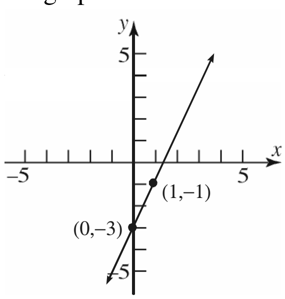
Chapter 3
Linear and Quadratic Functions

Section 3.1

1. From the equation $y = 2x - 3$, we see that the y-intercept is -3. Thus, the point $(0, -3)$ is on the graph. We can obtain a second point by choosing a value for x and finding the corresponding value for y. Let $x = 1$, then $y = 2(1) - 3 = -1$. Thus, the point $(1, -1)$ is also on the graph. Plotting the two points and connecting with a line yields the graph below.



2. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 5}{-1 - 2} = \frac{-2}{-3} = \frac{2}{3}$

3. $f(2) = 3(2)^2 - 2 = 10$

$f(4) = 3(4)^2 - 2 = 46$

$\frac{\Delta y}{\Delta x} = \frac{f(4) - f(2)}{4 - 2} = \frac{46 - 10}{4 - 2} = \frac{36}{2} = 18$

4. $60x - 900 = -15x + 2850$

$75x - 900 = 2850$

$75x = 3750$

$x = 50$

The solution set is {50}.

5. $f(-2) = (-2)^2 - 4 = 4 - 4 = 0$

6. True

7. slope; y-intercept

8. -4; 3

9. positive

10. True

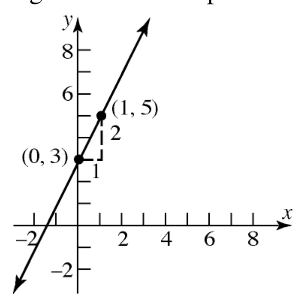
11. False. If x increases by 3, then y increases by 2.

12. False. The y-intercept is 8. The average rate of change is 2 (the slope).

13. $f(x) = 2x + 3$

a. Slope = 2; y-intercept = 3

b. Plot the point (0, 3). Use the slope to find an additional point by moving 1 unit to the right and 2 units up.



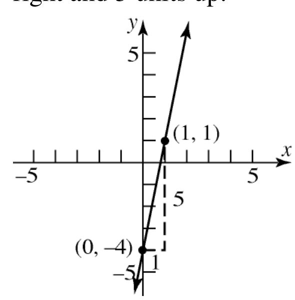
c. average rate of change = 2

d. increasing

14. $g(x) = 5x - 4$

a. Slope = 5; y-intercept = -4

b. Plot the point $(0, -4)$. Use the slope to find an additional point by moving 1 unit to the right and 5 units up.



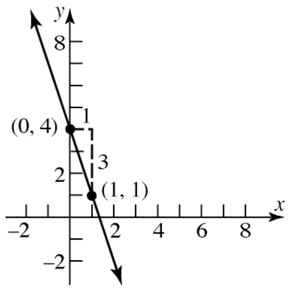
c. average rate of change = 5

d. increasing

15. $h(x) = -3x + 4$

a. Slope $= -3$; y-intercept = 4

b. Plot the point (0, 4). Use the slope to find an additional point by moving 1 unit to the right and 3 units down.



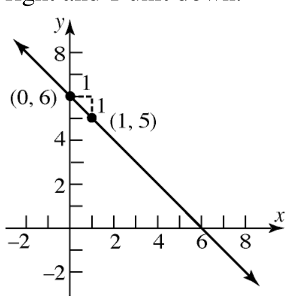
c. average rate of change $= -3$

d. decreasing

16. $p(x) = -x + 6$

a. Slope $= -1$; y-intercept = 6

b. Plot the point (0, 6). Use the slope to find an additional point by moving 1 unit to the right and 1 unit down.



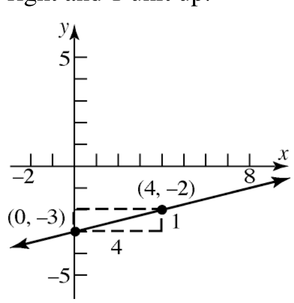
c. average rate of change $= -1$

d. decreasing

17. $f(x) = \frac{1}{4}x - 3$

a. Slope $= \frac{1}{4}$; y-intercept $= -3$

b. Plot the point $(0, -3)$. Use the slope to find an additional point by moving 4 units to the right and 1 unit up.



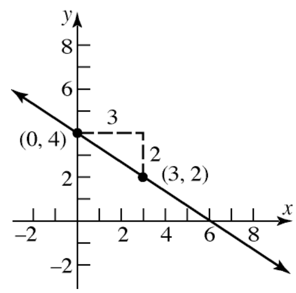
c. average rate of change $= \frac{1}{4}$

d. increasing

18. $h(x) = -\frac{2}{3}x + 4$

a. Slope $= -\frac{2}{3}$; y-intercept = 4

b. Plot the point (0, 4). Use the slope to find an additional point by moving 3 units to the right and 2 units down.



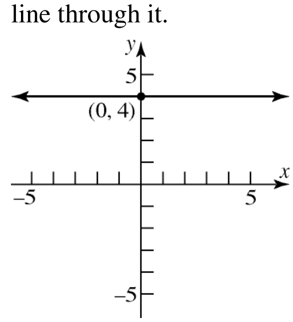
c. average rate of change $= -\frac{2}{3}$

d. decreasing

19. $F(x) = 4$

a. Slope = 0; y-intercept = 4

b. Plot the point (0, 4) and draw a horizontal line through it.



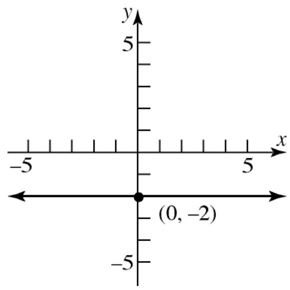
c. average rate of change = 0

d. constant

20. $G(x) = -2$

a. Slope = 0; y-intercept = -2

b. Plot the point $(0,-2)$ and draw a horizontal line through it.



c. average rate of change = 0

d. constant

21.

x	y	Avg. rate of change = $\frac{\Delta y}{\Delta x}$
-2	4	
-1	1	$\frac{1-4}{-1-(-2)} = \frac{-3}{1} = -3$
0	-2	$\frac{-2-1}{0-(-1)} = \frac{-3}{1} = -3$
1	-5	$\frac{-5-(-2)}{1-0} = \frac{-3}{1} = -3$
2	-8	$\frac{-8-(-5)}{2-1} = \frac{-3}{1} = -3$

Since the average rate of change is constant at -3, this is a linear function with slope = -3. The y-intercept is $(0,-2)$, so the equation of the line is $y = -3x - 2$.

22.

x	y	Avg. rate of change = $\frac{\Delta y}{\Delta x}$
-2	$\frac{1}{4}$	
-1	$\frac{1}{2}$	$\frac{\left(\frac{1}{2}-\frac{1}{4}\right)}{-1-(-2)} = \frac{\frac{1}{4}}{1} = \frac{1}{4}$
0	1	$\frac{\left(1-\frac{1}{2}\right)}{0-(-1)} = \frac{\frac{1}{2}}{1} = \frac{1}{2}$
1	2	
2	4	

Since the average rate of change is not constant, this is not a linear function.

23.

x	y	Avg. rate of change = $\frac{\Delta y}{\Delta x}$
-2	-8	
-1	-3	$\frac{-3-(-8)}{-1-(-2)} = \frac{5}{1} = 5$
0	0	$\frac{0-(-3)}{0-(-1)} = \frac{3}{1} = 3$
1	1	
2	0	

Since the average rate of change is not constant, this is not a linear function.

24.

x	y	Avg. rate of change = $\frac{\Delta y}{\Delta x}$
-2	-4	
-1	0	$\frac{0-(-4)}{-1-(-2)} = \frac{4}{1} = 4$
0	4	$\frac{4-0}{0-(-1)} = \frac{4}{1} = 4$
1	8	$\frac{8-4}{1-0} = \frac{4}{1} = 4$
2	12	$\frac{12-8}{2-1} = \frac{4}{1} = 4$

Since the average rate of change is constant at 4, this is a linear function with slope = 4. The y-intercept is $(0, 4)$, so the equation of the line is $y = 4x + 4$.

25.

x	y	Avg. rate of change = $\frac{\Delta y}{\Delta x}$
-2	-26	
-1	-4	$\frac{-4-(-26)}{-1-(-2)} = \frac{22}{1} = 22$
0	2	$\frac{2-(-4)}{0-(-1)} = \frac{6}{1} = 6$
1	-2	
2	-10	

Since the average rate of change is not constant, this is not a linear function.

26.

x	y	Avg. rate of change = $\frac{\Delta y}{\Delta x}$
–2	–4	
–1	–3.5	$\frac{-3.5-(-4)}{-1-(-2)}=\frac{0.5}{1}=0.5$
0	–3	$\frac{-3-(-3.5)}{0-(-1)}=\frac{0.5}{1}=0.5$
1	–2.5	$\frac{-2.5-(-3)}{1-0}=\frac{0.5}{1}=0.5$
2	–2	$\frac{-2-(-2.5)}{2-1}=\frac{0.5}{1}=0.5$

Since the average rate of change is constant at 0.5, this is a linear function with slope = 0.5. The y-intercept is $(0,-3)$, so the equation of the line is $y=0.5x-3$.

27.

x	y	Avg. rate of change = $\frac{\Delta y}{\Delta x}$
–2	8	
–1	8	$\frac{8-8}{-1-(-2)}=\frac{0}{1}=0$
0	8	$\frac{8-8}{0-(-1)}=\frac{0}{1}=0$
1	8	$\frac{8-8}{1-0}=\frac{0}{1}=0$
2	8	$\frac{8-8}{2-1}=\frac{0}{1}=0$

Since the average rate of change is constant at 0, this is a linear function with slope = 0. The y-intercept is $(0, 8)$, so the equation of the line is $y=0x+8$ or $y=8$.

28.

x	y	Avg. rate of change = $\frac{\Delta y}{\Delta x}$
–2	0	
–1	1	$\frac{1-0}{-1-(-2)}=\frac{1}{1}=1$
0	4	$\frac{4-1}{0-(-1)}=\frac{3}{1}=3$
1	9	
2	16	

Since the average rate of change is not constant, this is not a linear function.

29. $f(x)=4x-1; \quad g(x)=-2x+5$

a. $f(x)=0$

$4x-1=0$

$x=\frac{1}{4}$

b. $f(x)>0$

$4x-1>0$

$x>\frac{1}{4}$

The solution set is $\left\{x \middle| x>\frac{1}{4}\right\}$ or $\left(\frac{1}{4}, \infty\right)$.

c. $f(x)=g(x)$

$4x-1=-2x+5$

$6x=6$

$x=1$

d. $f(x)\le g(x)$

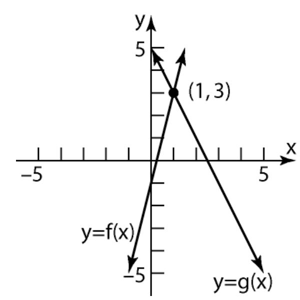
$4x-1\le -2x+5$

$6x\le 6$

$x\le 1$

The solution set is $\{x | x\le 1\}$ or $(-\infty, 1]$.

e.



30. $f(x)=3x+5; \quad g(x)=-2x+15$

a. $f(x)=0$

$3x+5=0$

$x=-\frac{5}{3}$

b. $f(x)<0$

$3x+5<0$

$x<-\frac{5}{3}$

The solution set is $\left\{x \middle| x<-\frac{5}{3}\right\}$ or $\left(-\infty,-\frac{5}{3}\right)$.

c. $f(x) = g(x)$

$3x + 5 = -2x + 15$

$5x = 10$

$x = 2$

d. $f(x) \geq g(x)$

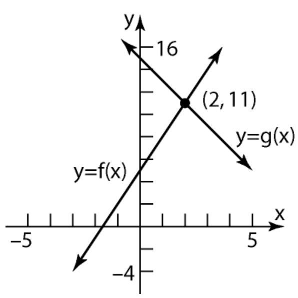
$3x + 5 \geq -2x + 15$

$5x \geq 10$

$x \geq 2$

The solution set is $\{x | x \geq 2\}$ or $[2, \infty)$.

e.



31. a. The point (40, 50) is on the graph of $y = f(x)$, so the solution to $f(x) = 50$ is $x = 40$.

b. The point (88, 80) is on the graph of $y = f(x)$, so the solution to $f(x) = 80$ is $x = 88$.

c. The point $(-40, 0)$ is on the graph of $y = f(x)$, so the solution to $f(x) = 0$ is $x = -40$.

d. The *y*-coordinates of the graph of $y = f(x)$ are above 50 when the *x*-coordinates are larger than 40. Thus, the solution to $f(x) > 50$ is $\{x | x > 40\}$ or $(40, \infty)$.

e. The *y*-coordinates of the graph of $y = f(x)$ are below 80 when the *x*-coordinates are smaller than 88. Thus, the solution to $f(x) \leq 80$ is $\{x | x \leq 88\}$ or $(-\infty, 88]$.

f. The *y*-coordinates of the graph of $y = f(x)$ are between 0 and 80 when the *x*-coordinates are between -40 and 88. Thus, the solution to $0 < f(x) < 80$ is $\{x | -40 < x < 88\}$ or $(-40, 88)$.

32. a. The point (5, 20) is on the graph of $y = g(x)$, so the solution to $g(x) = 20$ is $x = 5$.

b. The point $(-15, 60)$ is on the graph of $y = g(x)$, so the solution to $g(x) = 60$ is $x = -15$.

c. The point (15, 0) is on the graph of $y = g(x)$, so the solution to $g(x) = 0$ is $x = 15$.

d. The *y*-coordinates of the graph of $y = g(x)$ are above 20 when the *x*-coordinates are smaller than 5. Thus, the solution to $g(x) > 20$ is $\{x | x < 5\}$ or $(-\infty, 5)$.

e. The *y*-coordinates of the graph of $y = f(x)$ are below 60 when the *x*-coordinates are larger than -15. Thus, the solution to $g(x) \leq 60$ is $\{x | x \geq -15\}$ or $[-15, \infty)$.

f. The *y*-coordinates of the graph of $y = f(x)$ are between 0 and 60 when the *x*-coordinates are between -15 and 15. Thus, the solution to $0 < f(x) < 60$ is $\{x | -15 < x < 15\}$ or $(-15, 15)$.

33. a. $f(x) = g(x)$ when their graphs intersect. Thus, $x = -4$.

b. $f(x) \leq g(x)$ when the graph of *f* is above the graph of *g*. Thus, the solution is $\{x | \ x < -4\}$ or $(-\infty, -4)$.

34. a. $f(x) = g(x)$ when their graphs intersect. Thus, $x = 2$.

b. $f(x) \leq g(x)$ when the graph of *f* is below or intersects the graph of *g*. Thus, the solution is $\{x | \ x \leq 2\}$ or $(-\infty, 2]$.

35. a. $f(x) = g(x)$ when their graphs intersect. Thus, $x = -6$.

b. $g(x) \leq f(x) < h(x)$ when the graph of *f* is above or intersects the graph of *g* and below the graph of *h*. Thus, the solution is $\{x | -6 \leq x < 5\}$ or $[-6, 5)$.

36. **a.** $f(x) = g(x)$ when their graphs intersect. Thus, $x = 7$.

b. $g(x) \le f(x) < h(x)$ when the graph of *f* is above or intersects the graph of *g* and below the graph of *h*. Thus, the solution is $\{x | -4 \le x < 7\}$ or $[-4, 7)$.

37. $C(x) = 0.25x + 35$

a. $C(40) = 0.25(40) + 35 = \$45$.

b. Solve $C(x) = 0.25x + 35 = 80$

$$0.25x + 35 = 80$$
$$0.25x = 45$$
$$x = \frac{45}{0.25} = 180 \text{ miles}$$

c. Solve $C(x) = 0.25x + 35 \le 100$

$$0.25x + 35 \le 100$$
$$0.25x \le 65$$
$$x \le \frac{65}{0.25} = 260 \text{ miles}$$

d. The number of mile driven cannot be negative, so the implied domain for *C* is $\{x \mid x \ge 0\}$ or $[0, \infty)$.

e. The cost of renting the moving truck for a day increases \$0.25 for each mile driven, or there is a charge of \$0.25 per mile to rent the truck in addition to a fixed charge of \$35.

f. It costs \$35 to rent the moving truck if 0 miles are driven, or there is a fixed charge of \$35 to rent the truck in addition to a charge that depends on mileage.

38. $C(x) = 0.38x + 5$

a. $C(50) = 0.38(50) + 5 = \$24$.

b. Solve $C(x) = 0.38x + 5 = 29.32$

$$0.38x + 5 = 29.32$$
$$0.38x = 24.32$$
$$x = \frac{24.32}{0.38} = 64 \text{ minutes}$$

c. Solve $C(x) = 0.38x + 5 \le 60$

$$0.38x + 5 \le 60$$
$$0.38x \le 55$$
$$x \le \frac{55}{0.38} \approx 144 \text{ minutes}$$

d. The number of minutes cannot be negative, so $x \ge 0$. If there are 30 days in the month, then the number of minutes can be at most $30 \cdot 24 \cdot 60 = 43,200$. Thus, the implied domain for *C* is $\{x \mid 0 \le x \le 43,200\}$ or $[0, 43200]$.

e. The monthly cost of the plan increases \$0.38 for each minute used, or there is a charge of \$0.38 per minute to use the phone in addition to a fixed charge of \$5.

f. It costs \$5 per month for the plan if 0 minutes are used, or there is a fixed charge of \$5 per month for the plan in addition to a charge that depends on the number of minutes used.

39. $S(p) = -200 + 50p$; $D(p) = 1000 - 25p$

a. Solve $S(p) = D(p)$.

$$-200 + 50p = 1000 - 25p$$
$$75p = 1200$$
$$p = \frac{1200}{75} = 16$$
$$S(16) = -200 + 50(16) = 600$$

Thus, the equilibrium price is \$16, and the equilibrium quantity is 600 T-shirts.

b. Solve $D(p) > S(p)$.

$$1000 - 25p > -200 + 50p$$
$$1200 > 75p$$
$$\frac{1200}{75} > p$$
$$16 > p$$

The demand will exceed supply when the price is less than \$16 (but still greater than \$0). That is, $\$0 < p < \$16$.

c. The price will eventually be increased.

40. $S(p) = -2000 + 3000p$; $D(p) = 10000 - 1000p$

a. Solve $S(p) = D(p)$.

$$-2000 + 3000p = 10000 - 1000p$$
$$4000p = 12000$$
$$p = \frac{12000}{4000} = 3$$
$$S(3) = -2000 + 3000(3) = 7000$$

Thus, the equilibrium price is \$3, and the equilibrium quantity is 7000 hot dogs.

b. Solve $D(p) < S(p)$.

$$10000 - 1000p < -2000 + 3000p$$
$$12000 < 4000p$$
$$\frac{12000}{4000} < p$$
$$3 < p$$

The demand will be less than the supply when the price is greater than \$3.

c. The price will eventually be decreased.

41. a. We are told that the tax function T is for adjusted gross incomes x between \$8,350 and \$33,950, inclusive. Thus, the domain is $\{x \mid 8,350 \le x \le 33,950\}$ or $[8350,\ 33950]$.

b. $T(20,000) = 0.15(20,000 - 8350) + 835 = 2582.50$

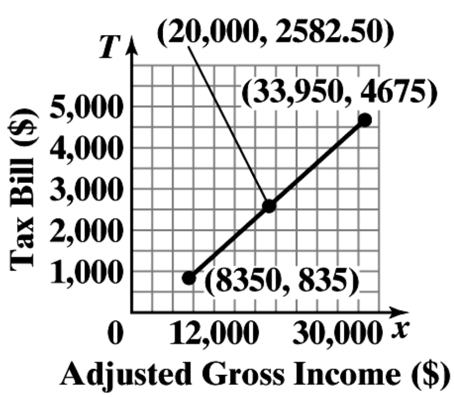
If a single filer's adjusted gross income is \$20,000, then his or her tax bill will be \$2582.50.

c. The independent variable is adjusted gross income, x. The dependent variable is the tax bill, T.

d. Evaluate T at $x = 8350,\ 20000,\ \text{and } 33950$.

$$T(8350) = 0.15(8350 - 8350) + 835 = 835$$
$$T(20,000) = 0.15(20,000 - 8350) + 835 = 2582.50$$
$$T(33,950) = 0.15(33,950 - 8350) + 835 = 4675$$

Thus, the points $(8350, 835)$, $(20000, 2582.5)$, and $(33950, 4675)$ are on the graph.



e. We must solve $T(x) = 3707.50$.

$$0.15(x - 8350) + 835 = 3707.50$$
$$0.15x - 1252.5 + 835 = 3707.50$$
$$0.15x - 417.5 = 3707.50$$
$$0.15x = 4125$$
$$x = 27,500$$

A single filer with an adjusted gross income of \$27,500 will have a tax bill of \$3707.50.

f. For each additional dollar of taxable income between \$8350 and \$33,950, the tax bill of a single person in 2009 increased by \$0.15.

42. a. The independent variable is payroll, p. The payroll tax only applies if the payroll exceeds \$136.5 million. Thus, the domain of T is $\{p \mid p > 136.5\}$ or $(136.5,\ \infty)$.

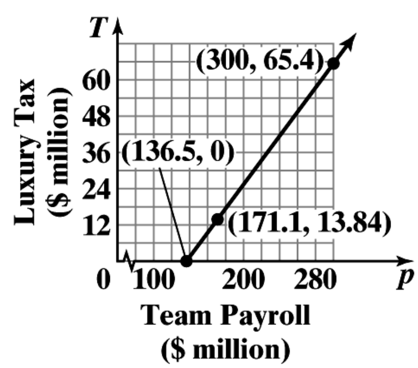
b. $T(171.1) = 0.40(171.1 - 136.5) = 13.84$

The luxury tax for the New York Yankees was \$13.84 million.

c. Evaluate T at $p = 136.5$, 171.1, and 300 million.

$$T(136.5) = 0.40(136.5 - 136.5) = 0 \text{ million}$$
$$T(171.1) = 0.40(171.1 - 136.5) = 13.84 \text{ million}$$
$$T(300) = 0.40(300 - 136.5) = 65.4 \text{ million}$$

Thus, the points $(136.5 \text{ million},\ 0 \text{ million})$, $(171.1 \text{ million},\ 13.84 \text{ million})$, and $(300 \text{ million},\ 65.4 \text{ million})$ are on the graph.



d. We must solve $T(p) = 11.7$.

$$0.40(p - 136.5) = 11.7$$
$$0.40p - 54.6 = 11.7$$
$$0.40p = 66.3$$
$$p = 165.75$$

If the luxury tax is \$11.7 million, then the payroll of the team is \$165.75 million.

e. For each additional million dollars of payroll in excess of \$136.5 million in 2006, the luxury tax of a team increased by \$0.40 million.

43. $R(x) = 8x$; $C(x) = 4.5x + 17,500$

a. Solve $R(x) = C(x)$.

$8x = 4.5x + 17,500$

$3.5x = 17,500$

$x = 5000$

The break-even point occurs when the company sells 5000 units.

b. Solve $R(x) > C(x)$

$8x > 4.5x + 17,500$

$3.5x > 17,500$

$x > 5000$

The company makes a profit if it sells more than 5000 units.

44. $R(x) = 12x$; $C(x) = 10x + 15,000$

a. Solve $R(x) = C(x)$

$12x = 10x + 15,000$

$2x = 15,000$

$x = 7500$

The break-even point occurs when the company sells 7500 units.

b. Solve $R(x) > C(x)$

$12x > 10x + 15,000$

$2x > 15,000$

$x > 7500$

The company makes a profit if it sells more than 7500 units.

45. a. Consider the data points (x, y), where x = the age in years of the computer and y = the value in dollars of the computer. So we have the points $(0, 3000)$ and $(3, 0)$. The slope formula yields:

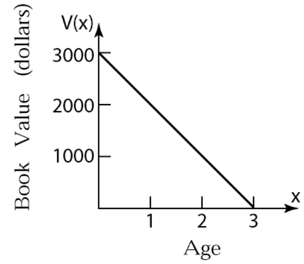
$$m = \frac{\Delta y}{\Delta x} = \frac{0 - 3000}{3 - 0} = \frac{-3000}{3} = -1000$$

The y-intercept is $(0, 3000)$, so $b = 3000$.

Therefore, the linear function is $V(x) = mx + b = -1000x + 3000$.

b. The age of the computer cannot be negative, and the book value of the computer will be \$0 after 3 years. Thus, the implied domain for V is $\{x \mid 0 \le x \le 3\}$ or [0, 3].

c. The graph of $V(x) = -1000x + 3000$



d. $V(2) = -1000(2) + 3000 = 1000$

The computer's book value after 2 years will be \$1000.

e. Solve $V(x) = 2000$

$-1000x + 3000 = 2000$

$-1000x = -1000$

$x = 1$

The computer will have a book value of \$2000 after 1 year.

46. a. Consider the data points (x, y), where x = the age in years of the machine and y = the value in dollars of the machine. So we have the points $(0, 120000)$ and $(10, 0)$. The slope formula yields:

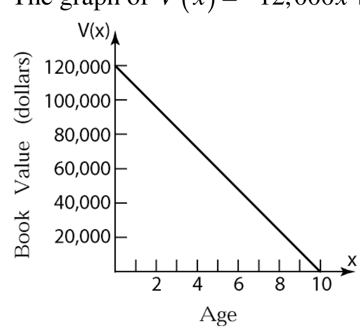
$$m = \frac{\Delta y}{\Delta x} = \frac{0 - 120000}{10 - 0} = \frac{-120000}{10} = -12000$$

The y-intercept is $(0, 120000)$, so $b = 120,000$.

Therefore, the linear function is $V(x) = mx + b = -12,000x + 120,000$.

b. The age of the machine cannot be negative, and the book value of the machine will be \$0 after 10 years. Thus, the implied domain for V is $\{x \mid 0 \le x \le 10\}$ or [0, 10].

c. The graph of $V(x) = -12,000x + 120,000$



d. $V(4) = -12000(4) + 120000 = 72000$

The machine's value after 4 years is given by \$72,000.

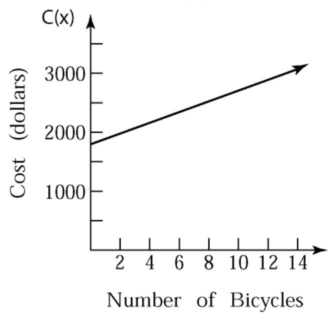
e. Solve $V(x) = 72000$.

$$-12000x + 120000 = 72000$$
$$-12000x = -48000$$
$$x = 4$$

The machine will be worth \$72,000 after 4 years.

47. a. Let x = the number of bicycles manufactured. We can use the cost function $C(x) = mx + b$, with m = 90 and b = 1800. Therefore $C(x) = 90x + 1800$

b. The graph of $C(x) = 90x + 1800$



c. The cost of manufacturing 14 bicycles is given by $C(14) = 90(14) + 1800 = \$3060$.

d. Solve $C(x) = 90x + 1800 = 3780$

$$90x + 1800 = 3780$$
$$90x = 1980$$
$$x = 22$$

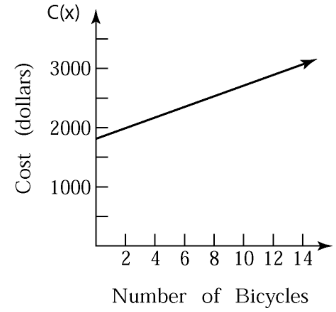
So 22 bicycles could be manufactured for \$3780.

48. a. The new daily fixed cost is

$$1800 + \frac{100}{20} = \$1805$$

b. Let x = the number of bicycles manufactured. We can use the cost function $C(x) = mx + b$, with m = 90 and b = 1805. Therefore $C(x) = 90x + 1805$

c. The graph of $C(x) = 90x + 1805$



d. The cost of manufacturing 14 bicycles is given by $C(14) = 90(14) + 1805 = \$3065$.

e. Solve $C(x) = 90x + 1805 = 3780$

$$90x + 1805 = 3780$$
$$90x = 1975$$
$$x \approx 21.94$$

So approximately 21 bicycles could be manufactured for \$3780.

49. a. Let x = number of miles driven, and let C = cost in dollars. Total cost = (cost per mile)(number of miles) + fixed cost

$C(x) = 0.07x + 29$

b. $C(110) = (0.07)(110) + 29 = \$36.70$

$C(230) = (0.07)(230) + 29 = \$45.10$

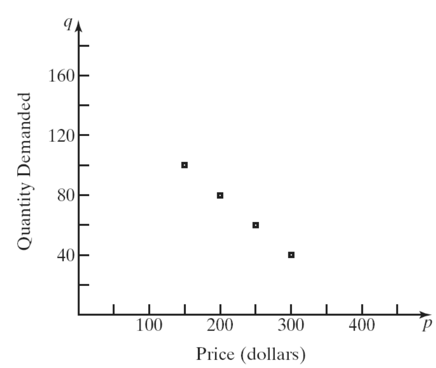
50. a. Let x = number of minutes used, and let C = cost in dollars. Total cost = (cost per minute)(number of minutes) + fixed cost

$C(x) = 0.05x + 5$

b. $C(105) = (0.05)(105) + 5 = \$10.25$

$C(180) = (0.05)(180) + 5 = \$14$

51. a.



b.

p	q	Avg. rate of change = $\dfrac{\Delta q}{\Delta p}$
150	100	
200	80	$\dfrac{80-100}{200-150}=\dfrac{-20}{50}=-0.4$
250	60	$\dfrac{60-80}{250-200}=\dfrac{-20}{50}=-0.4$
300	40	$\dfrac{40-60}{300-250}=\dfrac{-20}{50}=-0.4$

Since each input (price) corresponds to a single output (quantity demanded), we know that the quantity demanded is a function of price. Also, because the average rate of change is constant at $-\$0.4$ per LCD monitor, the function is linear.

c. From part (b), we know $m=-0.4$. Using $(p_1, q_1)=(150,100)$, we get the equation:

$$q-q_1=m(p-p_1)$$
$$q-100=-0.4(p-150)$$
$$q-100=-0.4p+60$$
$$q=-0.4p+160$$

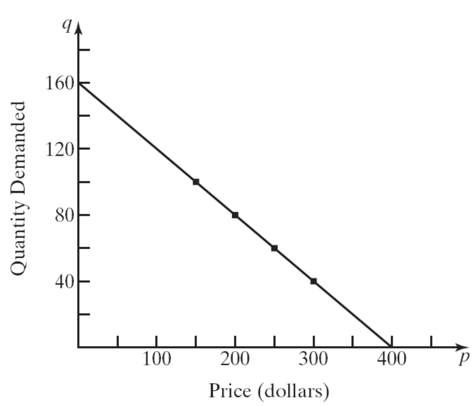
Using function notation, we have $q(p)=-0.4p+160$.

d. The price cannot be negative, so $p\ge 0$. Likewise, the quantity cannot be negative, so, $q(p)\ge 0$.

$$-0.4p+160\ge 0$$
$$-0.4p\ge -160$$
$$p\le 400$$

Thus, the implied domain for $q(p)$ is $\{p \mid 0\le p\le 400\}$ or $[0, 400]$.

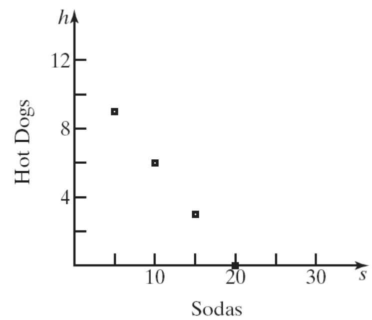
e.



f. If the price increases by \$1, then the quantity demanded of LCD monitors decreases by 0.4 monitor.

g. p-intercept: If the price is \$0, then 160 LCD monitors will be demanded.
q-intercept: There will be 0 LCD monitors demanded when the price is \$400.

52. a.



b.

s	h	Avg. rate of change = $\dfrac{\Delta h}{\Delta s}$
20	0	
15	3	$\dfrac{3-0}{15-20}=\dfrac{3}{-5}=-0.6$
10	6	$\dfrac{6-3}{10-15}=\dfrac{3}{-5}=-0.6$
5	9	$\dfrac{9-6}{5-10}=\dfrac{3}{-5}=-0.6$

Since each input (soda) corresponds to a single output (hot dogs), we know that number of hot dogs purchased is a function of number of sodas purchased. Also, because the average rate of change is constant at -0.6 hot dogs per soda, the function is linear.

c. From part (b), we know $m=-0.6$. Using $(s_1, h_1)=(20, 0)$, we get the equation:

$$h-h_1=m(s-s_1)$$
$$h-0=-0.6(s-20)$$
$$h=-0.6s+12$$

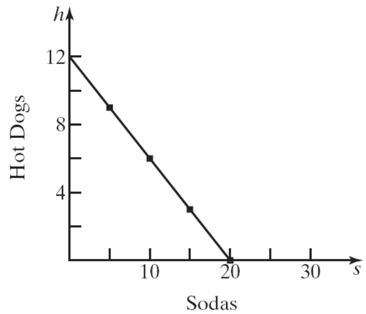
Using function notation, we have $h(s)=-0.6s+12$.

d. The number of sodas cannot be negative, so $s\ge 0$. Likewise, the number of hot dogs cannot be negative, so, $h(s)\ge 0$.

$$-0.6s+12\ge 0$$
$$-0.6s\ge -12$$
$$s\le 20$$

Thus, the implied domain for $h(s)$ is
$\{s \mid 0 \le s \le 20\}$ or $[0, 20]$.

e.



f. If the number of hot dogs purchased increases by \$1, then the number of sodas purchased decreases by 0.6.

g. *s*-intercept: If 0 hot dogs are purchased, then 20 sodas can be purchased.
h-intercept: If 0 sodas are purchased, then 12 hot dogs may be purchased.

53. The graph shown has a positive slope and a positive *y*-intercept. Therefore, the function from (d) and (e) might have the graph shown.

54. The graph shown has a negative slope and a positive *y*-intercept. Therefore, the function from (b) and (e) might have the graph shown.

55. A linear function $f(x) = mx + b$ will be odd provided $f(-x) = -f(x)$.
That is, provided $m(-x) + b = -(mx + b)$.

$$-mx + b = -mx - b$$
$$b = -b$$
$$2b = 0$$
$$b = 0$$

So a linear function $f(x) = mx + b$ will be odd provided $b = 0$.

A linear function $f(x) = mx + b$ will be even provided $f(-x) = f(x)$.
That is, provided $m(-x) + b = mx + b$.

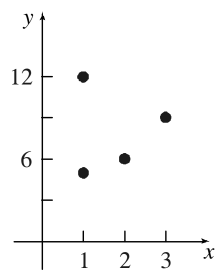
$$-mx + b = mx + b$$
$$-mxb = mx$$
$$0 = 2mx$$
$$m = 0$$

So, yes, a linear function $f(x) = mx + b$ cab be even provided $m = 0$.

56. If you solve the linear function $f(x) = mx + b$ for 0 you are actually finding the x-intercept. Therefore using x-intercept of the graph of $f(x) = mx + b$ would be same x-value as solving $mx + b > 0$ for x. Then the appropriate interval could be determined

Section 3.2

1.



No, the relation is not a function because an input, 1, corresponds to two different outputs, 5 and 12.

2. Let $(x_1, y_1) = (1, 4)$ and $(x_2, y_2) = (3, 8)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-4}{3-1} = \frac{4}{2} = 2$$
$$y - y_1 = m(x - x_1)$$
$$y - 4 = 2(x - 1)$$
$$y - 4 = 2x - 2$$
$$y = 2x + 2$$

3. scatter diagram

4. decrease; 0.008

5. Linear relation, $m > 0$

6. Nonlinear relation

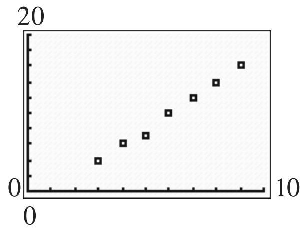
7. Linear relation, $m < 0$

8. Linear relation, $m > 0$

9. Nonlinear relation

10. Nonlinear relation

11. a.



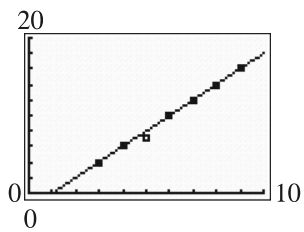
b. Answers will vary. We select (4, 6) and (8, 14). The slope of the line containing these points is:

$$m = \frac{14-6}{8-4} = \frac{8}{4} = 2$$

The equation of the line is:

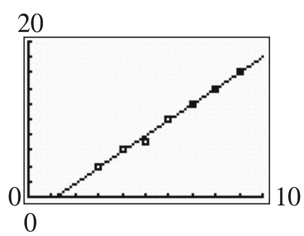
$$y - y_1 = m(x - x_1)$$
$$y - 6 = 2(x - 4)$$
$$y - 6 = 2x - 8$$
$$y = 2x - 2$$

c.

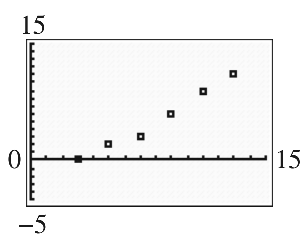


d. Using the LINear REGression program, the line of best fit is:
$y = 2.0357x - 2.3571$

e.



12. a.

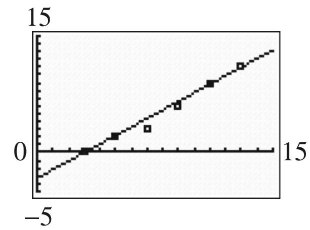


b. Answers will vary. We select (5, 2) and (11, 9). The slope of the line containing these points is: $m = \frac{9-2}{11-5} = \frac{7}{6}$

The equation of the line is:

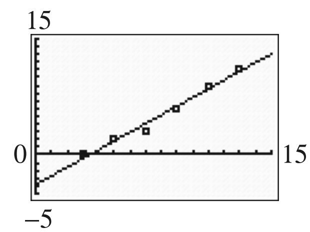
$$y - y_1 = m(x - x_1)$$
$$y - 2 = \frac{7}{6}(x - 5)$$
$$y - 2 = \frac{7}{6}x - \frac{35}{6}$$
$$y = \frac{7}{6}x - \frac{23}{6}$$

c.

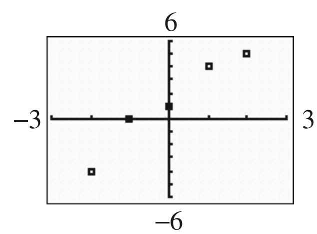


d. Using the LINear REGression program, the line of best fit is:
$y = 1.1286x - 3.8619$

e.



13. a.

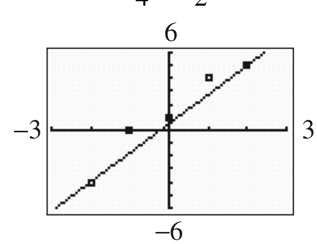


b. Answers will vary. We select (–2,–4) and (2, 5). The slope of the line containing these points is: $m = \frac{5-(-4)}{2-(-2)} = \frac{9}{4}$.

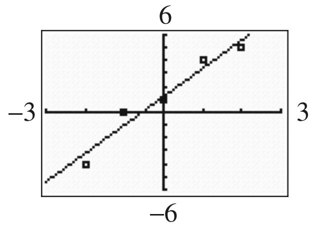
The equation of the line is:

$$y - y_1 = m(x - x_1)$$
$$y - (-4) = \frac{9}{4}(x - (-2))$$
$$y + 4 = \frac{9}{4}x + \frac{9}{2}$$
$$y = \frac{9}{4}x + \frac{1}{2}$$

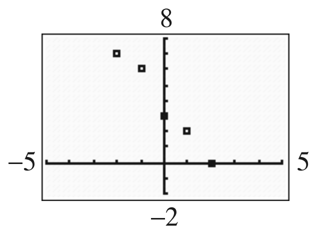
c.



e. Using the LINear REGression program, the line of best fit is:
$y = 2.2x + 1.2$



14. a.

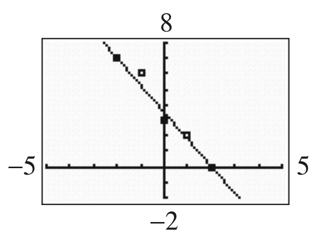


b. Answers will vary. We select (–2, 7) and (2, 0). The slope of the line containing these points is: $m = \frac{0-7}{2-(-2)} = \frac{-7}{4} = -\frac{7}{4}$.

The equation of the line is:

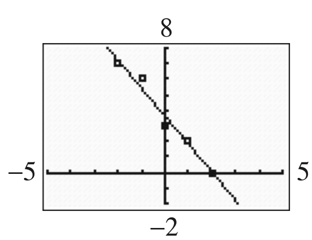
$$y - y_1 = m(x - x_1)$$
$$y - 7 = -\frac{7}{4}(x - (-2))$$
$$y - 7 = -\frac{7}{4}x - \frac{7}{2}$$
$$y = -\frac{7}{4}x + \frac{7}{2}$$

c.

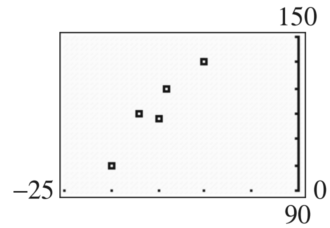


d. Using the LINear REGression program, the line of best fit is:
$y = -1.8x + 3.6$

e.



15. a.



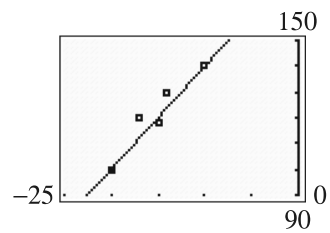
b. Answers will vary. We select (–20,100) and (–10,140). The slope of the line containing these points is:

$$m = \frac{140-100}{-10-(-20)} = \frac{40}{10} = 4$$

The equation of the line is:

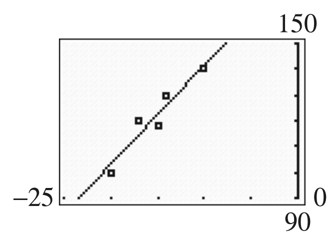
$$y - y_1 = m(x - x_1)$$
$$y - 100 = 4(x - (-20))$$
$$y - 100 = 4x + 80$$
$$y = 4x + 180$$

c.

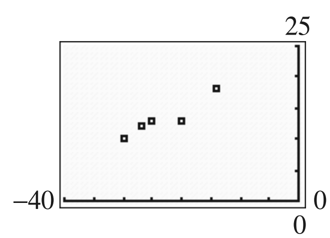


d. Using the LINear REGression program, the line of best fit is:
$y = 3.8613x + 180.2920$

e.



16. a.



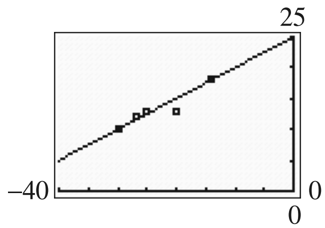
b. Selection of points will vary. We select (–30, 10) and (–14, 18). The slope of the line containing these points is:

$$m = \frac{18-10}{-14-(-30)} = \frac{8}{16} = \frac{1}{2}$$

The equation of the line is:

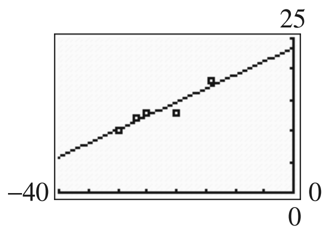
$$y - y_1 = m(x - x_1)$$
$$y - 10 = \frac{1}{2}\left(x - (-30)\right)$$
$$y - 10 = \frac{1}{2}x + 15$$
$$y = \frac{1}{2}x + 25$$

c.

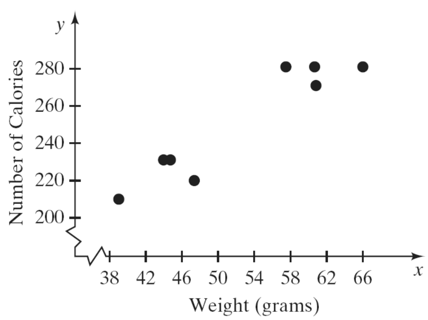


d. Using the LINear REGression program, the line of best fit is:
$y = 0.4421x + 23.4559$

e.



17. a.

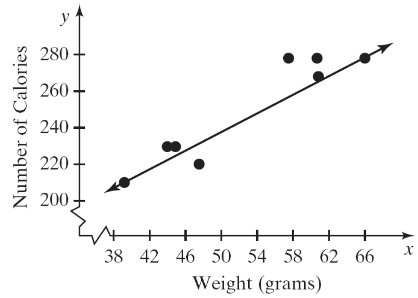


b. Linear.

c. Answers will vary. We will use the points $(39.52,\ 210)$ and $(66.45,\ 280)$.

$$m = \frac{280 - 210}{66.45 - 39.52} = \frac{70}{26.93} \approx 2.5993316$$
$$y - 210 = 2.5993316(x - 39.52)$$
$$y - 210 = 2.5993316x - 102.7255848$$
$$y = 2.599x + 107.274$$

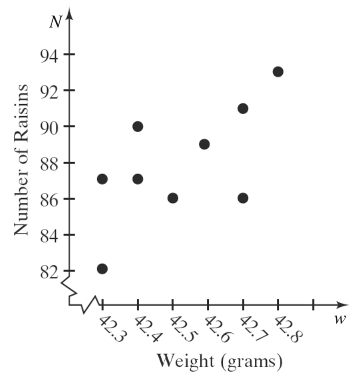
d.



e. $x = 62.3:\ y = 2.599(62.3) + 107.274 \approx 269$
We predict that a candy bar weighing 62.3 grams will contain 269 calories.

f. If the weight of a candy bar is increased by one gram, then the number of calories will increase by 2.599.

18. a.

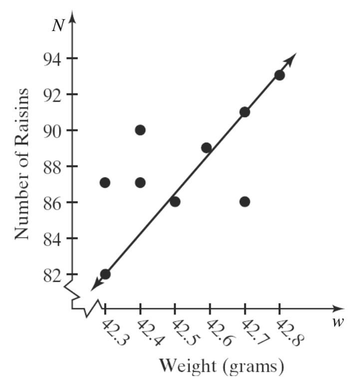


b. Linear with positive slope.

c. Answers will vary. We will use the points $(42.3,\ 82)$ and $(42.8,\ 93)$.

$$m = \frac{93 - 82}{42.8 - 42.3} = \frac{11}{0.5} = 22$$
$$N - N_1 = m\left(w - w_1\right)$$
$$N - 82 = 22\left(w - 42.3\right)$$
$$N - 82 = 22w - 930.6$$
$$N = 22w - 848.6$$

d.



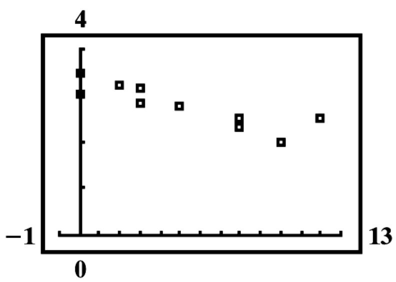
e. $N(42.5) = 22(42.5) - 848.6 = 86.4$

We predict that approximately 86 raisins will be in a box weighing 42.5 grams.

f. If the weight is increased by one gram, then the number of raisins will increase by 22.

19. a. The independent variable is the number of hours spent playing video games and cumulative grade-point average is the dependent variable because we are using number of hours playing video games to predict (or explain) cumulative grade-point average.

b.



c. Using the LINear REGression program, the line of best fit is: $G(h) = -0.0942h + 3.2763$

d. If the number of hours playing video games in a week increases by 1 hour, the cumulative grade-point average decreases 0.09, on average.

e. $G(8) = -0.0942(8) + 3.2763 = 2.52$

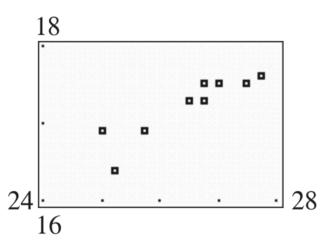
We predict a grade-point average of approximately 2.52 for a student who plays 8 hours of video games each week.

f.

$$\begin{aligned} 2.40 &= -0.0942(h) + 3.2763 \\ 2.40 - 3.2763 &= -0.0942h \\ -0.8763 &= -0.0942h \\ 9.3 &= h \end{aligned}$$

A student who has a grade-point average of 2.40 will have played approximately 9.3 hours of video games.

20. a.



b. Using the LINear REGression program, the line of best fit is: $C(H) = 0.3734H + 7.3268$

c. If height increases by one inch, the head circumference increases by 0.3734 inch.

d. $C(26) = 0.3734(26) + 7.3268 \approx 17.0$ inches

e. To find the height, we solve the following equation:

$$\begin{aligned} 17.4 &= 0.3734H + 7.3268 \\ 10.0732 &= 0.3734H \\ 26.98 &\approx H \end{aligned}$$

A child with a head circumference of 17.4 inches would have a height of about 26.98 inches.

21. a. The relation is not a function because 23 is paired with both 56 and 53.

b.

c. Using the LINear REGression program, the line of best fit is: $D = -1.3355p + 86.1974$.

The correlation coefficient is: $r \approx -0.9491$.

d. If the price of the jeans increases by \$1, the demand for the jeans decreases by about 1.34 pairs per day.

e. $D(p) = -1.3355p + 86.1974$

f. Domain: $\{p \mid 0 < p \le 64\}$

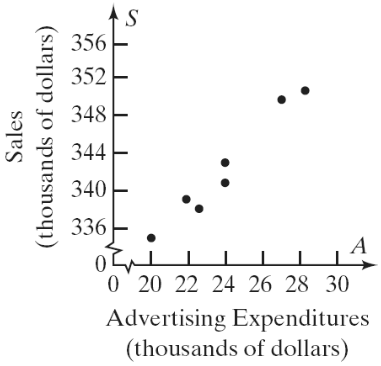
Note that the p-intercept is roughly 64.54 and that the number of pairs of jeans in demand cannot be negative.

g. $D(28) = -1.3355(28) + 86.1974 \approx 48.8034$

Demand is about 49 pairs.

22. a. The relation is not a function because 24 is paired with both 343 and 341.

b.



c. Using the LINear REGression program, the line of best fit is: $S = 2.0667A + 292.8869$.

The correlation coefficient is: $r \approx 0.9833$.

d. As the advertising expenditure increases by \$1000, the sales increase by about \$2067.

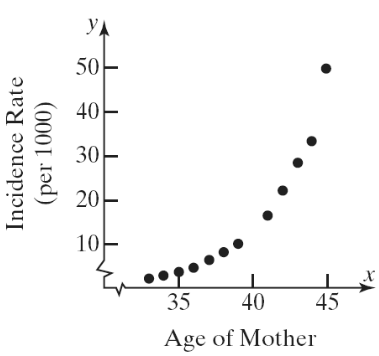
e. $S(A) = 2.0667A + 292.8869$

f. Domain: $\{A \mid A \geq 0\}$

g. $S(25) = 2.0667(25) + 292.8869 \approx 345$

Sales are about \$345 thousand.

23.



The data do not follow a linear pattern so it would not make sense to find the line of best fit.

24. The y-intercept would be the calories of a candy bar with weight 0 which would not be meaningful in this problem.

25. $G(0) = -0.0942(0) + 3.2763 = 3.2763$. The approximate grade-point average of a student who plays 0 hours of video games per week would be 3.28.

Section 3.3

1. $y = x^2 - 9$

To find the y-intercept, let $x = 0$:

$y = 0^2 - 9 = -9$.

To find the x-intercept(s), let $y = 0$:

$x^2 - 9 = 0$

$x^2 = 9$

$x = \pm\sqrt{9} = \pm 3$

The intercepts are $(0, -9), (-3, 0),$ and $(3, 0)$.

2. $2x^2 + 7x - 4 = 0$

$(2x-1)(x+4) = 0$

$2x - 1 = 0$ or $x + 4 = 0$

$2x = 1$ or $x = -4$

$x = \frac{1}{2}$ or $x = -4$

The solution set is $\left\{-4, \frac{1}{2}\right\}$..

3. $\left(\frac{1}{2}\cdot(-5)\right)^2 = \frac{25}{4}$

4. right; 4

5. parabola

6. axis (or axis of symmetry)

7. $-\frac{b}{2a}$

8. True; $a = 2 > 0$.

9. True; $-\frac{b}{2a} = -\frac{4}{2(-1)} = 2$

10. True

11. C

12. E

13. F

14. A

15. G

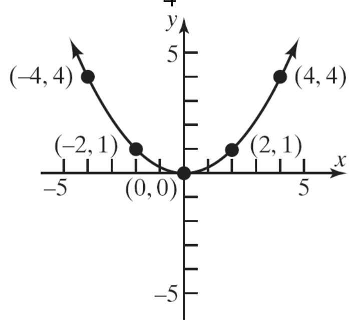
16. B

17. H

18. D

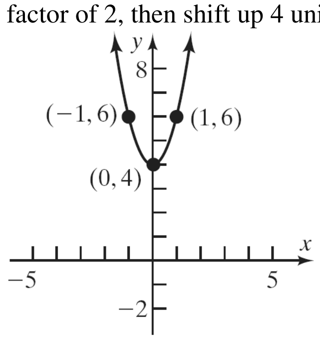
19. $f(x)=\frac{1}{4}x^2$

Using the graph of $y=x^2$, compress vertically by a factor of $\frac{1}{4}$.



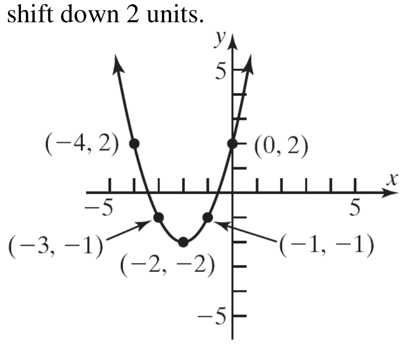
20. $f(x)=2x^2+4$

Using the graph of $y=x^2$, stretch vertically by a factor of 2, then shift up 4 units.



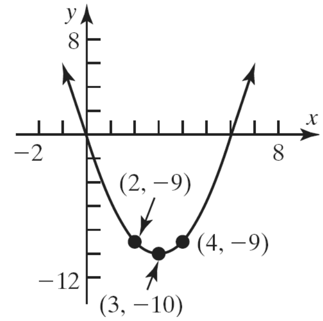
21. $f(x)=(x+2)^2-2$

Using the graph of $y=x^2$, shift left 2 units, then shift down 2 units.



22. $f(x)=(x-3)^2-10$

Using the graph of $y=x^2$, shift right 3 units, then shift down 10 units.

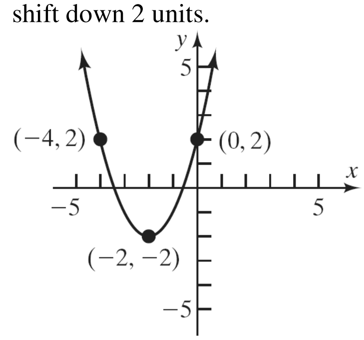


23. $f(x)=x^2+4x+2$

$=(x^2+4x+4)+2-4$

$=(x+2)^2-2$

Using the graph of $y=x^2$, shift left 2 units, then shift down 2 units.

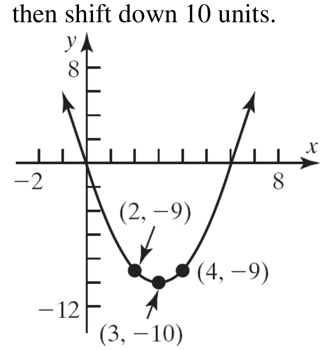


24. $f(x)=x^2-6x-1$

$=(x^2-6x+9)-1-9$

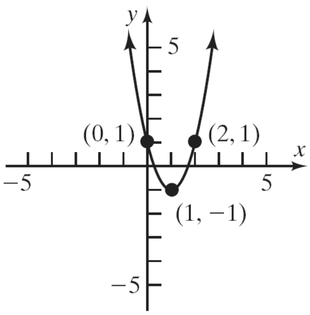
$=(x-3)^2-10$

Using the graph of $y=x^2$, shift right 3 units, then shift down 10 units.



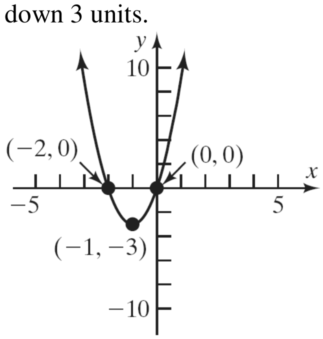
25. $f(x) = 2x^2 - 4x + 1$
$$= 2\left(x^2 - 2x\right) + 1$$
$$= 2(x^2 - 2x + 1) + 1 - 2$$
$$= 2(x-1)^2 - 1$$

Using the graph of $y = x^2$, shift right 1 unit, stretch vertically by a factor of 2, then shift down 1 unit.



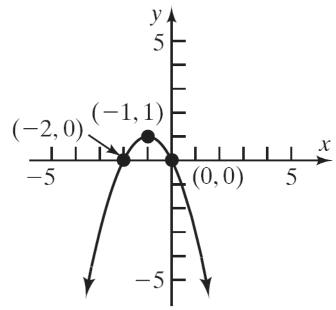
26. $f(x) = 3x^2 + 6x$
$$= 3\left(x^2 + 2x\right)$$
$$= 3(x^2 + 2x + 1) - 3$$
$$= 3(x+1)^2 - 3$$

Using the graph of $y = x^2$, shift left 1 unit, stretch vertically by a factor of 3, then shift down 3 units.



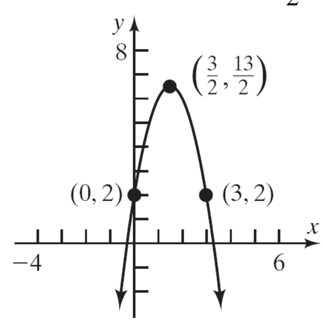
27. $f(x) = -x^2 - 2x$
$$= -\left(x^2 + 2x\right)$$
$$= -(x^2 + 2x + 1) + 1$$
$$= -(x+1)^2 + 1$$

Using the graph of $y = x^2$, shift left 1 unit, reflect across the *x*-axis, then shift up 1 unit.



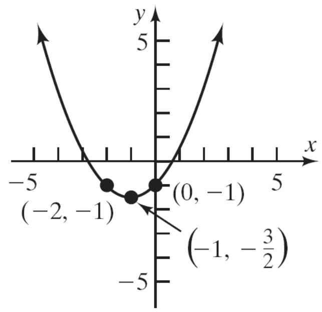
28. $f(x) = -2x^2 + 6x + 2$
$$= -2\left(x^2 - 3x\right) + 2$$
$$= -2\left(x^2 - 3x + \frac{9}{4}\right) + 2 + \frac{9}{2}$$
$$= -2\left(x - \frac{3}{2}\right)^2 + \frac{13}{2}$$

Using the graph of $y = x^2$, shift right $\frac{3}{2}$ units, reflect about the *x*-axis, stretch vertically by a factor of 2, then shift up $\frac{13}{2}$ units.



29. $f(x) = \frac{1}{2}x^2 + x - 1$
$$= \frac{1}{2}\left(x^2 + 2x\right) - 1$$
$$= \frac{1}{2}\left(x^2 + 2x + 1\right) - 1 - \frac{1}{2}$$
$$= \frac{1}{2}(x+1)^2 - \frac{3}{2}$$

Using the graph of $y = x^2$, shift left 1 unit, compress vertically by a factor of $\frac{1}{2}$, then shift down $\frac{3}{2}$ units.



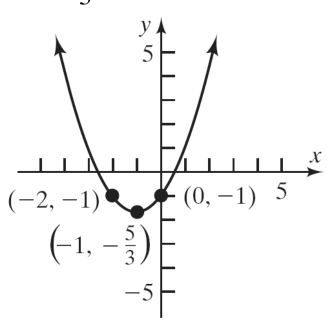
30. $f(x)=\frac{2}{3}x^2+\frac{4}{3}x-1$

$=\frac{2}{3}\left(x^2+2x\right)-1$

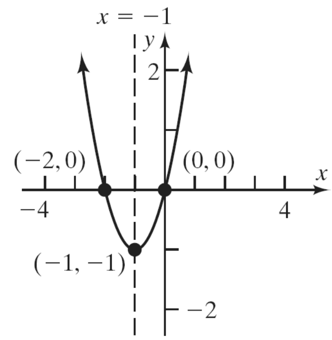
$=\frac{2}{3}\left(x^2+2x+1\right)-1-\frac{2}{3}$

$=\frac{2}{3}\left(x+1\right)^2-\frac{5}{3}$

Using the graph of $y=x^2$, shift left 1 unit, compress vertically by a factor of $\frac{2}{3}$, then shift down $\frac{5}{3}$ unit.



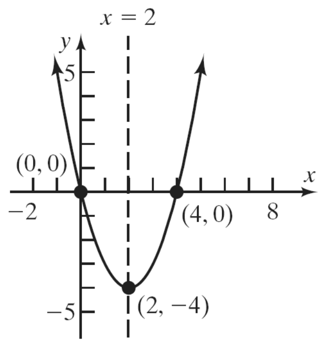
31. a. For $f(x)=x^2+2x$, $a=1$, $b=2$, $c=0.$
Since $a=1>0$, the graph opens up.
The x-coordinate of the vertex is
$x=\frac{-b}{2a}=\frac{-(2)}{2(1)}=\frac{-2}{2}=-1$.
The y-coordinate of the vertex is
$f\left(\frac{-b}{2a}\right)=f(-1)=(-1)^2+2(-1)=1-2=-1.$
Thus, the vertex is $(-1,-1)$.
The axis of symmetry is the line $x=-1$.
The discriminant is
$b^2-4ac=(2)^2-4(1)(0)=4>0$, so the graph has two x-intercepts.
The x-intercepts are found by solving:
$x^2+2x=0$
$x(x+2)=0$
$x=0$ or $x=-2$
The x-intercepts are -2 and 0.
The y-intercept is $f(0)=0$.



b. The domain is $(-\infty,\infty)$.
The range is $[-1,\infty)$.

c. Decreasing on $(-\infty,-1)$.
Increasing on $\left(-1,\infty\right)$.

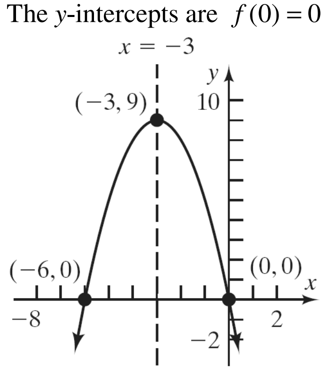
32. a. For $f(x)=x^2-4x$, $a=1$, $b=-4$, $c=0$.
Since $a=1>0$, the graph opens up.
The x-coordinate of the vertex is
$x=\frac{-b}{2a}=\frac{-(-4)}{2(1)}=\frac{4}{2}=2$.
The y-coordinate of the vertex is
$f\left(\frac{-b}{2a}\right)=f(2)=(2)^2-4(2)=4-8=-4.$
Thus, the vertex is $(2,-4)$.
The axis of symmetry is the line $x=2$.
The discriminant is:
$b^2-4ac=(-4)^2-4(1)(0)=16>0$, so the graph has two x-intercepts.
The x-intercepts are found by solving:
$x^2-4x=0$
$x(x-4)=0$
$x=0$ or $x=4.$
The x-intercepts are 0 and 4.
The y-intercept is $f(0)=0$.



b. The domain is $(-\infty, \infty)$.
The range is $[-4, \infty)$.

c. Decreasing on $(-\infty, 2)$.
Increasing on $(2, \infty)$.

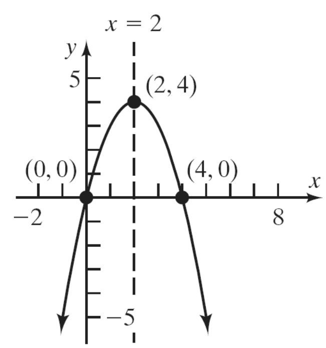
33. a. For $f(x) = -x^2 - 6x$, $a = -1$, $b = -6$, $c = 0$. Since $a = -1 < 0$, the graph opens down. The x-coordinate of the vertex is
$x = \frac{-b}{2a} = \frac{-(-6)}{2(-1)} = \frac{6}{-2} = -3.$
The y-coordinate of the vertex is
$f\left(\frac{-b}{2a}\right) = f(-3) = -(-3)^2 - 6(-3)$
$= -9 + 18 = 9.$
Thus, the vertex is $(-3, 9)$.
The axis of symmetry is the line $x = -3$.
The discriminant is:
$b^2 - 4ac = (-6)^2 - 4(-1)(0) = 36 > 0$,
so the graph has two x-intercepts.
The x-intercepts are found by solving:
$-x^2 - 6x = 0$
$-x(x + 6) = 0$
$x = 0$ or $x = -6$.
The x-intercepts are -6 and 0.
The y-intercepts are $f(0) = 0$.



b. The domain is $(-\infty, \infty)$.
The range is $(-\infty, 9]$.

c. Increasing on $(-\infty, -3)$.
Decreasing on $(-3, \infty)$.

34. a. For $f(x) = -x^2 + 4x$, $a = -1$, $b = 4$, $c = 0$.
Since $a = -1 < 0$, the graph opens down.
The x-coordinate of the vertex is
$x = \frac{-b}{2a} = \frac{-4}{2(-1)} = \frac{-4}{-2} = 2.$
The y-coordinate of the vertex is
$f\left(\frac{-b}{2a}\right) = f(2)$
$= -(2)^2 + 4(2)$
$= 4.$
Thus, the vertex is $(2, 4)$.
The axis of symmetry is the line $x = 2$.
The discriminant is:
$b^2 - 4ac = 4^2 - 4(-1)(0) = 16 > 0,$
so the graph has two x-intercepts.
The x-intercepts are found by solving:
$-x^2 + 4x = 0$
$-x(x - 4) = 0$
$x = 0$ or $x = 4$.
The x-intercepts are 0 and 4.
The y-intercept is $f(0) = 0$.



b. The domain is $(-\infty, \infty)$.
The range is $(-\infty, 4]$.

c. Increasing on $(-\infty, 2)$.
Decreasing on $(2, \infty)$.

35. a. For $f(x) = x^2 + 2x - 8$, $a = 1$, $b = 2$, $c = -8$.
Since $a = 1 > 0$, the graph opens up.
The x-coordinate of the vertex is

$x = \frac{-b}{2a} = \frac{-2}{2(1)} = \frac{-2}{2} = -1$.

The y-coordinate of the vertex is

$f\left(\frac{-b}{2a}\right) = f(-1) = (-1)^2 + 2(-1) - 8$

$= 1 - 2 - 8 = -9$.

Thus, the vertex is $(-1, -9)$.

The axis of symmetry is the line $x = -1$.

The discriminant is:

$b^2 - 4ac = 2^2 - 4(1)(-8) = 4 + 32 = 36 > 0$,

so the graph has two x-intercepts.

The x-intercepts are found by solving:

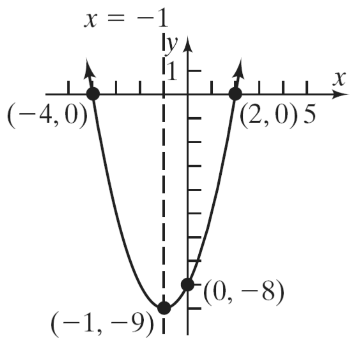
$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$x = -4$ or $x = 2$.

The x-intercepts are -4 and 2.

The y-intercept is $f(0) = -8$.



b. The domain is $(-\infty, \infty)$.

The range is $[-9, \infty)$.

c. Decreasing on $(-\infty, -1)$.

Increasing on $(-1, \infty)$.

36. a. For $f(x) = x^2 - 2x - 3$, $a = 1$, $b = -2$, $c = -3$.

Since $a = 1 > 0$, the graph opens up.

The x-coordinate of the vertex is

$x = \frac{-b}{2a} = \frac{-(-2)}{2(1)} = \frac{2}{2} = 1$.

The y-coordinate of the vertex is

$f\left(\frac{-b}{2a}\right) = f(1) = 1^2 - 2(1) - 3 = -4$.

Thus, the vertex is $(1, -4)$.

The axis of symmetry is the line $x = 1$.

The discriminant is:

$b^2 - 4ac = (-2)^2 - 4(1)(-3) = 4 + 12 = 16 > 0$,

so the graph has two x-intercepts.

The x-intercepts are found by solving:

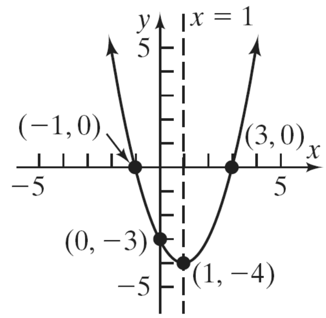
$x^2 - 2x - 3 = 0$

$(x+1)(x-3) = 0$

$x = -1$ or $x = 3$.

The x-intercepts are -1 and 3.

The y-intercept is $f(0) = -3$.



b. The domain is $(-\infty, \infty)$. The range is $[-4, \infty)$.

c. Decreasing on $(-\infty, 1)$. Increasing on $(1, \infty)$.

37. a. For $f(x) = x^2 + 2x + 1$, $a = 1$, $b = 2$, $c = 1$.

Since $a = 1 > 0$, the graph opens up.

The x-coordinate of the vertex is

$x = \frac{-b}{2a} = \frac{-2}{2(1)} = \frac{-2}{2} = -1$.

The y-coordinate of the vertex is

$f\left(\frac{-b}{2a}\right) = f(-1)$

$= (-1)^2 + 2(-1) + 1 = 1 - 2 + 1 = 0$.

Thus, the vertex is $(-1, 0)$.

The axis of symmetry is the line $x = -1$.

The discriminant is:

$b^2 - 4ac = 2^2 - 4(1)(1) = 4 - 4 = 0$,

so the graph has one x-intercept.

The x-intercept is found by solving:

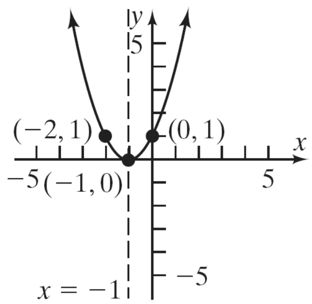
$x^2 + 2x + 1 = 0$

$(x+1)^2 = 0$

$x = -1$.

The x-intercept is -1.

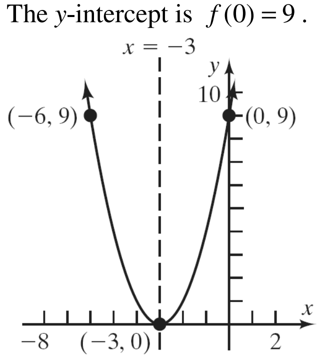
The y-intercept is $f(0)=1$.



b. The domain is $(-\infty, \infty)$.
The range is $[0, \infty)$.

c. Decreasing on $(-\infty, -1)$.
Increasing on $(-1, \infty)$.

38. a. For $f(x)=x^2+6x+9$, $a=1$, $b=6$, $c=9$.
Since $a=1>0$, the graph opens up.
The x-coordinate of the vertex is
$$x=\frac{-b}{2a}=\frac{-6}{2(1)}=\frac{-6}{2}=-3.$$
The y-coordinate of the vertex is
$$f\left(\frac{-b}{2a}\right)=f(-3)$$
$$=(-3)^2+6(-3)+9=9-18+9=0.$$
Thus, the vertex is $(-3, 0)$.
The axis of symmetry is the line $x=-3$.
The discriminant is:
$b^2-4ac=6^2-4(1)(9)=36-36=0$,
so the graph has one x-intercept.
The x-intercept is found by solving:
$$x^2+6x+9=0$$
$$(x+3)^2=0$$
$$x=-3.$$
The x-intercept is -3.
The y-intercept is $f(0)=9$.

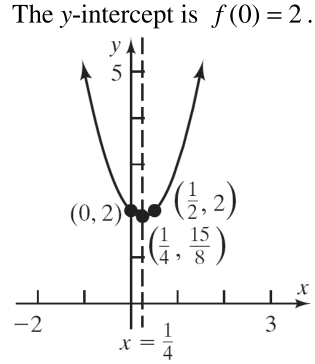


b. The domain is $(-\infty, \infty)$. The range is $[0, \infty)$.

c. Decreasing on $(-\infty, -3)$.
Increasing on $(-3, \infty)$.

39. a. For $f(x)=2x^2-x+2$, $a=2$, $b=-1$, $c=2$.
Since $a=2>0$, the graph opens up.
The x-coordinate of the vertex is
$$x=\frac{-b}{2a}=\frac{-(-1)}{2(2)}=\frac{1}{4}.$$
The y-coordinate of the vertex is
$$f\left(\frac{-b}{2a}\right)=f\left(\frac{1}{4}\right)=2\left(\frac{1}{4}\right)^2-\frac{1}{4}+2$$
$$=\frac{1}{8}-\frac{1}{4}+2=\frac{15}{8}.$$
Thus, the vertex is $\left(\frac{1}{4}, \frac{15}{8}\right)$.

The axis of symmetry is the line $x=\frac{1}{4}$.
The discriminant is:
$b^2-4ac=(-1)^2-4(2)(2)=1-16=-15$,
so the graph has no x-intercepts.
The y-intercept is $f(0)=2$.



b. The domain is $(-\infty, \infty)$. The range is $\left[\frac{15}{8}, \infty\right)$.

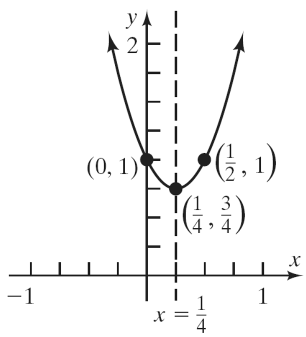
c. Decreasing on $\left(-\infty, \frac{1}{4}\right)$. Increasing on $\left(\frac{1}{4}, \infty\right)$.

40. a. For $f(x)=4x^2-2x+1$, $a=4$, $b=-2$, $c=1$.
Since $a=4>0$, the graph opens up.
The x-coordinate of the vertex is
$$x=\frac{-b}{2a}=\frac{-(-2)}{2(4)}=\frac{2}{8}=\frac{1}{4}.$$
The y-coordinate of the vertex is
$$f\left(\frac{-b}{2a}\right)=f\left(\frac{1}{4}\right)=4\left(\frac{1}{4}\right)^2-2\left(\frac{1}{4}\right)+1$$
$$=\frac{1}{4}-\frac{1}{2}+1=\frac{3}{4}.$$

Thus, the vertex is $\left(\frac{1}{4},\frac{3}{4}\right)$.

The axis of symmetry is the line $x=\frac{1}{4}$.

The discriminant is:
$b^2-4ac=(-2)^2-4(4)(1)=4-16=-12$,
so the graph has no x-intercepts.
The y-intercept is $f(0)=1$.



b. The domain is $(-\infty,\infty)$.

The range is $\left[\frac{3}{4},\infty\right)$.

c. Decreasing on $\left(-\infty,\frac{1}{4}\right)$.

Increasing on $\left(\frac{1}{4},\infty\right)$.

41. a. For $f(x)=-2x^2+2x-3$, $a=-2$, $b=2$, $c=-3$. Since $a=-2<0$, the graph opens down.

The x-coordinate of the vertex is
$x=\frac{-b}{2a}=\frac{-(2)}{2(-2)}=\frac{-2}{-4}=\frac{1}{2}$.

The y-coordinate of the vertex is

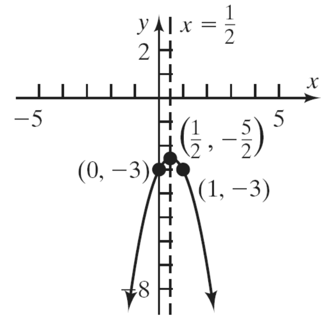
$$f\left(\frac{-b}{2a}\right)=f\left(\frac{1}{2}\right)=-2\left(\frac{1}{2}\right)^2+2\left(\frac{1}{2}\right)-3$$
$$=-\frac{1}{2}+1-3=-\frac{5}{2}.$$

Thus, the vertex is $\left(\frac{1}{2},-\frac{5}{2}\right)$.

The axis of symmetry is the line $x=\frac{1}{2}$.

The discriminant is:
$b^2-4ac=2^2-4(-2)(-3)=4-24=-20$,
so the graph has no x-intercepts.

The y-intercept is $f(0)=-3$.



b. The domain is $(-\infty,\infty)$.

The range is $\left(-\infty,-\frac{5}{2}\right]$.

c. Increasing on $\left(-\infty,\frac{1}{2}\right)$.

Decreasing on $\left(\frac{1}{2},\infty\right)$.

42. a. For $f(x)=-3x^2+3x-2$, $a=-3$, $b=3$, $c=-2$. Since $a=-3<0$, the graph opens down.

The x-coordinate of the vertex is
$x=\frac{-b}{2a}=\frac{-3}{2(-3)}=\frac{-3}{-6}=\frac{1}{2}$.

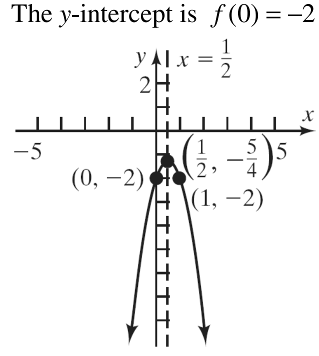
The y-coordinate of the vertex is

$$f\left(\frac{-b}{2a}\right)=f\left(\frac{1}{2}\right)=-3\left(\frac{1}{2}\right)^2+3\left(\frac{1}{2}\right)-2$$
$$=-\frac{3}{4}+\frac{3}{2}-2=-\frac{5}{4}.$$

Thus, the vertex is $\left(\frac{1}{2},-\frac{5}{4}\right)$.

The axis of symmetry is the line $x=\frac{1}{2}$.

The discriminant is:
$b^2-4ac=3^2-4(-3)(-2)=9-24=-15$,
so the graph has no x-intercepts.
The y-intercept is $f(0)=-2$.



b. The domain is $(-\infty, \infty)$.

The range is $\left(-\infty, -\frac{5}{4}\right]$.

c. Increasing on $\left(-\infty, \frac{1}{2}\right)$.

Decreasing on $\left(\frac{1}{2}, \infty\right)$.

43. a. For $f(x) = 3x^2 + 6x + 2$, $a = 3$, $b = 6$, $c = 2$. Since $a = 3 > 0$, the graph opens up.
The x-coordinate of the vertex is
$x = \frac{-b}{2a} = \frac{-6}{2(3)} = \frac{-6}{6} = -1$.
The y-coordinate of the vertex is

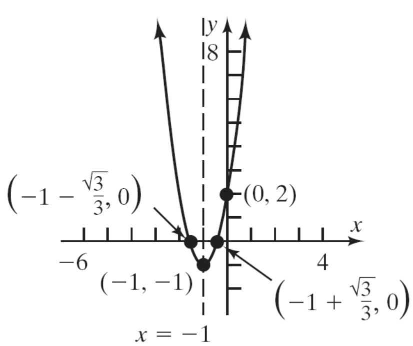
$$f\left(\frac{-b}{2a}\right) = f(-1) = 3(-1)^2 + 6(-1) + 2$$
$$= 3 - 6 + 2 = -1.$$

Thus, the vertex is $(-1, -1)$.
The axis of symmetry is the line $x = -1$.
The discriminant is:
$b^2 - 4ac = 6^2 - 4(3)(2) = 36 - 24 = 12$,
so the graph has two x-intercepts.
The x-intercepts are found by solving:

$$3x^2 + 6x + 2 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-6 \pm \sqrt{12}}{6} = \frac{-6 \pm 2\sqrt{3}}{6} = \frac{-3 \pm \sqrt{3}}{3}$$

The x-intercepts are $-1 - \frac{\sqrt{3}}{3}$ and $-1 + \frac{\sqrt{3}}{3}$.

The y-intercept is $f(0) = 2$.



b. The domain is $(-\infty, \infty)$.

The range is $[-1, \infty)$.

c. Decreasing on $(-\infty, -1)$.

Increasing on $(-1, \infty)$.

44. a. For $f(x) = 2x^2 + 5x + 3$, $a = 2$, $b = 5$, $c = 3$. Since $a = 2 > 0$, the graph opens up.
The x-coordinate of the vertex is
$x = \frac{-b}{2a} = \frac{-5}{2(2)} = -\frac{5}{4}$.
The y-coordinate of the vertex is

$$f\left(\frac{-b}{2a}\right) = f\left(-\frac{5}{4}\right)$$
$$= 2\left(-\frac{5}{4}\right)^2 + 5\left(-\frac{5}{4}\right) + 3$$
$$= \frac{25}{8} - \frac{25}{4} + 3$$
$$= -\frac{1}{8}.$$

Thus, the vertex is $\left(-\frac{5}{4}, -\frac{1}{8}\right)$.

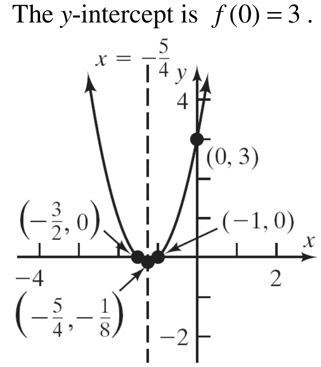
The axis of symmetry is the line $x = -\frac{5}{4}$.

The discriminant is:
$b^2 - 4ac = 5^2 - 4(2)(3) = 25 - 24 = 1$,
so the graph has two x-intercepts.
The x-intercepts are found by solving:

$$2x^2 + 5x + 3 = 0$$
$$(2x + 3)(x + 1) = 0$$
$$x = -\frac{3}{2} \text{ or } x = -1.$$

The x-intercepts are $-\frac{3}{2}$ and -1.

The y-intercept is $f(0) = 3$.



b. The domain is $(-\infty, \infty)$.

The range is $\left[-\frac{1}{8}, \infty\right)$.

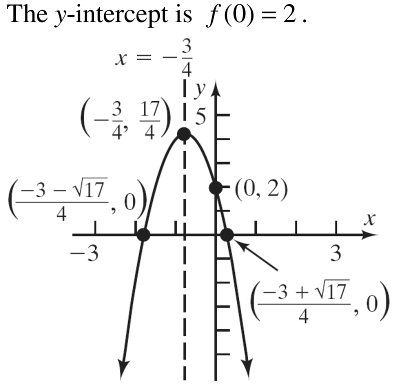
c. Decreasing on $\left(-\infty, -\frac{5}{4}\right)$.

Increasing on $\left(-\frac{5}{4}, \infty\right)$.

45. a. For $f(x)=-4x^2-6x+2$, $a=-4$, $b=-6$, $c=2$. Since $a=-4<0$, the graph opens down.
The x-coordinate of the vertex is
$$x=\frac{-b}{2a}=\frac{-(-6)}{2(-4)}=\frac{6}{-8}=-\frac{3}{4}.$$
The y-coordinate of the vertex is
$$f\left(\frac{-b}{2a}\right)=f\left(-\frac{3}{4}\right)=-4\left(-\frac{3}{4}\right)^2-6\left(-\frac{3}{4}\right)+2$$
$$=-\frac{9}{4}+\frac{9}{2}+2=\frac{17}{4}.$$
Thus, the vertex is $\left(-\frac{3}{4},\frac{17}{4}\right)$.

The axis of symmetry is the line $x=-\frac{3}{4}$.

The discriminant is:
$b^2-4ac=(-6)^2-4(-4)(2)=36+32=68$,
so the graph has two x-intercepts.
The x-intercepts are found by solving:
$$-4x^2-6x+2=0$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-6)\pm\sqrt{68}}{2(-4)}$$
$$=\frac{6\pm\sqrt{68}}{-8}=\frac{6\pm2\sqrt{17}}{-8}=\frac{3\pm\sqrt{17}}{-4}$$
The x-intercepts are $\frac{-3+\sqrt{17}}{4}$ and $\frac{-3-\sqrt{17}}{4}$.
The y-intercept is $f(0)=2$.



b. The domain is $(-\infty,\infty)$.

The range is $\left(-\infty,\frac{17}{4}\right]$.

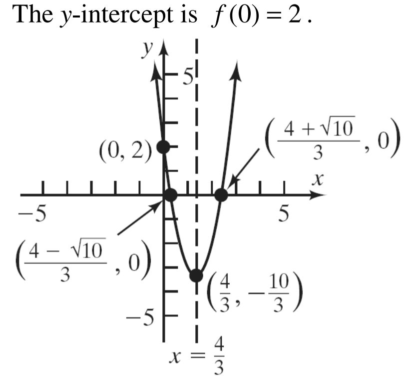
c. Decreasing on $\left(-\frac{3}{4},\infty\right)$.

Increasing on $\left(-\infty,-\frac{3}{4}\right)$.

46. a. For $f(x)=3x^2-8x+2$, $a=3$, $b=-8$, $c=2$. Since $a=3>0$, the graph opens up.
The x-coordinate of the vertex is
$$x=\frac{-b}{2a}=\frac{-(-8)}{2(3)}=\frac{8}{6}=\frac{4}{3}.$$
The y-coordinate of the vertex is
$$f\left(\frac{-b}{2a}\right)=f\left(\frac{4}{3}\right)=3\left(\frac{4}{3}\right)^2-8\left(\frac{4}{3}\right)+2$$
$$=\frac{16}{3}-\frac{32}{3}+2=-\frac{10}{3}.$$
Thus, the vertex is $\left(\frac{4}{3},-\frac{10}{3}\right)$.

The axis of symmetry is the line $x=\frac{4}{3}$.

The discriminant is:
$b^2-4ac=(-8)^2-4(3)(2)=64-24=40$,
so the graph has two x-intercepts.
The x-intercepts are found by solving:
$$3x^2-8x+2=0$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-8)\pm\sqrt{40}}{2(3)}$$
$$=\frac{8\pm\sqrt{40}}{6}=\frac{8\pm2\sqrt{10}}{6}=\frac{4\pm\sqrt{10}}{3}$$
The x-intercepts are $\frac{4+\sqrt{10}}{3}$ and $\frac{4-\sqrt{10}}{3}$.
The y-intercept is $f(0)=2$.



b. The domain is $(-\infty,\infty)$.

The range is $\left[-\frac{10}{3},\infty\right)$.

c. Decreasing on $\left(-\infty,\frac{4}{3}\right)$.

Increasing on $\left(\frac{4}{3},\infty\right)$.

47. Consider the form $y = a(x-h)^2 + k$. From the graph we know that the vertex is $(-1,-2)$ so we have $h = -1$ and $k = -2$. The graph also passes through the point $(x, y) = (0,-1)$. Substituting these values for *x, y, h,* and *k*, we can solve for *a*:

$-1 = a(0-(-1))^2 + (-2)$

$-1 = a(1)^2 - 2$

$-1 = a - 2$

$1 = a$

The quadratic function is

$f(x) = (x+1)^2 - 2 = x^2 + 2x - 1$.

48. Consider the form $y = a(x-h)^2 + k$. From the graph we know that the vertex is $(2,1)$ so we have $h = 2$ and $k = 1$. The graph also passes through the point $(x, y) = (0,5)$. Substituting these values for *x, y, h,* and *k*, we can solve for *a*:

$5 = a(0-2)^2 + 1$

$5 = a(-2)^2 + 1$

$5 = 4a + 1$

$4 = 4a$

$1 = a$

The quadratic function is

$f(x) = (x-2)^2 + 1 = x^2 - 4x + 5$.

49. Consider the form $y = a(x-h)^2 + k$. From the graph we know that the vertex is $(-3,5)$ so we have $h = -3$ and $k = 5$. The graph also passes through the point $(x, y) = (0,-4)$. Substituting these values for *x, y, h,* and *k*, we can solve for *a*:

$-4 = a(0-(-3))^2 + 5$

$-4 = a(3)^2 + 5$

$-4 = 9a + 5$

$-9 = 9a$

$-1 = a$

The quadratic function is

$f(x) = -(x+3)^2 + 5 = -x^2 - 6x - 4$.

50. Consider the form $y = a(x-h)^2 + k$. From the graph we know that the vertex is $(2,3)$ so we have $h = 2$ and $k = 3$. The graph also passes through the point $(x, y) = (0,-1)$. Substituting these values for *x, y, h,* and *k*, we can solve for *a*:

$-1 = a(0-2)^2 + 3$

$-1 = a(-2)^2 + 3$

$-1 = 4a + 3$

$-4 = 4a$

$-1 = a$

The quadratic function is

$f(x) = -(x-2)^2 + 3 = -x^2 + 4x - 1$.

51. Consider the form $y = ax^2 + bx + c$. Substituting the three points from the graph into the general form we have the following three equations.

$5 = a(-1)^2 + b(-1) + c \Rightarrow 5 = a - b + c$

and

$5 = a(3)^2 + b(3) + c \Rightarrow 5 = 9a + 3b + c$

and

$-1 = a(0)^2 + b(0) + c \Rightarrow -1 = c$

Since $-1 = c$, we have the following equations:

$5 = a - b - 1, \quad 5 = 9a + 3b - 1, \quad -1 = c$

Solving the first two simultaneously we have

$$\left.\begin{aligned} 5 &= a - b - 1 \\ 5 &= 9a + 3b - 1 \end{aligned}\right)$$

$$\left.\begin{aligned} 6 &= a - b \\ 6 &= 9a + 3b \end{aligned}\right) \rightarrow \quad a = 2, b = -4$$

The quadratic function is $f(x) = 2x^2 - 4x - 1$.

52. Consider the form $y = ax^2 + bx + c$. Substituting the three points from the graph into the general form we have the following three equations.

$-2 = a(-4)^2 + b(-4) + c \Rightarrow -2 = 16a - 4b + c$

and

$4 = a(-1)^2 + b(-1) + c \Rightarrow 4 = a - b + c$

and

$-2 = a(0)^2 + b(0) + c \Rightarrow -2 = c$

Since $-2 = c$, we have the following equations:

$-2 = 16a - 4b - 2, \quad 4 = a - b - 2, \quad -2 = c$

Solving the first two simultaneously we have

$$\left.\begin{aligned} -2 &= 16a-4b-2 \\ 4 &= a-b-2 \end{aligned}\right\}$$

$$\left.\begin{aligned} 0 &= 16a-4b \\ 6 &= a-b \end{aligned}\right\} \rightarrow \quad a=-2, b=-8$$

The quadratic function is $f(x)=-2x^2-8x-2$.

53. For $f(x)=2x^2+12x$, $a=2$, $b=12$, $c=0$. Since $a=2>0$, the graph opens up, so the vertex is a minimum point. The minimum occurs at $x=\frac{-b}{2a}=\frac{-12}{2(2)}=\frac{-12}{4}=-3$.
The minimum value is
$f(-3)=2(-3)^2+12(-3)=18-36=-18$.

54. For $f(x)=-2x^2+12x$, $a=-2$, $b=12$, $c=0$, . Since $a=-2<0$, the graph opens down, so the vertex is a maximum point. The maximum occurs at $x=\frac{-b}{2a}=\frac{-12}{2(-2)}=\frac{-12}{-4}=3$.
The maximum value is
$f(3)=-2(3)^2+12(3)=-18+36=18$.

55. For $f(x)=2x^2+12x-3$, $a=2$, $b=12$, $c=-3$. Since $a=2>0$, the graph opens up, so the vertex is a minimum point. The minimum occurs at $x=\frac{-b}{2a}=\frac{-12}{2(2)}=\frac{-12}{4}=-3$. The minimum value is
$f(-3)=2(-3)^2+12(-3)-3=18-36-3=-21$.

56. For $f(x)=4x^2-8x+3$, $a=4$, $b=-8$, $c=3$. Since $a=4>0$, the graph opens up, so the vertex is a minimum point. The minimum occurs at $x=\frac{-b}{2a}=\frac{-(-8)}{2(4)}=\frac{8}{8}=1$. The minimum value is
$f(1)=4(1)^2-8(1)+3=4-8+3=-1$.

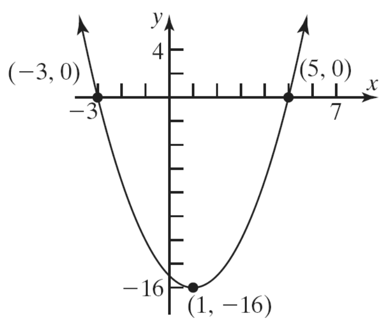
57. For $f(x)=-x^2+10x-4$, $a=-1$, $b=10$, $c=-4$. Since $a=-1<0$, the graph opens down, so the vertex is a maximum point. The maximum occurs at $x=\frac{-b}{2a}=\frac{-10}{2(-1)}=\frac{-10}{-2}=5$. The maximum value is
$f(5)=-(5)^2+10(5)-4=-25+50-4=21$.

58. For $f(x)=-2x^2+8x+3$, $a=-2$, $b=8$, $c=3$. Since $a=-2<0$, the graph opens down, so the vertex is a maximum point. The maximum occurs at $x=\frac{-b}{2a}=\frac{-8}{2(-2)}=\frac{-8}{-4}=2$. The maximum value is
$f(2)=-2(2)^2+8(2)+3=-8+16+3=11$.

59. For $f(x)=-3x^2+12x+1$, $a=-3$, $b=12$, $c=1$. Since $a=-3<0$, the graph opens down, so the vertex is a maximum point. The maximum occurs at $x=\frac{-b}{2a}=\frac{-12}{2(-3)}=\frac{-12}{-6}=2$. The maximum value is $f(2)=-3(2)^2+12(2)+1=-12+24+1=13$.

60. For $f(x)=4x^2-4x$, $a=4$, $b=-4$, $c=0$. Since $a=4>0$, the graph opens up, so the vertex is a minimum point. The minimum occurs at $x=\frac{-b}{2a}=\frac{-(-4)}{2(4)}=\frac{4}{8}=\frac{1}{2}$. The minimum value is
$f\left(\frac{1}{2}\right)=4\left(\frac{1}{2}\right)^2-4\left(\frac{1}{2}\right)=1-2=-1$.

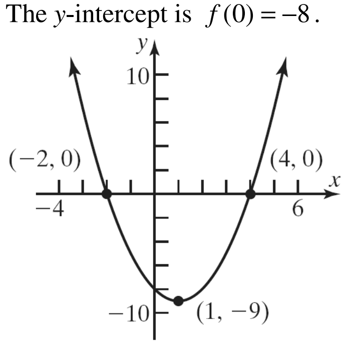
61. **a.** For $f(x)=x^2-2x-15$, $a=1$, $b=-2$, $c=-15$. Since $a=1>0$, the graph opens up. The x-coordinate of the vertex is
$x=\frac{-b}{2a}=\frac{-(-2)}{2(1)}=\frac{2}{2}=1$.
The y-coordinate of the vertex is
$$\begin{aligned} f\left(\frac{-b}{2a}\right) &= f(1)=(1)^2-2(1)-15 \\ &= 1-2-15=-16. \end{aligned}$$
Thus, the vertex is $(1,-16)$.
The discriminant is:
$b^2-4ac=(-2)^2-4(1)(-15)=4+60=64>0$, so the graph has two x-intercepts.
The x-intercepts are found by solving:
$$\begin{aligned} &x^2-2x-15=0 \\ &(x+3)(x-5)=0 \\ &x=-3 \text{ or } x=5 \end{aligned}$$
The x-intercepts are -3 and 5.
The y-intercept is $f(0)=-15$.



b. The domain is $(-\infty, \infty)$.
The range is $[-16, \infty)$.

c. Decreasing on $(-\infty, 1)$.
Increasing on $(1, \infty)$.

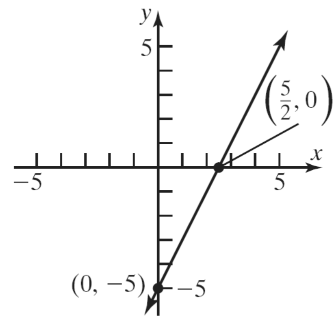
62. a. For $f(x) = x^2 - 2x - 8$, $a = 1$, $b = -2$, $c = -8$. Since $a = 1 > 0$, the graph opens up.
The x-coordinate of the vertex is
$x = \frac{-b}{2a} = \frac{-(-2)}{2(1)} = \frac{2}{2} = 1$.
The y-coordinate of the vertex is
$f\left(\frac{-b}{2a}\right) = f(1) = (1)^2 - 2(1) - 8 = 1 - 2 - 8 = -9.$
Thus, the vertex is $(1, -9)$.
The discriminant is:
$b^2 - 4ac = (-2)^2 - 4(1)(-8) = 4 + 32 = 36 > 0$,
so the graph has two x-intercepts.
The x-intercepts are found by solving:
$x^2 - 2x - 8 = 0$
$(x + 2)(x - 4) = 0$
$x = -2$ or $x = 4$
The x-intercepts are -2 and 4.
The y-intercept is $f(0) = -8$.



b. The domain is $(-\infty, \infty)$. The range is $[-9, \infty)$.

c. Decreasing on $(-\infty, 1)$. Increasing on $(1, \infty)$.

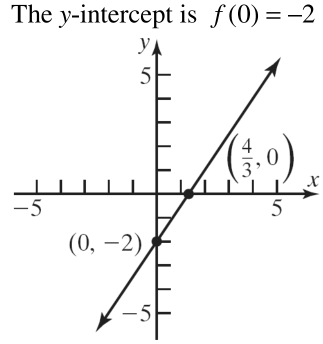
63. a. $F(x) = 2x - 5$ is a linear function.
The x-intercept is found by solving:
$2x - 5 = 0$
$2x = 5$
$x = \frac{5}{2}$
The x-intercept is $\frac{5}{2}$.
The y-intercept is $F(0) = -5$.



b. The domain is $(-\infty, \infty)$.
The range is $(-\infty, \infty)$.

c. Increasing on $(-\infty, \infty)$.

64. a. $f(x) = \frac{3}{2}x - 2$ is a linear function.
The x-intercept is found by solving:
$\frac{3}{2}x - 2 = 0$
$\frac{3}{2}x = 2$
$x = 2 \cdot \frac{2}{3} = \frac{4}{3}$
The x-intercept is $\frac{4}{3}$.
The y-intercept is $f(0) = -2$.

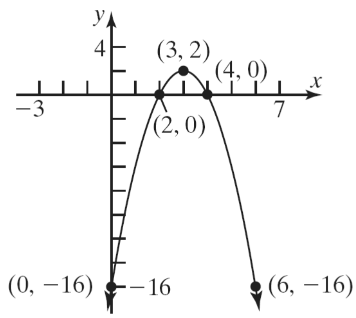


b. The domain is $(-\infty, \infty)$.
The range is $(-\infty, \infty)$.

c. Increasing on $(-\infty, \infty)$.

65. a. $g(x) = -2(x-3)^2 + 2$

Using the graph of $y = x^2$, shift right 3 units, reflect about the *x*-axis, stretch vertically by a factor of 2, then shift up 2 units.



b. The domain is $(-\infty, \infty)$.

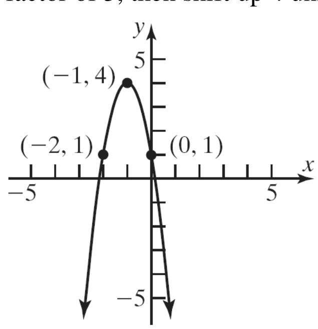
The range is $(-\infty, 2]$.

c. Increasing on $(-\infty, 3)$.

Decreasing on $(3, \infty)$.

66. a. $h(x) = -3(x+1)^2 + 4$

Using the graph of $y = x^2$, shift left 1 unit, reflect about the *x*-axis, stretch vertically by a factor of 3, then shift up 4 units.



b. The domain is $(-\infty, \infty)$.

The range is $(-\infty, 4]$.

c. Increasing on $(-\infty, -1)$.

Decreasing on $(-1, \infty)$.

67. a. For $f(x) = 2x^2 + x + 1$, $a = 2$, $b = 1$, $c = 1$.

Since $a = 2 > 0$, the graph opens up.

The *x*-coordinate of the vertex is

$$x = \frac{-b}{2a} = \frac{-1}{2(2)} = \frac{-1}{4} = -\frac{1}{4}.$$

The *y*-coordinate of the vertex is

$$f\left(\frac{-b}{2a}\right) = f\left(-\frac{1}{4}\right) = 2\left(-\frac{1}{4}\right)^2 + \left(-\frac{1}{4}\right) + 1$$

$$= \frac{1}{8} - \frac{1}{4} + 1 = \frac{7}{8}.$$

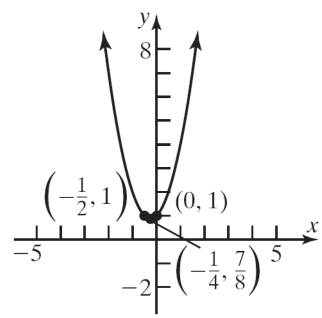
Thus, the vertex is $\left(-\frac{1}{4}, \frac{7}{8}\right)$.

The discriminant is:

$b^2 - 4ac = 1^2 - 4(2)(1) = 1 - 8 = -7$,

so the graph has no *x*-intercepts.

The *y*-intercept is $f(0) = 1$.



b. The domain is $(-\infty, \infty)$.

The range is $\left[\frac{7}{8}, \infty\right)$.

c. Decreasing on $\left(-\infty, -\frac{1}{4}\right)$.

Increasing on $\left(-\frac{1}{4}, \infty\right)$.

68. a. For $G(x) = 3x^2 + 2x + 5$, $a = 3$, $b = 2$, $c = 5$.

Since $a = 3 > 0$, the graph opens up.

The *x*-coordinate of the vertex is

$$x = \frac{-b}{2a} = \frac{-2}{2(3)} = \frac{-2}{6} = -\frac{1}{3}.$$

The *y*-coordinate of the vertex is

$$G\left(\frac{-b}{2a}\right) = G\left(-\frac{1}{3}\right) = 3\left(-\frac{1}{3}\right)^2 + 2\left(-\frac{1}{3}\right) + 5$$

$$= \frac{1}{3} - \frac{2}{3} + 5 = \frac{14}{3}.$$

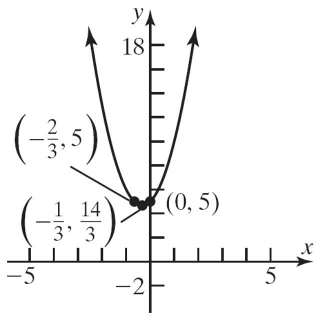
Thus, the vertex is $\left(-\frac{1}{3}, \frac{14}{3}\right)$.

The discriminant is:

$b^2 - 4ac = 2^2 - 4(3)(5) = 4 - 60 = -56$,

so the graph has no *x*-intercepts.

The *y*-intercept is $G(0) = 5$.



b. The domain is $(-\infty, \infty)$.

The range is $\left[\frac{14}{3}, \infty\right)$.

c. Decreasing on $\left(-\infty, -\frac{1}{3}\right)$.

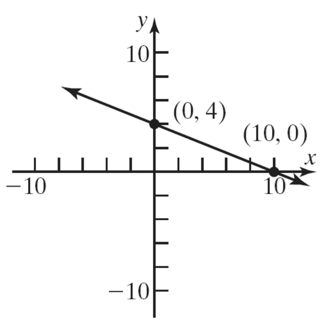
Increasing on $\left(-\frac{1}{3}, \infty\right)$.

69. a. $h(x) = -\frac{2}{5}x + 4$ is a linear function.

The x-intercept is found by solving:

$$-\frac{2}{5}x + 4 = 0$$
$$-\frac{2}{5}x = -4$$
$$x = -4\left(-\frac{5}{2}\right) = 10$$

The x-intercept is 10.
The y-intercept is $h(0) = 4$.



b. The domain is $(-\infty, \infty)$.

The range is $(-\infty, \infty)$.

c. Decreasing on $(-\infty, \infty)$.

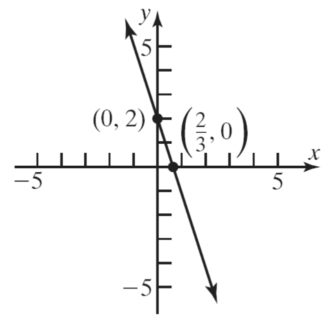
70. a. $f(x) = -3x + 2$ is a linear function.

The x-intercept is found by solving:

$$-3x + 2 = 0$$
$$-3x = -2$$
$$x = \frac{-2}{-3} = \frac{2}{3}$$

The x-intercept is $\frac{2}{3}$.

The y-intercept is $f(0) = 2$.



b. The domain is $(-\infty, \infty)$.

The range is $(-\infty, \infty)$.

c. Decreasing on $(-\infty, \infty)$.

71. a. For $H(x) = -4x^2 - 4x - 1$, $a = -4$, $b = -4$, $c = -1$. Since $a = -4 < 0$, the graph opens down. The x-coordinate of the vertex is $x = \frac{-b}{2a} = \frac{-(-4)}{2(-4)} = \frac{4}{-8} = -\frac{1}{2}$.

The y-coordinate of the vertex is

$$H\left(\frac{-b}{2a}\right) = H\left(-\frac{1}{2}\right) = -4\left(-\frac{1}{2}\right)^2 - 4\left(-\frac{1}{2}\right) - 1$$
$$= -1 + 2 - 1 = 0$$

Thus, the vertex is $\left(-\frac{1}{2}, 0\right)$.

The discriminant is:

$b^2 - 4ac = (-4)^2 - 4(-4)(-1) = 16 - 16 = 0$,

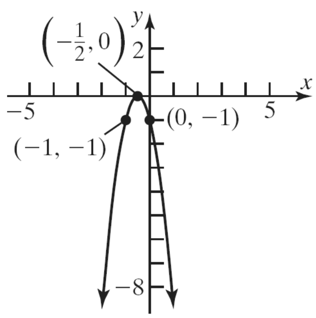
so the graph has one x-intercept.

The x-intercept is found by solving:

$$-4x^2 - 4x - 1 = 0$$
$$4x^2 + 4x + 1 = 0$$
$$(2x + 1)^2 = 0$$
$$2x + 1 = 0$$
$$x = -\frac{1}{2}$$

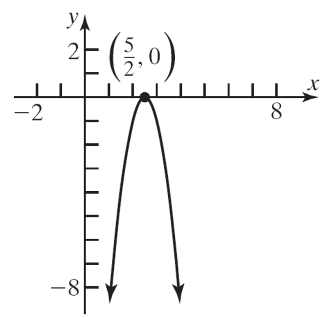
The x-intercept is $-\frac{1}{2}$.

The y-intercept is $H(0) = -1$.



b. The domain is $(-\infty, \infty)$.
The range is $(-\infty, 0]$.

c. Increasing on $\left(-\infty, -\frac{1}{2}\right)$.
Decreasing on $\left(-\frac{1}{2}, \infty\right)$.



b. The domain is $(-\infty, \infty)$.
The range is $(-\infty, 0]$.

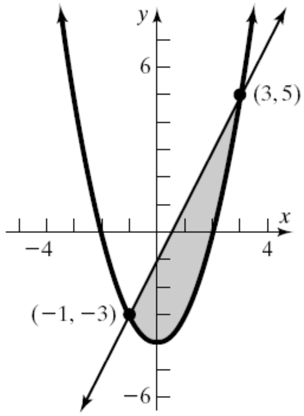
c. Increasing on $\left(-\infty, \frac{5}{2}\right)$.
Decreasing on $\left(\frac{5}{2}, \infty\right)$.

72. a. For $F(x) = -4x^2 + 20x - 25$, $a = -4$, $b = 20$, $c = -25$. Since $a = -4 < 0$, the graph opens down. The x-coordinate of the vertex is
$x = \frac{-b}{2a} = \frac{-20}{2(-4)} = \frac{-20}{-8} = \frac{5}{2}$.
The y-coordinate of the vertex is
$$F\left(\frac{-b}{2a}\right) = F\left(\frac{5}{2}\right) = -4\left(\frac{5}{2}\right)^2 + 20\left(\frac{5}{2}\right) - 25$$
$$= -25 + 50 - 25 = 0$$
Thus, the vertex is $\left(\frac{5}{2}, 0\right)$.
The discriminant is:
$$b^2 - 4ac = (20)^2 - 4(-4)(-25)$$
$$= 400 - 400 = 0,$$
so the graph has one x-intercept.
The x-intercept is found by solving:
$$-4x^2 + 20x - 25 = 0$$
$$4x^2 - 20x + 25 = 0$$
$$(2x - 5)^2 = 0$$
$$2x - 5 = 0$$
$$x = \frac{5}{2}$$
The x-intercept is $\frac{5}{2}$.
The y-intercept is $F(0) = -25$.

73. Use the form $f(x) = a(x - h)^2 + k$.
The vertex is $(0, 2)$, so $h = 0$ and $k = 2$.
$f(x) = a(x - 0)^2 + 2 = ax^2 + 2$.
Since the graph passes through $(1, 8)$, $f(1) = 8$.
$$f(x) = ax^2 + 2$$
$$8 = a(1)^2 + 2$$
$$8 = a + 2$$
$$6 = a$$
$f(x) = 6x^2 + 2$.
$a = 6, b = 0, c = 2$

74. Use the form $f(x) = a(x - h)^2 + k$.
The vertex is $(1, 4)$, so $h = 1$ and $k = 4$.
$f(x) = a(x - 1)^2 + 4$.
Since the graph passes through $(-1, -8)$, $f(-1) = -8$.
$$-8 = a(-1 - 1)^2 + 4$$
$$-8 = a(-2)^2 + 4$$
$$-8 = 4a + 4$$
$$-12 = 4a$$
$$-3 = a$$
$$f(x) = -3(x - 1)^2 + 4$$
$$= -3(x^2 - 2x + 1) + 4$$
$$= -3x^2 + 6x - 3 + 4$$
$$= -3x^2 + 6x + 1$$
$a = -3,\ b = 6,\ c = 1$

75. **a** and **d.**



b. $f(x) = g(x)$

$2x - 1 = x^2 - 4$

$0 = x^2 - 2x - 3$

$0 = (x+1)(x-3)$

$x + 1 = 0$ or $x - 3 = 0$

$x = -1$ $\quad$ $x = 3$

The solution set is $\{-1, 3\}$.

c. $f(-1) = 2(-1) - 1 = -2 - 1 = -3$

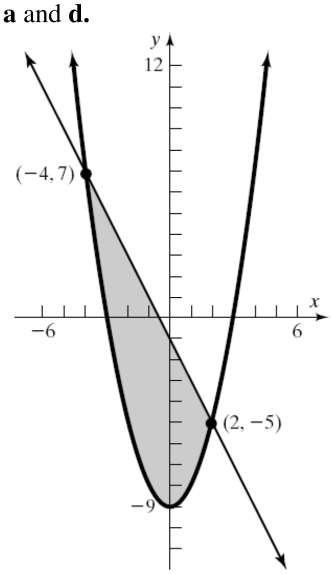
$g(-1) = (-1)^2 - 4 = 1 - 4 = -3$

$f(3) = 2(3) - 1 = 6 - 1 = 5$

$g(3) = (3)^2 - 4 = 9 - 4 = 5$

Thus, the graphs of *f* and *g* intersect at the points $(-1, -3)$ and $(3, 5)$.

76. **a** and **d.**



b. $f(x) = g(x)$

$-2x - 1 = x^2 - 9$

$0 = x^2 + 2x - 8$

$0 = (x+4)(x-2)$

$x + 4 = 0$ or $x - 2 = 0$

$x = -4$ $\quad$ $x = 2$

The solution set is $\{-4, 2\}$.

c. $f(-4) = -2(-4) - 1 = 8 - 1 = 7$

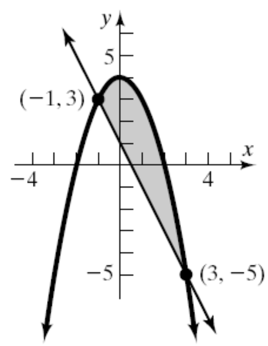
$g(-4) = (-4)^2 - 9 = 16 - 9 = 7$

$f(2) = -2(2) - 1 = -4 - 1 = -5$

$g(2) = (2)^2 - 9 = 4 - 9 = -5$

Thus, the graphs of *f* and *g* intersect at the points $(-4, 7)$ and $(2, -5)$.

77. **a** and **d.**



b. $f(x) = g(x)$

$-x^2 + 4 = -2x + 1$

$0 = x^2 - 2x - 3$

$0 = (x+1)(x-3)$

$x + 1 = 0$ or $x - 3 = 0$

$x = -1$ $\quad$ $x = 3$

The solution set is $\{-1, 3\}$.

c. $f(1) = -(-1)^2 + 4 = -1 + 4 = 3$

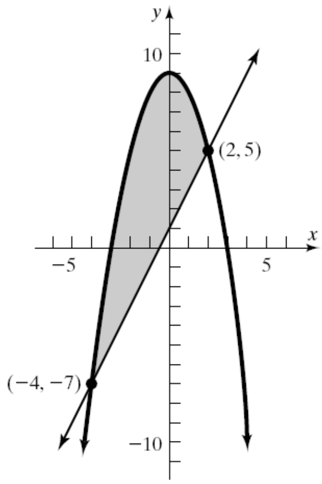
$g(1) = -2(-1) + 1 = 2 + 1 = 3$

$f(3) = -(3)^2 + 4 = -9 + 4 = -5$

$g(3) = -2(3) + 1 = -6 + 1 = -5$

Thus, the graphs of *f* and *g* intersect at the points $(-1, 3)$ and $(3, -5)$.

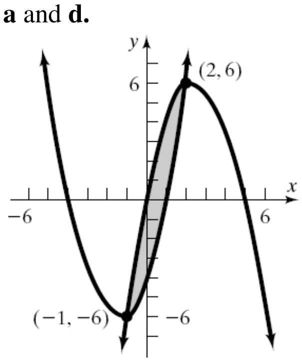
78. **a** and **d.**



b.
$$\begin{aligned} f(x) &= g(x) \\ -x^2+9 &= 2x+1 \\ 0 &= x^2+2x-8 \\ 0 &= (x+4)(x-2) \\ x+4=0 \quad &\text{or} \quad x-2=0 \\ x=-4 \quad & \quad\quad x=2 \end{aligned}$$
The solution set is $\{-4, 2\}$.

c.
$$f(-4) = -(-4)^2+9 = -16+9 = -7$$
$$g(-4) = 2(-4)+1 = -8+1 = -7$$
$$f(2) = -(2)^2+9 = -4+9 = 5$$
$$g(2) = 2(2)+1 = 4+1 = 5$$
Thus, the graphs of *f* and *g* intersect at the points $(-4,-7)$ and $(2,\ 5)$.

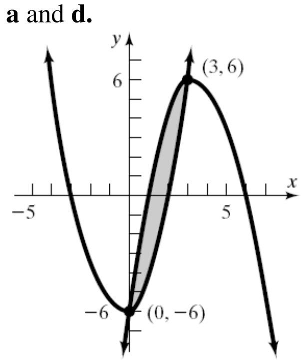
79. **a** and **d.**



b.
$$\begin{aligned} f(x) &= g(x) \\ -x^2+5x &= x^2+3x-4 \\ 0 &= 2x^2-2x-4 \\ 0 &= x^2-x-2 \\ 0 &= (x+1)(x-2) \\ x+1=0 \quad &\text{or} \quad x-2=0 \\ x=-1 \quad & \quad\quad x=2 \end{aligned}$$
The solution set is $\{-1, 2\}$.

c.
$$f(-1) = -(-1)^2+5(-1) = -1-5 = -6$$
$$g(-1) = (-1)^2+3(-1)-4 = 1-3-4 = -6$$
$$f(2) = -(2)^2+5(2) = -4+10 = 6$$
$$g(2) = 2^2+3(2)-4 = 4+6-4 = 6$$
Thus, the graphs of *f* and *g* intersect at the points $(-1,-6)$ and $(2,\ 6)$.

80. **a** and **d.**



b.
$$\begin{aligned} f(x) &= g(x) \\ -x^2+7x-6 &= x^2+x-6 \\ 0 &= 2x^2-6x \\ 0 &= 2x(x-3) \\ 2x=0 \quad &\text{or} \quad x-3=0 \\ x=0 \quad & \quad\quad x=3 \end{aligned}$$
The solution set is $\{0, 3\}$.

c.
$$f(0) = -(0)^2+7(0)-6 = -6$$
$$g(0) = 0^2+0-6 = -6$$
$$f(3) = -(3)^2+7(3)-6 = -9+21-6 = 6$$
$$g(3) = 3^2+3-6 = 9+3-6 = 6$$
Thus, the graphs of *f* and *g* intersect at the points $(0,-6)$ and $(3,\ 6)$.

81. a. For $a=1$:

$$f(x)=a(x-r_1)(x-r_2)$$
$$=1(x-(-3))(x-1)$$
$$=(x+3)(x-1)=x^2+2x-3$$

For $a=2$:

$$f(x)=2(x-(-3))(x-1)$$
$$=2(x+3)(x-1)$$
$$=2(x^2+2x-3)=2x^2+4x-6$$

For $a=-2$:

$$f(x)=-2(x-(-3))(x-1)$$
$$=-2(x+3)(x-1)$$
$$=-2(x^2+2x-3)=-2x^2-4x+6$$

For $a=5$:

$$f(x)=5(x-(-3))(x-1)$$
$$=5(x+3)(x-1)$$
$$=5(x^2+2x-3)=5x^2+10x-15$$

b. The *x*-intercepts are not affected by the value of *a*. The *y*-intercept is multiplied by the value of a.

c. The axis of symmetry is unaffected by the value of a. For this problem, the axis of symmetry is $x=-1$ for all values of *a*.

d. The *x*-coordinate of the vertex is not affected by the value of *a*. The *y*-coordinate of the vertex is multiplied by the value of a.

e. The *x*-coordinate of the vertex is the mean of the *x*-intercepts.

82. a. For $a=1$:

$$f(x)=1(x-(-5))(x-3)$$
$$=(x+5)(x-3)=x^2+2x-15$$

For $a=2$:

$$f(x)=2(x-(-5))(x-3)$$
$$=2(x+5)(x-3)$$
$$=2(x^2+2x-15)=2x^2+4x-30$$

For $a=-2$:

$$f(x)=-2(x-(-5))(x-3)$$
$$=-2(x+5)(x-3)$$
$$=-2(x^2+2x-15)=-2x^2-4x+30$$

For $a=5$:

$$f(x)=5(x-(-5))(x-3)$$
$$=5(x+5)(x-3)$$
$$=5(x^2+2x-15)=5x^2+10x-75$$

b. The *x*-intercepts are not affected by the value of *a*. The *y*-intercept is multiplied by the value of a.

c. The axis of symmetry is unaffected by the value of a. For this problem, the axis of symmetry is $x=-1$ for all values of *a*.

d. The *x*-coordinate of the vertex is not affected by the value of *a*. The *y*-coordinate of the vertex is multiplied by the value of a.

e. The *x*-coordinate of the vertex is the mean of the *x*-intercepts.

83. a. $x=-\dfrac{b}{2a}=-\dfrac{4}{2(1)}=-2$

$$y=f(-2)=(-2)^2+4(-2)-21=-25$$

The vertex is $(-2,-25)$.

b.

$$f(x)=0$$
$$x^2+4x-21=0$$
$$(x+7)(x-3)=0$$
$$x+7=0 \quad \text{or} \quad x-3=0$$
$$x=-7 \qquad\qquad x=3$$

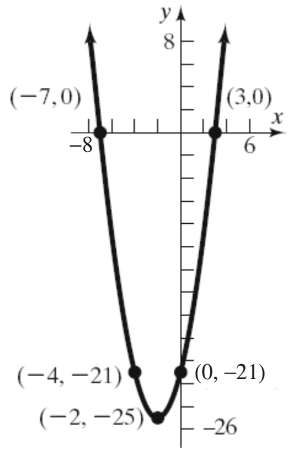
The *x*-intercepts of *f* are $(-7, 0)$ and $(3, 0)$.

c.

$$f(x)=-21$$
$$x^2+4x-21=-21$$
$$x^2+4x=0$$
$$x(x+4)=0$$
$$x=0 \quad \text{or} \quad x+4=0$$
$$x=-4$$

The solutions $f(x)=-21$ are -4 and 0. Thus, the points $(-4,-21)$ and $(0,-21)$ are on the graph of *f*.

d.



84. a. $x = -\frac{b}{2a} = -\frac{2}{2(1)} = -1$

$y = f(-1) = (-1)^2 + 2(-1) - 8 = -9$

The vertex is $(-1, -9)$.

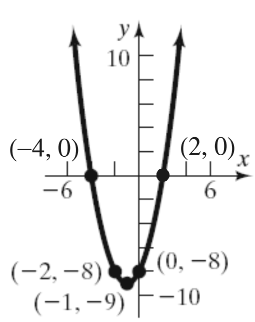
b.
$$f(x) = 0$$
$$x^2 + 2x - 8 = 0$$
$$(x+4)(x-2) = 0$$
$$x + 4 = 0 \quad \text{or} \quad x - 2 = 0$$
$$x = -4 \qquad x = 2$$

The x-intercepts of f are $(-4, 0)$ and $(2, 0)$.

c.
$$f(x) = -8$$
$$x^2 + 2x - 8 = -8$$
$$x^2 + 2x = 0$$
$$x(x+2) = 0$$
$$x = 0 \quad \text{or} \quad x + 2 = 0$$
$$x = -2$$

The solutions $f(x) = -8$ are -2 and 0. Thus, the points $(-2, -8)$ and $(0, -8)$ are on the graph of f.

d.



85. Let (x, y) represent a point on the line $y = x$. Then the distance from (x, y) to the point (3, 1) is $d = \sqrt{(x-3)^2 + (y-1)^2}$. Since $y = x$, we can replace the y variable with x so that we have the distance expressed as a function of x:

$$d(x) = \sqrt{(x-3)^2 + (x-1)^2}$$
$$= \sqrt{x^2 - 6x + 9 + x^2 - 2x + 1}$$
$$= \sqrt{2x^2 - 8x + 10}$$

Squaring both sides of this function, we obtain $[d(x)]^2 = 2x^2 - 8x + 10$.

Now, the expression on the right is quadratic. Since $a = 2 > 0$, it has a minimum. Finding the x-coordinate of the minimum point of $[d(x)]^2$ will also give us the x-coordinate of the minimum of $d(x)$: $x = \frac{-b}{2a} = \frac{-(-8)}{2(2)} = \frac{8}{4} = 2$. So, 2 is the x-coordinate of the point on the line $y = x$ that is closest to the point (3, 1). Since $y = x$, the y-coordinate is also 2. Thus, the point is (2, 2) is the point on the line $y = x$ that is closest to (3, 1).

86. Let (x, y) represent a point on the line $y = x + 1$. Then the distance from (x, y) to the point (4, 1) is $d = \sqrt{(x-4)^2 + (y-1)^2}$. Replacing the y variable with $x + 1$, we find the distance expressed as a function of x:

$$d(x) = \sqrt{(x-4)^2 + ((x+1)-1)^2}$$
$$= \sqrt{x^2 - 8x + 16 + x^2}$$
$$= \sqrt{2x^2 - 8x + 16}$$

Squaring both sides of this function, we obtain $[d(x)]^2 = 2x^2 - 8x + 16$.

Now, the expression on the right is quadratic. Since $a = 2 > 0$, it has a minimum. Finding the x-coordinate of the minimum point of $[d(x)]^2$ will also give us the x-coordinate of the minimum of $d(x)$: $x = \frac{-b}{2a} = \frac{-(-8)}{2(2)} = \frac{8}{4} = 2$. So, 2 is the x-coordinate of the point on the line $y = x + 1$ that is closest to the point (4, 1). The y-coordinate is $y = 2 + 1 = 3$. Thus, the point is (2, 3) is the point on the line $y = x + 1$ that is closest to (4, 1).

87. $R(p) = -4p^2 + 4000p$, $a = -4, b = 4000, c = 0.$ Since $a = -4 < 0$ the graph is a parabola that opens down, so the vertex is a maximum point. The maximum occurs at $p = \frac{-b}{2a} = \frac{-4000}{2(-4)} = 500$.

Thus, the unit price should be \$500 for maximum revenue. The maximum revenue is

$$R(500) = -4(500)^2 + 4000(500)$$
$$= -1000000 + 2000000$$
$$= \$1,000,000$$

88. $R(p) = -\frac{1}{2}p^2 + 1900p$, $a = -\frac{1}{2}, b = 1900, c = 0.$

Since $a = -\frac{1}{2} < 0,$ the graph is a parabola that opens down, so the vertex is a maximum point. The maximum occurs at

$p = \frac{-b}{2a} = \frac{-1900}{2(-1/2)} = \frac{-1900}{-1} = 1900$. Thus, the unit price should be \$1900 for maximum revenue. The maximum revenue is

$$R(1900) = -\frac{1}{2}(1900)^2 + 1900(1900)$$
$$= -1805000 + 3610000$$
$$= \$1,805,000$$

89. a. $C(x) = x^2 - 140x + 7400$,

$a = 1, b = -140, c = 7400.$ Since $a = 1 > 0,$ the graph opens up, so the vertex is a minimum point. The minimum marginal cost occurs at $x = \frac{-b}{2a} = \frac{-(-140)}{2(1)} = \frac{140}{2} = 70$,

70,000 mp3 players produced.

b. The minimum marginal cost is

$$f\left(\frac{-b}{2a}\right) = f(70) = (70)^2 - 140(70) + 7400$$
$$= 4900 - 9800 + 7400$$
$$= \$2500$$

90. a. $C(x) = 5x^2 - 200x + 4000$,

$a = 5, b = -200, c = 4000.$ Since $a = 5 > 0,$ the graph opens up, so the vertex is a minimum point. The minimum marginal cost occurs at $x = \frac{-b}{2a} = \frac{-(-200)}{2(5)} = \frac{200}{10} = 20$,

20,000 thousand cell phones manufactured.

b. The minimum marginal cost is

$$f\left(\frac{-b}{2a}\right) = f(20) = 5(20)^2 - 200(20) + 4000$$
$$= 2000 - 4000 + 4000$$
$$= \$2000$$

91. a. $R(x) = 75x - 0.2x^2$

$a = -0.2, b = 75, c = 0$

The maximum revenue occurs when

$$x = \frac{-b}{2a} = \frac{-75}{2(-0.2)} = \frac{-75}{-0.4} = 187.5$$

The maximum revenue occurs when $x = 187$ or $x = 188$ watches.

The maximum revenue is:

$$R(187) = 75(187) - 0.2(187)^2 = \$7031.20$$
$$R(188) = 75(188) - 0.2(188)^2 = \$7031.20$$

b. $P(x) = R(x) - C(x)$

$$= 75x - 0.2x^2 - (32x + 1750)$$
$$= -0.2x^2 + 43x - 1750$$

c. $P(x) = -0.2x^2 + 43x - 1750$

$a = -0.2, b = 43, c = -1750$

$$x = \frac{-b}{2a} = \frac{-43}{2(-0.2)} = \frac{-43}{-0.4} = 107.5$$

The maximum profit occurs when $x = 107$ or $x = 108$ watches.

The maximum profit is:

$$P(107) = -0.2(107)^2 + 43(107) - 1750$$
$$= \$561.20$$
$$P(108) = -0.2(108)^2 + 43(108) - 1750$$
$$= \$561.20$$

d. Answers will vary.

92. a. $R(x) = 9.5x - 0.04x^2$

$a = -0.04, b = 9.5, c = 0$

The maximum revenue occurs when

$$x = \frac{-b}{2a} = \frac{-9.5}{2(-0.04)} = \frac{-9.5}{-0.08}$$
$$= 118.75 \approx 119 \text{ boxes of candy}$$

The maximum revenue is:

$$R(119) = 9.5(119) - 0.04(119)^2 = \$564.06$$

b. $P(x) = R(x) - C(x)$
$= 9.5x - 0.04x^2 - (1.25x + 250)$
$= -0.04x^2 + 8.25x - 250$

c. $P(x) = -0.04x^2 + 8.25x - 250$
$a = -0.04,\ b = 8.25,\ c = -250$
The maximum profit occurs when
$x = \frac{-b}{2a} = \frac{-8.25}{2(-0.04)} = \frac{-8.25}{-0.08}$
$= 103.125 \approx 103$ boxes of candy
The maximum profit is:
$P(103) = -0.04(103)^2 + 8.25(103) - 250$
$= \$175.39$

d. Answers will vary.

93. a. $d(v) = 1.1v + 0.06v^2$
$d(45) = 1.1(45) + 0.06(45)^2$
$= 49.5 + 121.5 = 171$ ft.

b. $200 = 1.1v + 0.06v^2$
$0 = -200 + 1.1v + 0.06v^2$
$x = \frac{-(1.1) \pm \sqrt{(1.1)^2 - 4(0.06)(-200)}}{2(0.06)}$
$= \frac{-1.1 \pm \sqrt{49.21}}{0.12}$
$\approx \frac{-1.1 \pm 7.015}{0.12}$
$v \approx 49$ or $v \approx -68$
Disregard the negative value since we are talking about speed. So the maximum speed you can be traveling would be approximately 49 mph.

c. The 1.1v term might represent the reaction time.

94. a. $a = \frac{-b}{2a} = \frac{-14.23}{2(-0.27)} = \frac{-14.23}{-0.54} \approx 26.4$ years old

b. $B(26.4) = -0.27(26.4)^2 + 14.23(26.4) - 120.16$
≈ 67.3 births per 1000 unmarried women

c. $B(40) = -0.27(40)^2 + 14.23(40) - 120.16$
≈ 17 births per 1000 unmarried women

95. $f(x) = a(x - r_1)(x - r_2)$
$= a(x+4)(x-2)$
$= ax^2 + 2ax - 8a$
The x value of the vertex is $x = \frac{-b}{2a} = \frac{-2a}{2a} = -1$.
The y value of the vertex is 18.
$-18 = a(-1)^2 + 2a(-1) - 8a$
$-18 = -9a$
$a = 2$

So the function is $f(x) = 2(x+4)(x-2)$

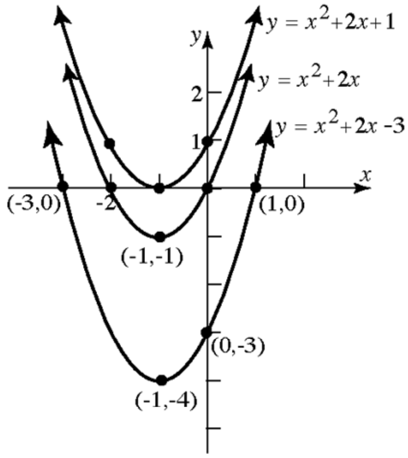
96. $f(x) = a(x - r_1)(x - r_2)$
$= a(x+1)(x-5)$
$= ax^2 - 4ax - 5a$
The x value of the vertex is
$x = \frac{-b}{2a} = \frac{-(-4a)}{2a} = 2$.
The y value of the vertex is 9.
$9 = a(2)^2 - 4a(2) - 5a$
$9 = -9a$
$a = -1$

So the function is $f(x) = -(x+1)(x-5)$

97. If x is even, then ax^2 and bx are even. When two even numbers are added to an odd number the result is odd. Thus, $f(x)$ is odd. If x is odd, then ax^2 and bx are odd. The sum of three odd numbers is an odd number. Thus, $f(x)$ is odd.

98. Answers will vary.

99. $y = x^2 + 2x - 3$; $y = x^2 + 2x + 1$; $y = x^2 + 2x$



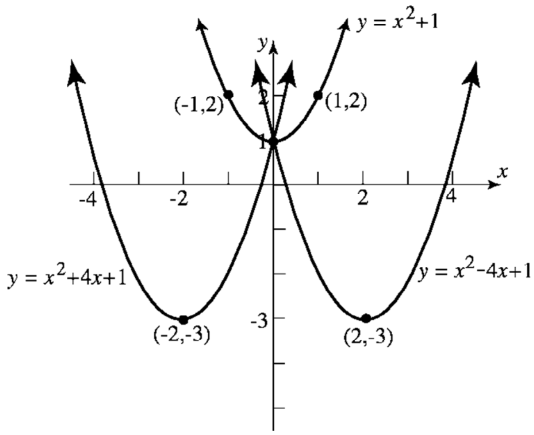
Each member of this family will be a parabola with the following characteristics:

(i) opens upwards since a > 0;

(ii) vertex occurs at $x = -\frac{b}{2a} = -\frac{2}{2(1)} = -1$;

(iii) There is at least one x-intercept since $b^2 - 4ac \geq 0$.

100. $y = x^2 - 4x + 1$; $y = x^2 + 1$; $y = x^2 + 4x + 1$



Each member of this family will be a parabola with the following characteristics:

(i) opens upwards since $a > 0$

(ii) y-intercept occurs at (0, 1).

101. The graph of the quadratic function $f(x) = ax^2 + bx + c$ will not have any x-intercepts whenever $b^2 - 4ac < 0$.

102. By completing the square on the quadratic function $f(x) = ax^2 + bx + c$ we obtain the equation $y = a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$. We can then draw the graph by applying transformations to the graph of the basic parabola $y = x^2$, which opens up. When $a > 0$, the basic parabola will either be stretched or compressed vertically. When $a < 0$, the basic parabola will either be stretched or compressed vertically as well as reflected across the x-axis. Therefore, when $a > 0$, the graph of $f(x) = ax^2 + bx + c$ will open up, and when $a < 0$, the graph of $f(x) = ax^2 + bx + c$ will open down.

103. No. We know that the graph of a quadratic function $f(x) = ax^2 + bx + c$ is a parabola with vertex $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$. If $a > 0$, then the vertex is a minimum point, so the range is $\left[f\left(-\frac{b}{2a}\right), \infty\right)$. If $a < 0$, then the vertex is a maximum point, so the range is $\left(-\infty, f\left(-\frac{b}{2a}\right)\right]$. Therefore, it is impossible for the range to be $(-\infty, \infty)$.

104. Two quadratic functions can intersect 0, 1, or 2 times.

Section 3.4

1. $R = 3x$

2. Use LIN REGression to get $y = 1.7826x + 4.0652$

3. **a.** $R(x) = x\left(-\frac{1}{6}x + 100\right) = -\frac{1}{6}x^2 + 100x$

   b. The quantity sold price cannot be negative, so $x \geq 0$. Similarly, the price should be positive, so $p > 0$.

$$-\frac{1}{6}x + 100 > 0$$
$$-\frac{1}{6}x > -100$$
$$x < 600$$

Thus, the implied domain for R is $\{x \mid 0 \leq x < 600\}$ or $[0, 600)$.

c. $R(200) = -\frac{1}{6}(200)^2 + 100(200)$
$= \frac{-20000}{3} + 20000$
$= \frac{40000}{3} \approx \$13,333.33$

d. $x = \frac{-b}{2a} = \frac{-100}{2\left(-\frac{1}{6}\right)} = \frac{-100}{\left(-\frac{1}{3}\right)} = \frac{300}{1} = 300$

The maximum revenue is
$R(300) = -\frac{1}{6}(300)^2 + 100(300)$
$= -15000 + 30000$
$= \$15,000$

e. $p = -\frac{1}{6}(300) + 100 = -50 + 100 = \$50$

4. a. $R(x) = x\left(-\frac{1}{3}x + 100\right) = -\frac{1}{3}x^2 + 100x$

b. The quantity sold price cannot be negative, so $x \ge 0$. Similarly, the price should be positive, so $p > 0$.

$-\frac{1}{3}x + 100 > 0$
$-\frac{1}{3}x > -100$
$x < 300$

Thus, the implied domain for R is $\{x \mid 0 \le x < 300\}$ or $[0, 300)$.

c. $R(100) = -\frac{1}{3}(100)^2 + 100(100)$
$= \frac{-10000}{3} + 10000$
$= \frac{20000}{3} \approx \$6,666.67$

d. $x = \frac{-b}{2a} = \frac{-100}{2\left(-\frac{1}{3}\right)} = \frac{-100}{\left(-\frac{2}{3}\right)} = \frac{300}{2} = 150$

The maximum revenue is
$R(150) = -\frac{1}{3}(150)^2 + 100(150)$
$= -7500 + 15000 = \$7,500$

e. $p = -\frac{1}{3}(150) + 100 = -50 + 100 = \$50$

5. a. If $x = -5p + 100$, then $p = \frac{100 - x}{5}$.
$R(x) = x\left(\frac{100 - x}{5}\right) = -\frac{1}{5}x^2 + 20x$

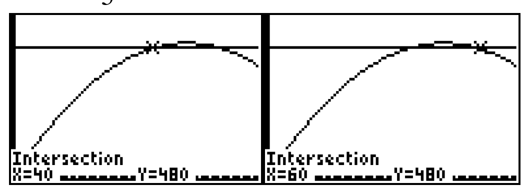
b. $R(15) = -\frac{1}{5}(15)^2 + 20(15)$
$= -45 + 300 = \$255$

c. $x = \frac{-b}{2a} = \frac{-20}{2\left(-\frac{1}{5}\right)} = \frac{-20}{\left(-\frac{2}{5}\right)} = \frac{100}{2} = 50$

The maximum revenue is
$R(50) = -\frac{1}{5}(50)^2 + 20(50)$
$= -500 + 1000 = \$500$

d. $p = \frac{100 - 50}{5} = \frac{50}{5} = \$10$

e. Graph $R = -\frac{1}{5}x^2 + 20x$ and $R = 480$. Find where the graphs intersect by solving $480 = -\frac{1}{5}x^2 + 20x$.



$\frac{1}{5}x^2 - 20x + 480 = 0$
$x^2 - 100x + 2400 = 0$
$(x - 40)(x - 60) = 0$
$x = 40, x = 60$

Solve for price.

$x = -5p + 100$
$40 = -5p + 100 \Rightarrow p = \$12$
$60 = -5p + 100 \Rightarrow p = \$8$

The company should charge between \$8 and \$12.

6. a. If $x = -20p + 500$, then $p = \frac{500 - x}{20}$.
$R(x) = x\left(\frac{500 - x}{20}\right) = -\frac{1}{20}x^2 + 25x$

b. $R(20) = -\frac{1}{20}(20)^2 + 25(20)$
$= -20 + 500 = \$480$

c. $x = \frac{-b}{2a} = \frac{-25}{2\left(-\frac{1}{20}\right)} = \frac{-25}{\left(-\frac{1}{10}\right)} = \frac{250}{1} = 250$.

The maximum revenue is

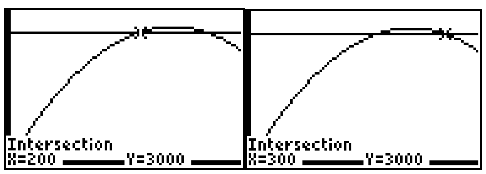
$R(250) = -\frac{1}{20}(250)^2 + 25(250)$
$= -3125 + 6250 = \$3125$

d. $p = \frac{500-250}{20} = \frac{250}{20} = \$12.50$

e. Graph $R = -\frac{1}{20}x^2 + 25x$ and $R = 3000$.

Find where the graphs intersect by solving

$3000 = -\frac{1}{20}x^2 + 25x$.



$\frac{1}{20}x^2 - 25x + 3000 = 0$
$x^2 - 500x + 60000 = 0$
$(x-200)(x-300) = 0$
$x = 200, x = 300$

Solve for price.

$x = -20p + 500$
$300 = -20p + 500 \Rightarrow p = \$10$
$200 = -20p + 500 \Rightarrow p = \$15$

The company should charge between \$10 and \$15.

7. a. Let w = width and l = length of the rectangular area.
Solving $P = 2w + 2l = 400$ for l:

$l = \frac{400-2w}{2} = 200 - w$.

Then $A(w) = (200-w)w = 200w - w^2$
$= -w^2 + 200w$

b. $w = \frac{-b}{2a} = \frac{-200}{2(-1)} = \frac{-200}{-2} = 100$ yards

c. $A(100) = -100^2 + 200(100)$
$= -10000 + 20000$
$= 10,000 \text{ yd}^2$

8. a. Let x = width and y = width of the rectangle.
Solving $P = 2x + 2y = 3000$ for y:

$y = \frac{3000-2x}{2} = 1500 - x.$

Then $A(x) = (1500-x)x$
$= 1500x - x^2$
$= -x^2 + 1500x.$

b. $x = \frac{-b}{2a} = \frac{-1500}{2(-1)} = \frac{-1500}{-2} = 750$ feet

c. $A(750) = -750^2 + 1500(750)$
$= -562500 + 1125000$
$= 562,500 \text{ ft}^2$

9. Let x = width and y = length of the rectangle.
Solving $P = 2x + y = 4000$ for y:
$y = 4000 - 2x$.
Then $A(x) = (4000-2x)x$
$= 4000x - 2x^2$
$= -2x^2 + 4000x$

$x = \frac{-b}{2a} = \frac{-4000}{2(-2)} = \frac{-4000}{-4} = 1000$ meters

maximizes area.

$A(1000) = -2(1000)^2 + 4000(1000)$.
$= -2000000 + 4000000$
$= 2,000,000$

The largest area that can be enclosed is 2,000,000 square meters.

10. Let x = width and y = length of the rectangle.
$2x + y = 2000$
$y = 2000 - 2x$
Then $A(x) = (2000-2x)x$
$= 2000x - 2x^2$
$= -2x^2 + 2000x$

$x = \frac{-b}{2a} = \frac{-2000}{2(-2)} = \frac{-2000}{-4} = 500$ meters

maximizes area.

$A(500) = -2(500)^2 + 2000(500)$
$= -500,000 + 1,000,000$
$= 500,000$

The largest area that can be enclosed is 500,000 square meters.

11. $h(x)=\dfrac{-32x^2}{(50)^2}+x+200=-\dfrac{8}{625}x^2+x+200$

a. $a=-\dfrac{8}{625},\ b=1,\ c=200.$

The maximum height occurs when

$x=\dfrac{-b}{2a}=\dfrac{-1}{2(-8/625)}=\dfrac{625}{16}\approx 39$ feet from base of the cliff.

b. The maximum height is

$$h\left(\frac{625}{16}\right)=\frac{-8}{625}\left(\frac{625}{16}\right)^2+\frac{625}{16}+200$$
$$=\frac{7025}{32}\approx 219.5 \text{ feet.}$$

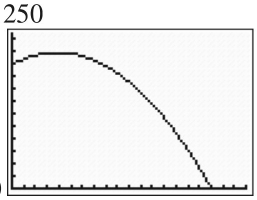
c. Solving when $h(x)=0$:

$$-\frac{8}{625}x^2+x+200=0$$
$$x=\frac{-1\pm\sqrt{1^2-4(-8/625)(200)}}{2(-8/625)}$$
$$x\approx\frac{-1\pm\sqrt{11.24}}{-0.0256}$$

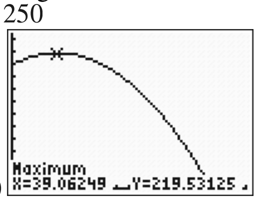
$x\approx -91.90$ or $x\approx 170$

Since the distance cannot be negative, the projectile strikes the water approximately 170 feet from the base of the cliff.

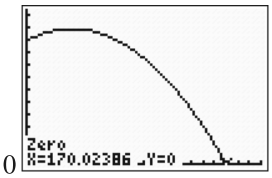
d.



e. Using the MAXIMUM function



Using the ZERO function



f. $-\dfrac{8}{625}x^2+x+200=100$

$$-\frac{8}{625}x^2+x+100=0$$
$$x=\frac{\sqrt{1^2-4(-8/625)(100)}}{2(-8/625)}=\frac{-1\pm\sqrt{6.12}}{-0.0256}$$

$x\approx -57.57$ or $x\approx 135.70$

Since the distance cannot be negative, the projectile is 100 feet above the water when it is approximately 135.7 feet from the base of the cliff.

12. a. $h(x)=\dfrac{-32x^2}{(100)^2}+x=-\dfrac{2}{625}x^2+x$

$a=-\dfrac{2}{625},\ b=1,\ c=0.$

The maximum height occurs when

$x=\dfrac{-b}{2a}=\dfrac{-1}{2(-2/625)}=\dfrac{625}{4}=156.25$ feet

b. The maximum height is

$$h\left(\frac{625}{4}\right)=\frac{-2}{625}\left(\frac{625}{4}\right)^2+\frac{625}{4}$$
$$=\frac{625}{8}=78.125 \text{ feet}$$

c. Solving when $h(x)=0$:

$$-\frac{2}{625}x^2+x=0$$
$$x\left(-\frac{2}{625}x+1\right)=0$$

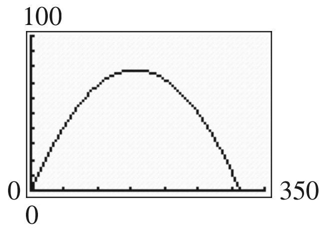
$x=0$ or $-\dfrac{2}{625}x+1=0$

$x=0$ or $1=\dfrac{2}{625}x$

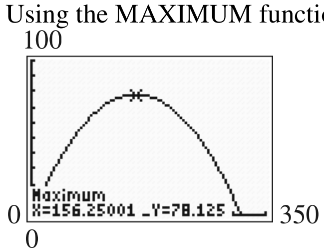
$x=0$ or $x=\dfrac{625}{2}=312.5$

Since the distance cannot be zero, the projectile lands 312.5 feet from where it was fired.

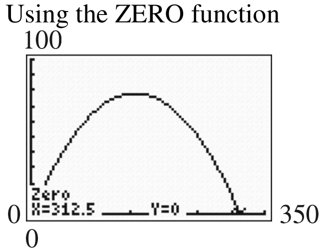
d.



e. Using the MAXIMUM function



Using the ZERO function



f. Solving when $h(x)=50$:

$$-\frac{2}{625}x^2+x=50$$

$$-\frac{2}{625}x^2+x-50=0$$

$$x=\frac{-1\pm\sqrt{1^2-4(-2/625)(-50)}}{2(-2/625)}$$

$$=\frac{-1\pm\sqrt{0.36}}{-0.0064}\approx\frac{-1\pm 0.6}{-0.0064}$$

$x=62.5$ or $x=250$

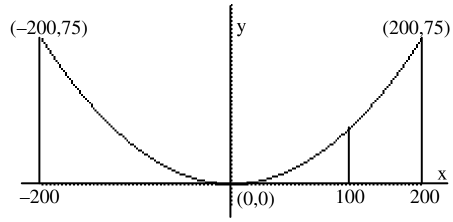
The projectile is 50 feet above the ground 62.5 feet and 250 feet from where it was fired.

13. Locate the origin at the point where the cable touches the road. Then the equation of the parabola is of the form: $y=ax^2$, where $a>0$. Since the point (200, 75) is on the parabola, we can find the constant a:

Since $75=a(200)^2$, then $a=\frac{75}{200^2}=0.001875$.

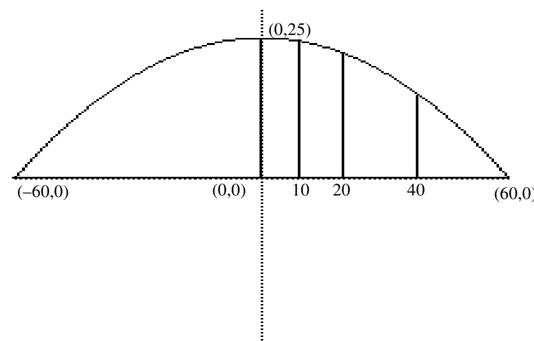
When $x=100$, we have:

$y=0.001875(100)^2=18.75$ meters.



14. Locate the origin at the point directly under the highest point of the arch. Then the equation of the parabola is of the form: $y=-ax^2+k$, where $a>0$. Since the maximum height is 25 feet, when $x=0$, $y=k=25$. Since the point (60, 0) is on the parabola, we can find the constant a: Since $0=-a(60)^2+25$ then $a=\frac{25}{60^2}$. The equation of the parabola is:

$$h(x)=-\frac{25}{60^2}x^2+25.$$



At $x=10$:

$$h(10)=-\frac{25}{60^2}(10)^2+25=-\frac{25}{36}+25\approx 24.3\text{ ft.}$$

At $x=20$:

$$h(20)=-\frac{25}{60^2}(20)^2+25=-\frac{25}{9}+25\approx 22.2\text{ ft.}$$

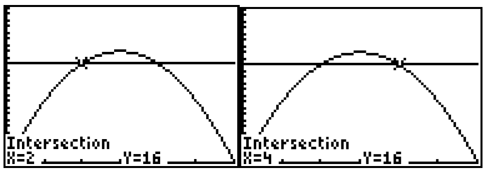
At $x=40$:

$$h(40)=-\frac{25}{60^2}(40)^2+25=-\frac{100}{9}+25\approx 13.9\text{ ft.}$$

15. a. Let x = the depth of the gutter and y the width of the gutter. Then $A=xy$ is the cross-sectional area of the gutter. Since the aluminum sheets for the gutter are 12 inches wide, we have $2x+y=12$. Solving for y: $y=12-2x$. The area is to be maximized, so: $A=xy=x(12-2x)=-2x^2+12x$. This equation is a parabola opening down; thus, it has a maximum when $x=\frac{-b}{2a}=\frac{-12}{2(-2)}=\frac{-12}{-4}=3$.

Thus, a depth of 3 inches produces a maximum cross-sectional area.

b. Graph $A=-2x^2+12x$ and $A=16$. Find where the graphs intersect by solving $16=-2x^2+12x$.



$$2x^2-12x+16=0$$
$$x^2-6x+8=0$$
$$(x-4)(x-2)=0$$
$$x=4, x=2$$

The graph of $A=-2x^2+12x$ is above the graph of $A=16$ where the depth is between 2 and 4 inches.

16. Let $x=$ width of the window and $y=$ height of the rectangular part of the window. The perimeter of the window is: $x+2y+\frac{\pi x}{2}=20.$

Solving for y: $y=\frac{40-2x-\pi x}{4}$.

The area of the window is:

$$A(x)=x\left(\frac{40-2x-\pi x}{4}\right)+\frac{1}{2}\pi\left(\frac{x}{2}\right)^2$$
$$=10x-\frac{x^2}{2}-\frac{\pi x^2}{4}+\frac{\pi x^2}{8}$$
$$=\left(-\frac{1}{2}-\frac{\pi}{8}\right)x^2+10x.$$

This equation is a parabola opening down; thus, it has a maximum when

$$x=\frac{-b}{2a}=\frac{-10}{2\left(-\frac{1}{2}-\frac{\pi}{8}\right)}=\frac{10}{\left(1+\frac{\pi}{4}\right)}\approx 5.6 \text{ feet}$$
$$y=\frac{40-2(5.60)-\pi(5.60)}{4}\approx 2.8 \text{ feet}$$

The width of the window is about 5.6 feet and the height of the rectangular part is approximately 2.8 feet. The radius of the semicircle is roughly 2.8 feet, so the total height is about 5.6 feet.

17. Let $x=$ the width of the rectangle or the diameter of the semicircle and let $y=$ the length of the rectangle. The perimeter of each semicircle is $\frac{\pi x}{2}$.

The perimeter of the track is given by: $\frac{\pi x}{2}+\frac{\pi x}{2}+y+y=1500$.

Solving for x:

$$\pi x+2y=1500$$
$$\pi x=1500-2y$$
$$x=\frac{1500-2y}{\pi}$$

The area of the rectangle is:

$$A=xy=\left(\frac{1500-2y}{\pi}\right)y=\frac{-2}{\pi}y^2+\frac{1500}{\pi}y.$$

This equation is a parabola opening down; thus, it has a maximum when

$$y=\frac{-b}{2a}=\frac{\frac{-1500}{\pi}}{2\left(\frac{-2}{\pi}\right)}=\frac{-1500}{-4}=375.$$

Thus, $x=\frac{1500-2(375)}{\pi}=\frac{750}{\pi}\approx 238.73$

The dimensions for the rectangle with maximum area are $\frac{750}{\pi}\approx 238.73$ meters by 375 meters.

18. Let $x=$ width of the window and $y=$ height of the rectangular part of the window. The perimeter of the window is:

$$3x+2y=16$$
$$y=\frac{16-3x}{2}$$

The area of the window is

$$A(x)=x\left(\frac{16-3x}{2}\right)+\frac{\sqrt{3}}{4}x^2$$
$$=8x-\frac{3}{2}x^2+\frac{\sqrt{3}}{4}x^2$$
$$=\left(-\frac{3}{2}+\frac{\sqrt{3}}{4}\right)x^2+8x$$

This equation is a parabola opening down; thus, it has a maximum when

$$x=\frac{-b}{2a}=\frac{-8}{2\left(-\frac{3}{2}+\frac{\sqrt{3}}{4}\right)}$$
$$=\frac{-8}{-3+\frac{\sqrt{3}}{2}}=\frac{-16}{-6+\sqrt{3}}\approx 3.75 \text{ ft.}$$

The window is approximately 3.75 feet wide.

$$y=\frac{16-3\left(\frac{-16}{-6+\sqrt{3}}\right)}{2}=\frac{16+\frac{48}{-6+\sqrt{3}}}{2}=8+\frac{24}{-6+\sqrt{3}}$$

The height of the equilateral triangle is

$\frac{\sqrt{3}}{2}\left(\frac{-16}{-6+\sqrt{3}}\right)=\frac{-8\sqrt{3}}{-6+\sqrt{3}}$ feet, so the total height is

$8+\frac{24}{-6+\sqrt{3}}+\frac{-8\sqrt{3}}{-6+\sqrt{3}}\approx 5.62$ feet.

19. We are given: $V(x)=kx(a-x)=-kx^2+akx$.
The reaction rate is a maximum when:
$x=\frac{-b}{2a}=\frac{-ak}{2(-k)}=\frac{ak}{2k}=\frac{a}{2}$.

20. We have:

$a(-h)^2+b(-h)+c=ah^2-bh+c=y_0$

$a(0)^2+b(0)+c=c=y_1$

$a(h)^2+b(h)+c=ah^2+bh+c=y_2$

Equating the two equations for the area, we have:

$$y_0+4y_1+y_2=ah^2-bh+c+4c+ah^2+bh+c$$
$$=2ah^2+6c.$$

Therefore,

Area $=\frac{h}{3}\left(2ah^2+6c\right)=\frac{h}{3}\left(y_0+4y_1+y_2\right)$ sq. units.

21. $f(x)=-5x^2+8,\ h=1$

Area $=\frac{h}{3}\left(2ah^2+6c\right)=\frac{1}{3}\left(2(-5)(1)^2+6(8)\right)$

$=\frac{1}{3}(-10+48)=\frac{38}{3}$ sq. units

22. $f(x)=2x^2+8,\quad h=2$

Area $=\frac{h}{3}(2ah^2+6c)=\frac{2}{3}\left(2(2)(2)^2+6(8)\right)$

$=\frac{2}{3}(16+48)=\frac{2}{3}(64)=\frac{128}{3}$ sq. units

23. $f(x)=x^2+3x+5,\ h=4$

Area $=\frac{h}{3}\left(2ah^2+6c\right)=\frac{4}{3}\left(2(1)(4)^2+6(5)\right)$

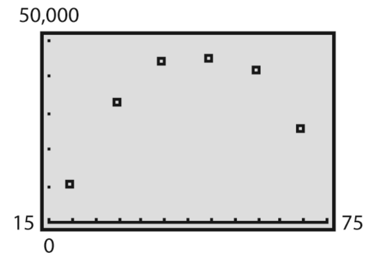
$=\frac{4}{3}(32+30)=\frac{248}{3}$ sq. units

24. $f(x)=-x^2+x+4,\quad h=1$

Area $=\frac{h}{3}(2ah^2+6c)=\frac{1}{3}\left(2(-1)(1)^2+6(4)\right)$

$=\frac{1}{3}\left(-2+24\right)=\frac{1}{3}(22)=\frac{22}{3}$ sq. units

25. **a.**



From the graph, the data appear to follow a quadratic relation with $a<0$.

b. Using the QUADratic REGression program

```
QuadReg
 y=ax²+bx+c
 a=-45.46571429
 b=4314.374286
 c=-55961.675
```

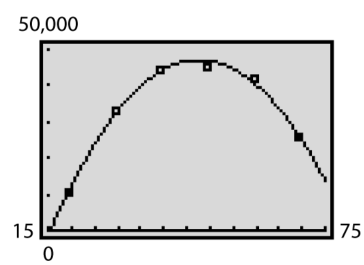
$I(x)=-45.122x^2+4301.575x-55,376.404$

c. $x=\frac{-b}{2a}=\frac{-4301.575}{2(-45.122)}\approx 47.7$

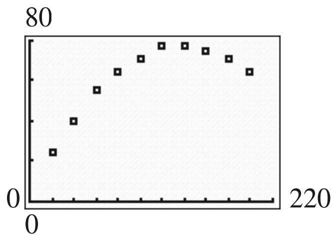
An individual will earn the most income at about 47.4 years of age.

d. The maximum income will be: $I(47.7)=$
$-45.122(47.7)^2+4301.575(47.7)-55,376.404$
$\approx \$47,143$

e.

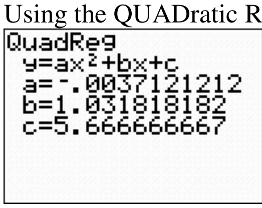


26. a.



From the graph, the data appear to follow a quadratic relation with $a < 0$.

b. Using the QUADratic REGression program



$h(x) = -0.0037x^2 + 1.0318x + 5.6667$

c. $x = \frac{-b}{2a} = \frac{-1.0318}{2(-0.0037)} \approx 139.4$

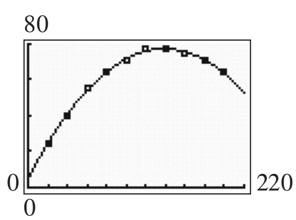
The ball will travel about 139.4 feet before it reaches its maximum height.

d. The maximum height will be: $h(139.4) =$

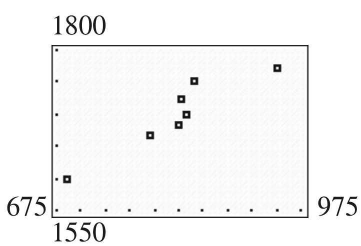
$-0.0037(139.4)^2 + 1.0318(139.4) + 5.6667$

≈ 77.6 feet

e.

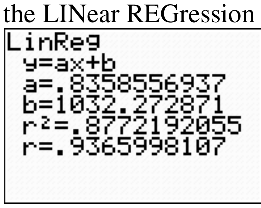


27. a.



From the graph, the data appear to be linearly related with $m > 0$.

b. Using the LINear REGression program

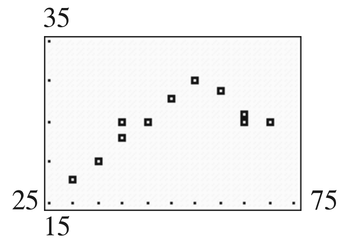


$R(x) = 0.836x + 1032.273$

c. $R(850) = 0.836(850) + 1032.273 \approx 1743$

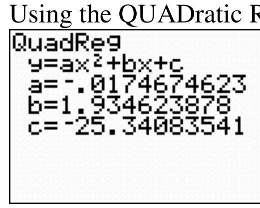
The rent for an 850 square-foot apartment in San Diego will be about \$1743 per month.

28. a.



From the graph, the data appear to follow a quadratic relation with $a < 0$.

b. Using the QUADratic REGression program



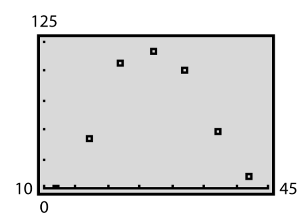
$M(s) = -0.017s^2 + 1.935s - 25.341$

c. $M(63) = -0.017(63)^2 + 1.935(63) - 25.341$

≈ 29.1

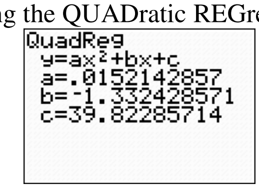
A Camry traveling 63 miles per hour will get about 29.1 miles per gallon.

29. a.



From the graph, the data appear to follow a quadratic relation with $a > 0$.

b. Using the QUADratic REGression program



$P(a) = -0.495a^2 + 26.928a - 256.398$

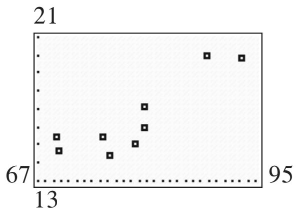
c.

$P(35) = -0.495(35)^2 + 26.928(35) - 256.398$

≈ 79.7

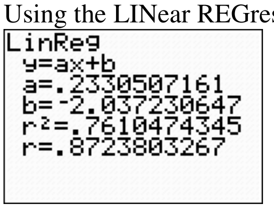
In 2007, the birth rate of women age 35 is about 79.7%.

30. a.



From the graph, the data appear to be linearly related with $m > 0$.

b. Using the LINear REGression program



$C(x) = 0.233x - 2.037$

c. $C(80) = 0.233(80) - 2.037 \approx 16.6$
When the temperature is $80°\text{F}$, there will be about 16.6 chirps per second.

31. Answers will vary. One possibility follows: If the price is \$140, no one will buy the calculators, thus making the revenue \$0.

Section 3.5

1. $-3x - 2 < 7$
$-3x < 9$
$x > -3$
The solution set is $\{x \mid x > -3\}$ or $(-3, \infty)$.

2. $(-2, 7]$ represents the numbers between -2 and 7, including 7 but not including -2. Using inequality notation, this is written as $-2 < x \le 7$.

3. a. $f(x) > 0$ when the graph of *f* is above the *x*-axis. Thus, $\{x \mid x < -2 \text{ or } x > 2\}$ or, using interval notation, $(-\infty, -2) \cup (2, \infty)$.

b. $f(x) \le 0$ when the graph of *f* is below or intersects the *x*-axis. Thus, $\{x \mid -2 \le x \le 2\}$ or, using interval notation, $[-2, 2]$.

4. a. $g(x) < 0$ when the graph of *g* is below the *x*-axis. Thus, $\{x \mid x < -1 \text{ or } x > 4\}$ or, using interval notation, $(-\infty, -1) \cup (4, \infty)$.

b. $g(x) \ge 0$ when the graph of *f* is above or intersects the *x*-axis. Thus, $\{x \mid -1 \le x \le 4\}$ or, using interval notation, $[-1, 4]$.

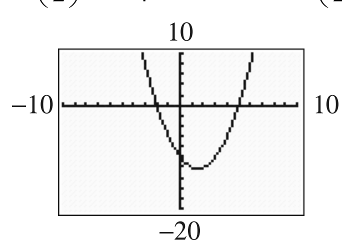
5. a. $g(x) \ge f(x)$ when the graph of *g* is above or intersects the graph of *f*. Thus $\{x \mid -2 \le x \le 1\}$ or, using interval notation, $[-2, 1]$.

b. $f(x) > g(x)$ when the graph of *f* is above the graph of *g*. Thus, $\{x \mid x < -2 \text{ or } x > 1\}$ or, using interval notation, $(-\infty, -2) \cup (1, \infty)$.

6. a. $f(x) < g(x)$ when the graph of *f* is below the graph of *g*. Thus, $\{x \mid x < -3 \text{ or } x > 1\}$ or, using interval notation, $(-\infty, -3) \cup (1, \infty)$.

b. $f(x) \ge g(x)$ when the graph of *f* is above or intersects the graph of *g*. Thus, $\{x \mid -3 \le x \le 1\}$ or, using interval notation, $[-3, 1]$.

7. $x^2 - 3x - 10 < 0$
We graph the function $f(x) = x^2 - 3x - 10$. The intercepts are
y-intercept: $f(0) = -10$
x-intercepts: $x^2 - 3x - 10 = 0$
$(x-5)(x+2) = 0$
$x = 5, x = -2$
The vertex is at $x = \frac{-b}{2a} = \frac{-(-3)}{2(1)} = \frac{3}{2}$. Since $f\left(\frac{3}{2}\right) = -\frac{49}{4}$, the vertex is $\left(\frac{3}{2}, -\frac{49}{4}\right)$.



The graph is below the *x*-axis for $-2 < x < 5$. Since the inequality is strict, the solution set is $\{x \mid -2 < x < 5\}$ or, using interval notation, $(-2, 5)$.

8. $x^2+3x-10>0$

We graph the function $f(x)=x^2+3x-10$. The intercepts are

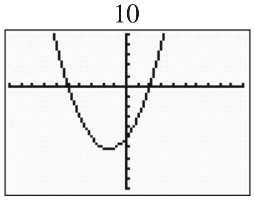
y-intercept: $f(0)=-10$

x-intercepts: $x^2+3x-10=0$

$(x+5)(x-2)=0$

$x=-5,\ x=2$

The vertex is at $x=\frac{-b}{2a}=\frac{-(3)}{2(1)}=-\frac{3}{2}$. Since $f\left(-\frac{3}{2}\right)=-\frac{49}{4}$, the vertex is $\left(-\frac{3}{2},-\frac{49}{4}\right)$.



The graph is above the x-axis when $x<-5$ or $x>2$. Since the inequality is strict, the solution set is $\{x \mid x<-5 \text{ or } x>2\}$ or, using interval notation, $(-\infty,-5)\cup(2,\infty)$.

9. $x^2-4x>0$

We graph the function $f(x)=x^2-4x$. The intercepts are

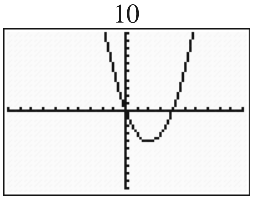
y-intercept: $f(0)=0$

x-intercepts: $x^2-4x=0$

$x(x-4)=0$

$x=0,\ x=4$

The vertex is at $x=\frac{-b}{2a}=\frac{-(-4)}{2(1)}=\frac{4}{2}=2$. Since $f(2)=-4$, the vertex is $(2,-4)$.



The graph is above the x-axis when $x<0$ or $x>4$. Since the inequality is strict, the solution set is $\{x \mid x<0 \text{ or } x>4\}$ or, using interval notation, $(-\infty,0)\cup(4,\infty)$.

10. $x^2+8x>0$

We graph the function $f(x)=x^2+8x$. The intercepts are

y-intercept: $f(0)=0$

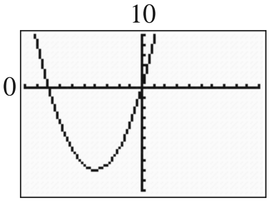
x-intercepts: $x^2+8x=0$

$x(x+8)=0$

$x=0,\ x=-8$

The vertex is at $x=\frac{-b}{2a}=\frac{-(8)}{2(1)}=\frac{-8}{2}=-4$.

Since $f(-4)=-16$, the vertex is $(-4,-16)$.



The graph is above the x-axis when $x<-8$ or $x>0$. Since the inequality is strict, the solution set is $\{x \mid x<-8 \text{ or } x>0\}$ or, using interval notation, $(-\infty,-8)\cup(0,\infty)$.

11. $x^2-9<0$

We graph the function $f(x)=x^2-9$. The intercepts are

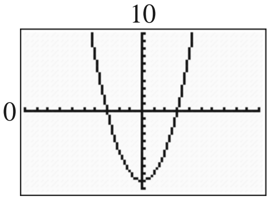
y-intercept: $f(0)=-9$

x-intercepts: $x^2-9=0$

$(x+3)(x-3)=0$

$x=-3,\ x=3$

The vertex is at $x=\frac{-b}{2a}=\frac{-(0)}{2(1)}=0$. Since $f(0)=-9$, the vertex is $(0,-9)$.



The graph is below the x-axis when $-3<x<3$. Since the inequality is strict, the solution set is $\{x \mid -3<x<3\}$ or, using interval notation, $(-3,3)$.

12. $x^2 - 1 < 0$

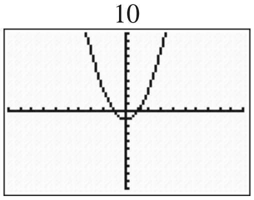
We graph the function $f(x) = x^2 - 1$. The intercepts are

y-intercept: $f(0) = -1$

x-intercepts:
$$x^2 - 1 = 0$$
$$(x+1)(x-1) = 0$$
$$x = -1,\ x = 1$$

The vertex is at $x = \dfrac{-b}{2a} = \dfrac{-(0)}{2(1)} = 0$. Since $f(0) = -1$, the vertex is $(0, -1)$.

10

−10  10

−10

The graph is below the x-axis when $-1 < x < 1$. Since the inequality is strict, the solution set is $\{\, x \mid -1 < x < 1 \,\}$ or, using interval notation, $(-1,\ 1)$.

13.
$$x^2 + x > 12$$
$$x^2 + x - 12 > 0$$

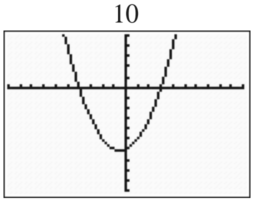
We graph the function $f(x) = x^2 + x - 12$.

y-intercept: $f(0) = -12$

x-intercepts:
$$x^2 + x - 12 = 0$$
$$(x+4)(x-3) = 0$$
$$x = -4,\ x = 3$$

The vertex is at $x = \dfrac{-b}{2a} = \dfrac{-(1)}{2(1)} = -\dfrac{1}{2}$. Since $f\left(-\dfrac{1}{2}\right) = -\dfrac{49}{4}$, the vertex is $\left(-\dfrac{1}{2}, -\dfrac{49}{4}\right)$.

10

−10  10

−20

The graph is above the x-axis when $x < -4$ or $x > 3$. Since the inequality is strict, the solution set is $\{\, x \mid x < -4 \text{ or } x > 3 \,\}$ or, using interval notation, $(-\infty, -4) \cup (3,\ \infty)$.

14.
$$x^2 + 7x < -12$$
$$x^2 + 7x + 12 < 0$$

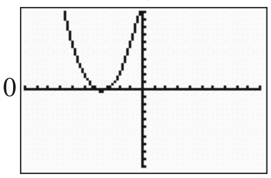
We graph the function $f(x) = x^2 + 7x + 12$.

y-intercept: $f(0) = 12$

x-intercepts:
$$x^2 + 7x + 12 = 0$$
$$(x+4)(x+3) = 0$$
$$x = -4,\ x = -3$$

The vertex is at $x = \dfrac{-b}{2a} = \dfrac{-(7)}{2(1)} = -\dfrac{7}{2}$. Since $f\left(-\dfrac{7}{2}\right) = -\dfrac{1}{4}$, the vertex is $\left(-\dfrac{1}{2}, -\dfrac{1}{4}\right)$.

10

−10  10

−10

The graph is below the x-axis when $-4 < x < -3$. Since the inequality is strict, the solution set is $\{x \mid -4 < x < -3\}$ or, using interval notation, $(-4, -3)$.

15.
$$2x^2 < 5x + 3$$
$$2x^2 - 5x - 3 < 0$$

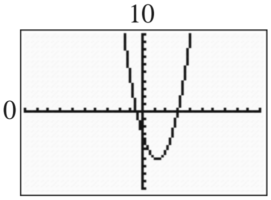
We graph the function $f(x) = 2x^2 - 5x - 3$. The intercepts are

y-intercept: $f(0) = -3$

x-intercepts:
$$2x^2 - 5x - 3 = 0$$
$$(2x+1)(x-3) = 0$$
$$x = -\frac{1}{2},\ x = 3$$

The vertex is at $x = \dfrac{-b}{2a} = \dfrac{-(-5)}{2(2)} = \dfrac{5}{4}$. Since $f\left(\dfrac{5}{4}\right) = -\dfrac{49}{8}$, the vertex is $\left(\dfrac{5}{4}, -\dfrac{49}{8}\right)$.

10

−10  10

−10

The graph is below the x-axis when $-\dfrac{1}{2} < x < 3$. Since the inequality is strict, the solution set is

$\left\{ x \middle| -\frac{1}{2} < x < 3 \right\}$ or, using interval notation, $\left(-\frac{1}{2},\ 3\right)$.

16. $6x^2 < 6 + 5x$

$6x^2 - 5x - 6 < 0$

We graph the function $f(x) = 6x^2 - 5x - 6$. The intercepts are

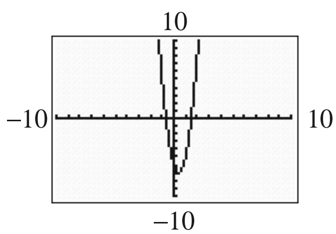
y-intercept: $f(0) = -6$

x-intercepts: $6x^2 - 5x - 6 = 0$

$(3x+2)(2x-3) = 0$

$x = -\frac{2}{3},\ x = \frac{3}{2}$

The vertex is at $x = \frac{-b}{2a} = \frac{-(-5)}{2(6)} = \frac{5}{12}$. Since $f\left(\frac{5}{12}\right) = -\frac{169}{24}$, the vertex is $\left(\frac{5}{12}, -\frac{169}{24}\right)$.



The graph is below the x-axis when $-\frac{2}{3} < x < \frac{3}{2}$. Since the inequality is strict, the solution set is $\left\{ x \middle| -\frac{2}{3} < x < \frac{3}{2} \right\}$ or, using interval notation, $\left(-\frac{2}{3},\ \frac{3}{2}\right)$.

17. $x^2 - x + 1 \le 0$

We graph the function $f(x) = x^2 - x + 1$. The intercepts are

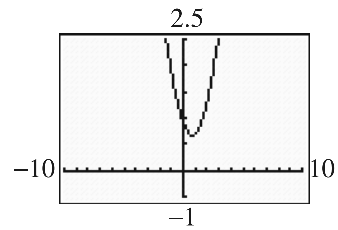
y-intercept: $f(0) = 1$

x-intercepts: $x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$

$= \frac{1 \pm \sqrt{-3}}{2}$ (not real)

Therefore, f has no x-intercepts.

The vertex is at $x = \frac{-b}{2a} = \frac{-(-1)}{2(1)} = \frac{1}{2}$. Since $f\left(\frac{1}{2}\right) = \frac{3}{4}$, the vertex is $\left(\frac{1}{2}, \frac{3}{4}\right)$.



The graph is never below the x-axis. Thus, there is no real solution.

18. $x^2 + 2x + 4 > 0$

We graph the function $f(x) = x^2 + 2x + 4$.

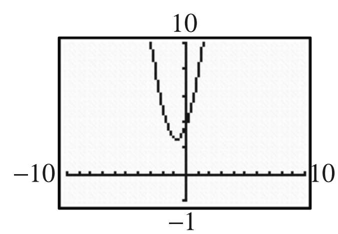
y-intercept: $f(0) = 4$

x-intercepts: $x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(4)}}{2(1)}$

$= \frac{-2 \pm \sqrt{-12}}{2}$ (not real)

Therefore, f has no x-intercepts.

The vertex is at $x = \frac{-b}{2a} = \frac{-(2)}{2(1)} = -1$. Since $f(-1) = 3$, the vertex is $(-1, 3)$.



The graph is always above the x-axis. Thus, the solution is all real numbers or using interval notation, $(-\infty,\ \infty)$.

19. $4x^2 + 9 < 6x$

$4x^2 - 6x + 9 < 0$

We graph the function $f(x) = 4x^2 - 6x + 9$.

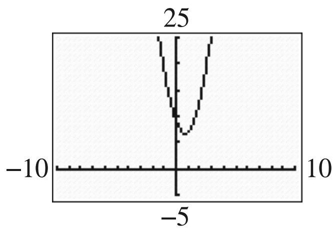
y-intercept: $f(0) = 9$

x-intercepts: $x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(4)(9)}}{2(4)}$

$= \frac{6 \pm \sqrt{-108}}{8}$ (not real)

Therefore, f has no x-intercepts.

The vertex is at $x = \frac{-b}{2a} = \frac{-(-6)}{2(4)} = \frac{6}{8} = \frac{3}{4}$. Since $f\left(\frac{3}{4}\right) = \frac{27}{4}$, the vertex is $\left(\frac{3}{4}, \frac{27}{4}\right)$.



The graph is never below the x-axis. Thus, there is no real solution.

20. $25x^2 + 16 < 40x$

$25x^2 - 40x + 16 < 0$

We graph the function $f(x) = 25x^2 - 40x + 16$.

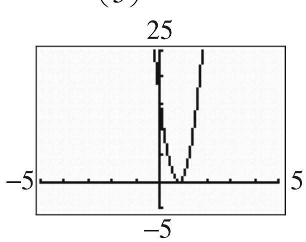
y-intercept: $f(0) = 16$

x-intercepts: $25x^2 - 40x + 16 = 0$

$$(5x-4)^2 = 0$$
$$5x - 4 = 0$$
$$x = \frac{4}{5}$$

The vertex is at $x = \frac{-b}{2a} = \frac{-(-40)}{2(25)} = \frac{40}{50} = \frac{4}{5}$.

Since $f\left(\frac{4}{5}\right) = 0$, the vertex is $\left(\frac{4}{5}, 0\right)$.



The graph is never below the x-axis. Thus, there is no real solution.

21. $6\left(x^2 - 1\right) > 5x$

$6x^2 - 6 > 5x$

$6x^2 - 5x - 6 > 0$

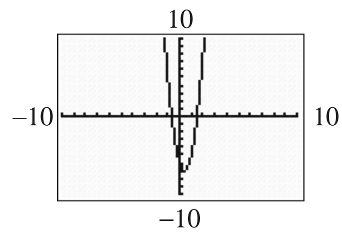
We graph the function $f(x) = 6x^2 - 5x - 6$.

y-intercept: $f(0) = -6$

x-intercepts: $6x^2 - 5x - 6 = 0$

$$(3x+2)(2x-3) = 0$$
$$x = -\frac{2}{3},\ x = \frac{3}{2}$$

The vertex is at $x = \frac{-b}{2a} = \frac{-(-5)}{2(6)} = \frac{5}{12}$. Since $f\left(\frac{5}{12}\right) = -\frac{169}{24}$, the vertex is $\left(\frac{5}{12}, -\frac{169}{24}\right)$.



The graph is above the x-axis when $x < -\frac{2}{3}$ or $x > \frac{3}{2}$. Since the inequality is strict, solution set is $\left\{ x \,\middle|\, x < -\frac{2}{3} \text{ or } x > \frac{3}{2} \right\}$ or, using interval notation, $\left(-\infty, -\frac{2}{3}\right) \cup \left(\frac{3}{2}, \infty\right)$.

22. $2\left(2x^2 - 3x\right) > -9$

$4x^2 - 6x > -9$

$4x^2 - 6x + 9 > 0$

We graph the function $f(x) = 4x^2 - 6x + 9$.

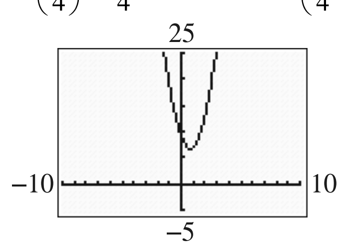
y-intercept: $f(0) = 9$

x-intercepts: $x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(4)(9)}}{2(4)}$

$$= \frac{6 \pm \sqrt{-108}}{8} \quad \text{(not real)}$$

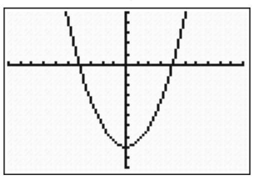
Therefore, f has no x-intercepts.

The vertex is at $x = \frac{-b}{2a} = \frac{-(-6)}{2(4)} = \frac{6}{8} = \frac{3}{4}$. Since $f\left(\frac{3}{4}\right) = \frac{27}{4}$, the vertex is $\left(\frac{3}{4}, \frac{27}{4}\right)$.



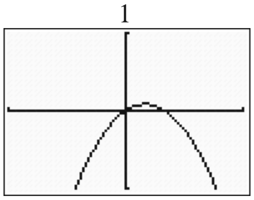
The graph is always above the x-axis. Thus, the solution set is all real numbers or, using interval notation, $(-\infty, \infty)$.

23. The domain of the expression $f(x)=\sqrt{x^2-16}$ includes all values for which $x^2-16\ge 0$.
We graph the function $p(x)=x^2-16$. The intercepts of p are
y-intercept: $p(0)=-6$
x-intercepts:
$$x^2-16=0$$
$$(x+4)(x-4)=0$$
$$x=-4,\ x=4$$
The vertex of p is at $x=\frac{-b}{2a}=\frac{-(0)}{2(1)}=0$. Since $p(0)=-16$, the vertex is $(0,-16)$.



The graph of p is above the x-axis when $x<-4$ or $x>4$. Since the inequality is not strict, the solution set of $x^2-16\ge 0$ is $\{x\mid x\le -4 \text{ or } x\ge 4\}$.
Thus, the domain of f is also $\{x\mid x\le -4 \text{ or } x\ge 4\}$ or, using interval notation, $(-\infty,-4]\cup[4,\ \infty)$.

24. The domain of the expression $f(x)=\sqrt{x-3x^2}$ includes all values for which $x-3x^2\ge 0$.
We graph the function $p(x)=x-3x^2$. The intercepts of p are
y-intercept: $p(0)=-6$
x-intercepts:
$$x-3x^2=0$$
$$x(1-3x)=0$$
$$x=0,\ x=\frac{1}{3}.$$
The vertex of p is at $x=\frac{-b}{2a}=\frac{-(1)}{2(-3)}=\frac{-1}{-6}=\frac{1}{6}$.
Since $p\left(\frac{1}{6}\right)=\frac{1}{12}$, the vertex is $\left(\frac{1}{6},\frac{1}{12}\right)$.



The graph of p is above the x-axis when $0<x<\frac{1}{3}$. Since the inequality is not strict, the solution set of $x-3x^2\ge 0$ is $\left\{x\,\middle|\,0\le x\le\frac{1}{3}\right\}$.
Thus, the domain of f is also $\left\{x\,\middle|\,0\le x\le\frac{1}{3}\right\}$ or, using interval notation, $\left[0,\ \frac{1}{3}\right]$.

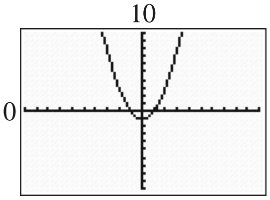
25. $f(x)=x^2-1;\quad g(x)=3x+3$

a.
$$f(x)=0$$
$$x^2-1=0$$
$$(x-1)(x+1)=0$$
$$x=1;\ x=-1$$
Solution set: $\{-1,1\}$.

b.
$$g(x)=0$$
$$3x+3=0$$
$$3x=-3$$
$$x=-1$$
Solution set: $\{-1\}$.

c.
$$f(x)=g(x)$$
$$x^2-1=3x+3$$
$$x^2-3x-4=0$$
$$(x-4)(x+1)=0$$
$$x=4;\ x=-1$$
Solution set: $\{-1,4\}$.

d. $f(x)>0$
We graph the function $f(x)=x^2-1$.
y-intercept: $f(0)=-1$
x-intercepts:
$$x^2-1=0$$
$$(x+1)(x-1)=0$$
$$x=-1,\ x=1$$
The vertex is at $x=\frac{-b}{2a}=\frac{-(0)}{2(1)}=0$. Since $f(0)=-1$, the vertex is $(0,-1)$.



The graph is above the x-axis when $x<-1$

or $x > 1$. Since the inequality is strict, the solution set is $\{x \mid x < -1 \text{ or } x > 1\}$ or, using interval notation, $(-\infty, -1) \cup (1, \infty)$.

e. $g(x) \le 0$

$3x + 3 \le 0$

$3x \le -3$

$x \le -1$

The solution set is $\{x \mid x \le -1\}$ or, using interval notation, $(-\infty, -1]$.

f. $f(x) > g(x)$

$x^2 - 1 > 3x + 3$

$x^2 - 3x - 4 > 0$

We graph the function $p(x) = x^2 - 3x - 4$.
The intercepts of p are
y-intercept: $p(0) = -4$

x-intercepts: $x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = 4,\ x = -1$

The vertex is at $x = \frac{-b}{2a} = \frac{-(-3)}{2(1)} = \frac{3}{2}$. Since $p\left(\frac{3}{2}\right) = -\frac{25}{4}$, the vertex is $\left(\frac{3}{2}, -\frac{25}{4}\right)$.

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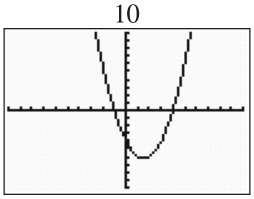
−10  10

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The graph of p is above the x-axis when $x < -1$ or $x > 4$. Since the inequality is strict, the solution set is $\{x \mid x < -1 \text{ or } x > 4\}$ or, using interval notation, $(-\infty, -1) \cup (4, \infty)$.

g. $f(x) \ge 1$

$x^2 - 1 \ge 1$

$x^2 - 2 \ge 0$

We graph the function $p(x) = x^2 - 2$. The intercepts of p are
y-intercept: $p(0) = -2$

x-intercepts: $x^2 - 2 = 0$

$x^2 = 2$

$x = \pm\sqrt{2}$

The vertex is at $x = \frac{-b}{2a} = \frac{-(0)}{2(1)} = 0$. Since $p(0) = -2$, the vertex is $(0, -2)$.

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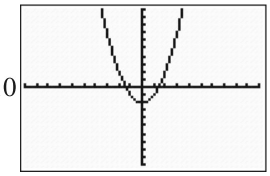
−10  10

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The graph of p is above the x-axis when $x < -\sqrt{2}$ or $x > \sqrt{2}$. Since the inequality is not strict, the solution set is $\{x \mid x \le -\sqrt{2} \text{ or } x \ge \sqrt{2}\}$ or, using interval notation, $\left(-\infty, -\sqrt{2}\right] \cup \left[\sqrt{2}, \infty\right)$.

26. $f(x) = -x^2 + 3$; $g(x) = -3x + 3$

a. $f(x) = 0$

$-x^2 + 3 = 0$

$x^2 = 3$

$x = \pm\sqrt{3}$

Solution set: $\{-\sqrt{3}, \sqrt{3}\}$.

b. $g(x) = 0$

$-3x + 3 = 0$

$-3x = -3$

$x = 1$

Solution set: $\{1\}$.

c. $f(x) = g(x)$

$-x^2 + 3 = -3x + 3$

$0 = x^2 - 3x$

$0 = x(x-3)$

$x = 0; x = 3$

Solution set: $\{0, 3\}$.

d. $f(x) > 0$

We graph the function $f(x) = -x^2 + 3$.

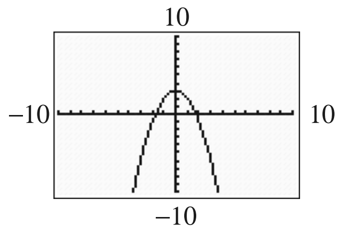
y-intercept: $f(0) = 3$

x-intercepts: $-x^2 + 3 = 0$

$x^2 = 3$

$x = \pm\sqrt{3}$

The vertex is at $x = \frac{-b}{2a} = \frac{-(0)}{2(-1)} = 0$. Since $f(0) = 3$, the vertex is $(0, 3)$.



The graph is above the x-axis when $-\sqrt{3} < x < \sqrt{3}$. Since the inequality is strict, the solution set is $\left\{ x \middle| -\sqrt{3} < x < \sqrt{3} \right\}$ or, using interval notation, $\left(-\sqrt{3}, \sqrt{3}\right)$.

e. $g(x) \le 0$

$-3x + 3 \le 0$

$-3x \le -3$

$x \ge 1$

The solution set is $\{ x \mid x \ge 1\}$ or, using interval notation, $[1, \infty)$.

f. $f(x) > g(x)$

$-x^2 + 3 > -3x + 3$

$-x^2 + 3x > 0$

We graph the function $p(x) = -x^2 + 3x$. The intercepts of p are

y-intercept: $p(0) = 0$

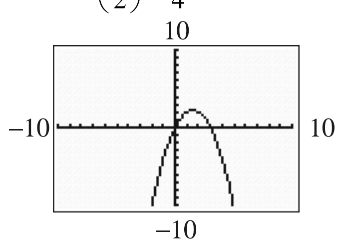
x-intercepts: $-x^2 + 3x = 0$

$-x(x-3) = 0$

$x = 0;\ x = 3$

The vertex is at $x = \dfrac{-b}{2a} = \dfrac{-(3)}{2(-1)} = \dfrac{-3}{-2} = \dfrac{3}{2}$.

Since $p\left(\dfrac{3}{2}\right) = \dfrac{9}{4}$, the vertex is $\left(\dfrac{3}{2}, \dfrac{9}{4}\right)$.



The graph of p is above the x-axis when $0 < x < 3$. Since the inequality is strict, the solution set is $\{ x \mid 0 < x < 3\}$ or, using interval notation, $(0, 3)$.

g. $f(x) \ge 1$

$-x^2 + 3 \ge 1$

$-x^2 + 2 \ge 0$

We graph the function $p(x) = -x^2 + 2$. The intercepts of p are

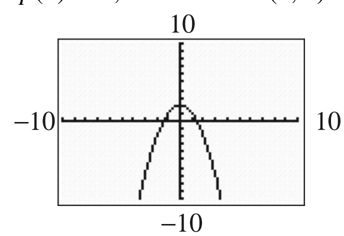
y-intercept: $p(0) = 2$

x-intercepts: $-x^2 + 2 = 0$

$x^2 = 2$

$x = \pm\sqrt{2}$

The vertex is at $x = \dfrac{-b}{2a} = \dfrac{-(0)}{2(-1)} = 0$. Since $p(0) = 2$, the vertex is $(0, 2)$.



The graph of p is above the x-axis when $-\sqrt{2} < x < \sqrt{2}$. Since the inequality is not strict, the solution set is $\left\{ x \middle| -\sqrt{2} \le x \le \sqrt{2} \right\}$ or, using interval notation, $\left[-\sqrt{2}, \sqrt{2}\right]$.

27. $f(x) = -x^2 + 1; \quad g(x) = 4x + 1$

a. $f(x) = 0$

$-x^2 + 1 = 0$

$1 - x^2 = 0$

$(1-x)(1+x) = 0$

$x = 1;\ x = -1$

Solution set: $\{-1, 1\}$.

b. $g(x) = 0$

$4x + 1 = 0$

$4x = -1$

$x = -\dfrac{1}{4}$

Solution set: $\left\{-\dfrac{1}{4}\right\}$.

c. $f(x) = g(x)$

$-x^2 + 1 = 4x + 1$

$0 = x^2 + 4x$

$0 = x(x+4)$

$x = 0; x - 4$

Solution set: $\{-4, 0\}$.

d. $f(x) > 0$

We graph the function $f(x) = -x^2 + 1$.

y-intercept: $f(0) = 1$

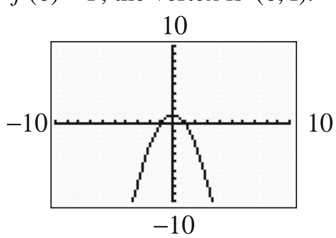
x-intercepts: $-x^2 + 1 = 0$

$x^2 - 1 = 0$

$(x+1)(x-1) = 0$

$x = -1; x = 1$

The vertex is at $x = \frac{-b}{2a} = \frac{-(0)}{2(-1)} = 0$. Since $f(0) = 1$, the vertex is $(0, 1)$.



The graph is above the x-axis when $-1 < x < 1$. Since the inequality is strict, the solution set is $\{x \mid -1 < x < 1\}$ or, using interval notation, $(-1, 1)$.

e. $g(x) \le 0$

$4x + 1 \le 0$

$4x \le -1$

$x \le -\frac{1}{4}$

The solution set is $\left\{x \middle| x \le -\frac{1}{4}\right\}$ or, using interval notation, $\left(-\infty, -\frac{1}{4}\right]$.

f. $f(x) > g(x)$

$-x^2 + 1 > 4x + 1$

$-x^2 - 4x > 0$

We graph the function $p(x) = -x^2 - 4x$.

The intercepts of p are

y-intercept: $p(0) = 0$

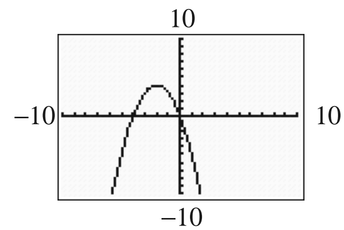
x-intercepts: $-x^2 - 4x = 0$

$-x(x+4) = 0$

$x = 0; x = -4$

The vertex is at $x = \frac{-b}{2a} = \frac{-(-4)}{2(-1)} = \frac{4}{-2} = -2$.

Since $p(-2) = 4$, the vertex is $(-2, 4)$.



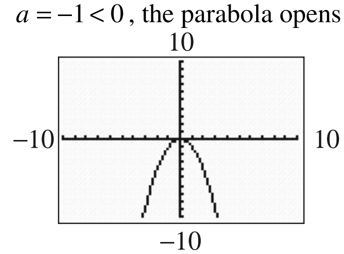
The graph of p is above the x-axis when $-4 < x < 0$. Since the inequality is strict, the solution set is $\{x \mid -4 < x < 0\}$ or, using interval notation, $(-4, 0)$.

g. $f(x) \ge 1$

$-x^2 + 1 \ge 1$

$-x^2 \ge 0$

We graph the function $p(x) = -x^2$. The vertex is at $x = \frac{-b}{2a} = \frac{-(0)}{2(-1)} = 0$. Since $p(0) = 0$, the vertex is $(0, 0)$. Since $a = -1 < 0$, the parabola opens downward.



The graph of p is never above the x-axis, but it does touch the x-axis at $x = 0$. Since the inequality is not strict, the solution set is $\{0\}$.

28. $f(x) = -x^2 + 4$; $g(x) = -x - 2$

a. $f(x) = 0$

$-x^2 + 4 = 0$

$x^2 - 4 = 0$

$(x+2)(x-2) = 0$

$x = -2; x = 2$

Solution set: $\{-2, 2\}$.

b. $g(x)=0$

$-x-2=0$

$-2=x$

Solution set: $\{-2\}$.

c. $f(x)=g(x)$

$-x^2+4=-x-2$

$0=x^2-x-6$

$0=(x-3)(x+2)$

$x=3; x=-2$

Solution set: $\{-2, 3\}$.

d. $f(x)>0$

$-x^2+4>0$

We graph the function $f(x)=-x^2+4$.

y-intercept: $f(0)=4$

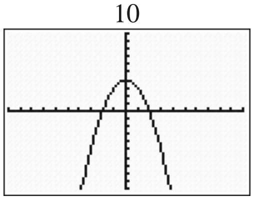
x-intercepts: $-x^2+4=0$

$x^2-4=0$

$(x+2)(x-2)=0$

$x=-2; x=2$

The vertex is at $x=\frac{-b}{2a}=\frac{-(0)}{2(-1)}=0$. Since $f(0)=4$, the vertex is $(0, 4)$.



The graph is above the x-axis when $-2<x<2$. Since the inequality is strict, the solution set is $\{x \mid -2<x<2\}$ or, using interval notation, $(-2, 2)$.

e. $g(x)\le 0$

$-x-2\le 0$

$-x\le 2$

$x\ge -2$

The solution set is $\{x \mid x\ge -2\}$ or, using interval notation, $[-2, \infty)$.

f. $f(x)>g(x)$

$-x^2+4>-x-2$

$-x^2+x+6>0$

We graph the function $p(x)=-x^2+x+6$. The intercepts of p are

y-intercept: $p(0)=6$

x-intercepts: $-x^2+x+6=0$

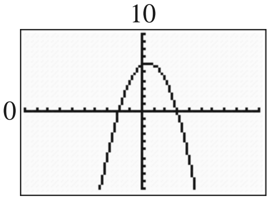
$x^2-x-6=0$

$(x+2)(x-3)=0$

$x=-2; x=3$

The vertex is at $x=\frac{-b}{2a}=\frac{-(1)}{2(-1)}=\frac{-1}{-2}=\frac{1}{2}$.

Since $p\left(\frac{1}{2}\right)=\frac{25}{4}$, the vertex is $\left(\frac{1}{2}, \frac{25}{4}\right)$.



The graph of p is above the x-axis when $-2<x<3$. Since the inequality is strict, the solution set is $\{x \mid -2<x<3\}$ or, using interval notation, $(-2, 3)$.

g. $f(x)\ge 1$

$-x^2+4>1$

$-x^2+3>0$

We graph the function $p(x)=-x^2+3$. The intercepts of p are

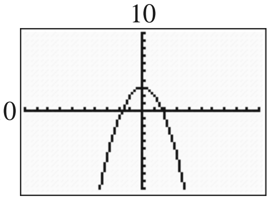
y-intercept: $p(0)=3$

x-intercepts: $-x^2+3=0$

$x^2=3$

$x=\pm\sqrt{3}$

The vertex is at $x=\frac{-b}{2a}=\frac{-(0)}{2(-1)}=0$. Since $p(0)=3$, the vertex is $(0, 3)$.



The graph of p is above the x-axis when $-\sqrt{3}<x<\sqrt{3}$. Since the inequality is not strict, the solution set is $\{x \mid -\sqrt{3}\le x\le\sqrt{3}\}$ or, using interval notation, $[-\sqrt{3}, \sqrt{3}]$.

29. $f(x) = x^2 - 4; \quad g(x) = -x^2 + 4$

a.
$$f(x) = 0$$
$$x^2 - 4 = 0$$
$$(x-2)(x+2) = 0$$
$$x = 2; x = -2$$
Solution set: $\{-2, 2\}$.

b.
$$g(x) = 0$$
$$-x^2 + 4 = 0$$
$$x^2 - 4 = 0$$
$$(x+2)(x-2) = 0$$
$$x = -2; x = 2$$
Solution set: $\{-2, 2\}$.

c.
$$f(x) = g(x)$$
$$x^2 - 4 = -x^2 + 4$$
$$2x^2 - 8 = 0$$
$$2(x-2)(x+2) = 0$$
$$x = 2; x = -2$$
Solution set: $\{-2, 2\}$.

d.
$$f(x) > 0$$
$$x^2 - 4 > 0$$
We graph the function $f(x) = x^2 - 4$.

y-intercept: $f(0) = -4$

x-intercepts: $x^2 - 4 = 0$
$$(x+2)(x-2) = 0$$
$$x = -2; x = 2$$

The vertex is at $x = \frac{-b}{2a} = \frac{-(0)}{2(-1)} = 0$. Since $f(0) = -4$, the vertex is $(0, -4)$.

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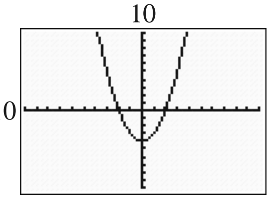
−10  10

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The graph is above the x-axis when $x < -2$ or $x > 2$. Since the inequality is strict, the solution set is $\{\, x \mid x < -2 \text{ or } x > 2 \,\}$ or, using interval notation, $(-\infty, -2) \cup (2, \infty)$.

e.
$$g(x) \le 0$$
$$-x^2 + 4 \le 0$$
We graph the function $g(x) = -x^2 + 4$.

y-intercept: $g(0) = 4$

x-intercepts: $-x^2 + 4 = 0$
$$x^2 - 4 = 0$$
$$(x+2)(x-2) = 0$$
$$x = -2; x = 2$$

The vertex is at $x = \frac{-b}{2a} = \frac{-(0)}{2(-1)} = 0$. Since $g(0) = 4$, the vertex is $(0, 4)$.

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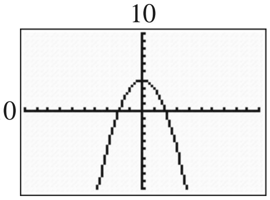
−10  10

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The graph is below the x-axis when $x < -2$ or $x > 2$. Since the inequality is not strict, the solution set is $\{\, x \mid x \le -2 \text{ or } x \ge 2 \,\}$ or, using interval notation, $(-\infty, -2] \cup [2, \infty)$.

f.
$$f(x) > g(x)$$
$$x^2 - 4 > -x^2 + 4$$
$$2x^2 - 8 > 0$$
We graph the function $p(x) = 2x^2 - 8$.

y-intercept: $p(0) = -8$

x-intercepts: $2x^2 - 8 = 0$
$$2(x+2)(x-2) = 0$$
$$x = -2; x = 2$$

The vertex is at $x = \frac{-b}{2a} = \frac{-(0)}{2(2)} = 0$. Since $p(0) = -8$, the vertex is $(0, -8)$.

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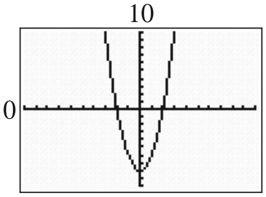
−10  10

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The graph is above the x-axis when $x < -2$ or $x > 2$. Since the inequality is strict, the solution set is $\{\, x \mid x < -2 \text{ or } x > 2 \,\}$ or, using interval notation, $(-\infty, -2) \cup (2, \infty)$.

g.
$$f(x) \ge 1$$
$$x^2 - 4 \ge 1$$
$$x^2 - 5 \ge 0$$
We graph the function $p(x) = x^2 - 5$.

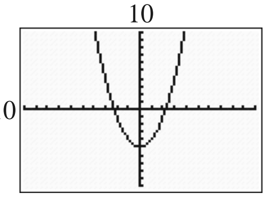
y-intercept: $p(0)=-5$

x-intercepts: $x^2-5=0$

$x^2=5$

$x=\pm\sqrt{5}$

The vertex is at $x=\frac{-b}{2a}=\frac{-(0)}{2(1)}=0$. Since $p(0)=-5$, the vertex is $(0,-5)$.



The graph of p is above the x-axis when $x<-\sqrt{5}$ or $x>\sqrt{5}$. Since the inequality is not strict, the solution set is $\left\{x \mid x\le-\sqrt{5} \text{ or } x\ge\sqrt{5}\right\}$ or, using interval notation, $\left(-\infty,-\sqrt{5}\right]\cup\left[\sqrt{5},\infty\right)$.

30. $f(x)=x^2-2x+1$; $g(x)=-x^2+1$

a. $f(x)=0$

$x^2-2x+1=0$

$(x-1)^2=0$

$x-1=0$

$x=1$

Solution set: $\{1\}$.

b. $g(x)=0$

$-x^2+1=0$

$x^2-1=0$

$(x+1)(x-1)=0$

$x=-1; x=1$

Solution set: $\{-1,1\}$.

c. $f(x)=g(x)$

$x^2-2x+1=-x^2+1$

$2x^2-2x=0$

$2x(x-1)=0$

$x=0, x=1$

Solution set: $\{0,1\}$.

d. $f(x)>0$

$x^2-2x+1>0$

We graph the function $f(x)=x^2-2x+1$.

y-intercept: $f(0)=1$

x-intercepts: $x^2-2x+1=0$

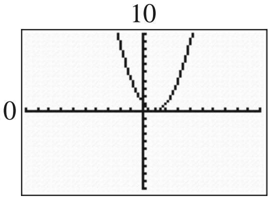
$(x-1)^2=0$

$x-1=0$

$x=1$

The vertex is at $x=\frac{-b}{2a}=\frac{-(-2)}{2(1)}=\frac{2}{2}=1$.

Since $f(1)=0$, the vertex is $(1,0)$.



The graph is above the x-axis when $x<1$ or $x>1$. Since the inequality is strict, the solution set is $\{x \mid x<1 \text{ or } x>1\}$ or, using interval notation, $(-\infty,1)\cup(1,\infty)$.

e. $g(x)\le 0$

$-x^2+1\le 0$

We graph the function $g(x)=-x^2+1$.

y-intercept: $g(0)=1$

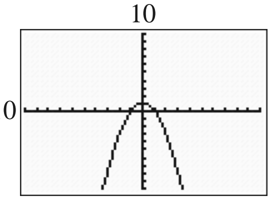
x-intercepts: $-x^2+1=0$

$x^2-1=0$

$(x+1)(x-1)=0$

$x=-1; x=1$

The vertex is at $x=\frac{-b}{2a}=\frac{-(0)}{2(-1)}=0$. Since $g(0)=1$, the vertex is $(0,1)$.



The graph is below the x-axis when $x<-1$ or $x>1$. Since the inequality is not strict, the solution set is $\{x \mid x\le-1 \text{ or } x\ge 1\}$ or, using interval notation, $(-\infty,-1]\cup[1,\infty)$.

f. $f(x)>g(x)$

$x^2-2x+1>-x^2+1$

$2x^2-2x>0$

We graph the function $p(x)=2x^2-2x$.

y-intercept: $p(0)=0$

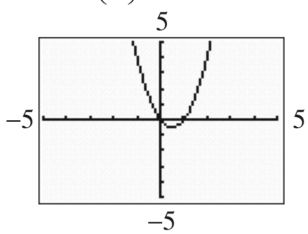
x-intercepts: $2x^2 - 2x = 0$

$2x(x-1) = 0$

$x = 0; x = 1$

The vertex is at $x = \frac{-b}{2a} = \frac{-(-2)}{2(2)} = \frac{2}{4} = \frac{1}{2}$.

Since $p\left(\frac{1}{2}\right) = \frac{1}{2}$, the vertex is $\left(\frac{1}{2}, \frac{1}{2}\right)$.



The graph is above the x-axis when $x < 0$ or $x > 1$. Since the inequality is strict, the solution set is $\{x \mid x < 0 \text{ or } x > 1\}$ or, using interval notation, $(-\infty, 0) \cup (1, \infty)$.

g. $f(x) \ge 1$

$x^2 - 2x + 1 \ge 1$

$x^2 - 2x \ge 0$

We graph the function $p(x) = x^2 - 2x$.

y-intercept: $p(0) = 0$

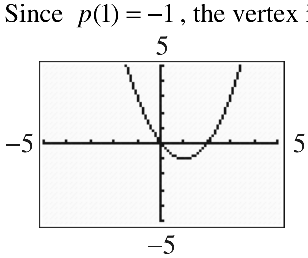
x-intercepts: $x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0; x = 2$

The vertex is at $x = \frac{-b}{2a} = \frac{-(-2)}{2(1)} = \frac{2}{2} = 1$.

Since $p(1) = -1$, the vertex is $(1, -1)$.



The graph of p is above the x-axis when $x < 0$ or $x > 2$. Since the inequality is not strict, the solution set is $\{x \mid x \le 0 \text{ or } x \ge 2\}$ or, using interval notation, $(-\infty, 0] \cup [2, \infty)$.

31. $f(x) = x^2 - x - 2$; $g(x) = x^2 + x - 2$

a. $f(x) = 0$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2, x = -1$

Solution set: $\{-1, 2\}$.

b. $g(x) = 0$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2; x = 1$

Solution set: $\{-2, 1\}$.

c. $f(x) = g(x)$

$x^2 - x - 2 = x^2 + x - 2$

$-2x = 0$

$x = 0$

Solution set: $\{0\}$.

d. $f(x) > 0$

$x^2 - x - 2 > 0$

We graph the function $f(x) = x^2 - x - 2$.

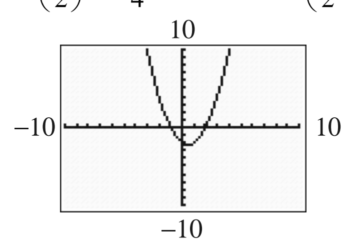
y-intercept: $f(0) = -2$

x-intercepts: $x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2; x = -1$

The vertex is at $x = \frac{-b}{2a} = \frac{-(-1)}{2(1)} = \frac{1}{2}$. Since $f\left(\frac{1}{2}\right) = -\frac{9}{4}$, the vertex is $\left(\frac{1}{2}, -\frac{9}{4}\right)$.



The graph is above the x-axis when $x < -1$ or $x > 2$. Since the inequality is strict, the solution set is $\{x \mid x < -1 \text{ or } x > 2\}$ or, using interval notation, $(-\infty, -1) \cup (2, \infty)$.

e. $g(x) \le 0$

$x^2 + x - 2 \le 0$

We graph the function $g(x) = x^2 + x - 2$.

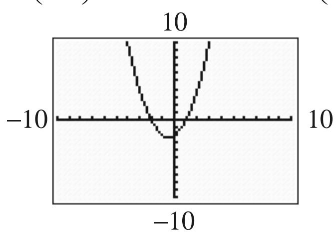
y-intercept: $g(0) = -2$

x-intercepts: $x^2+x-2=0$

$$(x+2)(x-1)=0$$

$$x=-2; x=1$$

The vertex is at $x=\frac{-b}{2a}=\frac{-(1)}{2(1)}=-\frac{1}{2}$. Since $f\left(-\frac{1}{2}\right)=-\frac{7}{4}$, the vertex is $\left(-\frac{1}{2},-\frac{7}{4}\right)$.



The graph is below the x-axis when $-2<x<1$. Since the inequality is not strict, the solution set is $\{x \mid -2\le x\le 1\}$ or, using interval notation, $[-2,1]$.

f.
$$f(x)>g(x)$$
$$x^2-x-2>x^2+x-2$$
$$-2x>0$$
$$x<0$$

The solution set is $\{x \mid x<0\}$ or, using interval notation, $(-\infty,0)$.

g.
$$f(x)\ge 1$$
$$x^2-x-2\ge 1$$
$$x^2-x-3\ge 0$$

We graph the function $p(x)=x^2-x-3$.

y-intercept: $p(0)=-3$

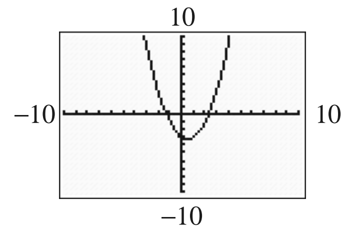
x-intercepts: $x^2-x-3=0$

$$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(-3)}}{2(1)}$$

$$=\frac{1\pm\sqrt{1+12}}{2}=\frac{1\pm\sqrt{13}}{2}$$

$$x\approx -1.30 \text{ or } x\approx 2.30$$

The vertex is at $x=\frac{-b}{2a}=\frac{-(-1)}{2(1)}=\frac{1}{2}$. Since $p\left(\frac{1}{2}\right)=-\frac{13}{4}$, the vertex is $\left(\frac{1}{2},-\frac{13}{4}\right)$.



The graph of p is above the x-axis when $x<\frac{1-\sqrt{13}}{2}$ or $x>\frac{1+\sqrt{13}}{2}$. Since the inequality is not strict, the solution set is $\left\{x \,\middle|\, x\le\frac{1-\sqrt{13}}{2} \text{ or } x\ge\frac{1+\sqrt{13}}{2}\right\}$ or, using interval notation,

$$\left(-\infty,\frac{1-\sqrt{13}}{2}\right]\cup\left[\frac{1+\sqrt{13}}{2},\infty\right).$$

32. $f(x)=-x^2-x+1$; $g(x)=-x^2+x+6$

a.
$$f(x)=0$$
$$-x^2-x+1=0$$
$$x^2+x-1=0$$
$$x=\frac{-(1)\pm\sqrt{(1)^2-4(1)(-1)}}{2(1)}$$
$$=\frac{-1\pm\sqrt{1+4}}{2}=\frac{-1\pm\sqrt{5}}{2}$$

Solution set: $\left\{\frac{-1-\sqrt{5}}{2},\frac{-1+\sqrt{5}}{2}\right\}$.

b.
$$g(x)=0$$
$$-x^2+x+6=0$$
$$x^2-x-6=0$$
$$(x-3)(x+2)=0$$
$$x=3; x=-2$$

Solution set: $\{-2,3\}$.

c.
$$f(x)=g(x)$$
$$-x^2-x+1=-x^2+x+6$$
$$-2x-5=0$$
$$-2x=5$$
$$x=-\frac{5}{2}$$

Solution set: $\left\{-\frac{5}{2}\right\}$.

d. $f(x) > 0$

$-x^2 - x + 1 > 0$

We graph the function $f(x) = -x^2 - x + 1$.

y-intercept: $f(0) = -1$

x-intercepts: $-x^2 - x + 2 = 0$

$$x^2 + x - 2 = 0$$

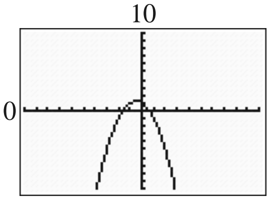
$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2}$$

$$x \approx -1.62 \text{ or } x \approx 0.62$$

The vertex is at $x = \frac{-b}{2a} = \frac{-(-1)}{2(-1)} = \frac{1}{-2} = -\frac{1}{2}$.

Since $f\left(-\frac{1}{2}\right) = \frac{5}{4}$, the vertex is $\left(-\frac{1}{2}, \frac{5}{4}\right)$.



The graph is above the x-axis when $\frac{-1-\sqrt{5}}{2} < x < \frac{-1+\sqrt{5}}{2}$. Since the inequality is strict, the solution set is $\left\{ x \middle| \frac{-1-\sqrt{5}}{2} < x < \frac{-1+\sqrt{5}}{2} \right\}$ or, using interval notation, $\left(\frac{-1-\sqrt{5}}{2}, \frac{-1+\sqrt{5}}{2} \right)$.

e. $g(x) \le 0$

$-x^2 + x + 6 \le 0$

We graph the function $g(x) = -x^2 + x + 6$.

y-intercept: $g(0) = 6$

x-intercepts: $-x^2 + x + 6 = 0$

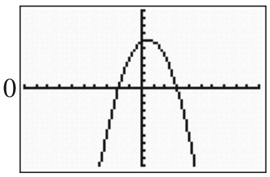
$$x^2 - x - 6 = 0$$

$$(x-3)(x+2) = 0$$

$$x = 3; x = -2$$

The vertex is at $x = \frac{-b}{2a} = \frac{-(1)}{2(-1)} = \frac{-1}{-2} = \frac{1}{2}$.

Since $f\left(\frac{1}{2}\right) = \frac{25}{4}$, the vertex is $\left(\frac{1}{2}, \frac{25}{4}\right)$.



The graph is below the x-axis when $x < -2$ or $x > 3$. Since the inequality is not strict, the solution set is $\{ x | x \le -2 \text{ or } x \ge 3 \}$ or, using interval notation, $(-\infty, 2] \cup [3, \infty)$.

f. $f(x) > g(x)$

$-x^2 - x + 1 > -x^2 + x + 6$

$-2x > 5$

$x < -\frac{5}{2}$

The solution set is $\left\{ x \middle| x < -\frac{5}{2} \right\}$ or, using interval notation, $\left(-\infty, -\frac{5}{2}\right)$.

g. $f(x) \ge 1$

$-x^2 - x + 1 \ge 1$

$-x^2 - x \ge 0$

We graph the function $p(x) = -x^2 - x$.

y-intercept: $p(0) = 0$

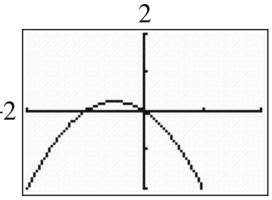
x-intercepts: $-x^2 - x = 0$

$$-x(x+1) = 0$$

$$x = 0; x = -1$$

The vertex is at $x = \frac{-b}{2a} = \frac{-(-1)}{2(-1)} = \frac{1}{-2} = -\frac{1}{2}$.

Since $p\left(-\frac{1}{2}\right) = \frac{1}{4}$, the vertex is $\left(-\frac{1}{2}, \frac{1}{4}\right)$.



The graph of p is above the x-axis when $-1 < x < 0$. Since the inequality is not strict, the solution set is $\{ x | -1 \le x \le 0 \}$ or, using interval notation, $[-1, 0]$.

33. a. The ball strikes the ground when $s(t) = 80t - 16t^2 = 0$.

$80t - 16t^2 = 0$

$16t(5 - t) = 0$

$t = 0, t = 5$

The ball strikes the ground after 5 seconds.

b. Find the values of t for which

$80t - 16t^2 > 96$

$-16t^2 + 80t - 96 > 0$

We graph the function $f(t) = -16t^2 + 80t - 96$. The intercepts are

y-intercept: $f(0) = -96$

t-intercepts: $-16t^2 + 80t - 96 = 0$

$-16(t^2 - 5t + 6) = 0$

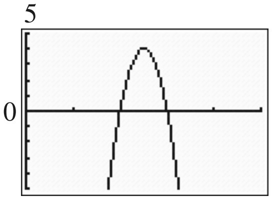
$16(t - 2)(t - 3) = 0$

$t = 2, t = 3$

The vertex is at $t = \dfrac{-b}{2a} = \dfrac{-(80)}{2(-16)} = 2.5$.

Since $f(2.5) = 4$, the vertex is $(2.5, 4)$.

5

0  5

–5

The graph of f is above the t-axis when $2 < t < 3$. Since the inequality is strict, the solution set is $\{t \mid 2 < t < 3\}$ or, using interval notation, $(2, 3)$. The ball is more than 96 feet above the ground for times between 2 and 3 seconds.

34. a. The ball strikes the ground when $s(t) = 96t - 16t^2 = 0$.

$96t - 16t^2 = 0$

$16t(6 - t) = 0$

$t = 0, t = 6$

The ball strikes the ground after 6 seconds.

b. Find the values of t for which

$96t - 16t^2 > 128$

$-16t^2 + 96t - 128 > 0$

We graph $f(t) = -16t^2 + 96t - 128$. The intercepts are

y-intercept: $f(0) = -128$

t-intercepts: $-16t^2 + 96t - 128 = 0$

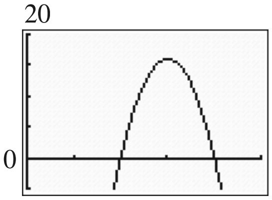
$16(t^2 - 6t + 8) = 0$

$-16(t - 4)(t - 2) = 0$

$t = 4, \; t = 2$

The vertex is at $t = \dfrac{-b}{2a} = \dfrac{-(96)}{2(-16)} = 3$. Since $f(3) = 16$, the vertex is $(3, 16)$.

20

0  5

–5

The graph of f is above the t-axis when $2 < t < 4$. Since the inequality is strict, the solution set is $\{t \mid 2 < t < 4\}$ or, using interval notation, $(2, 4)$. The ball is more than 128 feet above the ground for times between 2 and 4 seconds.

35. a. $R(p) = -4p^2 + 4000p = 0$

$-4p(p - 1000) = 0$

$p = 0, p = 1000$

Thus, the revenue equals zero when the price is \$0 or \$1000.

b. Find the values of p for which

$-4p^2 + 4000p > 800,000$

$-4p^2 + 4000p - 800,000 > 0$

We graph $f(p) = -4p^2 + 4000p - 800,000$. The intercepts are

y-intercept: $f(0) = -800,000$

p-intercepts:

$-4p^2 + 4000p - 800000 = 0$

$p^2 - 1000p + 200000 = 0$

$$p = \frac{-(-1000) \pm \sqrt{(-1000)^2 - 4(1)(200000)}}{2(1)}$$

$$= \frac{1000 \pm \sqrt{200000}}{2}$$

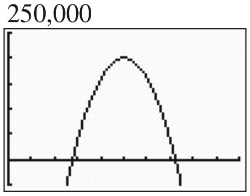
$$= \frac{1000 \pm 200\sqrt{5}}{2}$$

$$= 500 \pm 100\sqrt{5}$$

$p \approx 276.39; \; p \approx 723.61$.

The vertex is at $p = \frac{-b}{2a} = \frac{-(4000)}{2(-4)} = 500$.

Since $f(500) = 200{,}000$, the vertex is $(500, 200000)$.



The graph of f is above the p-axis when $276.39 < p < 723.61$. Since the inequality is strict, the solution set is $\{p \mid 276.39 < p < 723.61\}$ or, using interval notation, $(276.39, 723.61)$. The revenue is more than \$800,000 for prices between \$276.39 and \$723.61.

36. **a.** $R(p) = -\frac{1}{2}p^2 + 1900p = 0$

$-\frac{1}{2}p(p - 3800) = 0$

$p = 0, p = 3800$

Thus, the revenue equals zero when the price is \$0 or \$3800.

b. Find the values of p for which

$$-\frac{1}{2}p^2 + 1900p > 1200000$$

$$-\frac{1}{2}p^2 + 1900p - 1200000 > 0$$

We graph $f(p) = -\frac{1}{2}p^2 + 1900p - 1200000$.

The intercepts are

y-intercept: $f(0) = -1{,}200{,}000$

p-intercepts: $-\frac{1}{2}p^2 + 1900p - 1200000 = 0$

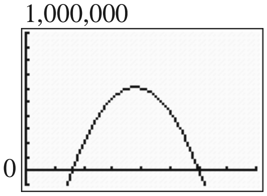
$$p^2 - 3800p + 2400000 = 0$$

$$(p - 800)(p - 3000) = 0$$

$$p = 800;\ p = 3000$$

The vertex is at $p = \frac{-b}{2a} = \frac{-(-1900)}{2(1/2)} = 1900$.

Since $f(1900) = 605{,}000$, the vertex is $(1900, 605000)$.



The graph of f is above the p-axis when $800 < p < 3000$. Since the inequality is strict, the solution set is $\{p \mid 800 < p < 3000\}$ or, using interval notation, $(800, 3000)$. The revenue is more than \$1,200,000 for prices between \$800 and \$3000.

37. $y = cx - (1 + c^2)\left(\frac{g}{2}\right)\left(\frac{x}{v}\right)^2$

a. Since the round must clear a hill 200 meters high, this mean $y > 200$.

Now $x = 2000$, $v = 897$, and $g = 9.81$.

$$c(2000) - (1 + c^2)\left(\frac{9.81}{2}\right)\left(\frac{2000}{897}\right)^2 > 200$$

$$2000c - 24.3845(1 + c^2) > 200$$

$$2000c - 24.3845 - 24.3845c^2 > 200$$

$$-24.3845c^2 + 2000c - 224.3845 > 0$$

We graph $f(c) = -24.3845c^2 + 2000c - 224.3845$.

The intercepts are

y-intercept: $f(0) = -224.3845$

c-intercepts:

$-24.3845c^2 + 2000c - 224.3845 = 0$

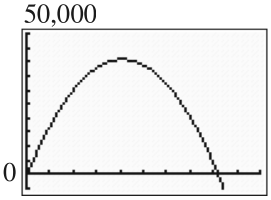
$$c = \frac{-2000 \pm \sqrt{(2000)^2 - 4(-24.3845)(-224.3845)}}{2(-24.3845)}$$

$$= \frac{-2000 \pm \sqrt{3{,}978{,}113.985}}{-48.769}$$

$c \approx 0.112$ or $c \approx 81.907$

The vertex is at $c = \frac{-b}{2a} = \frac{-(2000)}{2(-24.3845)} = 41.010$. Since $f(41.010) \approx 40{,}785.273$, the vertex is $(41.010, 40785.273)$.

50,000



0 100

−5000

The graph of f is above the c-axis when $0.112 < c < 81.907$. Since the inequality is strict, the solution set is $\{c \mid 0.112 < c < 81.907\}$ or, using interval notation, $(0.112, 81.907)$.

b. Since the round is to be on the ground $y = 0$. Note, 75 km = 75,000 m. So, $x = 75,000,\ v = 897, \text{ and } g = 9.81$.

$$c(75,000) - \left(1 + c^2\right)\left(\frac{9.81}{2}\right)\left(\frac{75,000}{897}\right)^2 = 0$$
$$75,000c - 34,290.724\left(1 + c^2\right) = 0$$
$$75,000c - 34,290.724 - 34,290.724c^2 = 0$$
$$-34,290.724c^2 + 75,000c - 34,290.724 = 0$$

We graph

$f(c) = -34,290.724c^2 + 75,000c - 34,290.724$.

The intercepts are

y-intercept: $f(0) = -34,290.724$

c-intercepts:

$-34,290.724c^2 + 75,000c - 34,290.724 = 0$

$$c = \frac{-(75,000) \pm \sqrt{(75,000)^2 - 4(-34,290.724)(-34,290.724)}}{2(-34,290.724)}$$
$$= \frac{-75,000 \pm \sqrt{921,584,990.2}}{-68,581.448}$$

$c \approx 0.651$ or $c \approx 1.536$

It is possible to hit the target 75 kilometers away so long as $c \approx 0.651$ or $c \approx 1.536$.

38. $W = \frac{1}{2}kx^2;\ \tilde{W} = \frac{w}{2g}v^2;\ x \ge 0$

Note v = 25 mph = $\frac{110}{3}$ ft/sec. For $k = 9450$, $w = 4000,\ g = 32.2, \text{ and } v = \frac{110}{3}$, we solve

$$W > \tilde{W}$$
$$\frac{1}{2}(9450)x^2 > \frac{4000}{2(32.2)}\left(\frac{110}{3}\right)^2$$
$$4725x^2 > 83,505.866$$
$$x^2 > 17.6732$$
$$x^2 - 17.6732 > 0$$

We graph $f(x) = x^2 - 17.6732$. The intercepts are

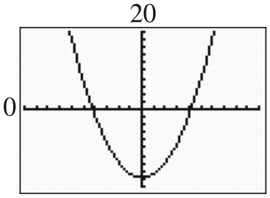
y-intercept: $f(0) = -17.6732$

x-intercepts: $x^2 - 17.6732 = 0$

$$x^2 = 17.6732$$
$$x = \pm\sqrt{17.6732}$$
$$x \approx \pm 4.2$$

The vertex is at $x = \frac{-b}{2a} = \frac{-(0)}{2(1)} = 0$. Since $f(0) = -17.6732$, the vertex is $(0, -17.6732)$.

20

−10  10

−20

The graph of f is above the x-axis when $x < -4.2$ or $x > 4.2$. Since we are restricted to $x \ge 0$, we disregard $x < -4.2$, so the solution is $x > 4.2$. Therefore, the spring must be able to compress at least 4.3 feet in order to stop the car safely.

39. $(x-4)^2 \le 0$

We graph the function $f(x) = (x-4)^2$.

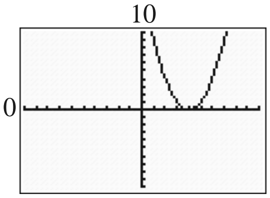
y-intercept: $f(0) = 16$

x-intercepts: $(x-4)^2 = 0$

$$x - 4 = 0$$
$$x = 4$$

The vertex is the vertex is $(4, 0)$.

10

−10  10

−10

The graph is never below the x-axis. Since the inequality is not strict, the only solution comes from the x-intercept. Therefore, the given inequality has exactly one real solution, namely $x = 4$.

40. $(x-2)^2 > 0$

We graph the function $f(x) = (x-2)^2$.

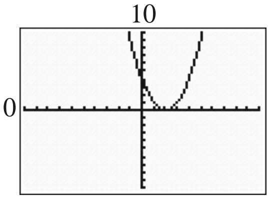
y-intercept: $f(0) = 4$

x-intercepts: $(x-2)^2 = 0$

$x - 2 = 0$

$x = 2$

The vertex is the vertex is $(2, 0)$.



The graph is above the x-axis when $x < 2$ or $x > 2$. Since the inequality is strict, the solution set is $\{x \mid x < 2 \text{ or } x > 2\}$. Therefore, the given inequality has exactly one real number that is not a solution, namely $x \neq 2$.

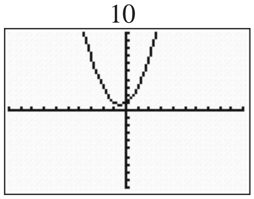
41. Solving $x^2 + x + 1 > 0$

We graph the function $f(x) = x^2 + x + 1$.

y-intercept: $f(0) = 1$

x-intercepts: $b^2 - 4ac = 1^2 - 4(1)(1) = -3$, so f has no x-intercepts.

The vertex is at $x = \frac{-b}{2a} = \frac{-(1)}{2(1)} = -\frac{1}{2}$. Since $f\left(-\frac{1}{2}\right) = \frac{3}{4}$, the vertex is $\left(-\frac{1}{2}, \frac{3}{4}\right)$.



The graph is always above the x-axis. Thus, the solution is the set of all real numbers or, using interval notation, $(-\infty, \infty)$.

42. Solving $x^2 - x + 1 < 0$

We graph the function $f(x) = x^2 - x + 1$.

y-intercept: $f(0) = 1$

x-intercepts: $b^2 - 4ac = (-1)^2 - 4(1)(1) = -3$, so f has no x-intercepts.

The vertex is at $x = \frac{-b}{2a} = \frac{-(-1)}{2(1)} = \frac{1}{2}$. Since $f\left(-\frac{1}{2}\right) = \frac{3}{4}$, the vertex is $\left(-\frac{1}{2}, \frac{3}{4}\right)$.

The graph is never below the x-axis. Thus, the inequality has no solution. That is, the solution set is { } or $\varnothing$.

43. The x-intercepts are included when the original inequality is not strict (when it contains an equal sign with the inequality).

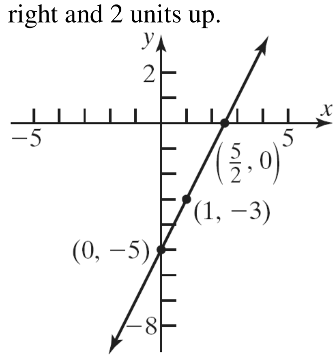
Chapter 3 Review Exercises

1. $f(x) = 2x - 5$

a. Slope = 2; y-intercept = -5

b. average rate of change = 2

c. Plot the point $(0, -5)$. Use the slope to find an additional point by moving 1 unit to the right and 2 units up.



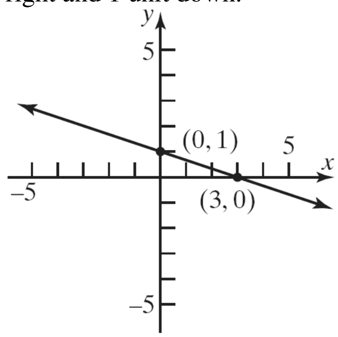
d. increasing

2. $F(x) = -\frac{1}{3}x + 1$

a. Slope = $-\frac{1}{3}$; y-intercept = 1

b. average rate of change = $-\frac{1}{3}$

c. Plot the point $(0, 1)$. Use the slope to find an additional point by moving 3 units to the right and 1 unit down.



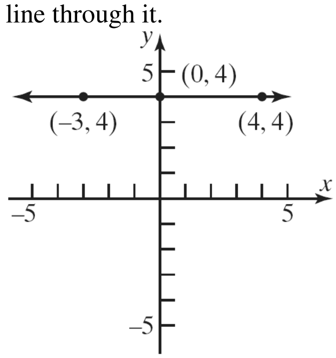
d. decreasing

3. $G(x) = 4$

a. Slope = 0; y-intercept = 4

b. average rate of change = 0

c. Plot the point (0, 4) and draw a horizontal line through it.



d. constant

4.

x	$y = f(x)$	Avg. rate of change = $\frac{\Delta y}{\Delta x}$
–1	–2	
0	3	$\frac{3-(-2)}{0-(-1)} = \frac{5}{1} = 5$
1	8	$\frac{8-3}{1-0} = \frac{5}{1} = 5$
2	13	$\frac{13-8}{2-1} = \frac{5}{1} = 5$
3	18	$\frac{18-13}{3-2} = \frac{5}{1} = 5$

Since the average rate of change is constant at 5, this is a linear function with slope = 5. The y-intercept is $(0, 3)$, so the equation of the line is $y = 5x + 3$.

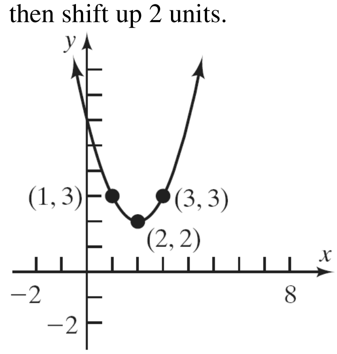
5.

x	$y = f(x)$	Avg. rate of change = $\frac{\Delta y}{\Delta x}$
–1	–3	
0	4	$\frac{4-(-3)}{0-(-1)} = \frac{7}{1} = 7$
1	7	$\frac{7-4}{1-0} = \frac{3}{1} = 3$
2	6	
3	1	

This is not a linear function, since the average rate of change is not constant.

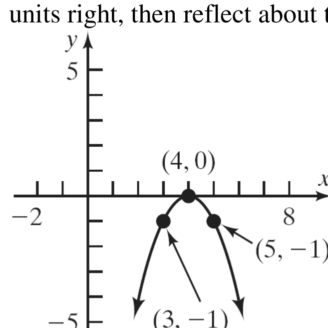
6. $f(x) = (x-2)^2 + 2$

Using the graph of $y = x^2$, shift right 2 units, then shift up 2 units.



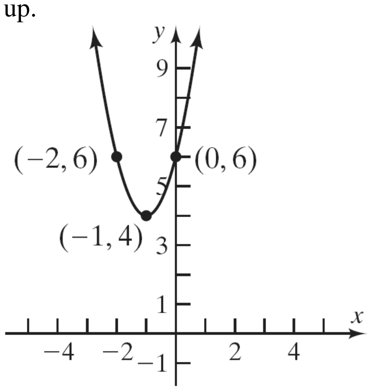
7. $f(x) = -(x-4)^2$

Using the graph of $y = x^2$, shift the graph 4 units right, then reflect about the x-axis.



8. $f(x) = 2(x+1)^2 + 4$

Using the graph of $y = x^2$, stretch vertically by a factor of 2, then shift 1 unit left, then shift 4 units up.



9. a. $f(x) = x^2 - 4x + 6$

$a = 1, b = -4, c = 6$. Since $a = 1 > 0$, the graph opens up. The x-coordinate of the vertex is $x = -\frac{b}{2a} = -\frac{-4}{2(1)} = \frac{4}{2} = 2$.

The y-coordinate of the vertex is

$f\left(-\frac{b}{2a}\right) = f(2) = (2)^2 - 4(2) + 6 = 2$.

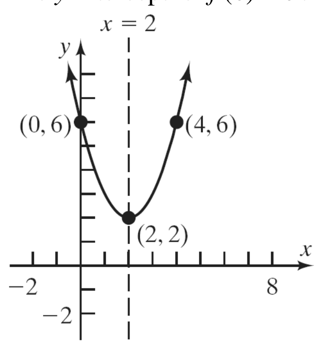
Thus, the vertex is (2, 2).

The axis of symmetry is the line $x = 2$.

The discriminant is:

$b^2 - 4ac = (-4)^2 - 4(1)(6) = -8 < 0$, so the graph has no x-intercepts.

The y-intercept is $f(0) = 6$.



b. The domain is $(-\infty, \infty)$.

The range is $[2, \infty)$.

c. Decreasing on $(-\infty, 2)$.

Increasing on $(2, \infty)$.

10. a. $f(x) = -\frac{1}{2}x^2 + 2$

$a = -\frac{1}{2}, b = 0, c = 2$. Since $a = -\frac{1}{2} < 0$, the graph opens down. The x-coordinate of the vertex is $x = -\frac{b}{2a} = -\frac{0}{2\left(-\frac{1}{2}\right)} = -\frac{0}{-1} = 0$.

The y-coordinate of the vertex is

$f\left(-\frac{b}{2a}\right) = f(0) = -\frac{1}{2}(0)^2 + 2 = 2$.

The axis of symmetry is the line $x = 0$.

he discriminant is:

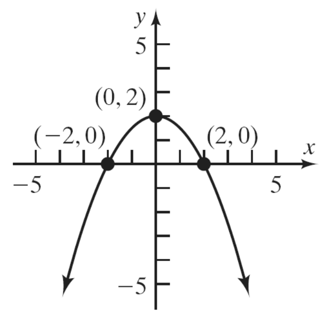
$b^2 - 4ac = (0)^2 - 4\left(-\frac{1}{2}\right)(2) = 4 > 0$, so the graph has two x-intercepts.

The x-intercepts are found by solving:

$$-\frac{1}{2}x^2 + 2 = 0$$
$$x^2 - 4 = 0$$
$$x^2 = 4$$
$$x = -2 \text{ or } x = 2$$

The x-intercepts are –2 and 2.

The y-intercept is $f(0) = 2$.



b. The domain is $(-\infty, \infty)$.

The range is $(-\infty, 2]$.

c. Increasing on $(-\infty, 0)$

Decreasing on $(0, \infty)$.

11. a. $f(x) = -4x^2 + 4x$

$a = -4, b = 4, c = 0$. Since $a = -4 < 0$, the graph opens down. The x-coordinate of the vertex is $x = -\frac{b}{2a} = -\frac{4}{2(-4)} = -\frac{4}{-8} = \frac{1}{2}$.

The y-coordinate of the vertex is

$$f\left(-\frac{b}{2a}\right) = f\left(\frac{1}{2}\right) = -4\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right)$$
$$= -1 + 2 = 1$$

Thus, the vertex is $\left(\frac{1}{2}, 1\right)$.

The axis of symmetry is the line $x = \frac{1}{2}$.

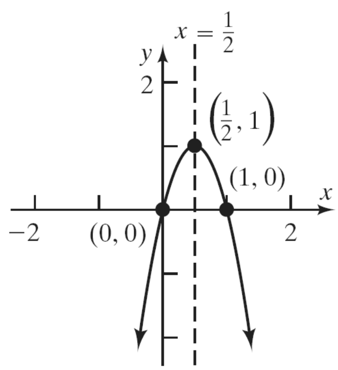
The discriminant is:
$b^2 - 4ac = 4^2 - 4(-4)(0) = 16 > 0$, so the graph has two x-intercepts.

The x-intercepts are found by solving:

$$-4x^2 + 4x = 0$$
$$-4x(x-1) = 0$$
$$x = 0 \text{ or } x = 1$$

The x-intercepts are 0 and 1.

The y-intercept is $f(0) = -4(0)^2 + 4(0) = 0$.



b. The domain is $(-\infty, \infty)$.
The range is $(-\infty, 1]$.

c. Increasing on $\left(-\infty, \frac{1}{2}\right)$

Decreasing on $\left(\frac{1}{2}, \infty\right)$.

12. a. $f(x) = 9x^2 - 6x + 3$

$a = 9, b = -6, c = 3$. Since $a = 9 > 0$, the graph opens up. The x-coordinate of the vertex is $x = -\frac{b}{2a} = -\frac{-6}{2(9)} = \frac{6}{18} = \frac{1}{3}$. The y-coordinate of the vertex is

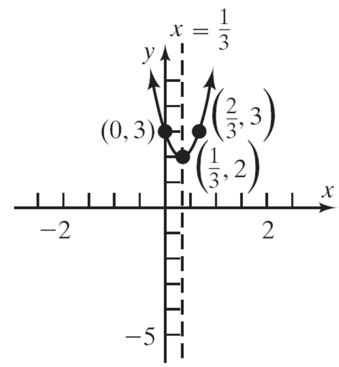
$$f\left(-\frac{b}{2a}\right) = f\left(\frac{1}{3}\right) = 9\left(\frac{1}{3}\right)^2 - 6\left(\frac{1}{3}\right) + 3$$
$$= 1 - 2 + 3 = 2$$

Thus, the vertex is $\left(\frac{1}{3}, 2\right)$.

The axis of symmetry is the line $x = \frac{1}{3}$.

The discriminant is:
$b^2 - 4ac = (-6)^2 - 4(9)(3) = -72 < 0$, so the graph has no x-intercepts.

The y-intercept is
$f(0) = 9(0)^2 - 6(0) + 3 = 3$.



b. The domain is $(-\infty, \infty)$.
The range is $[2, \infty)$.

c. Decreasing on $\left(-\infty, \frac{1}{3}\right)$.

Increasing on $\left(\frac{1}{3}, \infty\right)$.

13. a. $f(x) = -x^2 + x + \frac{1}{2}$

$a = -1, b = 1, c = \frac{1}{2}$. Since $a = -1 < 0$, the graph opens down. The x-coordinate of the vertex is $x = -\frac{b}{2a} = -\frac{1}{2(-1)} = -\frac{1}{-2} = \frac{1}{2}$.

The y-coordinate of the vertex is

$$f\left(-\frac{b}{2a}\right) = f\left(\frac{1}{2}\right) = -\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right) + \frac{1}{2}$$
$$= -\frac{1}{4} + 1 = \frac{3}{4}$$

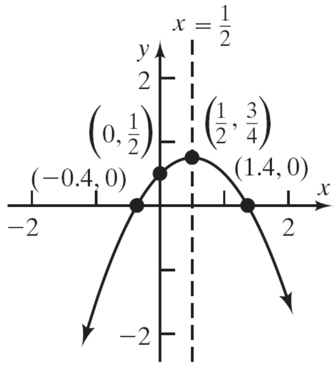
Thus, the vertex is $\left(\frac{1}{2}, \frac{3}{4}\right)$. The axis of symmetry is the line $x = \frac{1}{2}$. The discriminant is: $b^2 - 4ac = 1^2 - 4(-1)\left(\frac{1}{2}\right) = 3 > 0$, so the graph has two x-intercepts. The x-intercepts are found by solving: $-x^2 + x + \frac{1}{2} = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{3}}{2(-1)} = \frac{-1 \pm \sqrt{3}}{-2} = \frac{1 \pm \sqrt{3}}{2}$$

The x-intercepts are $\frac{1-\sqrt{3}}{2} \approx -0.4$ and

$\frac{1+\sqrt{3}}{2} \approx 1.4$.

The y-intercept is $f(0) = -(0)^2 + (0) + \frac{1}{2} = \frac{1}{2}$.



b. The domain is $(-\infty, \infty)$.

The range is $\left(-\infty, \frac{3}{4}\right]$.

c. Increasing on $\left(-\infty, \frac{1}{2}\right)$.

Decreasing on $\left(\frac{1}{2}, \infty\right)$.

14. a. $f(x) = 3x^2 + 4x - 1$

$a = 3, b = 4, c = -1$. Since $a = 3 > 0$, the graph opens up. The x-coordinate of the vertex is $x = -\frac{b}{2a} = -\frac{4}{2(3)} = -\frac{4}{6} = -\frac{2}{3}$.

The y-coordinate of the vertex is

$$f\left(-\frac{b}{2a}\right) = f\left(-\frac{2}{3}\right) = 3\left(-\frac{2}{3}\right)^2 + 4\left(-\frac{2}{3}\right) - 1$$
$$= \frac{4}{3} - \frac{8}{3} - 1 = -\frac{7}{3}$$

Thus, the vertex is $\left(-\frac{2}{3}, -\frac{7}{3}\right)$.

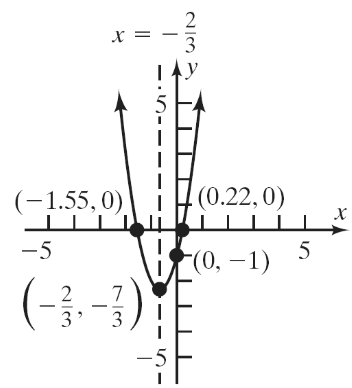
The axis of symmetry is the line $x = -\frac{2}{3}$.

The discriminant is:
$b^2 - 4ac = (4)^2 - 4(3)(-1) = 28 > 0$, so the graph has two x-intercepts.
The x-intercepts are found by solving:
$3x^2 + 4x - 1 = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{28}}{2(3)}$$
$$= \frac{-4 \pm 2\sqrt{7}}{6} = \frac{-2 \pm \sqrt{7}}{3}$$

The x-intercepts are $\frac{-2-\sqrt{7}}{3} \approx -1.55$ and $\frac{-2+\sqrt{7}}{3} \approx 0.22$.

The y-intercept is $f(0) = 3(0)^2 + 4(0) - 1 = -1$.



b. The domain is $(-\infty, \infty)$.

The range is $\left[-\frac{7}{3}, \infty\right)$.

c. Decreasing on $\left(-\infty, -\frac{2}{3}\right)$

Increasing on $\left(-\frac{2}{3}, \infty\right)$.

15. $f(x) = 3x^2 - 6x + 4$

$a = 3, b = -6, c = 4$. Since $a = 3 > 0$, the graph opens up, so the vertex is a minimum point.
The minimum occurs at
$x = -\frac{b}{2a} = -\frac{-6}{2(3)} = \frac{6}{6} = 1$.
The minimum value is

$$f\left(-\frac{b}{2a}\right) = f(1) = 3(1)^2 - 6(1) + 4$$
$$= 3 - 6 + 4 = 1$$

16. $f(x) = -x^2 + 8x - 4$

$a = -1, b = 8, c = -4$. Since $a = -1 < 0$, the graph opens down, so the vertex is a maximum point. The maximum occurs at

$x = -\frac{b}{2a} = -\frac{8}{2(-1)} = -\frac{8}{-2} = 4$.

The maximum value is

$f\left(-\frac{b}{2a}\right) = f(4) = -(4)^2 + 8(4) - 4$

$= -16 + 32 - 4 = 12$

17. $f(x) = -2x^2 + 4$

$a = -2, b = 0, c = 4$. Since $a = -2 < 0$, the graph opens down, so the vertex is a maximum point. The maximum occurs at

$x = -\frac{b}{2a} = -\frac{0}{2(-2)} = 0$.

The maximum value is

$f\left(-\frac{b}{2a}\right) = f(0) = -2(0)^2 + 4 = 4$.

18. $x^2 + 6x - 16 < 0$

We graph the function $f(x) = x^2 + 6x - 16$. The intercepts are

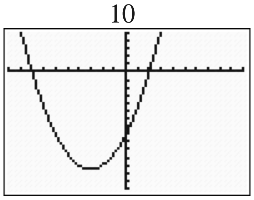
y-intercept: $f(0) = -16$

x-intercepts: $x^2 + 6x - 16 = 0$

$(x+8)(x-2) = 0$

$x = -8, x = 2$

The vertex is at $x = \frac{-b}{2a} = \frac{-(6)}{2(1)} = -3$. Since $f(-3) = -25$, the vertex is $(-3, -25)$.



The graph is below the x-axis when $-8 < x < 2$. Since the inequality is strict, the solution set is $\{x \mid -8 < x < 2\}$ or, using interval notation, $(-8, 2)$.

19. $3x^2 \geq 14x + 5$

$3x^2 - 14x - 5 \geq 0$

We graph the function $f(x) = 3x^2 - 14x - 5$. The intercepts are

y-intercept: $f(0) = -5$

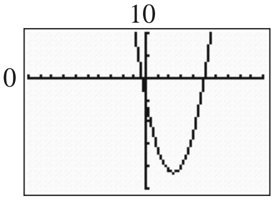
x-intercepts: $3x^2 - 14x - 5 = 0$

$(3x+1)(x-5) = 0$

$x = -\frac{1}{3}, x = 5$

The vertex is at $x = \frac{-b}{2a} = \frac{-(-14)}{2(3)} = \frac{14}{6} = \frac{7}{3}$.

Since $f\left(\frac{7}{3}\right) = -\frac{64}{3}$, the vertex is $\left(\frac{7}{3}, -\frac{64}{3}\right)$.



The graph is above the x-axis when $x < -\frac{1}{3}$ or $x > 5$. Since the inequality is not strict, the solution set is $\left\{x \,\middle|\, x \leq -\frac{1}{3} \text{ or } x \geq 5\right\}$ or, using interval notation, $\left(-\infty, -\frac{1}{3}\right] \cup [5, \infty)$.

20. Use the form $f(x) = a(x-h)^2 + k$.

The vertex is $(-1, 2)$, so $h = -1$ and $k = 2$.

$f(x) = a(x+1)^2 + 2$.

Since the graph passes through $(1, 6)$, $f(1) = 6$.

$6 = a(1+1)^2 + 2$

$6 = a(2)^2 + 2$

$6 = 4a + 2$

$4 = 4a$

$1 = a$

$f(x) = 1(x+1)^2 + 2 = (x^2 + 2x + 1) + 2$

$= x^2 + 2x + 3$

21. Consider the form $y = ax^2 + bx + c$. Substituting the three points from the graph into the general form we have the following three equations.

$2 = a(1)^2 + b(1) + c \Rightarrow 2 = a + b + c$

and

$2 = a(3)^2 + b(3) + c \Rightarrow 2 = 9a + 3b + c$

and

$5 = a(0)^2 + b(0) + c \Rightarrow 5 = c$

Since $5 = c$, we have the following equations:

$2 = a + b + 5,\ \ 2 = 9a + 3b + 5,\ \ 5 = c$

Solving the first two simultaneously we have

$$\left.\begin{aligned} 2 &= a + b + 5 \\ 2 &= 9a + 3b + 5 \end{aligned}\right)$$

$$\left.\begin{aligned} -3 &= a + b \\ -3 &= 9a + 3b \end{aligned}\right) \rightarrow\ a = 1, b = -4$$

The quadratic function is $f(x) = x^2 - 4x + 5$.

22. **a.** Company A: $C(x) = 0.06x + 7.00$

Company B: $C(x) = 0.08x$

b. $0.06x + 7.00 = 0.08x$

$7.00 = 0.02x$

$350 = x$

The bill from Company A will equal the bill from Company B if 350 minutes are used.

c. $0.08x < 0.06x + 7.00$

$0.02x < 7.00$

$x < 350$

The bill from Company B will be less than the bill from Company A if fewer than 350 minutes are used. That is, $0 \le x < 350$.

23. **a.** The revenue will equal the quantity x sold times the price p. That is, $R = xp$. Thus,

$$R(x) = x\left(-\frac{1}{10}x + 150\right) = -\frac{1}{10}x^2 + 150x$$

b. $R(100) = -\frac{1}{10}(100)^2 + 150(100) = 14{,}000$

The revenue is \$14,000 if 100 units are sold.

c. $a = -\frac{1}{10}, b = 150, c = 0.$ Since $a = -\frac{1}{10} < 0,$ the graph opens down, so the vertex is a maximum point. The maximum occurs at $x = \frac{-b}{2a} = \frac{-(150)}{2(-1/10)} = \frac{-150}{-1/5} = 750$. Thus, the quantity that maximizes revenue is 750 units. The maximum revenue is

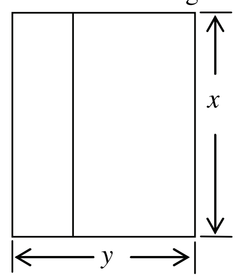
$$\begin{aligned} R(750) &= -\frac{1}{10}(750)^2 + 150(750) \\ &= -56{,}250 + 112{,}500 \\ &= \$56{,}250 \end{aligned}$$

d. From part (c), we know revenue is maximizes when $x = 750$ units are sold.

The price that should be charged for this is

$$p = -\frac{1}{10}(750) + 150 = \$75.$$

24. Consider the diagram



Total amount of fence = $3x + 2y = 10{,}000$

$$y = \frac{10{,}000 - 3x}{2} = 5000 - \frac{3}{2}x$$

Total area enclosed = $(x)(y) = (x)\left(5000 - \frac{3}{2}x\right)$

$A(x) = 5000x - \frac{3}{2}x^2 = -\frac{3}{2}x^2 + 5000x$ is a quadratic function with $a = -\frac{3}{2} < 0$.

So the vertex corresponds to the maximum value for this function. The vertex occurs when

$$x = -\frac{b}{2a} = -\frac{5000}{2(-3/2)} = \frac{5000}{3}.$$

The maximum area is:

$$\begin{aligned} A\left(\frac{5000}{3}\right) &= -\frac{3}{2}\left(\frac{5000}{3}\right)^2 + 5000\left(\frac{5000}{3}\right) \\ &= -\frac{3}{2}\left(\frac{25{,}000{,}000}{9}\right) + \frac{25{,}000{,}000}{3} \\ &= -\frac{12{,}500{,}000}{3} + \frac{25{,}000{,}000}{3} \\ &= \frac{12{,}500{,}000}{3} \\ &\approx 4{,}166{,}666.67 \text{ square meters} \end{aligned}$$

25. $C(x) = 4.9x^2 - 617.4x + 19{,}600$;

$a = 4.9, b = -617.4, c = 19{,}600.$ Since $a = 4.9 > 0,$ the graph opens up, so the vertex is a minimum point.

a. The minimum marginal cost occurs at

$$x = -\frac{b}{2a} = -\frac{-617.40}{2(4.9)} = \frac{617.40}{9.8} = 63.$$

Thus, 63 golf clubs should be manufactured in order to minimize the marginal cost.

b. The minimum marginal cost is

$$C(63) = 4.9(63)^2 - (617.40)(63) + 19600$$
$$= \$151.90$$

26. The area function is:

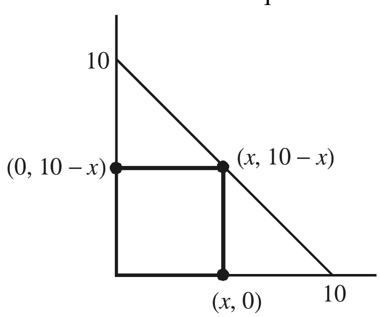
$$A(x) = x(10 - x) = -x^2 + 10x$$

The maximum value occurs at the vertex:

$$x = -\frac{b}{2a} = -\frac{10}{2(-1)} = -\frac{10}{-2} = 5$$

The maximum area is:

$$A(5) = -(5)^2 + 10(5)$$
$$= -25 + 50 = 25 \text{ square units}$$



27. Locate the origin at the point directly under the highest point of the arch. Then the equation is in the form: $y = -ax^2 + k$, where $a > 0$. Since the maximum height is 10 feet, when $x = 0$, $y = k = 10$. Since the point (10, 0) is on the parabola, we can find the constant:

$$0 = -a(10)^2 + 10$$
$$a = \frac{10}{10^2} = \frac{1}{10} = 0.10$$

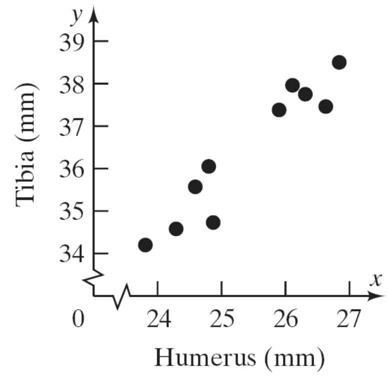
The equation of the parabola is:

$$y = -\frac{1}{10}x^2 + 10$$

At $x = 8$:

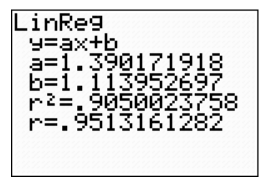
$$y = -\frac{1}{10}(8)^2 + 10 = -6.4 + 10 = 3.6 \text{ feet}$$

28. a.



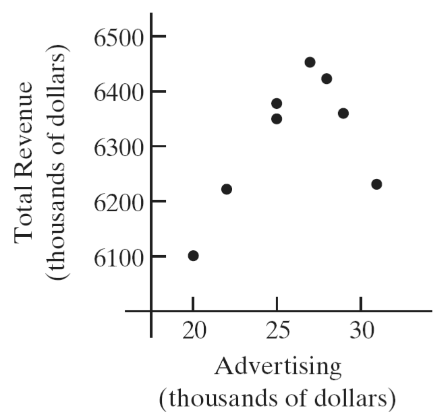
b. Yes, the two variables appear to have a linear relationship.

c. Using the LINear REGression program, the line of best fit is: $y = 1.3902x + 1.1140$



d. $y = 1.39017(26.5) + 1.11395 \approx 37.95$ mm

29. a.



The data appear to be quadratic with $a < 0$.

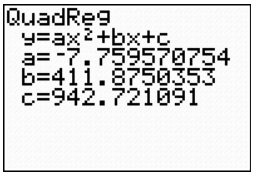
b. The maximum revenue occurs at

$$A = \frac{-b}{2a} = \frac{-(411.88)}{2(-7.76)}$$
$$= \frac{-411.88}{-15.52} \approx \$26.5 \text{ thousand}$$

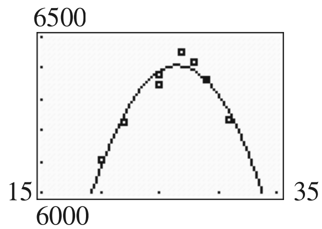
c. The maximum revenue is

$$R\left(\frac{-b}{2a}\right) = R(26.53866)$$
$$= -7.76(26.5)^2 + (411.88)(26.5) + 942.72$$
$$\approx \$6408 \text{ thousand}$$

d. Using the QUADratic REGression program, the quadratic function of best fit is:
$y = -7.76x^2 + 411.88x + 942.72$.



e.



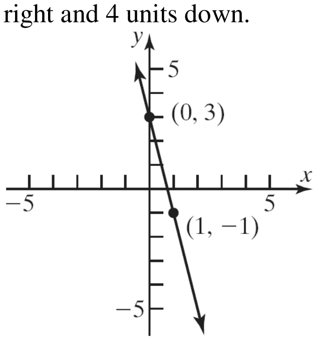
Chapter 3 Test

1. $f(x) = -4x + 3$

a. Slope = -4 ; y-intercept = 3.

b. The slope is negative, so the graph is decreasing.

c. Plot the point $(0, 3)$. Use the slope to find an additional point by moving 1 unit to the right and 4 units down.



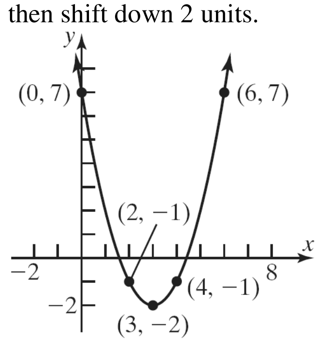
2.

x	y	Avg. rate of change $= \dfrac{\Delta y}{\Delta x}$
-2	12	
-1	7	$\dfrac{7-12}{-1-(-2)} = \dfrac{-5}{1} = -5$
0	2	$\dfrac{2-7}{0-(-1)} = \dfrac{-5}{1} = -5$
1	-3	$\dfrac{-3-2}{1-0} = \dfrac{-5}{1} = -5$
2	-8	$\dfrac{-8-(-3)}{2-1} = \dfrac{-5}{1} = -5$

Since the average rate of change is constant at -5 , this is a linear function with slope = -5 . The y-intercept is $(0, 2)$, so the equation of the line is $y = -5x + 2$.

3. $f(x) = (x-3)^2 - 2$

Using the graph of $y = x^2$, shift right 3 units, then shift down 2 units.



4. a. $f(x) = 3x^2 - 12x + 4$

$a = 3, b = -12, c = 4.$ Since $a = 3 > 0,$ the graph opens up.

b. The x-coordinate of the vertex is

$$x = -\frac{b}{2a} = -\frac{-12}{2(3)} = -\frac{-12}{6} = 2 .$$

The y-coordinate of the vertex is

$$f\left(-\frac{b}{2a}\right) = f(2) = 3(2)^2 - 12(2) + 4$$
$$= 12 - 24 + 4 = -8$$

Thus, the vertex is $(2, -8)$.

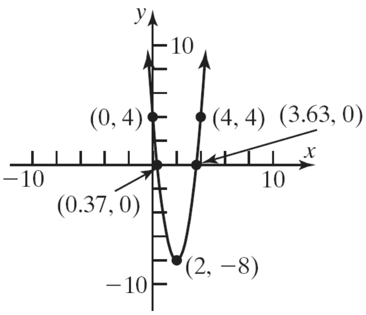
c. The axis of symmetry is the line $x = 2$.

d. The discriminant is:

$b^2 - 4ac = (-12)^2 - 4(3)(4) = 96 > 0$, so the graph has two x-intercepts. The x-intercepts are found by solving: $3x^2 - 12x + 4 = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-12) \pm \sqrt{96}}{2(3)}$$
$$= \frac{12 \pm 4\sqrt{6}}{6} = \frac{6 \pm 2\sqrt{6}}{3}$$

The x-intercepts are $\frac{6 - 2\sqrt{6}}{3} \approx 0.37$ and $\frac{6 \pm 2\sqrt{6}}{3} \approx 3.63$. The y-intercept is $f(0) = 3(0)^2 - 12(0) + 4 = 4$.

e. 

f. The domain is $(-\infty, \infty)$.
The range is $[-8, \infty)$.

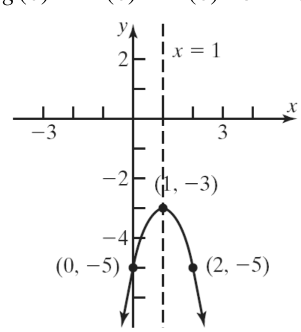
g. Decreasing on $(-\infty, 2)$.
Increasing on $(2, \infty)$.

5. a. $g(x) = -2x^2 + 4x - 5$
$a = -2, b = 4, c = -5.$ Since $a = -2 < 0,$ the graph opens down.

b. The x-coordinate of the vertex is
$x = -\frac{b}{2a} = -\frac{4}{2(-2)} = -\frac{4}{-4} = 1.$
The y-coordinate of the vertex is
$$g\left(-\frac{b}{2a}\right) = g(1) = -2(1)^2 + 4(1) - 5$$
$$= -2 + 4 - 5 = -3$$
Thus, the vertex is $(1, -3)$.

c. The axis of symmetry is the line $x = 1$.

d. The discriminant is:
$b^2 - 4ac = (4)^2 - 4(-2)(-5) = -24 < 0$, so the graph has no x-intercepts. The y-intercept is $g(0) = -2(0)^2 + 4(0) - 5 = -5$.

e. 

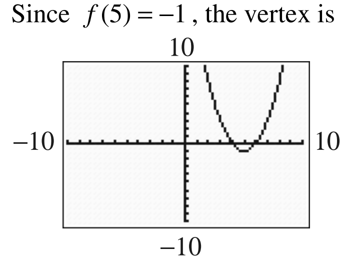
f. The domain is $(-\infty, \infty)$.
The range is $(-\infty, -3]$.

g. Increasing on $(-\infty, 1)$.
Decreasing on $(1, \infty)$.

6. Consider the form $y = a(x-h)^2 + k$. From the graph we know that the vertex is $(1, -32)$ so we have $h = 1$ and $k = -32$. The graph also passes through the point $(x, y) = (0, -30)$. Substituting these values for *x, y, h,* and *k*, we can solve for *a*:
$$-30 = a(0-1)^2 + (-32)$$
$$-30 = a(-1)^2 - 32$$
$$-30 = a - 32$$
$$2 = a$$
The quadratic function is
$f(x) = 2(x-1)^2 - 32 = 2x^2 - 4x - 30$.

7. $f(x) = -2x^2 + 12x + 3$
$a = -2, b = 12, c = 3.$ Since $a = -2 < 0,$ the graph opens down, so the vertex is a maximum point. The maximum occurs at
$x = -\frac{b}{2a} = -\frac{12}{2(-2)} = -\frac{12}{-4} = 3$.
The maximum value is
$f(3) = -2(3)^2 + 12(3) + 3 = -18 + 36 + 3 = 21$.

8. $x^2 - 10x + 24 \ge 0$
We graph the function $f(x) = x^2 - 10x + 24$.
The intercepts are
y-intercept: $f(0) = 24$
x-intercepts: $x^2 - 10x + 24 = 0$
$$(x-4)(x-6) = 0$$
$$x = 4, x = 6$$
The vertex is at $x = \frac{-b}{2a} = \frac{-(-10)}{2(1)} = \frac{10}{2} = 5$.
Since $f(5) = -1$, the vertex is $(5, -1)$.



The graph is above the x-axis when $x < 4$ or $x > 6$. Since the inequality is not strict, the solution set is $\{x \mid x \le 4 \text{ or } x \ge 6\}$ or, using interval notation, $(-\infty, 4] \cup [6, \infty)$.

9. a. $C(m) = 0.15m + 129.50$

b. $C(860) = 0.15(860) + 129.50$
$= 129 + 129.50 = 258.50$

If 860 miles are driven, the rental cost is \$258.50.

c. $C(m) = 213.80$
$0.15m + 129.50 = 213.80$
$0.15m = 84.30$
$m = 562$

The rental cost is \$213.80 if 562 miles were driven.

10. a. $R(x) = x\left(-\frac{1}{10}x + 1000\right) = -\frac{1}{10}x^2 + 1000x$

b. $R(400) = -\frac{1}{10}(400)^2 + 1000(400)$
$= -16,000 + 400,000$
$= \$384,000$

c. $x = \frac{-b}{2a} = \frac{-1000}{2\left(-\frac{1}{10}\right)} = \frac{-1000}{\left(-\frac{1}{5}\right)} = 5000$

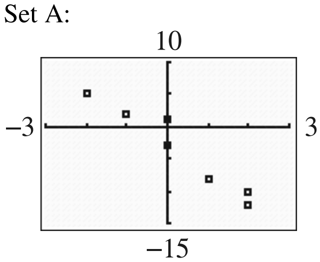
The maximum revenue is

$R(5000) = -\frac{1}{10}(5000)^2 + 1000(5000)$
$= -250,000 + 5,000,000$
$= \$2,500,000$

Thus, 5000 units maximizes revenue at \$2,500,000.

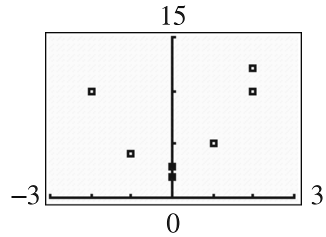
d. $p = -\frac{1}{10}(5000) + 1000$
$= -500 + 1000$
$= \$500$

11. a. Set A:



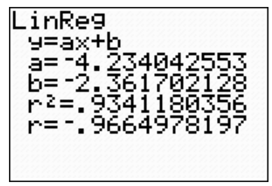
The data appear to be linear with a negative slope.

Set B:

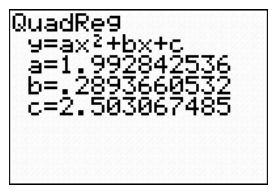


The data appear to be quadratic and opens up.

b. Using the LINear REGression program, the linear function of best fit is:
$y = -4.234x - 2.362$.



c. Using the QUADratic REGression program, the quadratic function of best fit is:
$y = 1.993x^2 + 0.289x + 2.503$.



Chapter 3 Cumulative Review

1. $P = (-1, 3); Q = (4, -2)$

Distance between *P* and *Q*:

$d(P,Q) = \sqrt{(4-(-1))^2 + (-2-3)^2}$
$= \sqrt{(5)^2 + (-5)^2}$
$= \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}$

Midpoint between *P* and *Q*:

$\left(\frac{-1+4}{2}, \frac{3-2}{2}\right) = \left(\frac{3}{2}, \frac{1}{2}\right) = (1.5, 0.5)$

2. $y = x^3 - 3x + 1$

a. $(-2,-1)$: $-1 = (-2)^3 - 3(-2) + 1$
$-1 = -8 + 6 + 1$
$-1 = -1$

Yes, $(-2,-1)$ is on the graph.

b. $(2,3)$: $3=(2)^3-3(2)+1$

$3=8-6+1$

$3=3$

Yes, $(2,3)$ is on the graph.

c. $(3,1)$: $1=(3)^3-3(3)+1$

$1=-27-9+1$

$1\neq -35$

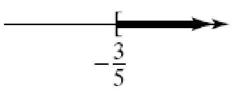
No, $(3,1)$ is not on the graph.

3. $5x+3\geq 0$

$5x\geq -3$

$x\geq -\frac{3}{5}$

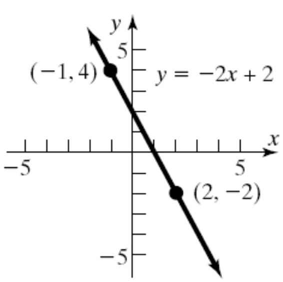
The solution set is $\left\{x \middle| x\geq -\frac{3}{5}\right\}$ or $\left[-\frac{3}{5},\infty\right)$.



4. (–1,4) and (2,–2) are points on the line.

$$\text{Slope}=\frac{-2-4}{2-(-1)}=\frac{-6}{3}=-2$$

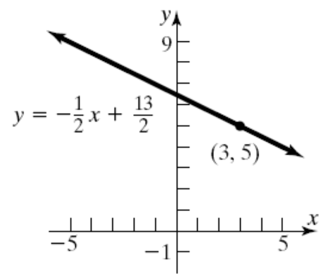
$$y-y_1=m(x-x_1)$$
$$y-4=-2(x-(-1))$$
$$y-4=-2(x+1)$$
$$y-4=-2x-2$$
$$y=-2x+2$$



5. Perpendicular to $y=2x+1$;
Containing (3,5)

Slope of perpendicular = $-\frac{1}{2}$

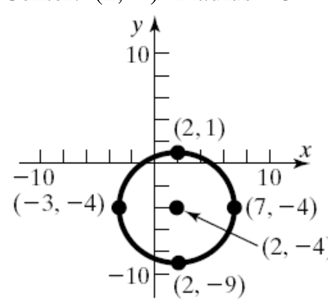
$$y-y_1=m(x-x_1)$$
$$y-5=-\frac{1}{2}(x-3)$$
$$y-5=-\frac{1}{2}x+\frac{3}{2}$$
$$y=-\frac{1}{2}x+\frac{13}{2}$$



6. $x^2+y^2-4x+8y-5=0$

$$x^2-4x+y^2+8y=5$$
$$(x^2-4x+4)+(y^2+8y+16)=5+4+16$$
$$(x-2)^2+(y+4)^2=25$$
$$(x-2)^2+(y+4)^2=5^2$$

Center: (2,–4) Radius = 5



7. Yes, this is a function since each x-value is paired with exactly one y-value.

8. $f(x)=x^2-4x+1$

a. $f(2)=2^2-4(2)+1=4-8+1=-3$

b. $f(x)+f(2)=x^2-4x+1+(-3)$
$=x^2-4x-2$

c. $f(-x)=(-x)^2-4(-x)+1=x^2+4x+1$

d. $-f(x)=-(x^2-4x+1)=-x^2+4x-1$

e. $f(x+2)=(x+2)^2-4(x+2)+1$

$=x^2+4x+4-4x-8+1$

$=x^2-3$

f. $\dfrac{f(x+h)-f(x)}{h}$

$=\dfrac{(x+h)^2-4(x+h)+1-(x^2-4x+1)}{h}$

$=\dfrac{x^2+2xh+h^2-4x-4h+1-x^2+4x-1}{h}$

$=\dfrac{2xh+h^2-4h}{h}$

$=\dfrac{h(2x+h-4)}{h}=2x+h-4$

9. $h(z)=\dfrac{3z-1}{6z-7}$

The denominator cannot be zero:

$6z-7\neq 0$

$6z\neq 7$

$z\neq \dfrac{7}{6}$

Domain: $\left\{z \,\middle|\, z\neq \dfrac{7}{6}\right\}$

10. Yes, the graph represents a function since it passes the Vertical Line Test.

11. $f(x)=\dfrac{x}{x+4}$

a. $f(1)=\dfrac{1}{1+4}=\dfrac{1}{5}\neq\dfrac{1}{4}$, so $\left(1,\dfrac{1}{4}\right)$ is not on the graph of *f*.

b. $f(-2)=\dfrac{-2}{-2+4}=\dfrac{-2}{2}=-1$, so $(-2,-1)$ is a point on the graph of *f*.

c. Solve for *x*:

$2=\dfrac{x}{x+4}$

$2x+8=x$

$x=-8$

So, $(-8, 2)$ is a point on the graph of *f*.

12. $f(x)=\dfrac{x^2}{2x+1}$

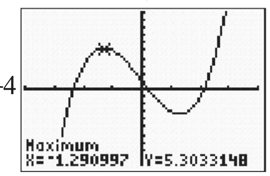
$f(-x)=\dfrac{(-x)^2}{2(-x)+1}=\dfrac{x^2}{-2x+1}\neq f(x)$ or $-f(x)$

Therefore, *f* is neither even nor odd.

13. $f(x)=x^3-5x+1$ on the interval $(-4,4)$

Use MAXIMUM and MINIMUM on the graph of $y_1=x^3-5x+4$.

10

−4  4

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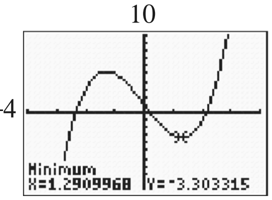
10

−4  4

−10

Local maximum is 5.30 and occurs at $x\approx -1.29$;
Local minimum is –3.30 and occurs at $x\approx 1.29$;
f is increasing on $(-4,-1.29)$ or $(1.29,4)$;
f is decreasing on $(-1.29,1.29)$.

14. $f(x)=3x+5;\quad g(x)=2x+1$

a. $f(x)=g(x)$

$3x+5=2x+1$

$3x+5=2x+1$

$x=-4$

b. $f(x)>g(x)$

$3x+5>2x+1$

$3x+5>2x+1$

$x>-4$

The solution set is $\{x|x>-4\}$ or $(-4,\infty)$.

15. a. Domain: $\{x\,|\,-4\leq x\leq 4\}$ or $[-4, 4]$

Range: $\{y\,|\,-1\leq y\leq 3\}$ or $[-1, 3]$

b. Intercepts: $(-1,0)$, $(0,-1)$, $(1,0)$

x-intercepts: $-1, 1$

y-intercept: -1

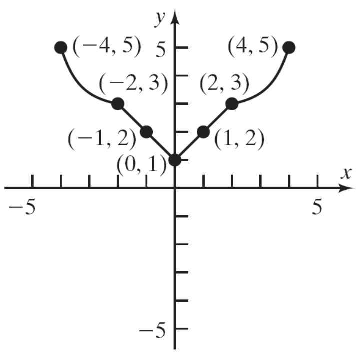
c. The graph is symmetric with respect to the y-axis.

d. When $x = 2$, the function takes on a value of 1. Therefore, $f(2) = 1$.

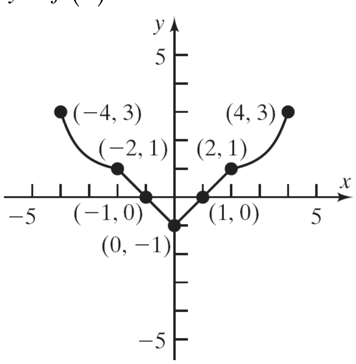
e. The function takes on the value 3 at $x = -4$ and $x = 4$.

f. $f(x) < 0$ means that the graph lies below the x-axis. This happens for x values between -1 and 1. Thus, the solution set is $\{x \mid -1 < x < 1\}$ or $(-1, 1)$.

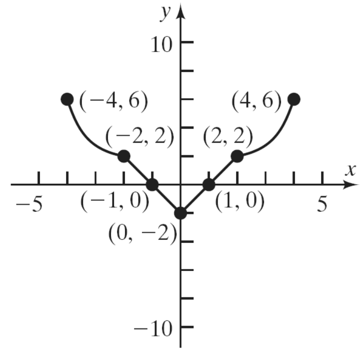
g. The graph of $y = f(x) + 2$ is the graph of $y = f(x)$ but shifted up 2 units.



h. The graph of $y = f(-x)$ is the graph of $y = f(x)$ but reflected about the y-axis.



i. The graph of $y = 2f(x)$ is the graph of $y = f(x)$ but stretched vertically by a factor of 2. That is, the coordinate of each point is multiplied by 2.



j. Since the graph is symmetric about the y-axis, the function is even.

k. The function is increasing on the open interval $(0, 4)$.

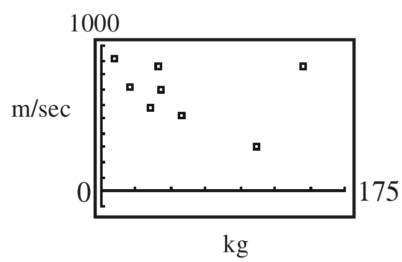
Chapter 3 Projects

Project I – Internet-based Project

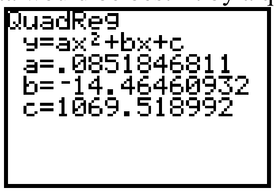
Answers will vary.

Project II

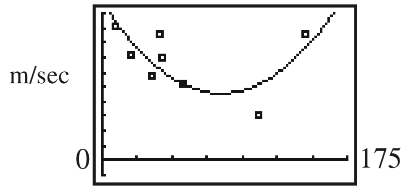
a.



b. The data would be best fit by a quadratic function.



$y = 0.085x^2 - 14.46x + 1069.52$



These results seem reasonable since the function fits the data well.

c. $s_0 = 0\text{m}$

Type	Weight kg	Velocity m/sec	Equation in the form: $s(t) = -4.9t^2 + \frac{\sqrt{2}}{2} v_0 t + s_0$
MG 17	10.2	905	$s(t) = -4.9t^2 + 639.93t$ Best. (It goes the highest)
MG 131	19.7	710	$s(t) = -4.9t^2 + 502.05t$
MG 151	41.5	850	$s(t) = -4.9t^2 + 601.04t$
MG 151/20	42.3	695	$s(t) = -4.9t^2 + 491.44t$
MG/FF	35.7	575	$s(t) = -4.9t^2 + 406.59t$
MK 103	145	860	$s(t) = -4.9t^2 + 608.11t$
MK 108	58	520	$s(t) = -4.9t^2 + 367.70t$
WGr 21	111	315	$s(t) = -4.9t^2 + 222.74t$

$s_0 = 200\text{m}$

Type	Weight kg	Velocity m/sec	Equation in the form: $s(t) = -4.9t^2 + \frac{\sqrt{2}}{2} v_0 t + s_0$
MG 17	10.2	905	$s(t) = -4.9t^2 + 639.93t + 200$ Best. (It goes the highest)
MG 131	19.7	710	$s(t) = -4.9t^2 + 502.05t + 200$
MG 151	41.5	850	$s(t) = -4.9t^2 + 601.04t + 200$
MG 151/20	42.3	695	$s(t) = -4.9t^2 + 491.44t + 200$
MG/FF	35.7	575	$s(t) = -4.9t^2 + 406.59t + 200$
MK 103	145	860	$s(t) = -4.9t^2 + 608.11t + 200$
MK 108	58	520	$s(t) = -4.9t^2 + 367.70t + 200$
WGr 21	111	315	$s(t) = -4.9t^2 + 222.74t + 200$

$s_0 = 30\text{m}$

Type	Weight kg	Velocity m/sec	Equation in the form: $s(t) = -4.9t^2 + \frac{\sqrt{2}}{2} v_0 t + s_0$
MG 17	10.2	905	$s(t) = -4.9t^2 + 639.93t + 30$ Best. (It goes the highest)
MG 131	19.7	710	$s(t) = -4.9t^2 + 502.05t + 30$
MG 151	41.5	850	$s(t) = -4.9t^2 + 601.04t + 30$

MG 151/20	42.3	695	$s(t) = -4.9t^2 + 491.44t + 30$
MG/FF	35.7	575	$s(t) = -4.9t^2 + 406.59t + 30$
MK 103	145	860	$s(t) = -4.9t^2 + 608.11t + 30$
MK 108	58	520	$s(t) = -4.9t^2 + 367.70t + 30$
WGr 21	111	315	$s(t) = -4.9t^2 + 222.74t + 30$

Notice that the gun is what makes the difference, not how high it is mounted necessarily. The only way to change the true maximum height that the projectile can go is to change the angle at which it fires.

Project III

a.

x	1	2	3	4	5
$y = -2x + 5$	3	1	−1	−3	−5

b. $\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-3}{1} = -2$

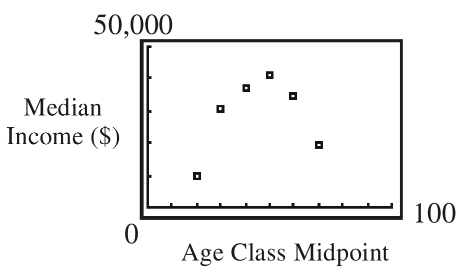
$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-1}{1} = -2$

$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-(-1)}{1} = -2$

$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5-(-3)}{1} = -2$

All of the values of $\frac{\Delta y}{\Delta x}$ are the same.

c.



d. $\frac{\Delta I}{\Delta x} = \frac{30633 - 9548}{10} = 2108.50$

$\frac{\Delta I}{\Delta x} = \frac{37088 - 30633}{10} = 645.50$

$\frac{\Delta I}{\Delta x} = \frac{41072 - 37088}{10} = 398.40$

$\frac{\Delta I}{\Delta x} = \frac{34414 - 41072}{10} = -665.80$

$\frac{\Delta I}{\Delta x} = \frac{19167 - 34414}{10} = -1524.70$

These $\frac{\Delta I}{\Delta x}$ values are not all equal. The data are not linearly related.

e.

x	−2	−1	0	1	2	3	4
y	23	9	3	5	15	33	59
$\frac{\Delta y}{\Delta x}$		−14	−6	2	10	18	26

As x increases, $\frac{\Delta y}{\Delta x}$ increases. This makes sense because the parabola is increasing (going up) steeply as x increases.

f.

x	−2	−1	0	1	2	3	4
y	23	9	3	5	15	33	59
$\frac{\Delta^2 y}{\Delta x^2}$			8	8	8	8	8

The second differences are all the same.

g. The paragraph should mention at least two observations:

1. The first differences for a linear function are all the same.
2. The second differences for a quadratic function are the same.

Project IV

a. – i. Answers will vary , depending on where the CBL is located above the bouncing ball.

j. The ratio of the heights between bounces will be the same.