Date ____________

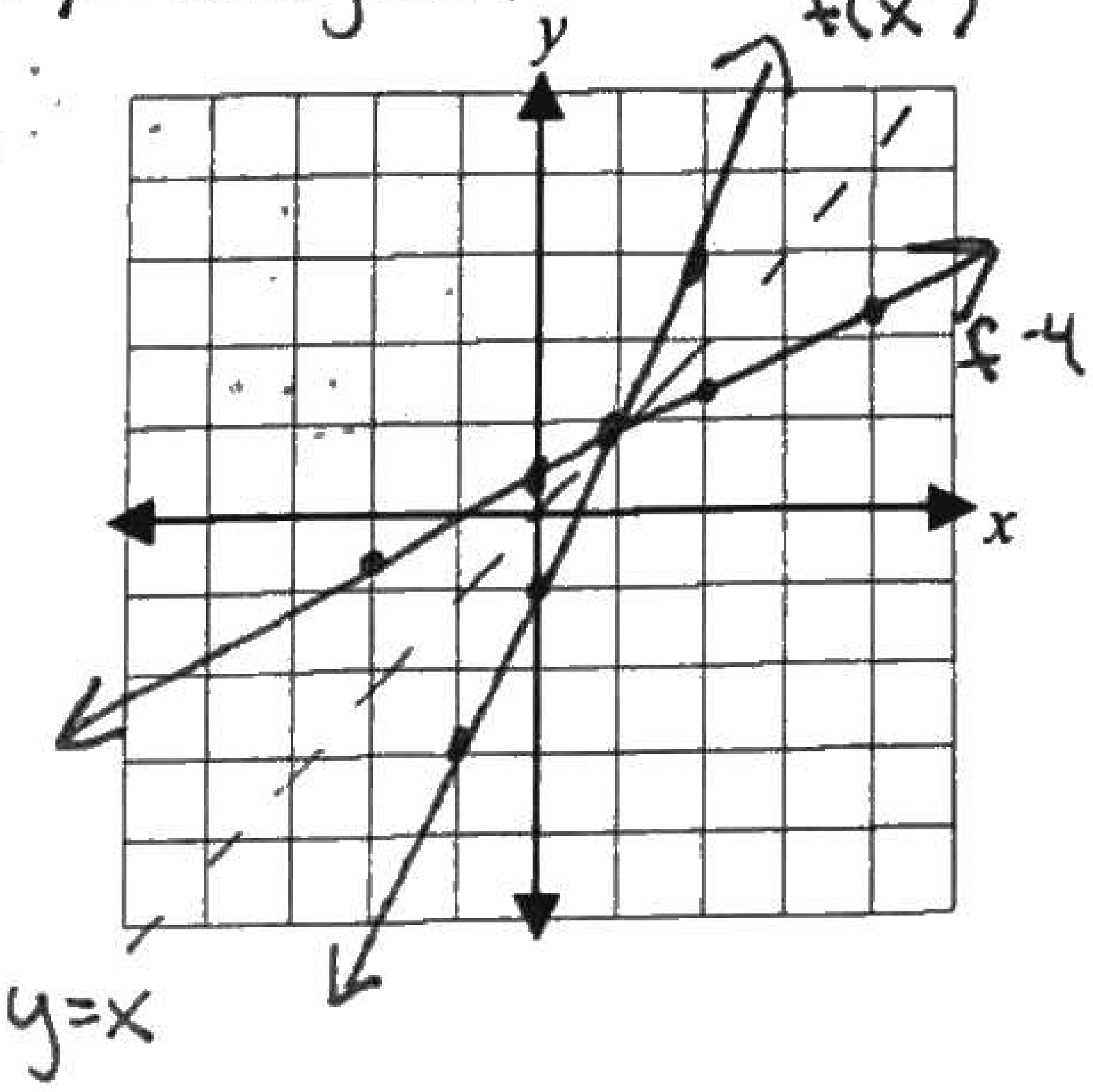
7.1 Notes Inverse Sine/Cosine/Tangent

Review: Find $f^{-1}(x)$ for $f(x)=2x-1$

$y=2x-1$ → $x+1=2y$

$x=2y-1$ → $\frac{x+1}{2}=y$

$f^{-1}(x)=\frac{1}{2}x+\frac{1}{2}$



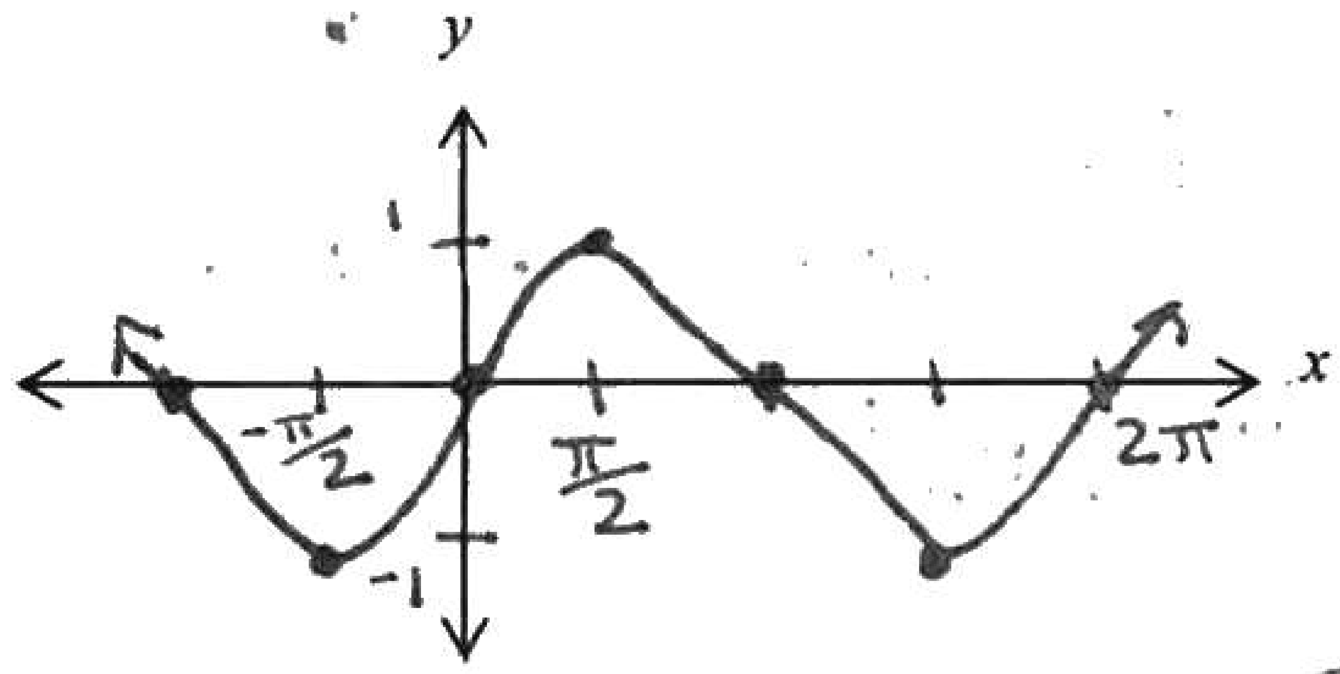
Properties of functions and their inverses.

1. Graphically: reflection thru y=x
2. Algebraically: domain & range switch
3. You can predict whether the *inverse* of a function will also be a function if original funct. passes HLT.

Application to trig functions. Will the inverse of $f(x)=\sin x$ be a function? NO Why not?

$f(x)=\sin x$ does not pass the HLT.

Graph $f(x)=\sin x$ in pencil. Then, using color, *restrict* the domain so that the restricted sine function will pass the horizontal line test.



For the *restricted sine function*:

Domain = $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

Range = $[-1,1]$

Because the *restricted sine function* is one -to- one, then its inverse will also be a function.

Use the graph of the restricted sine function to draw the inverse sine function or the arcsine function.

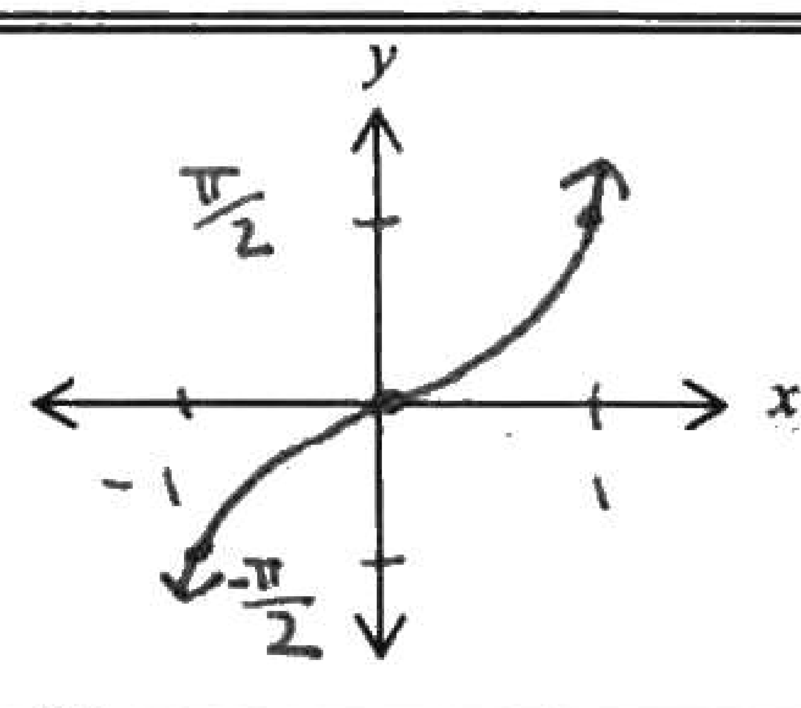
Inverse Sine Function

$g(x)=\sin^{-1}x$ or $g(x)=\arcsin x$

Domain = $[-1,1]$

Range = $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ → outputs in QI, IV only

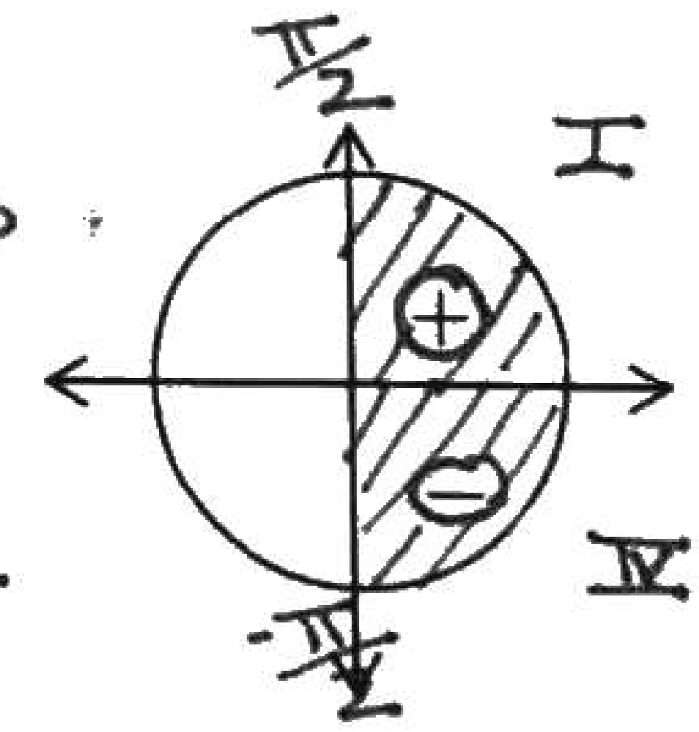
* Switch!



Reflection in the line y=x causes the concavity to switch.

[Check with your TI.]

Evaluate. [Use the unit circle to solve but restrict your answers to quadrants I and IV.]



Example: $\sin^{-1}\frac{1}{2}=\frac{\pi}{6}$. Hint: Find the angle whose sine is $\frac{1}{2}$.

a) $\sin^{-1}\left(-\frac{1}{2}\right)=-\frac{\pi}{6}$

b) $\sin^{-1}1=\frac{\pi}{2}$

c) $\arcsin\left(-\frac{\sqrt{2}}{2}\right)=-\frac{\pi}{4}$

Find an approximate value using your TI.

d) $\arcsin\frac{1}{3}=.3398$

e) $\sin^{-1}\left(-\frac{1}{4}\right)=-.2527$

f) $\sin^{-1}2=$ error not in domain!

What conclusions can you make from the problems below?

g) $\sin\left(\sin^{-1}\frac{\sqrt{2}}{2}\right) = \sin\left(\pi/4\right) = \boxed{\frac{\sqrt{2}}{2}}$

h) $\sin^{-1}\left(\sin\left(-\frac{\pi}{3}\right)\right) = \sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) = \boxed{\frac{-\pi}{3}}$

i) $\sin^{-1}\left(\sin\frac{3\pi}{4}\right) = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \boxed{\frac{\pi}{4}}$

* different b/c $\frac{3\pi}{4}$ is not in the domain of arcsin!

Conclusion – Property of Inverses!

$$f^{-1}(f(x)) = \sin^{-1}(\sin x) = x \quad \text{where D} = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] = D$$

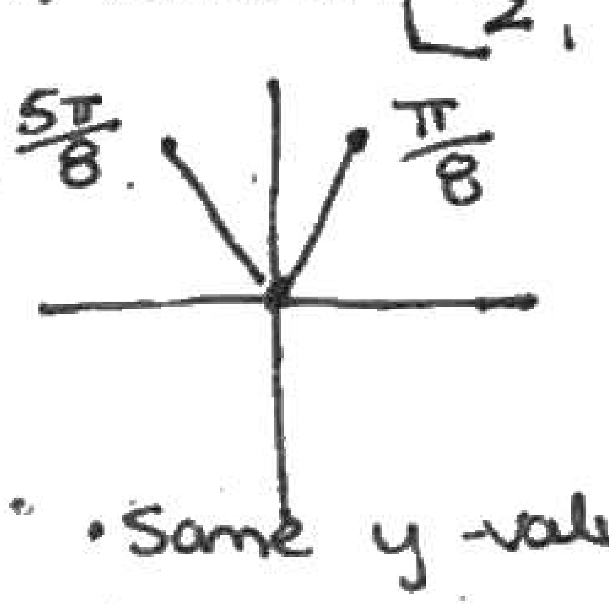
$$f(f^{-1}(x)) = \sin(\sin^{-1} x) = x \quad \text{where D} = [-1,1] = D$$

Find the exact value of each of the following composite functions. Verify with your TI. [Mode: rads]

j) $\sin^{-1}\left(\sin\frac{\pi}{8}\right) = \boxed{\frac{\pi}{8}}$

k) $\sin^{-1}\left(\sin\frac{5\pi}{8}\right) = \rightarrow$ not in domain $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$\sin^{-1}\left(\sin\frac{3\pi}{8}\right) = \boxed{\frac{3\pi}{8}}$



• Same y-values

l) $\sin\left(\sin^{-1} 0.8\right) = \boxed{.8}$

m) $\sin\left(\sin^{-1} 1.8\right) =$ error

not in $[-1,1]$!

*BTW: Does $\sin^{-1} x = \frac{1}{\sin x}$? NO Explain. inverse vs. reciprocal

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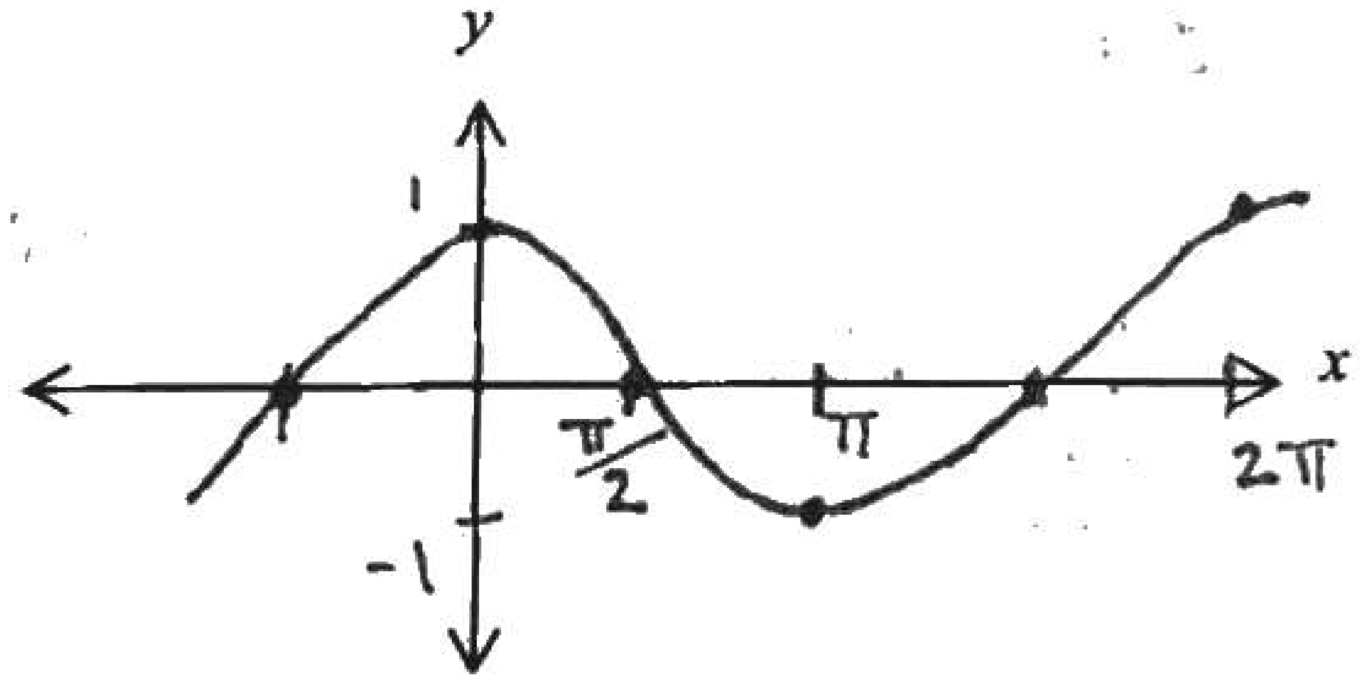
Will the inverse of $f(x) = \cos x$ be a function? NO Why not? $f(x) = \cos x$ does not pass the HLT.

Graph $f(x) = \cos x$ in pencil. Then, using color, *restrict* the domain so that the restricted cosine function will pass the horizontal line test.



For the *restricted cosine function*:

Domain = $[0, \pi]$

Range = $[-1, 1]$

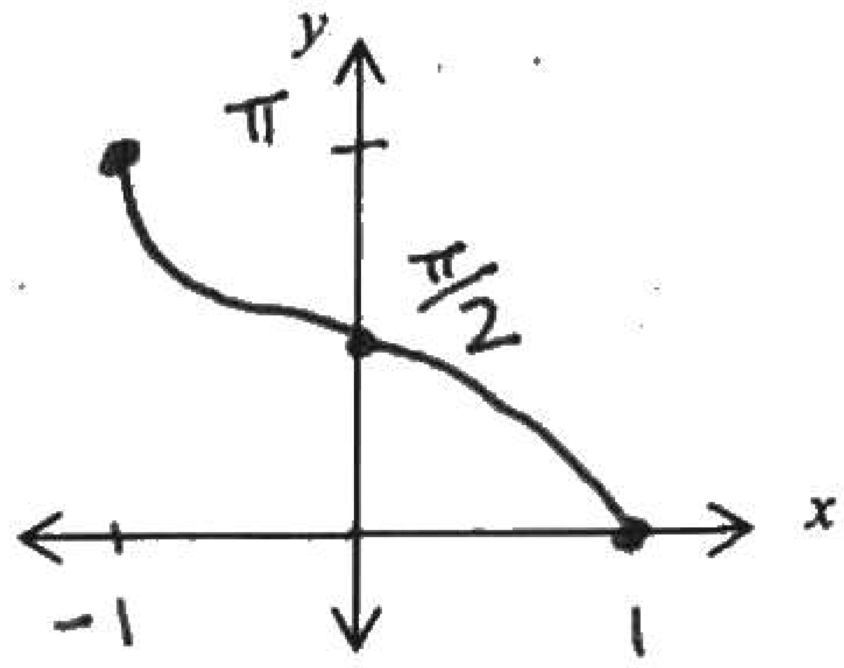
Because the *restricted cosine function* is one -to- one, then its inverse will also be a function.
Use the graph of the restricted cosine function to draw the inverse cosine function or the arccosine function.

Inverse Cosine Function

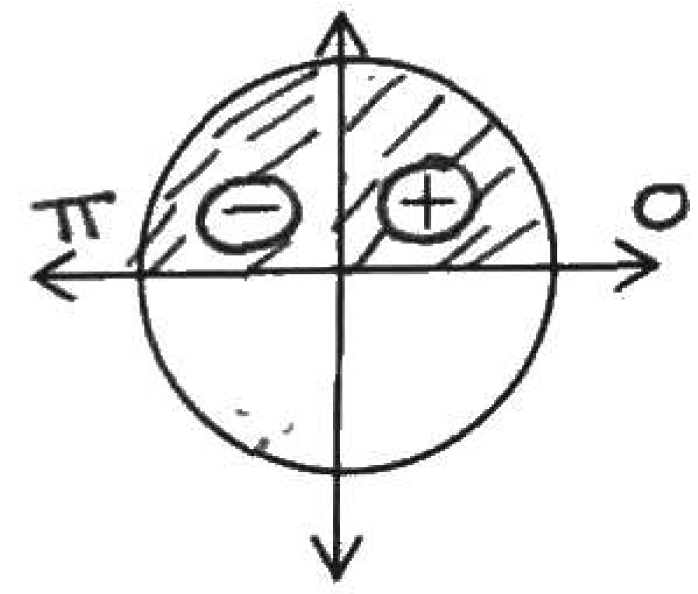
$h(x) = \cos^{-1} x$ or $h(x) = \arccos x$

Domain = $[-1, 1]$

Range = $[0, \pi]$ → outputs in QI, II only



[Check with your TI.]



Evaluate. [Hint: Use the unit circle to solve but restrict your answers to quadrants I and II].

a) $\cos^{-1}\frac{1}{2} = \boxed{\frac{\pi}{3}}$

b) $\cos^{-1}\left(-\frac{1}{2}\right) = \boxed{\frac{2\pi}{3}}$

c) $\cos^{-1}\frac{\sqrt{2}}{2} = \boxed{\frac{\pi}{4}}$

d) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \boxed{\frac{3\pi}{4}}$

e) $\arccos\frac{\sqrt{3}}{2} = \boxed{\frac{\pi}{6}}$

f) $\cos^{-1} 1 = \boxed{0}$

g) $\arccos(-1) = \boxed{\pi}$

h) $\cos^{-1}(-0.567) = \boxed{2.174}$ (use TI)

Property of Inverses for Cosine Composite Functions:

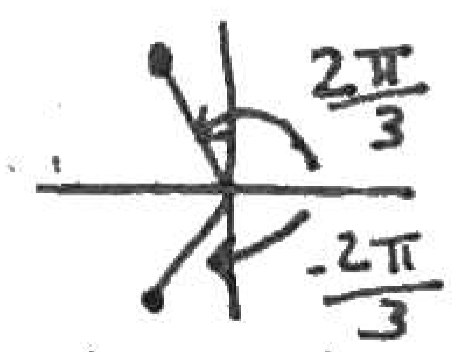
$$f^{-1}(f(x)) = \cos^{-1}(\cos x) = x \quad \text{where D} = [0, \pi] = D$$

$$f(f^{-1}(x)) = \cos(\cos^{-1} x) = x \quad \text{where D} = [-1, 1] = D$$

Find the exact value of each of the following composite functions. Verify with your TI. [Mode: radians]

i) $\cos^{-1}\left(\cos\frac{\pi}{12}\right) = \boxed{\frac{\pi}{12}}$

j) $\cos^{-1}\left(\cos\left(-\frac{2\pi}{3}\right)\right) = \cos^{-1}\left(\cos\frac{2\pi}{3}\right) = \cos^{-1}\left(-\frac{1}{2}\right)$



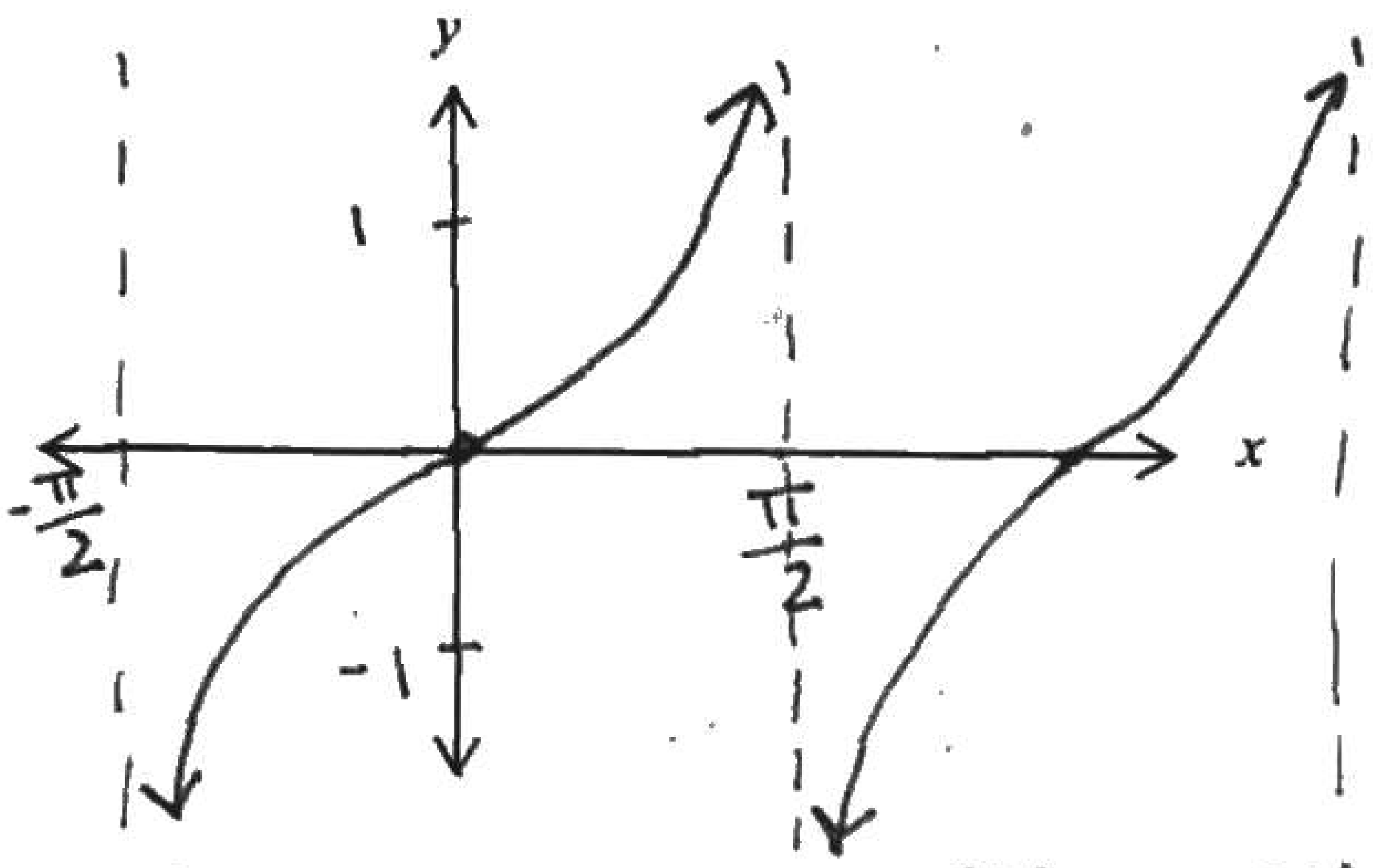
$= \boxed{\frac{2\pi}{3}}$

k) $\cos(\cos^{-1}(-0.4)) = \boxed{-.4}$

l) $\cos(\cos^{-1}\pi) =$ cos (cos^{-1} 3.14) = error

3.14 not in $[-1, 1]$

Graph $f(x) = \tan x$ in pencil. Then, using color, *restrict* the domain so that the restricted tangent function will pass the horizontal line test.



For the *restricted tangent function*:

Domain = $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Range = $(-\infty, \infty)$

Because the *restricted tangent function* is one -to- one, then its inverse will also be a function.

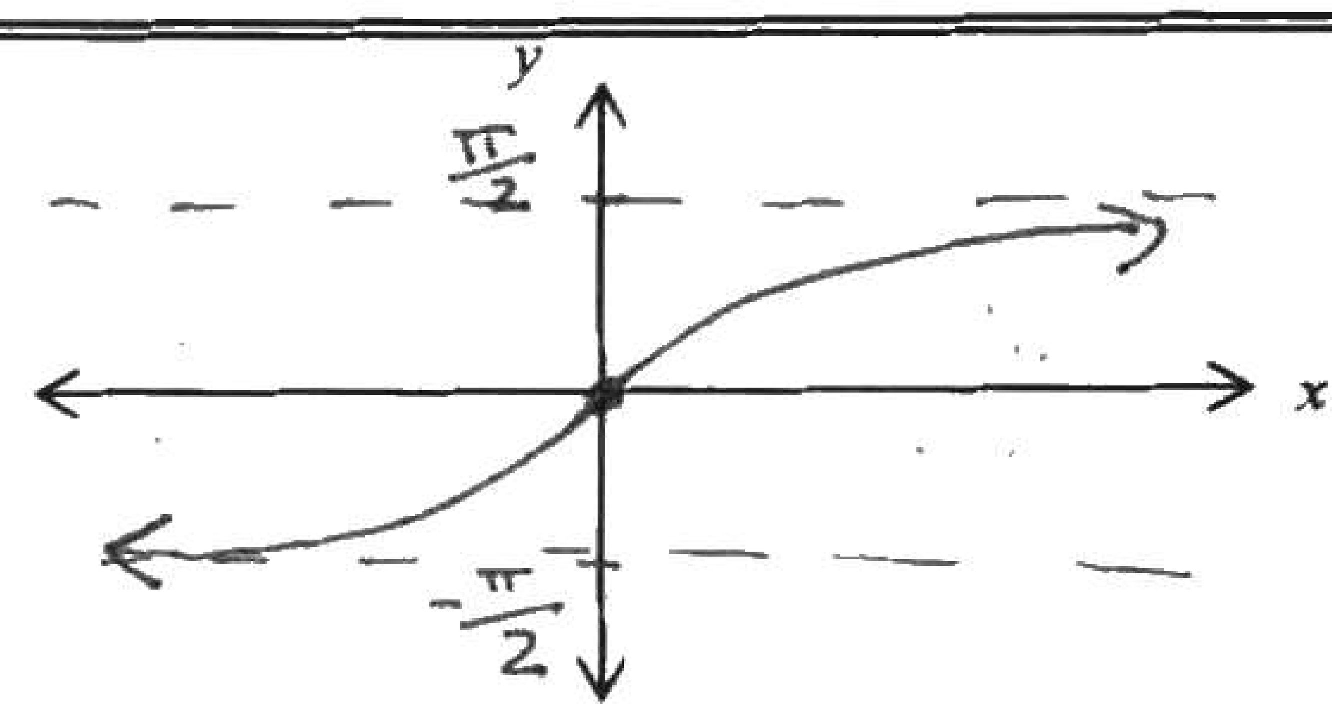
Use the graph of the restricted tangent function to draw the inverse tangent function or the arctangent function.

Inverse Tangent Function

$h(x) = \tan^{-1} x$ or $h(x) = \arctan x$

Domain = $(-\infty, \infty)$

Range = $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ → outputs in QI, QIV only



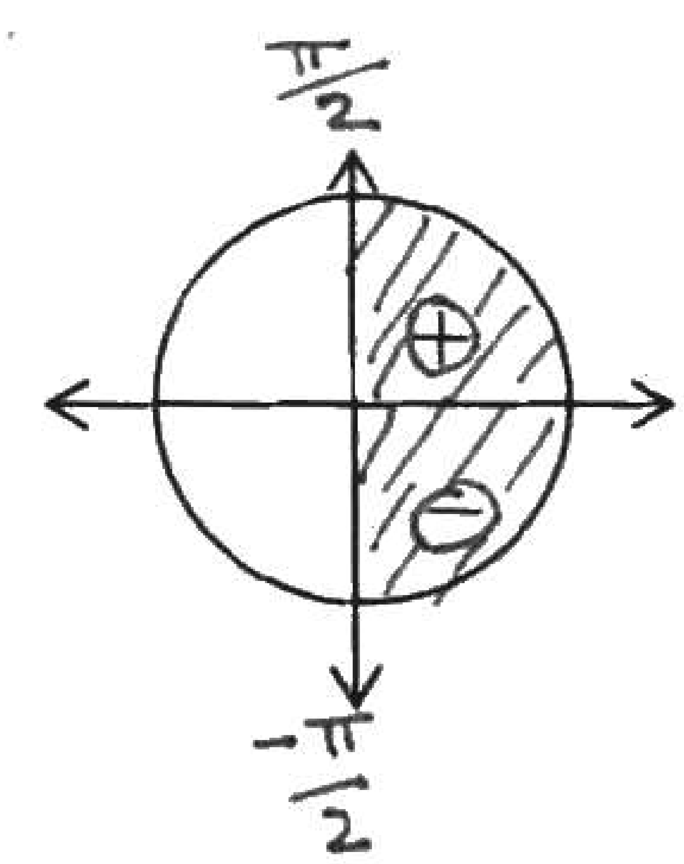
[Check w/ your TI.]

Evaluate. [Hint: Use the unit circle to solve but restrict your answers to quadrants I and IV].

a) $\tan^{-1} 1 = \frac{\pi}{4}$

b) $\tan^{-1}(-1) = -\frac{\pi}{4}$

c) $\arctan \frac{\sqrt{3}}{3} = \frac{\pi}{6}$



d) $\arctan\left(-\sqrt{3}\right) = -\frac{\pi}{3}$

e) $\tan^{-1} 0 = 0$

f) $\tan^{-1} 0.395 = .376$ (use TI)

g) $\tan^{-1} 395 = 1.568$

Property of Inverses for Tangent Composite Functions:

$$f^{-1}(f(x)) = \tan^{-1}(\tan x) = x \quad \text{where } D = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) = D$$

$$f(f^{-1}(x)) = \tan(\tan^{-1} x) = x \quad \text{where } D = (-\infty, \infty) = D$$

Finding the Inverse Function of a Trigonometric Function

Find the inverse function of $f(x) = 2\sin x - 1$. Find the range of $f(x)$. Then find the domain and range of $f^{-1}(x)$.

$x = 2\sin y - 1$

$x + 1 = 2\sin y$

$\frac{x+1}{2} = \sin y$

$y = \sin^{-1}\left(\frac{x+1}{2}\right)$

$\boxed{f^{-1}(x) = \sin^{-1}\left(\frac{x+1}{2}\right)}$

$y = 2\sin x - 1$

$\frac{y+1}{2} = \sin x \longrightarrow [-1, 1]$ must be between

$-1 \le \frac{y+1}{2} \le 1$

$-2 \le y + 1 \le 2$

$-3 \le y \le 1$

$\boxed{f(x)\ R = [-3, 1]}$

$f^{-1}(x)$

$\boxed{\begin{array}{l} D = [-3, 1] \\ R = [-\frac{\pi}{2}, \frac{\pi}{2}] \end{array}}$

(domain & range switch!)

Solving Inverse Trig Equations Algebraically

Solve for x. (Make sure your solutions are within the restricted range of the inverse trig. function).

1. $3\sin^{-1} x = \pi$

$\sin^{-1} x = \frac{\pi}{3}$

$x = \sin \frac{\pi}{3}$

$\boxed{x = \frac{\sqrt{3}}{2}}$

2. $-4\tan^{-1} x = \pi$

$\tan^{-1} x = \frac{-\pi}{4}$

$x = \tan\left(\frac{-\pi}{4}\right)$

$\boxed{x = -1}$

3. $3\cos^{-1}(2x) = 2\pi$

$\cos^{-1}(2x) = \frac{2\pi}{3}$

$2x = \cos \frac{2\pi}{3}$

$2x = \frac{-1}{2}$

$\boxed{x = \frac{-1}{4}}$