Date ______________ 6.6 Notes Phase Shift & Sinusoids

Describe the transformation from the parent function $f(t) = \cos t$ to $g(t) = \cos(t - \frac{\pi}{4})$.

horiz rt $\pi/4$

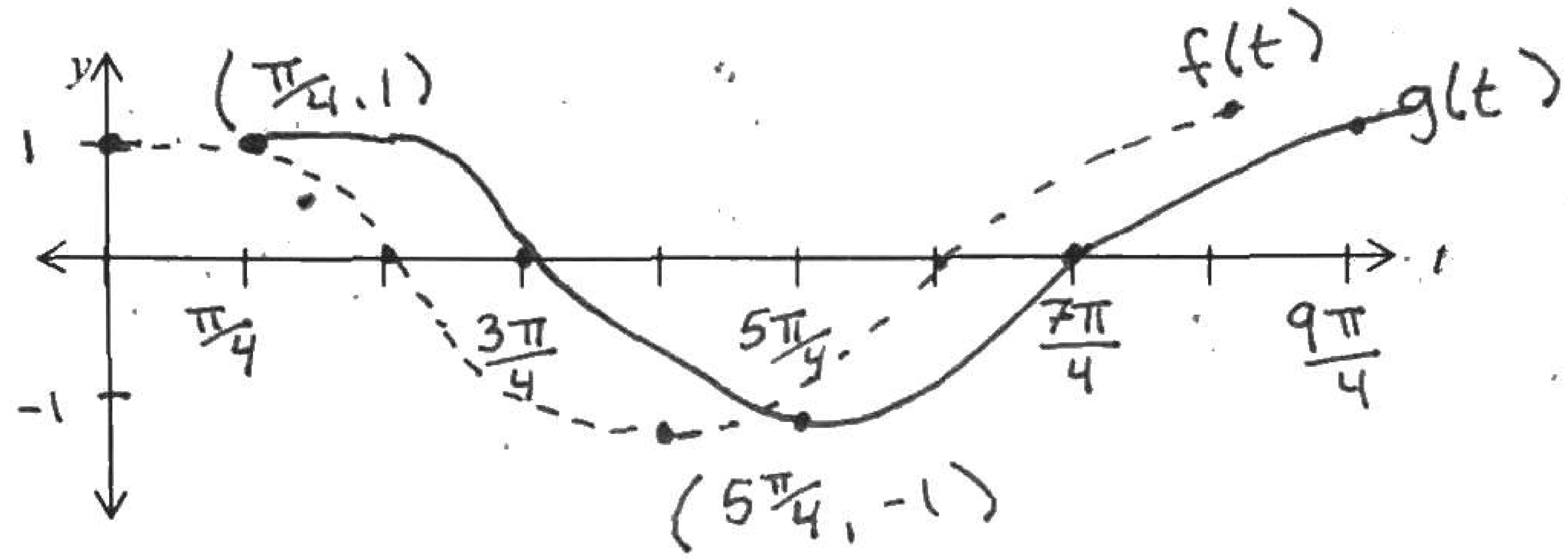
What values of t should you use to generate the "key" points of the cosine graph? Show those values in your chart.

t	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π		t	$\frac{\pi}{4}$	$\frac{3\pi}{4}$	$\frac{5\pi}{4}$	$\frac{7\pi}{4}$	$\frac{9\pi}{4}$
$f(t) = \cos t$	1	0	-1	0	1		$g(t) = \cos(t - \frac{\pi}{4})$	1	0	-1	0	1

add $\pi/4$ to t

Stays same

Use different colors and label each graph.



Describe the transformation from the parent graph $f(t) = \sin t$ to $g(t) = \sin(t + \frac{\pi}{6})$.

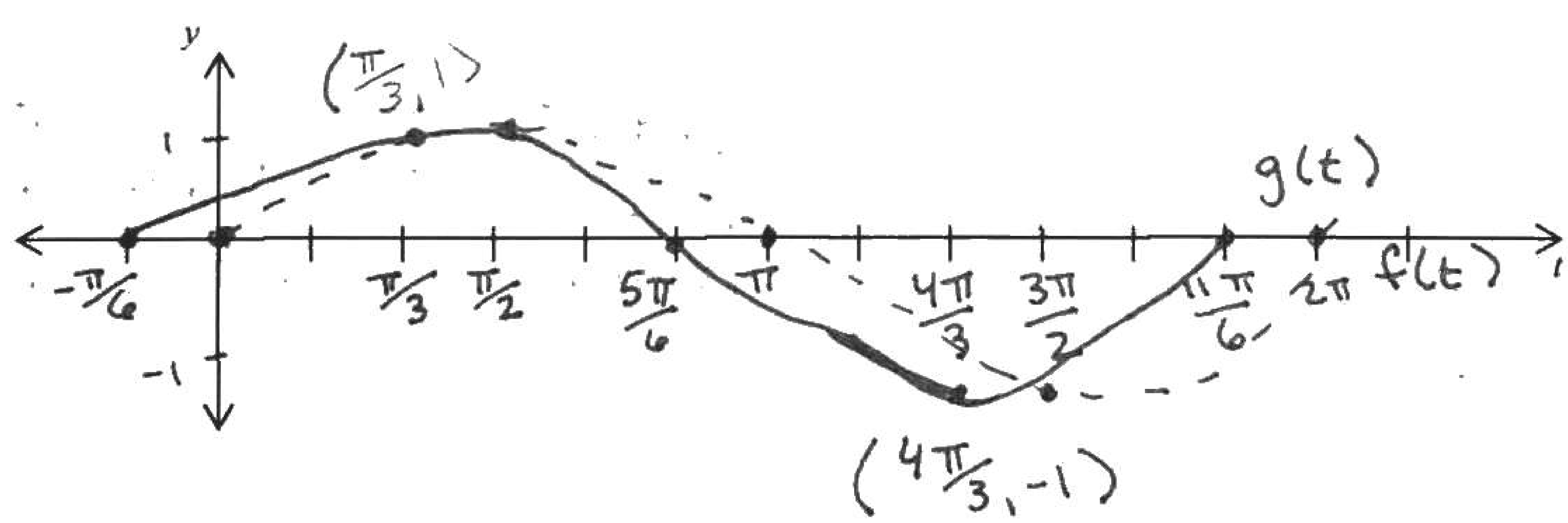
left $\pi/6$

Complete the chart with the appropriate values.

t	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π		t	$-\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{5\pi}{6}$	$\frac{4\pi}{3}$	$\frac{11\pi}{6}$
$f(t) = \sin t$	0	1	0	-1	0		$g(t) = \sin(t + \frac{\pi}{6})$	0	1	0	-1	0

Subtract $\pi/6$

Stays same



A horizontal translation of a trig. function is called a phase shift.

Examples: $g(t) = \sin(t + c) \Rightarrow$ $c > 0$, left c

$h(t) = \cos(t - c) \Rightarrow$ $c > 0$, right c

A vertical translation is of the form:

$j(t) = \sin t \underline{+d}$ or $k(t) = \cos t \underline{+d}$

Identify the amplitude, the phase shift and the vertical translation of each function from its parent function.

1. $k(t) = 4\sin\left(t-\frac{\pi}{2}\right)+3$

$\boxed{a = 4}$ $\boxed{p.s = rt\ \frac{\pi}{2}}$

$\boxed{up\ 3}$

2. $p(t) = -2\cos\left(t+\frac{\pi}{3}\right)-1$

$\boxed{a=2}$ $\boxed{p.s = left\ \frac{\pi}{3}}$

$\boxed{down\ 1}$

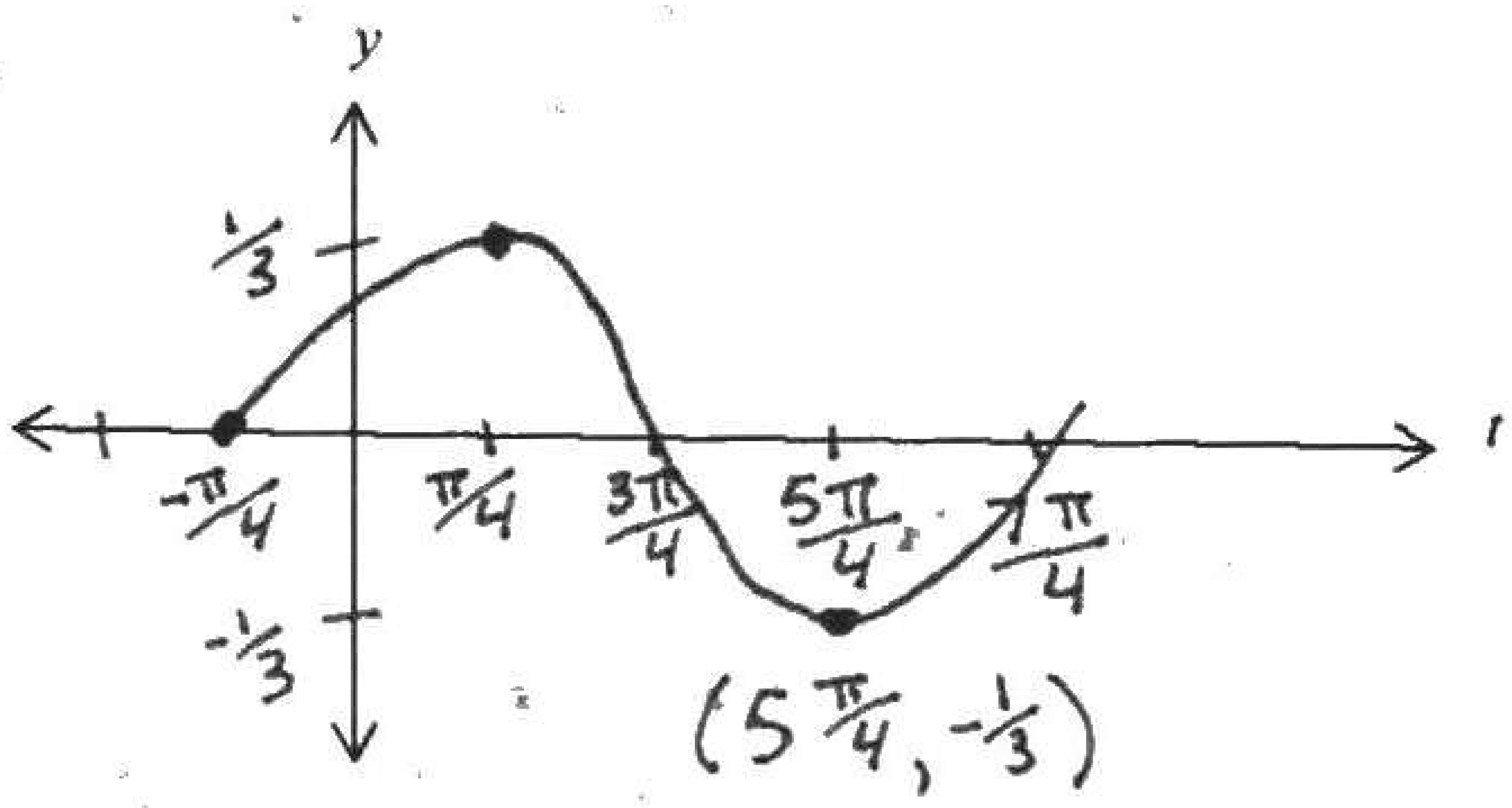
3. $v(t) = \frac{1}{3}\tan\left(t+\frac{\pi}{4}\right)$

$\boxed{no\ amp}$

$\boxed{left\ \frac{\pi}{4}}$

4. Sketch the graph for at least one period for $q(t) = \frac{1}{3}\sin\left(t+\frac{\pi}{4}\right)$.

	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
left $\pi/4$	$-\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{3\pi}{4}$	$\frac{5\pi}{4}$	$\frac{7\pi}{4}$
	0	1	0	-1	0
$a=\frac{1}{3}$	0	$\frac{1}{3}$	0	$-\frac{1}{2}$	0

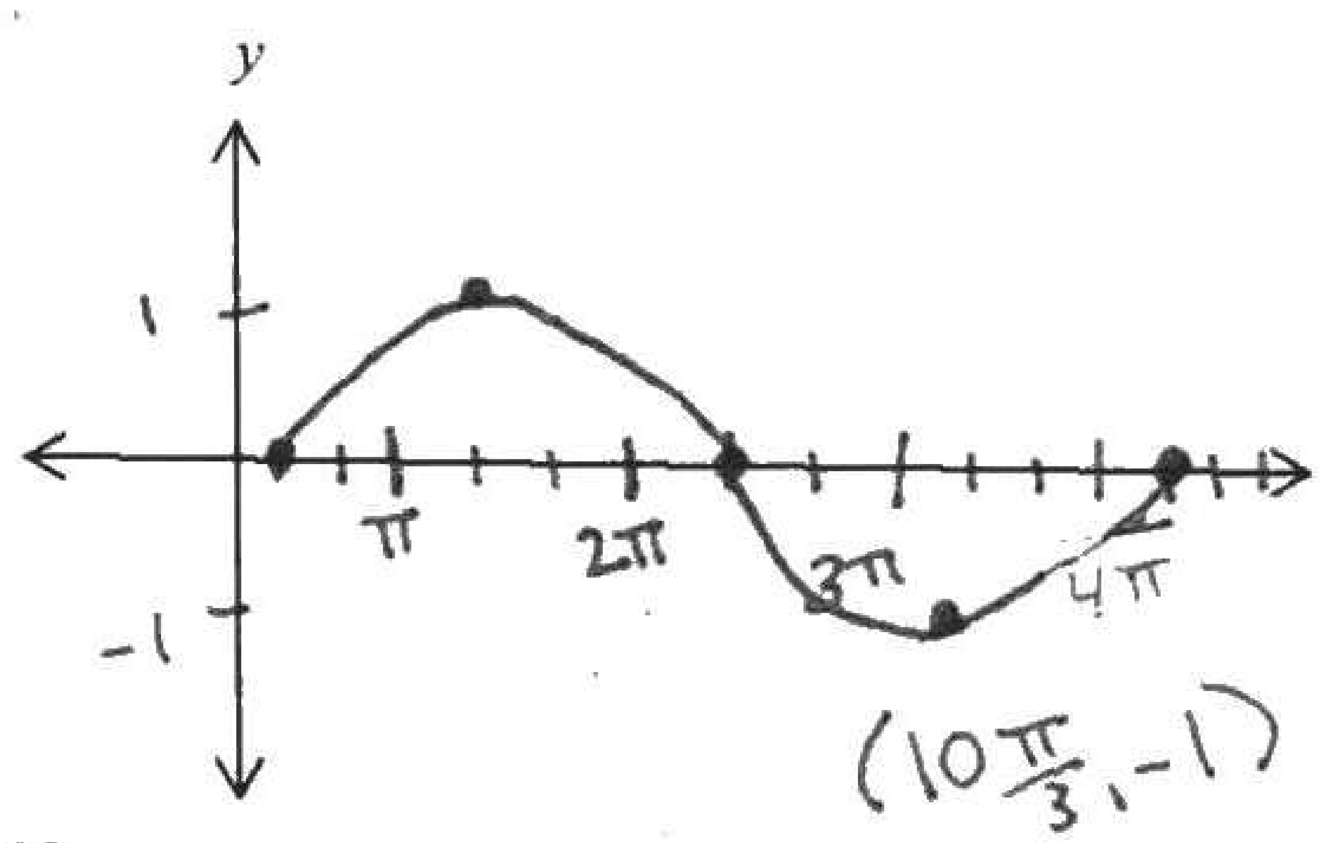


Combination of phase shift and horizontal compression/stretch:

Identify the phase shift and the period of $g(t) = \sin\left[\frac{1}{2}\left(t-\frac{\pi}{3}\right)\right]$.

$\boxed{P.S = rt\ \frac{\pi}{3}}$ $per. = \frac{2\pi}{1/2} = \boxed{4\pi}$

Sketch the graph.



Hints:

- Identify the key points of the parent graph due to the period change.
- Determine how those key points change due to the phase shift.
- * Organize your information in a table.
- Be careful how you set up the units on your axes.
- Label all key values.

per = 4π	0	π	2π	3π	4π
p.s + $\frac{\pi}{3}$	$\frac{\pi}{3}$	$\frac{4\pi}{3}$	$\frac{7\pi}{3}$	$\frac{10\pi}{3}$	$\frac{15\pi}{3}$
y-vals	0	1	0	-1	0

Identify the amplitude, phase shift, and period of $h(t) = \frac{1}{3}\cos\left(2t+\frac{\pi}{4}\right)$.

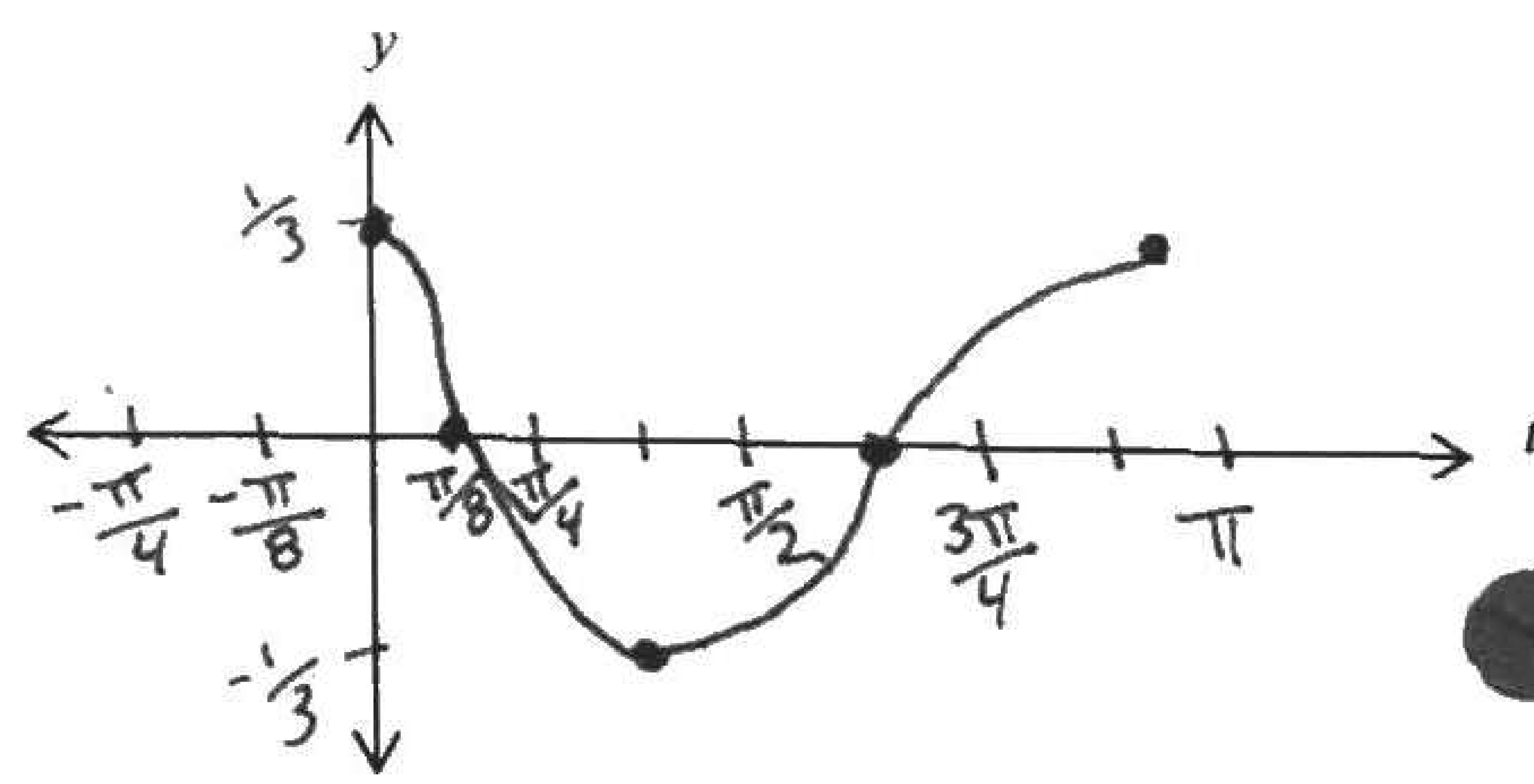
Hint: first rewrite in factored form.

$h(t) = \frac{1}{3}\cos\left[2\left(t+\frac{\pi}{8}\right)\right]$ $\boxed{a=\frac{1}{3}}$

$\boxed{ps:\ left\ \frac{\pi}{8}}$

Sketch the graph.

$per = \frac{2\pi}{2} = \boxed{\pi}$



	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$-\frac{\pi}{8}$	$-\frac{\pi}{8}$	$-\frac{\pi}{8}$	$\frac{3\pi}{8}$	$\frac{5\pi}{8}$	$\frac{7\pi}{8}$
	1	0	-1	0	1
	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	$\frac{1}{3}$

In general, given a function with $a \neq$ 0 and $b >$ 0, that is of the form:

- $f(t) = a\sin(bt + c) + d$
- $f(t) = a\cos(bt + c) + d$

amplitude = $|a|$ period = $\frac{2\pi}{b}$ vertical shift = d phase shift = $\frac{-c}{b}$

- $f(t) = a\tan(bt + c) + d$ = $a\tan\left[b\left(t + \frac{c}{b}\right)\right] + d$

amplitude = none period = $\frac{\pi}{b}$ vertical shift = d phase shift = $-\frac{c}{b}$

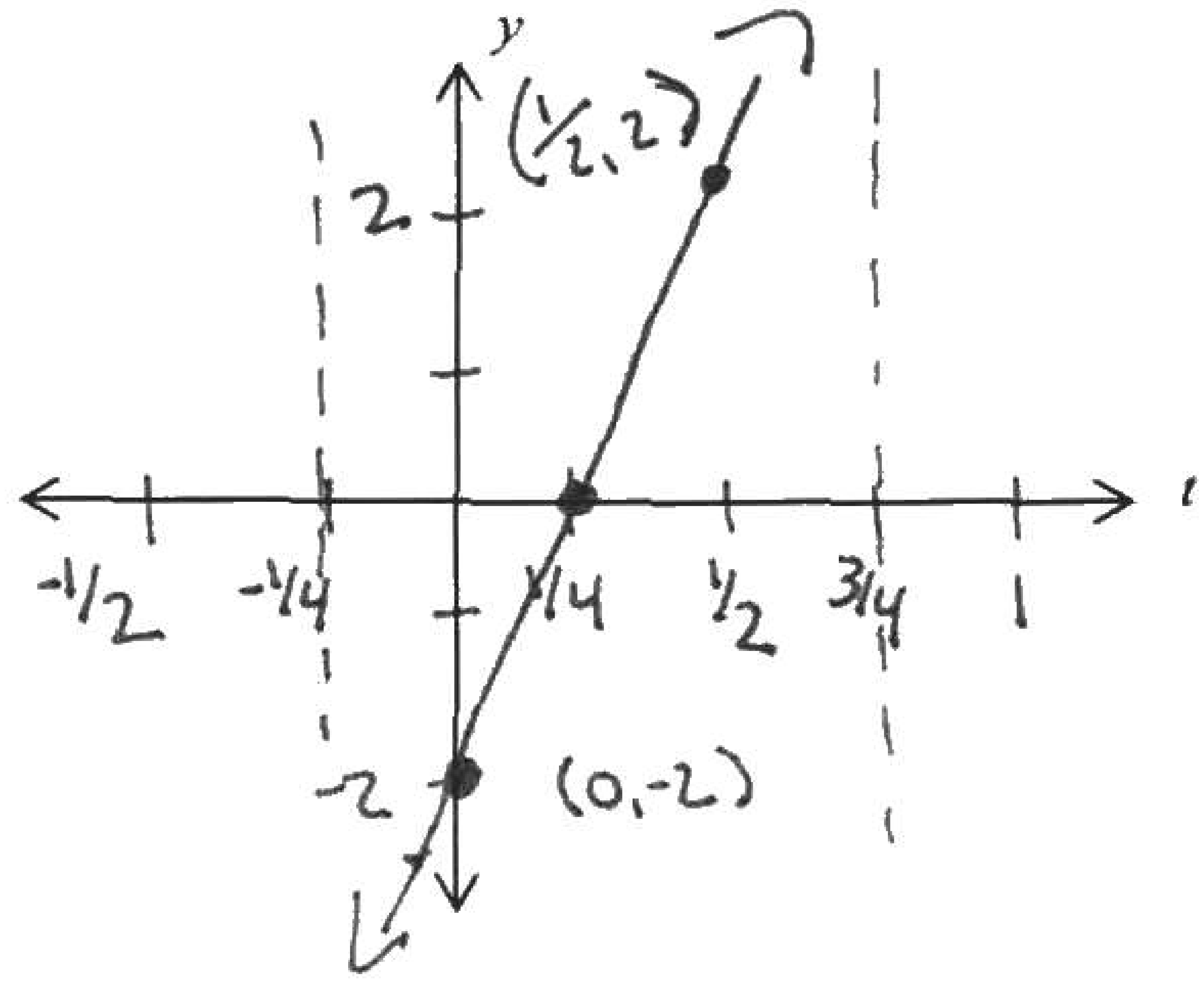
[Hint: Always factor $(bt + c)$ first.]

Sketch the graph for one complete period $f(t) = 2\tan\left(\pi t - \frac{\pi}{4}\right)$

Clearly label key points and equations of asymptotes. per $= \frac{\pi}{\pi} = 1$

$f(t) = 2\tan\left[\pi\left(t - \frac{1}{4}\right)\right]$

	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
per	$-1/2$	$-1/4$	0	$1/4$	$1/2$
rt $1/4$	$-1/4$	0	$1/4$	$1/2$	$3/4$
	und	-1	0	1	und
$a=2$	und	-2	0	2	und



State the rule of a cosine function with amplitude 2, period π, phase shift $-\frac{\pi}{6}$, and vertical shift -3.

$\frac{2\pi}{b} = \pi$

$\boxed{b=2}$

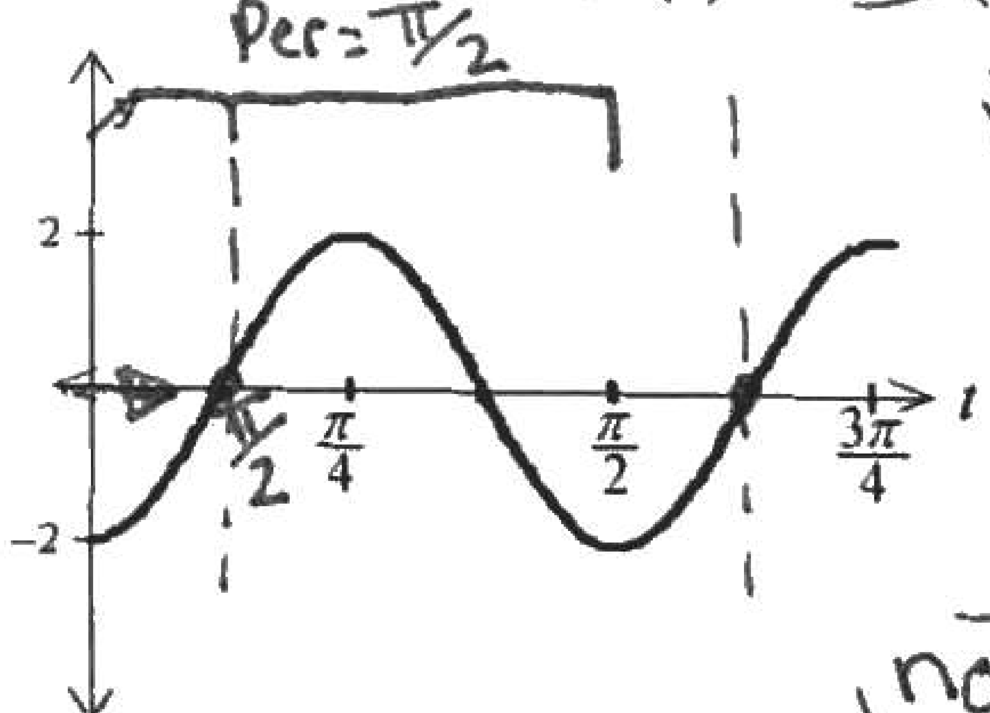
$\boxed{f(t) = \pm 2\cos\left[2\left(t + \frac{\pi}{6}\right)\right] - 3}$

or

$\boxed{f(t) = \pm 2\cos\left(2t + \frac{\pi}{3}\right) - 3}$

State the rule for the graph in the form $f(t) = a\sin(bt + c) + d$.

Per $= \pi/2$



$\boxed{a=2}$

$\boxed{PS = rt\ \frac{\pi}{8}}$

no vert. shift

per $= \frac{\pi}{2} = \frac{2\pi}{b}$ $\boxed{b=4}$

$\boxed{f(t) = 2\sin\left[4\left(t - \frac{\pi}{8}\right)\right]}$

or

$\boxed{f(t) = 2\sin\left(4t - \frac{\pi}{2}\right)}$

Finding the Sine Function of Best Fit

Use your TI to find the sine function of best fit for the data in the table below. The data below represents the average temperatures, each month, in Denver, Colorado.

Month, x	Jan (1)	Feb (2)	Mar (3)	Apr (4)	May (5)	June (6)	July (7)	Aug (8)	Sept (9)	Oct (10)	Nov (11)	Dec (12)
Avg. Monthly Temp, y	29.7 °F	33.4 °F	39.0 °F	48.2 °F	57.2 °F	66.9 °F	73.5 °F	71.4 °F	62.3 °F	51.4 °F	39.0 °F	31.0 °F

Enter the data:
Stat 1:Edit Enter. *Enter Month, x, into* L1. *Enter Average Monthly Temperature, y, into* L2.

Find the equation that best fits the data:
STAT → CALC ↓ C:SinReg ENTER ENTER Sine Reg. Eq.: $y = 21.1\sin(.55x - 2.4) + 51.2$

Plot the points and show a sketch of the best fit curve.