Chapter 10 review answers

1. $16x^2 = y \quad \rightarrow \quad x^2 = \frac{1}{16}y$

This is a parabola.

$a = \frac{1}{64}$

Vertex: (0, 0)

Focus: $\left(0, \frac{1}{64}\right)$

Directrix: $y = -\frac{1}{64}$

2. $9x^2 + 4y^2 = 36$

This is an ellipse. Write in standard form:

$\frac{x^2}{4} + \frac{y^2}{9} = 1$

$a = 3, \; b = 2$ .Find the value of c:

$c^2 = a^2 - b^2 = 9 - 4 = 5$

$c = \sqrt{5}$

Center: (0, 0)

Vertices: (0, 3), (0, –3)

Foci: $\left(0, \sqrt{5}\right), \left(0, -\sqrt{5}\right)$

3. $\frac{x^2}{9} + \frac{y^2}{16} = 1$ This is an ellipse.

$a = 4, \; b = 3$. Find the value of c:

$c^2 = a^2 - b^2 = 16 - 9 = 7 \Rightarrow c = \sqrt{7}$

Center: (0, 0) Vertices: (0, 4), (0, –4)

Foci: $\left(0, \sqrt{7}\right), \left(0, -\sqrt{7}\right)$

4. $3y^2 - x^2 = 9$

This is a hyperbola. Write in standard form:

$\frac{y^2}{3} - \frac{x^2}{9} = 1$

$a = \sqrt{3}, \; b = 3$

Find the value of c:

$c^2 = a^2 + b^2 = 3 + 9 = 12$

$c = \sqrt{12} = 2\sqrt{3}$

Center: (0, 0)

Vertices: $\left(0, \sqrt{3}\right), \left(0, -\sqrt{3}\right)$

Foci: $\left(0, 2\sqrt{3}\right), \left(0, -2\sqrt{3}\right)$

Asymptotes: $y = \frac{\sqrt{3}}{3}x; \quad y = -\frac{\sqrt{3}}{3}x$

5. $4x^2 + 9y^2 - 16x + 18y = 11$

This is an ellipse.
Write in standard form:

$$4x^2 + 9y^2 - 16x + 18y = 11$$

$$4(x^2 - 4x + 4) + 9(y^2 + 2y + 1) = 11 + 16 + 9$$

$$4(x-2)^2 + 9(y+1)^2 = 36$$

$$\frac{(x-2)^2}{9} + \frac{(y+1)^2}{4} = 1$$

$a = 3, \; b = 2$. Find the value of c:

$c^2 = a^2 - b^2 = 9 - 4 = 5$

$c = \sqrt{5}$

Center: (2, –1); Vertices: (–1, –1), (5, –1)

Foci: $\left(2 - \sqrt{5}, -1\right), \left(2 + \sqrt{5}, -1\right)$

6. $x^2 - y^2 - 2x - 2y = 1$

This is a hyperbola. Write in standard form:

$$(x^2 - 2x + 1) - (y^2 + 2y + 1) = 1 + 1 - 1$$

$$(x-1)^2 - (y+1)^2 = 1$$

$a = 1, \; b = 1$.

Find the value of c:

$c^2 = a^2 + b^2 = 1 + 1 = 2$

$c = \sqrt{2}$

Center: (1, –1)

Vertices: (0, –1), (2, –1)

Foci: $\left(1 + \sqrt{2}, -1\right), \left(1 - \sqrt{2}, -1\right)$

Asymptotes: $y + 1 = x - 1; \quad y + 1 = -(x - 1)$

7. $4y^2 + 3x - 16y + 19 = 0$

This is a parabola.
Write in standard form:

$$4(y^2 - 4y + 4) = -3x - 19 + 16$$

$$4(y-2)^2 = -3(x+1)$$

$$(y-2)^2 = -\frac{3}{4}(x+1)$$

$a = \frac{3}{16}$

Vertex: (–1, 2);

Focus: $\left(-\frac{19}{16}, 2\right)$

Directrix: $x = -\frac{13}{16}$

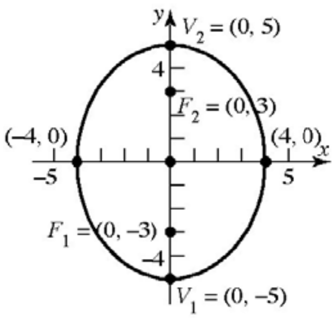
8. Ellipse: The center is (0, 0), a focus is (0, 3), and a vertex is (0, 5). The major axis is $x = 0$. $a = 5$, $c = 3$.

Find b: $b^2 = a^2 - c^2 = 25 - 9 = 16$. So, $b = 4$. The equation of the ellipse is:

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

$$\frac{x^2}{4^2} + \frac{y^2}{5^2} = 1$$

$$\frac{x^2}{16} + \frac{y^2}{25} = 1$$

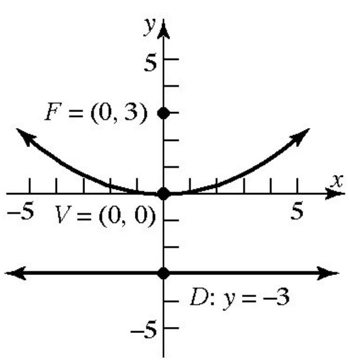


9. Parabola: Vertex: (0, 0); Directrix: $y = -3$; $a = 3$; the focus is the point $(0, 3)$; the graph opens up. The equation of the parabola is:

$$x^2 = 4ay$$

$$x^2 = 4(3)y$$

$$x^2 = 12y$$



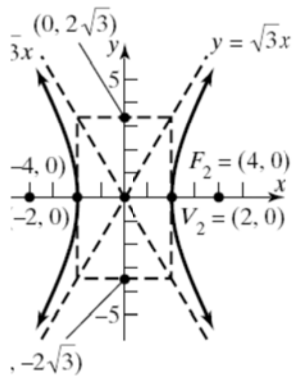
10. Hyperbola: Vertices: (–2, 0), (2, 0); Focus: (4, 0); Center: (0, 0); Transverse axis is the x-axis; $a = 2$; $c = 4$.

Find b:

$$b^2 = c^2 - a^2 = 16 - 4 = 12$$

$$b = \sqrt{12} = 2\sqrt{3}$$

Write the equation: $\frac{x^2}{4} - \frac{y^2}{12} = 1$

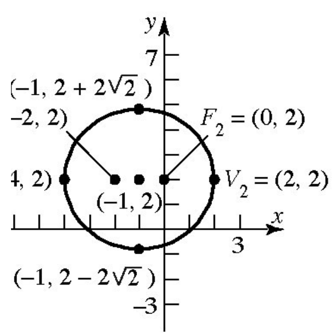


11. Ellipse: Center: (–1, 2); Focus: (0, 2); Vertex: (2, 2). Major axis: $y = 2$. $a = 3$; $c = 1$. Find b:

$$b^2 = a^2 - c^2 = 9 - 1 = 8$$

$$b = \sqrt{8} = 2\sqrt{2}$$

Write the equation: $\frac{(x+1)^2}{9} + \frac{(y-2)^2}{8} = 1$

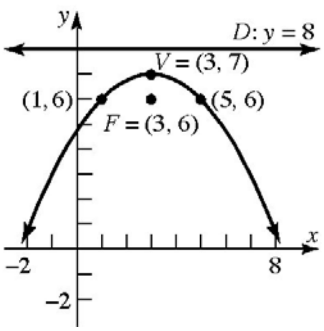


12. Parabola: Focus: (3, 6); Directrix: $y = 8$; Parabola opens down. Vertex: (3, 7) $a = 1$. The equation of the parabola is:

$$(x-h)^2 = -4a(y-k)$$

$$(x-3)^2 = -4(1)(y-7)$$

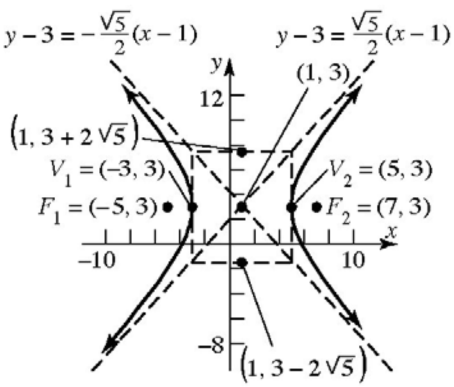
$$(x-3)^2 = -4(y-7)$$



13. Hyperbola: Vertices: (–3, 3), (5, 3); Focus: (7, 3); Center: (1, 3); Major axis is parallel to the x-axis; $a = 4$; $c = 6$. Find b:

$$b^2 = c^2 - a^2 = 36 - 16 = 20 \quad \rightarrow \quad b = \sqrt{20} = 2\sqrt{5}$$

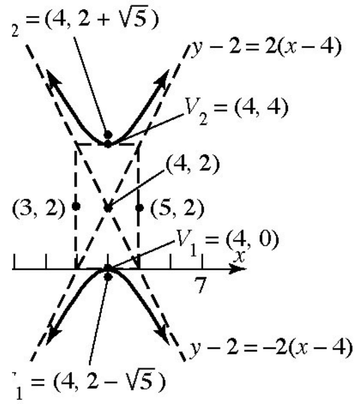
Write the equation: $\dfrac{(x-1)^2}{16} - \dfrac{(y-3)^2}{20} = 1$



14. Hyperbola: Vertices: (4, 0), (4, 4); Asymptote: $y + 2x - 10 = 0$; Center: (4, 2); Transverse axis is parallel to the y-axis; $a = 2$; The slope of the asymptote is -2; Find b:

$$\frac{-a}{b} = \frac{-2}{b} = -2 \;\rightarrow\; -2b = -2 \;\rightarrow\; b = 1$$

Write the equation: $\dfrac{(y-2)^2}{4} - (x-4)^2 = 1$



15. $2x^2 - y + 8x = 0$

$A = 2$ and $C = 0$; $AC = (2)(0) = 0$. Since $AC = 0$, the equation defines a parabola.

16. $x^2 - 8y^2 - x - 2y = 0$

$A = 1$ and $C = -8$; $AC = (1)(-8) = -8$. Since $AC < 0$, the equation defines a hyperbola.

17. $4x^2 + 12xy - 10y^2 + x + y - 10 = 0$

$A = 4, \; B = 12, \; C = -10$

$B^2 - 4AC = 12^2 - 4(4)(-10) = 304 > 0$

Hyperbola

18. $4x^2 - 10xy + 4y^2 - 9 = 0$

$A = 4, \; B = -10, \; C = 4$

$B^2 - 4AC = (-10)^2 - 4(4)(4) = 36 > 0$

Hyperbola

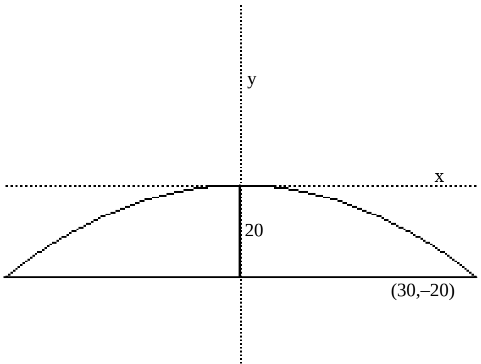
The set of points is a hyperbola.

19. Set up the problem so that the vertex of the parabola is at (0, 0) and it opens down. Then the equation of the parabola has the form: $x^2 = cy$.

The point (30, –20) is a point on the parabola. Solve for c and find the equation:

$30^2 = c(-20) \rightarrow c = -45$

$x^2 = -45y$



To find the height of the bridge, 5 feet from the center, the point (5, *y*) is a point on the parabola. Solve for *y*:

$5^2 = -45y \rightarrow 25 = -45y \rightarrow y \approx -0.56$

The height of the bridge, 5 feet from the center, is 20 – 0.56 = 19.44 feet.

To find the height of the bridge, 10 feet from the center, the point (10, *y*) is a point on the parabola. Solve for *y*:

$10^2 = -45y \rightarrow 100 = -45y \rightarrow y \approx -2.22$

The height of the bridge, 10 feet from the center, is 20 – 2.22 = 17.78 feet.

To find the height of the bridge, 20 feet from the center, the point (20, *y*) is a point on the parabola. Solve for *y*:

$20^2 = -45y \rightarrow 400 = -45y \rightarrow y \approx -8.89$

The height of the bridge, 20 feet from the center, is 20 – 8.89 = 11.11 feet.