2\left(x^2-3x+2\right)$ is $\{x \mid x<1 \text{ or } x>2\}$ or $(-\infty,1)\cup(2,\infty)$.

19. $\log_2\left(\frac{1}{8}\right)=\log_2 2^{-3}=-3\log_2 2=-3$

20. $\ln e^{\sqrt{2}}=\sqrt{2}$

21. $2^{\log_2 0.4}=0.4$

22. $\log_3\left(\frac{uv^2}{w}\right)=\log_3 uv^2-\log_3 w$

$=\log_3 u+\log_3 v^2-\log_3 w$

$=\log_3 u+2\log_3 v-\log_3 w$

23. $\log_2\left(a^2\sqrt{b}\right)^4=4\log_2\left(a^2\sqrt{b}\right)$

$=4\left(\log_2 a^2+\log_2 b^{1/2}\right)$

$=4\left(2\log_2 a+\frac{1}{2}\log_2 b\right)$

$=8\log_2 a+2\log_2 b$

24. $\log\left(x^2\sqrt{x^3+1}\right)=\log x^2+\log\left(x^3+1\right)^{1/2}$

$=2\log x+\frac{1}{2}\log\left(x^3+1\right)$

25. $\ln\left(\frac{2x+3}{x^2-3x+2}\right)^2$

$=2\ln\left(\frac{2x+3}{x^2-3x+2}\right)$

$=2\left(\ln(2x+3)-\ln\left[(x-1)(x-2)\right]\right)$

$=2\left(\ln(2x+3)-\ln(x-1)-\ln(x-2)\right)$

$=2\ln(2x+3)-2\ln(x-1)-2\ln(x-2)$

26. $3\log_4 x^2+\frac{1}{2}\log_4\sqrt{x}=\log_4\left(x^2\right)^3+\log_4\left(x^{1/2}\right)^{1/2}$

$=\log_4 x^6+\log_4 x^{1/4}$

$=\log_4\left(x^6\cdot x^{1/4}\right)$

$=\log_4 x^{25/4}$

$=\frac{25}{4}\log_4 x$

27. $\ln\left(\frac{x-1}{x}\right)+\ln\left(\frac{x}{x+1}\right)-\ln\left(x^2-1\right)$

$=\ln\left(\frac{x-1}{x}\cdot\frac{x}{x+1}\right)-\ln\left(x^2-1\right)$

$=\ln\left[\frac{\frac{x-1}{x+1}}{x^2-1}\right]$

$=\ln\left(\frac{x-1}{x+1}\cdot\frac{1}{(x-1)(x+1)}\right)$

$=\ln\frac{1}{(x+1)^2}$

$=\ln(x+1)^{-2}$

$=-2\ln(x+1)$

28. $\frac{1}{2}\ln\left(x^2+1\right)-4\ln\frac{1}{2}-\frac{1}{2}\left[\ln(x-4)+\ln x\right]$

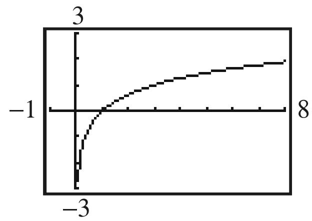
$=\ln\left(x^2+1\right)^{1/2}-\ln\left(\frac{1}{2}\right)^4-\ln\left(x(x-4)\right)^{1/2}$

$=\ln\left(\frac{\left(x^2+1\right)^{1/2}}{\frac{1}{16}\left[x(x-4)\right]^{1/2}}\right)$

$=\ln\left(\frac{16\sqrt{x^2+1}}{\sqrt{x(x-4)}}\right)$

29. $\log_4 19=\frac{\ln 19}{\ln 4}\approx 2.124$

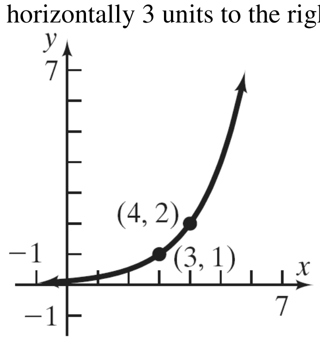
30. $Y_1 = \log_3 x = \dfrac{\ln x}{\ln 3}$



31. $f(x) = 2^{x-3}$

a. Domain: $(-\infty,\ \infty)$

b. Using the graph of $y = 2^x$, shift the graph horizontally 3 units to the right.



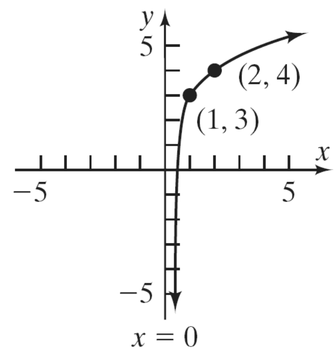
c. Range: $(0, \infty)$
Horizontal Asymptote: $y = 0$

d.
$$
\begin{aligned}
f(x) &= 2^{x-3} \\
y &= 2^{x-3} \\
x &= 2^{y-3} \quad \text{Inverse} \\
y-3 &= \log_2 x \\
y &= 3+\log_2 x \\
f^{-1}(x) &= 3+\log_2 x
\end{aligned}
$$

e. Range of f = Domain f^{-1}: $(0, \infty)$

Domain of f = Range of f^{-1}: $(-\infty,\ \infty)$

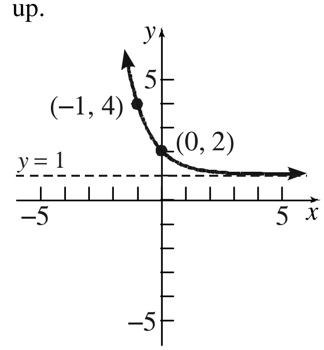
f. Using the graph of $y = \log_2 x$, shift the graph vertically 3 units up.



32. $f(x) = 1 + 3^{-x}$

a. Domain: $(-\infty, \infty)$

b. Using the graph of $y = 3^x$, reflect the graph about the *y*-axis, and shift vertically 1 unit up.

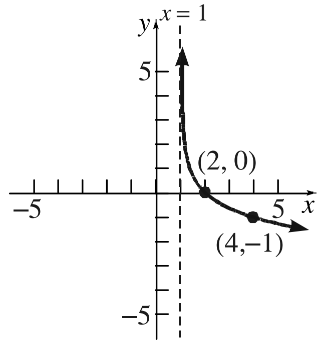


c. Range: $(1, \infty)$
Horizontal Asymptote: $y = 1$

d.
$$
\begin{aligned}
f(x) &= 1+3^{-x} \\
y &= 1+3^{-x} \\
x &= 1+3^{-y} \quad \text{Inverse} \\
x-1 &= 3^{-y} \\
-y &= \log_3(x-1) \\
y &= -\log_3(x-1) \\
f^{-1}(x) &= -\log_3(x-1)
\end{aligned}
$$

e. $x - 1 > 0$
$x > 1$
Range of f = Domain f^{-1}: $(1, \infty)$
Domain of f = Range of f^{-1}: $(-\infty, \infty)$

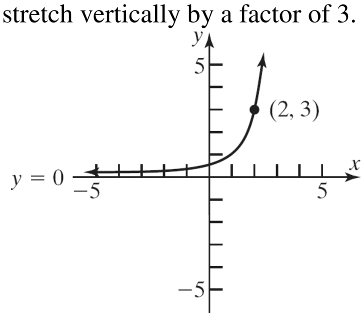
f. Using the graph of $y = \log_3 x$, shift the graph horizontally to the right 1 unit, and reflect vertically about the x-axis.



33. $f(x) = 3e^{x-2}$

a. Domain: $(-\infty, \infty)$

b. Using the graph of $y = e^x$, shift the graph two units horizontally to the right, and stretch vertically by a factor of 3.

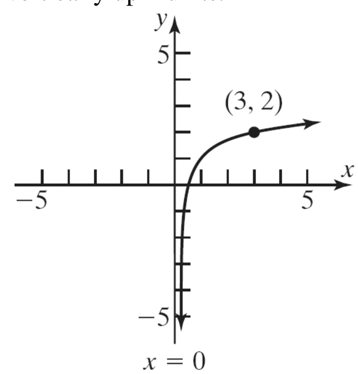


c. Range: $(0, \infty)$
Horizontal Asymptote: $y = 0$

d.
$$f(x) = 3e^{x-2}$$
$$y = 3e^{x-2}$$
$$x = 3e^{y-2} \quad \text{Inverse}$$
$$\frac{x}{3} = e^{y-2}$$
$$y - 2 = \ln\left(\frac{x}{3}\right)$$
$$y = 2 + \ln\left(\frac{x}{3}\right)$$
$$f^{-1}(x) = 2 + \ln\left(\frac{x}{3}\right)$$

e. Range of f = Domain f^{-1}: $(0, \infty)$
Domain of f = Range of f^{-1}: $(-\infty, \infty)$

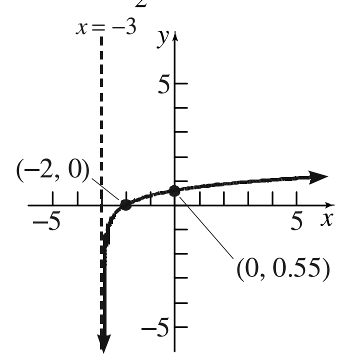
f. Using the graph of $y = \ln x$, stretch horizontally by a factor of 3, and shift vertically up 2 units.



34. $f(x) = \frac{1}{2}\ln(x+3)$

a. Domain: $(-3, \infty)$

b. Using the graph of $y = \ln x$, shift the graph to the left 3 units and compress vertically by a factor of $\frac{1}{2}$.

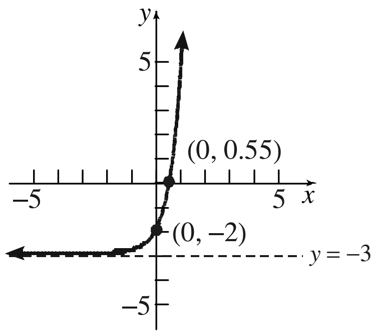


c. Range: $(-\infty, \infty)$
Vertical Asymptote: $x = -3$

d.
$$f(x) = \frac{1}{2}\ln(x+3)$$
$$y = \frac{1}{2}\ln(x+3)$$
$$x = \frac{1}{2}\ln(y+3) \quad \text{Inverse}$$
$$2x = \ln(y+3)$$
$$y + 3 = e^{2x}$$
$$y = e^{2x} - 3$$
$$f^{-1}(x) = e^{2x} - 3$$

e. Range of f = Domain f^{-1}: $(-\infty, \infty)$
Domain of f = Range of f^{-1}: $(-3, \infty)$

f. Using the graph of $y = e^x$, compress horizontally by a factor of $\frac{1}{2}$, and shift down 3 units.



35.
$$8^{6+3x} = 4$$
$$\left(2^3\right)^{6+3x} = 2^2$$
$$2^{18+9x} = 2^2$$
$$18 + 9x = 2$$
$$9x = -16$$
$$x = -\frac{16}{9}$$
The solution set is $\left\{-\frac{16}{9}\right\}$.

36.
$$3^{x^2+x} = \sqrt{3}$$
$$3^{x^2+x} = 3^{1/2}$$
$$x^2 + x = \frac{1}{2}$$
$$2x^2 + 2x - 1 = 0$$
$$x = \frac{-2 \pm \sqrt{2^2 - 4(2)(-1)}}{2(2)}$$
$$= \frac{-2 \pm \sqrt{12}}{4} = \frac{-2 \pm 2\sqrt{3}}{4} = \frac{-1 \pm \sqrt{3}}{2}$$
The solution is $\left\{\frac{-1-\sqrt{3}}{2}, \ \frac{-1+\sqrt{3}}{2}\right\} \approx \{-1.366, 0.366\}$.

37.
$$\log_x 64 = -3$$
$$x^{-3} = 64$$
$$\left(x^{-3}\right)^{-1/3} = 64^{-1/3}$$
$$x = \frac{1}{\sqrt[3]{64}} = \frac{1}{4}$$
The solution set is $\left\{\frac{1}{4}\right\}$.

38.
$$5^x = 3^{x+2}$$
$$\ln\left(5^x\right) = \ln\left(3^{x+2}\right)$$
$$x \ln 5 = (x+2)\ln 3$$
$$x \ln 5 = x \ln 3 + 2\ln 3$$
$$x \ln 5 - x \ln 3 = 2\ln 3$$
$$x(\ln 5 - \ln 3) = 2\ln 3$$
$$x = \frac{2\ln 3}{\ln 5 - \ln 3} \approx 4.301$$
The solution set is $\left\{\frac{2\ln 3}{\ln 5 - \ln 3}\right\} \approx \{4.301\}$.

39.
$$25^{2x} = 5^{x^2-12}$$
$$\left(5^2\right)^{2x} = 5^{x^2-12}$$
$$5^{4x} = 5^{x^2-12}$$
$$4x = x^2 - 12$$
$$x^2 - 4x - 12 = 0$$
$$(x-6)(x+2) = 0$$
$$x = 6 \ \text{ or } \ x = -2$$
The solution set is $\{-2, 6\}$.

40.
$$\log_3 \sqrt{x-2} = 2$$
$$\sqrt{x-2} = 3^2$$
$$\sqrt{x-2} = 9$$
$$x - 2 = 9^2$$
$$x - 2 = 81$$
$$x = 83$$
Check: $\log_3 \sqrt{83-2} = \log_3 \sqrt{81}$
$= \log_3 9$
$= 2$

The solution set is $\{83\}$.

41. $8 = 4^{x^2} \cdot 2^{5x}$

$2^3 = \left(2^2\right)^{x^2} \cdot 2^{5x}$

$2^3 = 2^{2x^2+5x}$

$3 = 2x^2 + 5x$

$0 = 2x^2 + 5x - 3$

$0 = (2x-1)(x+3)$

$x = \frac{1}{2}$ or $x = -3$

The solution set is $\left\{-3, \frac{1}{2}\right\}$.

42. $2^x \cdot 5 = 10^x$

$\ln\left(2^x \cdot 5\right) = \ln 10^x$

$\ln 2^x + \ln 5 = \ln 10^x$

$x \ln 2 + \ln 5 = x \ln 10$

$\ln 5 = x \ln 10 - x \ln 2$

$\ln 5 = x(\ln 10 - \ln 2)$

$\frac{\ln 5}{\ln 10 - \ln 2} = x$

$x = \frac{\ln 5}{\ln \frac{10}{2}} = \frac{\ln 5}{\ln 5} = 1$

The solution set is $\{1\}$.

43. $\log_6(x+3) + \log_6(x+4) = 1$

$\log_6\left((x+3)(x+4)\right) = 1$

$(x+3)(x+4) = 6^1$

$x^2 + 7x + 12 = 6$

$x^2 + 7x + 6 = 0$

$(x+6)(x+1) = 0$

$x = -6$ or $x = -1$

Since $\log_6(-6+3) = \log_6(-3)$ is undefined, the solution set is $\{-1\}$.

44. $e^{1-x} = 5$

$1 - x = \ln 5$

$-x = -1 + \ln 5$

$x = 1 - \ln 5 \approx -0.609$

The solution set is $\{1 - \ln 5\} \approx \{-0.609\}$.

45. $9^x + 4 \cdot 3^x - 3 = 0$

$\left(3^2\right)^x + 4 \cdot 3^x - 3 = 0$

$\left(3^x\right)^2 + 4 \cdot 3^x - 3 = 0$

Let $u = 3^x$.

$u^2 + 4u - 3 = 0$

$a = 1, b = 4, c = -3$

$u = \frac{-(4) \pm \sqrt{(4)^2 - 4(1)(-3)}}{2(1)}$

$= \frac{-4 \pm \sqrt{28}}{2} = \frac{-4 \pm 2\sqrt{7}}{2} = -2 \pm \sqrt{7}$

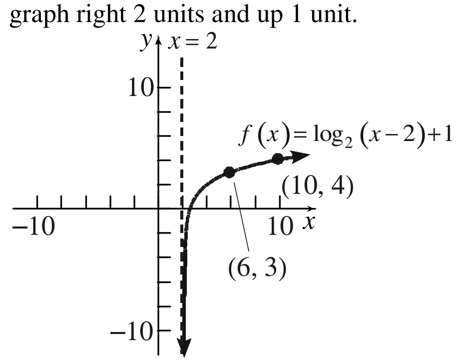
~~$3^x = -2 - \sqrt{7}$~~ or $3^x = -2 + \sqrt{7}$

3^x can't be negative $\quad x = \log_3\left(-2 + \sqrt{7}\right)$

The solution set is $\left\{\log_3\left(-2 + \sqrt{7}\right)\right\} \approx \{-0.398\}$.

46. a. $f(x) = \log_2(x-2) + 1$

Using the graph of $y = \log_2 x$, shift the graph right 2 units and up 1 unit.



b. $f(6) = \log_2(6-2) + 1$

$= \log_2(4) + 1 = 2 + 1 = 3$

The point $(6, 3)$ is on the graph of *f*.

c. $f(x) = 4$

$\log_2(x-2) + 1 = 4$

$\log_2(x-2) = 3$

$x - 2 = 2^3$

$x - 2 = 8$

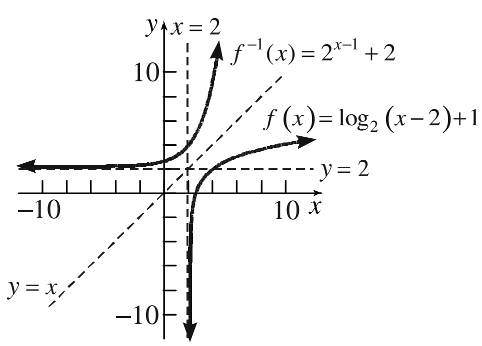
$x = 10$

The solution set is $\{10\}$. The point $(10, 4)$ is on the graph of *f*.

d. $$f(x)=0$$
$$\log_2(x-2)+1=0$$
$$\log_2(x-2)=-1$$
$$x-2=2^{-1}$$
$$x-2=\frac{1}{2}$$
$$x=\frac{5}{2}$$

Based on the graph drawn in part (a), $f(x)>0$ when $x>\frac{5}{2}$. The solution set is $\left\{x \mid x>\frac{5}{2}\right\}$ or $\left(\frac{5}{2},\infty\right)$.

e. $$f(x)=\log_2(x-2)+1$$
$$y=\log_2(x-2)+1$$
$$x=\log_2(y-2)+1 \quad \text{Inverse}$$
$$x-1=\log_2(y-2)$$
$$y-2=2^{x-1}$$
$$y=2^{x-1}+2$$
$$f^{-1}(x)=2^{x-1}+2$$



47. $P=25e^{0.1d}$

a. $P=25e^{0.1(4)}=25e^{0.4}\approx 37.3$ watts

b. $$50=25e^{0.1d}$$
$$2=e^{0.1d}$$
$$\ln 2=0.1d$$
$$d=\frac{\ln 2}{0.1}\approx 6.9 \text{ decibels}$$

48. $L=9+(5.1)\log d$

a. $L=9+(5.1)\log 3.5\approx 11.77$

b. $$14=9+(5.1)\log d$$
$$5=(5.1)\log d$$
$$\log d=\frac{5}{5.1}\approx 0.9804$$
$$d\approx 10^{0.9804}\approx 9.56 \text{ inches}$$

49. a. $n=\dfrac{\log 10{,}000-\log 90{,}000}{\log(1-0.20)}\approx 9.85$ years

b. $$n=\frac{\log(0.5i)-\log(i)}{\log(1-0.15)}=\frac{\log\left(\frac{0.5i}{i}\right)}{\log 0.85}=\frac{\log 0.5}{\log 0.85}\approx 4.27 \text{ years}$$

50. In 18 years, $A=10{,}000\left(1+\frac{0.04}{2}\right)^{(2)(18)}$
$$=10{,}000(1.02)^{36}$$
$$\approx \$20{,}398.87$$

The effective interest rate is computed as follows:

When $t=1$, $A=10{,}000\left(1+\frac{0.04}{2}\right)^{(2)(1)}$
$$=10{,}000(1.02)^2$$
$$=\$10{,}404$$

Note, $\dfrac{10{,}404-10{,}000}{10{,}000}=\dfrac{404}{10{,}000}=0.0404$, so the effective interest rate is 4.04%.

In order for the bond to double in value, we have the equation: $A=2P$.
$$10{,}000\left(1+\frac{0.04}{2}\right)^{2t}=20{,}000$$
$$(1.02)^{2t}=2$$
$$2t\ln 1.02=\ln 2$$
$$t=\frac{\ln 2}{2\ln 1.02}\approx 17.5 \text{ years}$$

51. $P = A\left(1+\frac{r}{n}\right)^{-nt} = 85,000\left(1+\frac{0.04}{2}\right)^{-2(18)}$
$\approx \$41,668.97$

52.
$$\begin{aligned}
A &= A_0 e^{kt}\\
0.5A_0 &= A_0 e^{k(5700)}\\
0.5 &= e^{5700k}\\
\ln 0.5 &= 5700k\\
k &= \frac{\ln 0.5}{5700}\\
0.05A_0 &= A_0 e^{\left(\frac{\ln 0.5}{5700}\right)t}\\
0.05 &= e^{\left(\frac{\ln 0.5}{5700}\right)t}\\
\ln 0.05 &= \left(\frac{\ln 0.5}{5700}\right)t\\
t &= \frac{\ln 0.05}{\left(\frac{\ln 0.5}{5700}\right)} \approx 24,635
\end{aligned}$$
The man died approximately 24,635 years ago.

53. Using $u = T + (u_0 - T)e^{kt}$, with $t = 5$, $T = 70$, $u_0 = 450$, and $u = 400$.
$$\begin{aligned}
400 &= 70 + (450-70)e^{k(5)}\\
330 &= 380e^{5k}\\
\frac{330}{380} &= e^{5k}\\
\ln\left(\frac{330}{380}\right) &= 5k\\
k &= \frac{\ln(330/380)}{5}
\end{aligned}$$

Find time for temperature of 150˚F:
$$\begin{aligned}
150 &= 70 + (450-70)e^{\left(\frac{\ln(330/380)}{5}\right)t}\\
80 &= 380e^{\left(\frac{\ln(330/380)}{5}\right)t}\\
\frac{80}{380} &= e^{\left(\frac{\ln(330/380)}{5}\right)t}\\
\ln\left(\frac{80}{380}\right) &= \left(\frac{\ln(330/380)}{5}\right)t\\
t &= \frac{\ln\left(\frac{80}{380}\right)}{\frac{\ln(330/380)}{5}} \approx 55.22
\end{aligned}$$
The temperature of the skillet will be 150˚F after approximately 55.22 minutes (or 55 minutes, 13 seconds).

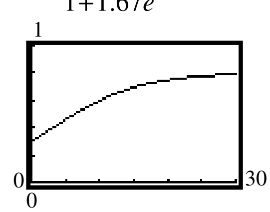
54. $P_0 = 6,852,472,823$, $k = 0.0115$, and $t = 2015 - 2010 = 5$
$P = P_0 e^{kt} = 6,852,472,823e^{0.0115(5)}$
$\approx 7,258,038,282$ people

55. $A = P\left(1+\frac{r}{n}\right)^{nt}$
$A = 1300\left(1+\frac{0.0335}{1}\right)^{1(10)}$
$= 1300(1.0335)^{10} \approx 1807.4$
The government would have to pay back approximately \$1.807 trillion in 2020. The interest would be 1.807 – 1.3 = .507 or \$507 billion dollars.

56. **a.** $P(0) = \frac{0.8}{1+1.67e^{-0.16(0)}} = \frac{0.8}{1+1.67} \approx 0.3$
In 2006, about 30% of cars had a GPS.

b. The maximum proportion is the carrying capacity, $c = 0.8 = 80\%$.

c. $Y_1 = \frac{0.8}{1+1.67e^{-0.16x}}$

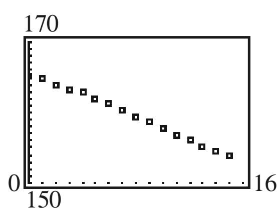


d. Find t such that $P(t) = 0.75$.

$$\frac{0.8}{1+1.67e^{-0.16t}} = 0.75$$
$$0.8 = 0.75\left(1+1.67e^{-0.16t}\right)$$
$$\frac{0.8}{0.75} = 1+1.67e^{-0.16t}$$
$$\frac{0.8}{0.75} - 1 = 1.67e^{-0.16t}$$
$$\frac{\frac{0.8}{0.75}-1}{1.67} = e^{-0.16t}$$
$$\ln\left(\frac{\frac{0.8}{0.75}-1}{1.67}\right) = -0.16t$$
$$t = \frac{\ln\left(\frac{\frac{0.8}{0.75}-1}{1.67}\right)}{-0.16} \approx 20.13$$

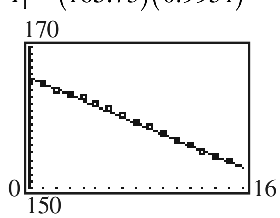
Note that $2006 + 20.13 = 2026.13$, so 75% of new cars will have GPS in 2026.

57. a.



b. Using EXPonential REGression on the data yields: $y = (165.73)(0.9951)^x$

c. $Y_1 = (165.73)(0.9951)^x$

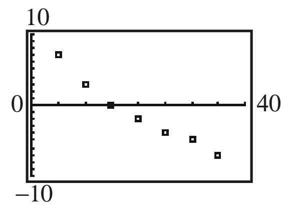


d. Find x when $y = 110$.

$$(165.73)(0.9951)^x = 110$$
$$(0.9951)^x = \frac{110}{165.73}$$
$$x\ln 0.9951 = \ln\left(\frac{110}{165.73}\right)$$
$$x = \frac{\ln\left(\frac{110}{165.73}\right)}{\ln 0.9951} \approx 83$$

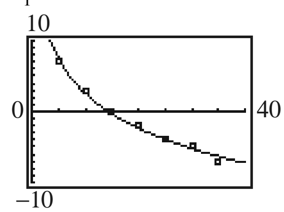
Therefore, it will take approximately 83 seconds for the probe to reach a temperature of 110˚F.

58. a.



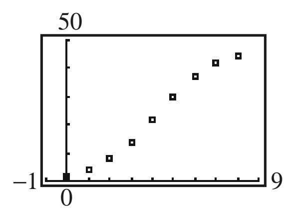
b. Using LnREGression on the data yields: $y = 18.9028 - 7.0963\ln x$ where y = wind chill and x = wind speed.

c. $Y_1 = 18.9028 - 7.0963\ln x$



d. If $x = 23$, then $y = 18.9028 - 7.0963\ln 23 \approx -3^{\circ}\text{F}$.

59. a.

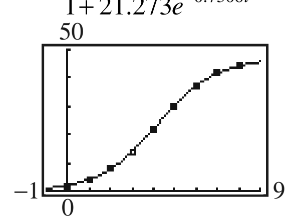


The data appear to have a logistic relation.

b. Using LOGISTIC REGression on the data yields:

$$C = \frac{46.93}{1+21.273e^{-0.7306t}}$$

c. $Y_1 = \dfrac{46.93}{1+21.273e^{-0.7306t}}$



d. As $t \to \infty$, $21.2733e^{-0.7306t} \to 0$, which means $1+21.2733e^{-0.7306t} \to 1$, so

$$C = \frac{46.9292}{1+21.2733e^{-0.7306t}} \to 46.9292$$

Therefore, according to the function, a

maximum of about 47 people can catch the cold.

In reality, all 50 people living in the town might catch the cold.

e. Find t when $C = 10$.

$$\frac{46.9292}{1+21.2733e^{-0.7306t}} = 10$$
$$46.9292 = 10\left(1+21.2733e^{-0.7306t}\right)$$
$$\frac{46.9292}{10} = 1+21.2733e^{-0.7306t}$$
$$\frac{46.9292}{10} - 1 = 21.2733e^{-0.7306t}$$
$$3.69292 = 21.2733e^{-0.7306t}$$
$$\frac{3.69292}{21.2733} = e^{-0.7306t}$$
$$\ln\left(\frac{3.69292}{21.2733}\right) = -0.7306t$$
$$\frac{\ln\left(\frac{3.69292}{21.2733}\right)}{-0.7306} = t$$
$$t \approx 2.4$$

Therefore, after approximately 2.4 days (during the 10th hour on the 3rd day), 10 people had caught the cold.

f. Find t when $C = 46$.

$$\frac{46.9292}{1+21.2733e^{-0.7306t}} = 46$$
$$46.9292 = 46\left(1+21.2733e^{-0.7306t}\right)$$
$$\frac{46.9292}{46} = 1+21.2733e^{-0.7306t}$$
$$\frac{46.9292}{46} - 1 = 21.2733e^{-0.7306t}$$
$$0.0202 = 21.2733e^{-0.7306t}$$
$$\frac{0.0202}{21.2733} = e^{-0.7306t}$$
$$\frac{0.0202}{21.2733} = e^{-0.7306t}$$
$$\ln\left(\frac{0.0202}{21.2733}\right) = -0.7306t$$
$$\frac{\ln\left(\frac{0.0202}{21.2733}\right)}{-0.7306} = t \approx 9.5$$

Therefore, after approximately 9.5 days (during the 12th hour on the 10th day), 46 people had caught the cold.

Chapter 5 Test

1. $f(x) = \dfrac{x+2}{x-2}$ $\qquad g(x) = 2x+5$

The domain of f is $\{x \mid x \neq 2\}$.

The domain of g is all real numbers.

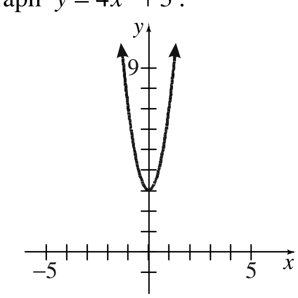
a.
$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f(2x+5) \\ &= \frac{(2x+5)+2}{(2x+5)-2} \\ &= \frac{2x+7}{2x+3}\end{aligned}$$

Domain $\left\{x \,\middle|\, x \neq -\dfrac{3}{2}\right\}$.

b.
$$\begin{aligned}(g \circ f)(-2) &= g(f(-2)) \\ &= g\left(\frac{-2+2}{-2-2}\right) \\ &= g(0) \\ &= 2(0)+5 \\ &= 5\end{aligned}$$

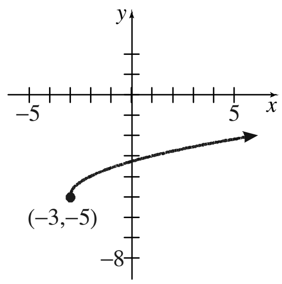
c.
$$\begin{aligned}(f \circ g)(-2) &= f(g(-2)) = f(2(-2)+5) \\ &= f(1) = \frac{1+2}{1-2} = \frac{3}{-1} = -3\end{aligned}$$

2. a. Graph $y = 4x^2 + 3$:



The function is not one-to-one because it fails the horizontal line test. A horizontal line (for example, $y = 4$) intersects the graph twice.

b. Graph $y=\sqrt{x+3}-5$:



The function is one-to-one because it passes the horizontal line test. Every horizontal line intersects the graph at most once.

3.
$$f(x)=\frac{2}{3x-5}$$
$$y=\frac{2}{3x-5}$$
$$x=\frac{2}{3y-5} \quad \text{Inverse}$$
$$x(3y-5)=2$$
$$3xy-5x=2$$
$$3xy=5x+2$$
$$y=\frac{5x+2}{3x}$$
$$f^{-1}(x)=\frac{5x+2}{3x}$$

Domain of f = Range of f^{-1} $=\left\{x \mid x\neq \frac{5}{3}\right\}$.

Range of f = Domain of f^{-1} $=\{x \mid x\neq 0\}$

4. If the point $(3,-5)$ is on the graph of *f*, then the point $(-5, 3)$ must be on the graph of f^{-1}.

5.
$$3^x=243$$
$$3^x=3^5$$
$$x=5$$

6.
$$\log_b 16=2$$
$$b^2=16$$
$$b=\pm\sqrt{16}=\pm 4$$

Since the base of a logarithm must be positive, the only viable solution is $b=4$.

7.
$$\log_5 x=4$$
$$x=5^4$$
$$x=625$$

8. $e^3+2\approx 22.086$

9. $\log 20\approx 1.301$

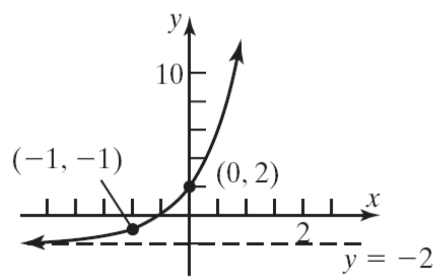
10. $\log_3 21=\frac{\ln 21}{\ln 3}\approx 2.771$

11. $\ln 133\approx 4.890$

12. $f(x)=4^{x+1}-2$

a. Domain: $(-\infty,\infty)$

b. Using the graph of $y=4^x$, shift the graph 1 unit to the left, and shift 2 units down.

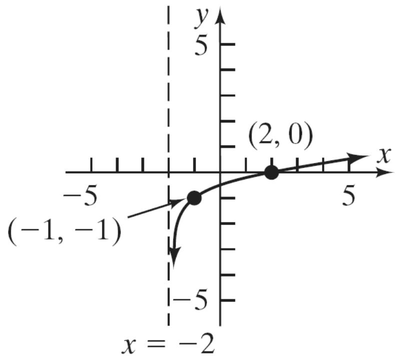


c. Range: $(-2,\infty)$
Horizontal Asymptote: $y=-2$

d.
$$f(x)=4^{x+1}-2$$
$$y=4^{x+1}-2$$
$$x=4^{y+1}-2 \quad \text{Inverse}$$
$$x+2=4^{y+1}$$
$$y+1=\log_4(x+2)$$
$$y=\log_4(x+2)-1$$
$$f^{-1}(x)=\log_4(x+2)-1$$

e. Range of f = Domain f^{-1}: $(-2,\infty)$
Domain of f = Range of f^{-1}: $(-\infty,\infty)$

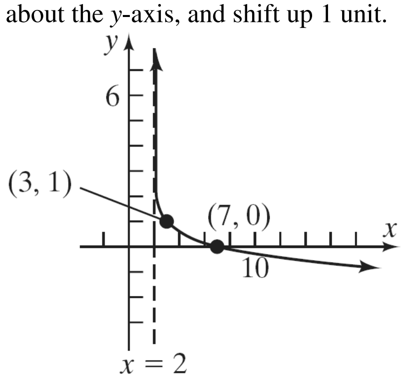
f. Using the graph of $y=\log_4 x$, shift the graph 2 units to the left, and shift down 1 unit.



13. $f(x) = 1 - \log_5(x-2)$

a. Domain: $(2, \infty)$

b. Using the graph of $y = \log_5 x$, shift the graph to the right 2 units, reflect vertically about the *y*-axis, and shift up 1 unit.

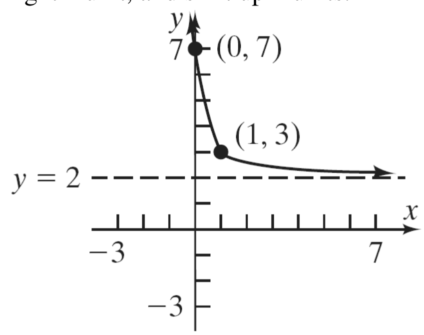


c. Range: $(-\infty, \infty)$
Vertical Asymptote: $x = 2$

d.
$$
\begin{aligned}
f(x) &= 1 - \log_5(x-2) \\
y &= 1 - \log_5(x-2) \\
x &= 1 - \log_5(y-2) \quad \text{Inverse} \\
x - 1 &= -\log_5(y-2) \\
1 - x &= \log_5(y-2) \\
y - 2 &= 5^{1-x} \\
y &= 5^{1-x} + 2 \\
f^{-1}(x) &= 5^{1-x} + 2
\end{aligned}
$$

e. Range of f = Domain f^{-1}: $(-\infty, \infty)$
Domain of f = Range of f^{-1}: $(2, \infty)$

f. Using the graph of $y = 5^x$, reflect the graph horizontally about the *y*-axis, shift to the right 1 unit, and shift up 2 units.



14.
$$
\begin{aligned}
5^{x+2} &= 125 \\
5^{x+2} &= 5^3 \\
x + 2 &= 3 \\
x &= 1
\end{aligned}
$$
The solution set is $\{1\}$.

15.
$$
\begin{aligned}
\log(x+9) &= 2 \\
x + 9 &= 10^2 \\
x + 9 &= 100 \\
x &= 91
\end{aligned}
$$
The solution set is $\{91\}$.

16.
$$
\begin{aligned}
8 - 2e^{-x} &= 4 \\
-2e^{-x} &= -4 \\
e^{-x} &= 2 \\
-x &= \ln 2 \\
x &= -\ln 2 \approx -0.693
\end{aligned}
$$
The solution set is $\{-\ln 2\} \approx \{-0.693\}$.

17.
$$
\begin{aligned}
\log(x^2+3) &= \log(x+6) \\
x^2 + 3 &= x + 6 \\
x^2 - x - 3 &= 0 \\
x &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-3)}}{2(1)} = \frac{1 \pm \sqrt{13}}{2}
\end{aligned}
$$
The solution set is $\left\{\frac{1-\sqrt{13}}{2}, \frac{1+\sqrt{13}}{2}\right\}$
$\approx \{-1.303, 2.303\}$.

18.
$$7^{x+3} = e^x$$
$$\ln 7^{x+3} = \ln e^x$$
$$(x+3)\ln 7 = x$$
$$x\ln 7 + 3\ln 7 = x$$
$$x\ln 7 - x = -3\ln 7$$
$$x(\ln 7 - 1) = -3\ln 7$$
$$x = \frac{-3\ln 7}{\ln 7 - 1} = \frac{3\ln 7}{1-\ln 7} \approx -6.172$$

The solution set is $\left\{\frac{3\ln 7}{1-\ln 7}\right\} \approx \{-6.172\}$.

19.
$$\log_2(x-4) + \log_2(x+4) = 3$$
$$\log_2\left[(x-4)(x+4)\right] = 3$$
$$\log_2\left(x^2-16\right) = 3$$
$$x^2 - 16 = 2^3$$
$$x^2 - 16 = 8$$
$$x^2 = 24$$
$$x = \pm\sqrt{24} = \pm 2\sqrt{6}$$

Because $x = -2\sqrt{6}$ results in a negative arguments for the original logarithms, the only viable solution is $x = 2\sqrt{6}$. That is, the solution set is $\left\{2\sqrt{6}\right\} \approx \{4.899\}$.

20.
$$\log_2\left(\frac{4x^3}{x^2-3x-18}\right)$$
$$= \log_2\left(\frac{2^2 x^3}{(x+3)(x-6)}\right)$$
$$= \log_2\left(2^2 x^3\right) - \log_2\left[(x-6)(x+3)\right]$$
$$= \log_2 2^2 + \log_2 x^3 - \left[\log_2(x-6) + \log_2(x+3)\right]$$
$$= 2 + 3\log_2 x - \log_2(x-6) - \log_2(x+3)$$

21.
$$A = A_0 e^{kt}$$
$$34 = 50e^{k(30)}$$
$$0.68 = e^{30k}$$
$$\ln 0.68 = 30k$$
$$k = \frac{\ln 0.68}{30}$$

Thus, the decay model is $A = 50e^{\left(\frac{\ln 0.68}{30}\right)t}$.
We need to find t when $A = 2$:
$$2 = 50e^{\left(\frac{\ln 0.68}{30}\right)t}$$
$$0.04 = e^{\left(\frac{\ln 0.68}{30}\right)t}$$
$$\ln 0.04 = \left(\frac{\ln 0.68}{30}\right)t$$
$$t = \frac{\ln 0.04}{\left(\frac{\ln 0.68}{30}\right)} \approx 250.39$$

There will be 2 mg of the substance remaining after about 250.39 days.

22. **a.** Note that 8 months = $\frac{2}{3}$ year. Thus,

$P = 1000$, $r = 0.05$, $n = 12$, and $t = \frac{2}{3}$.

So, $A = 1000\left(1+\frac{0.05}{12}\right)^{(12)(2/3)}$
$$= 1000\left(1+\frac{0.05}{12}\right)^8$$
$$\approx \$1033.82$$

b. Note that 9 months = $\frac{3}{4}$ year. Thus,

$A = 1000$, $r = 0.05$, $n = 4$, and $t = \frac{3}{4}$. So,
$$1000 = A_0\left(1+\frac{0.05}{4}\right)^{(4)(3/4)}$$
$$1000 = A_0(1.0125)^3$$
$$A_0 = \frac{1000}{(1.0125)^3} \approx \$963.42$$

c. $r = 0.06$ and $n = 1$. So,
$$2A_0 = A_0\left(1+\frac{0.06}{1}\right)^{(1)t}$$
$$2A_0 = A_0(1.06)^t$$
$$2 = (1.06)^t$$
$$t = \log_{1.06} 2 = \frac{\ln 2}{\ln 1.06} \approx 11.9$$

It will take about 11.9 years to double your money under these conditions.

23. a. $80 = 10\log\left(\frac{I}{10^{-12}}\right)$

$8 = \log\left(\frac{I}{10^{-12}}\right)$

$8 = \log I - \log 10^{-12}$

$8 = \log I - (-12)$

$8 = \log I + 12$

$-4 = \log I$

$I = 10^{-4} = 0.0001$

If one person shouts, the intensity is 10^{-4} watts per square meter. Thus, if two people shout at the same time, the intensity will be 2×10^{-4} watts per square meter. Thus, the loudness will be

$D = 10\log\left(\frac{2\times10^{-4}}{10^{-12}}\right) = 10\log\left(2\times10^{8}\right) \approx 83$

decibels

b. Let n represent the number of people who must shout. Then the intensity will be $n\times10^{-4}$. If $D = 125$, then

$125 = 10\log\left(\frac{n\times10^{-4}}{10^{-12}}\right)$

$125 = 10\log\left(n\times10^{8}\right)$

$12.5 = \log\left(n\times10^{8}\right)$

$n\times10^{8} = 10^{12.5}$

$n = 10^{4.5} \approx 31,623$

About 31,623 people would have to shout at the same time in order for the resulting sound level to meet the pain threshold.

Chapter 5 Cumulative Review

1. a. The graph represents a function since it passes the Vertical Line Test.

The function is not a one-to-one function since the graph fails the Horizontal Line Test.

b. The graph appears to be the graph of a polynomial function. It has no gaps, holes or cusps. It has two turning points and three real zeroes. The degree must be at least three.

2. $f(x) = 2x^2 - 3x + 1$

a. $f(3) = 2(3)^2 - 3(3) + 1 = 18 - 9 + 1 = 10$

b. $f(-x) = 2(-x)^2 - 3(-x) + 1 = 2x^2 + 3x + 1$

c. $f(x+h) = 2(x+h)^2 - 3(x+h) + 1$

$= 2\left(x^2 + 2xh + h^2\right) - 3x - 3h + 1$

$= 2x^2 + 4xh + 2h^2 - 3x - 3h + 1$

3. $x^2 + y^2 = 1$

a. $\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \neq 1$; $\left(\frac{1}{2}, \frac{1}{2}\right)$ is not on the graph.

b. $\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} + \frac{3}{4} = 1$; $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ is on the graph.

4. $3(x-2) = 4(x+5)$

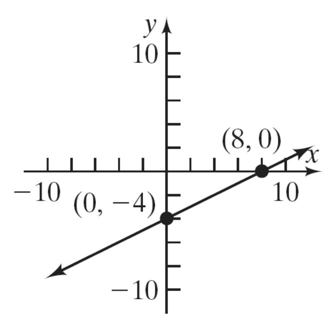
$3x - 6 = 4x + 20$

$-26 = x$

The solution set is $\{-26\}$.

5. $2x - 4y = 16$

x-intercept:	*y*-intercept:
$2x - 4(0) = 16$	$2(0) - 4y = 16$
$2x = 16$	$-4y = 16$
$x = 8$	$y = -4$



6. a. $f(x) = -x^2 + 2x - 3$; $a = -1, b = 2, c = -3$.

Since $a = -1 < 0$, the graph opens down.

The x-coordinate of the vertex is

$x = -\frac{b}{2a} = -\frac{2}{2(-1)} = -\frac{2}{-2} = 1$.

The y-coordinate of the vertex is

$$f\left(-\frac{b}{2a}\right) = f(1)$$
$$= -1^2 + 2(1) - 3$$
$$= -1 + 2 - 3$$
$$= -2$$

Thus, the vertex is $(1, -2)$.

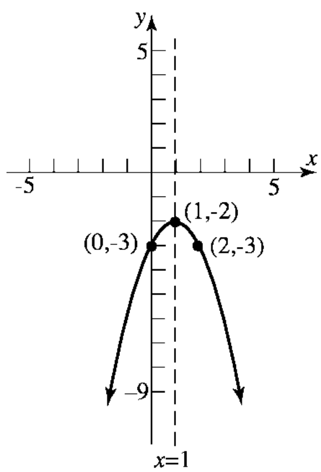
The axis of symmetry is the line $x = 1$.

The discriminant is:

$b^2 - 4ac = 2^2 - 4(-1)(-3) = 4 - 12 = -8 < 0$.

The graph has no x-intercepts.

The y-intercept is $f(0) = -0^2 + 2(0) - 3 = -3$.



b. The graph of $f(x) = -x^2 + 2x - 3$ indicates that $f(x) \le 0$ for all values of x. Thus, the solution to $f(x) \le 0$ is $(-\infty, \infty)$.

7. Given that the graph of $f(x) = ax^2 + bx + c$ has vertex $(4, -8)$ and passes through the point $(0, 24)$, we can conclude $-\frac{b}{2a} = 4$, $f(4) = -8$, and $f(0) = 24$. Notice that

$$f(0) = 24$$
$$a(0)^2 + b(0) + c = 24$$
$$c = 24$$

Therefore, $f(x) = ax^2 + bx + c = ax^2 + bx + 24$.

Furthermore, $-\frac{b}{2a} = 4$, so that $b = -8a$, and

$$f(4) = -8$$
$$a(4)^2 + b(4) + 24 = -8$$
$$16a + 4b + 24 = -8$$
$$16a + 4b = -32$$
$$4a + b = -8$$

Replacing b with $-8a$ in this equation yields

$$4a - 8a = -8$$
$$-4a = -8$$
$$a = 2$$

So $b = -8a = -8(2) = -16$.

Therefore, we have the function

$f(x) = 2x^2 - 16x + 24$.

8.

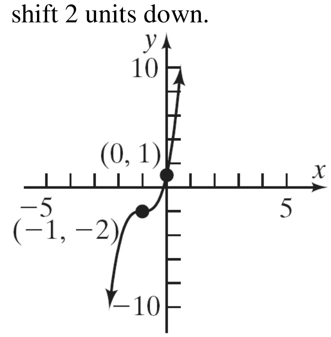
x	$y = f(x)$	$\frac{f(x+1)}{f(x)}$
-1	$\frac{2}{3}$	$\frac{2}{(2/3)} = 3$
0	2	$\frac{6}{2} = 3$
1	6	

The ratio of consecutive outputs is a constant, 3. This is an exponential function with growth factor $a = 3$. The initial value of the exponential function is $C = 2$. Therefore, the exponential function that models the data is

$f(x) = Ca^x = 2 \cdot (3)^x = 2 \cdot 3^x$.

9. $f(x) = 3(x+1)^3 - 2$

Using the graph of $y = x^3$, shift the graph 1 unit to the left, stretch vertically by a factor of 3, and shift 2 units down.



10. a. $f(x)=x^2+2 \qquad g(x)=\dfrac{2}{x-3}$

$$f(g(x))=f\left(\frac{2}{x-3}\right)=\left(\frac{2}{x-3}\right)^2+2=\frac{4}{(x-3)^2}+2$$

The domain of *f* is $\{x \mid x \text{ is any real number}\}$.

The domain of g is $\{x \mid x\neq 3\}$.

So, the domain of $f(g(x))$ is $\{x \mid x\neq 3\}$.

$$f(g(5))=\frac{4}{(5-3)^2}+2=\frac{4}{2^2}+2=\frac{4}{4}+2=3$$

b. $f(x)=x+2$; $g(x)=\log_2 x$

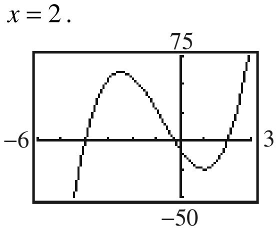
$f(g(x))=f(\log_2 x)=\log_2 x+2$

Domain: $\{x \mid x>0\}$ or $(0,\infty)$

$f(g(14))=f(\log_2 14)=\log_2 14+2$

11. $f(x)=4x^3+9x^2-30x-8$

a. The graph of $Y_1=4x^3+9x^2-30x-8$ appears to indicate zeros at $x=-4$ and $x=2$.



$$f(-4)=4(-4)^3+9(-4)^2-30(-4)-8=-256+144+120-8=0$$

$$f(2)=4(2)^3+9(2)^2-30(2)-8=32+36-60-8=0$$

Therefore, $x=-4$ and $x=2$ are real zeros for f.

Using synthetic division:

$$\begin{array}{r|rrrr} 2 & 4 & 9 & -30 & -8 \\ & & 8 & 34 & 8 \\ \hline & 4 & 17 & 4 & 0 \end{array}$$

$$f(x)=4x^3+9x^2-30x-8=(x-2)(4x^2+17x+4)=(x-2)(x+4)(4x+1)$$

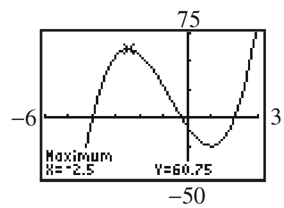
Therefore, $x=2$, $x=-\dfrac{1}{4}$ and $x=-4$ are real zeros of *f*.

b. f has *x*-intercepts at $x=2$, $x=-\dfrac{1}{4}$ and $x=-4$.

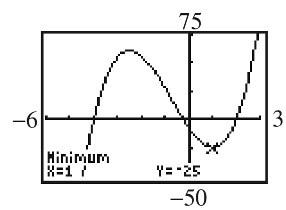
f has *y*-intercept at

$f(0)=4\cdot 0^3+9\cdot 0^2-30\cdot 0-8=-8$

c. Use MAXIMUM to determine that *f* has a local maximum at the point $(-2.5, 60.75)$.



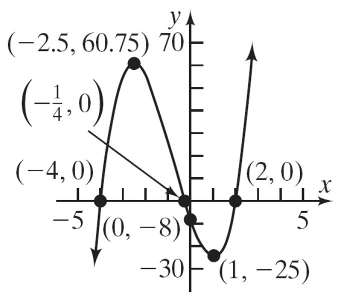
Use MINIMUM to determine that *f* has a local minimum at the point $(1,-25)$.



Thus, *f* has a local maximum of 60.75 that occurs at $x=-2.5$, and *f* has a local minimum of -25 that occurs at $x=1$.

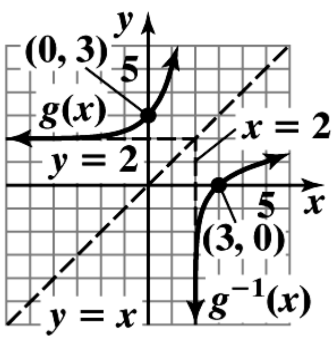
d. Graphing by hand:
The graph of f is above the *x*-axis for $\left(-4,-\dfrac{1}{4}\right)$ and $(2,\infty)$.
The graph of f is below the *x*-axis for

$(-\infty, -4)$.



12. a. $g(x) = 3^x + 2$

Using the graph of $y = 3^x$, shift up 2 units.



Domain of g: $(-\infty, \infty)$

Range of g: $(2, \infty)$

Horizontal Asymptote for g: $y = 2$

b.

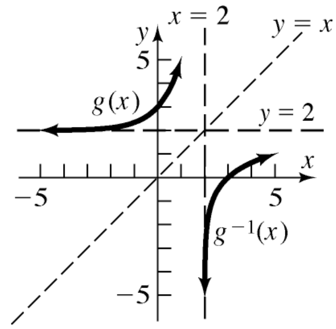
$$g(x) = 3^x + 2$$
$$y = 3^x + 2$$
$$x = 3^y + 2 \quad \text{Inverse}$$
$$x - 2 = 3^y$$
$$y = \log_3(x-2)$$
$$g^{-1}(x) = \log_3(x-2)$$

Domain of g^{-1}: $(2, \infty)$

Range of g^{-1}: $(-\infty, \infty)$

Vertical Asymptote for g^{-1}: $x = 2$

c.



13.

$$4^{x-3} = 8^{2x}$$
$$(2^2)^{x-3} = (2^3)^{2x}$$
$$2^{2x-6} = 2^{6x}$$
$$2x - 6 = 6x$$
$$-6 = 4x$$
$$x = -\frac{6}{4} = -\frac{3}{2}$$

The solution set is $\left\{-\frac{3}{2}\right\}$.

14. $\log_3(x+1) + \log_3(2x-3) = \log_9 9$

$$\log_3((x+1)(2x-3)) = 1$$
$$(x+1)(2x-3) = 3^1$$
$$2x^2 - x - 3 = 3$$
$$2x^2 - x - 6 = 0$$
$$(2x+3)(x-2) = 0$$
$$x = -\frac{3}{2} \quad \text{or} \quad x = 2$$

Since $\log_3\left(-\frac{3}{2}+1\right) = \log_3\left(-\frac{1}{2}\right)$ is undefined the solution set is $\{2\}$.

15. a. $\log_3(x+2) = 0$

$$x + 2 = 3^0$$
$$x + 2 = 1$$
$$x = -1$$

The solution set is $\{-1\}$.

b. $\log_3(x+2) > 0$

$$x + 2 > 3^0$$
$$x + 2 > 1$$
$$x > -1$$

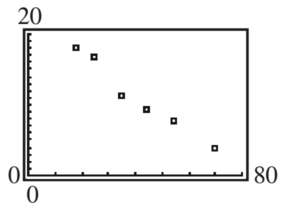
The solution set is $\{x \mid x > -1\}$ or $(-1, \infty)$.

c. $\log_3(x+2) = 3$

$$x+2 = 3^3$$
$$x+2 = 27$$
$$x = 25$$

The solution set is $\{25\}$.

16. a.



b. Logarithmic: $y = 49.293 - 10.563\ln x$

c. Answers will vary.

Chapter 5 Projects

Project I – Internet-based Project - Ans will vary

Project II

a. Newton's Law of Cooling:
$u(t) = T + (u_0 - T)e^{kt}$, k < 0

Container 1: u_0 = 200ºF, T = 70ºF, u(30)=100ºF, t = 30 mins.

$$100 = 70 + (200-70)e^{30k}$$
$$30 = 130e^{30k}$$
$$\frac{30}{130} = e^{30k}$$
$$30k = \ln\left(\frac{30}{130}\right)$$
$$k = \frac{1}{30}\ln\left(\frac{30}{130}\right) \approx -0.04888$$
$$u_1(t) = 70 + 130e^{-0.04888t}$$

Container 2: u_0 = 200ºF, T = 60ºF, u(25)=110ºF, t = 25 mins.

$$100 = 60 + (200-60)e^{25k}$$
$$50 = 140e^{25k}$$
$$\frac{50}{140} = e^{25k}$$
$$25k = \ln\left(\frac{50}{140}\right)$$
$$k = \frac{\ln\left(\frac{50}{140}\right)}{25} \approx -0.04118$$
$$u_2(t) = 60 + 140e^{-0.04118t}$$

Container 3: u_0 = 200ºF, T = 65ºF, u(20)=120ºF, t = 20 mins.

$$100 = 65 + (200-65)e^{20k}$$
$$55 = 135e^{20k}$$
$$\frac{55}{135} = e^{20k}$$
$$20k = \ln\left(\frac{55}{135}\right)$$
$$k = \frac{\ln\left(\frac{55}{135}\right)}{20} \approx -0.04490$$
$$u_3(t) = 65 + 135e^{-0.04490t}$$

b. We need time for each of the problems, so solve for t first then substitute the specific values for each container:

$$u = T + (u_0 - T)e^{kt}$$
$$u - T = (u_0 - T)e^{kt} \Rightarrow \frac{u-T}{u_0-T} = e^{kt}$$
$$kt = \ln\left(\frac{u-T}{u_0-T}\right) \Rightarrow t = \frac{\ln\left(\frac{u-T}{u_0-T}\right)}{k}$$

Container 1:

$$t = \frac{\ln\left(\frac{130-70}{200-70}\right)}{-0.04888} \approx 15.82 \text{ minutes}$$

Container 2:

$$t = \frac{\ln\left(\frac{130-60}{200-60}\right)}{-0.04118} \approx 16.83 \text{ minutes}$$

Container 3:

$$t = \frac{\ln\left(\frac{130-65}{200-65}\right)}{-0.04490} \approx 16.28 \text{ minutes}$$

c. Container 1:

$$t = \frac{\ln\left(\frac{110-70}{130-70}\right)}{-0.04888} \approx 8.295$$

It will remain between 110º and 130º for about 8.3 minutes.

Container 2:

$$t = \frac{\ln\left(\frac{110-60}{130-60}\right)}{-0.04118} \approx 8.171$$

It will remain between 110º and 130º for about 8.17 minutes

Container 3:

$$t = \frac{\ln\left(\frac{110-65}{130-65}\right)}{-0.04490} \approx 8.190$$

It will remain between 110º and 130º for about 8.19 minutes.

d. All three graphs basically lie on top of each other.

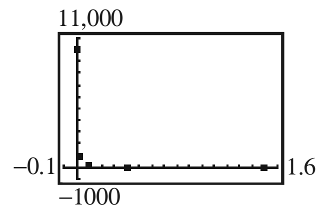
e. Container 1 would be the best. It cools off the quickest but it stays in a warm beverage range the longest.

f. Since all three containers are within seconds of each other in cooling and staying warm, the cost would have an effect. The cheaper one would be the best recommendation.

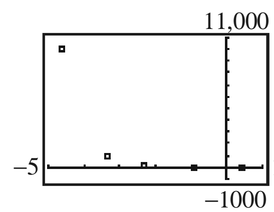
Project III

Solder Joint Strain, εp	X=ln(εp)	Fatigue Cycles, Nf	Y=ln(Nf)
0.01	–4.605	10,000	9.210
0.035	–3.352	1000	6.908
0.1	–2.303	100	4.605
0.4	–0.916	10	2.303
1.5	0.405	1	0

1.

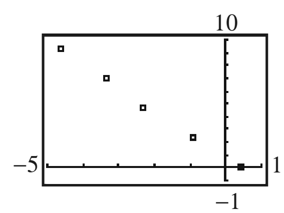


2.



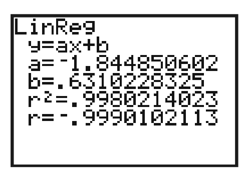
The shape becomes exponential.

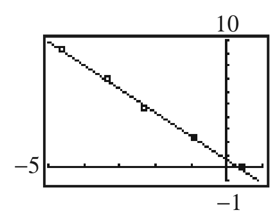
3.



The shape became linear.

4.





$Y = -1.84X + 0.63$

5. $Y = -1.84X + 0.63$

$$\ln(Nf) = -1.84\ln(\varepsilon p) + 0.63$$

$$\ln(Nf) = \ln\left((\varepsilon p)^{-1.84}\right) + \ln(e^{0.63})$$

$$\ln(Nf) = \ln\left(\left((\varepsilon p)^{-1.84}\right)(e^{0.63})\right)$$

$$Nf = \left((\varepsilon p)^{-1.84}\right)(e^{0.63})$$

$$Nf = e^{0.63}(\varepsilon p)^{-1.84}$$

6. $Nf = e^{0.63}(0.02)^{-1.84}$

$Nf = 2510.21\, cycles$

$Nf = e^{0.63}(\varepsilon p)^{-1.84}$

$3000 = e^{0.63}(\varepsilon p)^{-1.84}$

$\frac{3000}{e^{0.63}} = (\varepsilon p)^{-1.84}$

$\varepsilon p = \left(\frac{3000}{e^{0.63}}\right)^{-1/1.84}$

$\varepsilon p = 0.018$

7. $Nf = e^{0.63}(\varepsilon p)^{-1.84}$

$Nf = 1.88(\varepsilon p)^{-1.84}$

$\frac{Nf}{1.88} = (\varepsilon p)^{-1.84}$

$\varepsilon p = (0.53Nf)^{-1/1.84}$

$\varepsilon p = (0.53Nf)^{-.543}$

$\varepsilon p = 1.41(Nf)^{-.543}$

$\varepsilon p = 1.41(Nf)^{-.543}$

$\varepsilon p = 1.41(3000)^{-.543}$

$\varepsilon p = 0.018$