

# NOT Consumable

BC Review Wk 15 Block

DO NOT WRITE ON THIS SHEET Show all work on another sheet

1. If  $f(x) = (\ln x)^2$ , then  $f''(\sqrt{e}) =$

- A)  $\frac{1}{e}$       B)  $\frac{2}{e}$       C)  $\frac{1}{2\sqrt{e}}$       D)  $\frac{1}{\sqrt{e}}$       E)  $\frac{2}{\sqrt{e}}$

2. What are all the values of  $x$  for which the series  $\sum_{n=1}^{\infty} \left(\frac{2}{x^2+1}\right)^n$  converges?

- A)  $-1 < x < 1$       B)  $x > 1$  only      C)  $x \geq 1$  only  
 D)  $x < -1$  and  $x > 1$  only      E)  $x \leq -1$  and  $x \geq 1$

3. Let  $h$  be a differentiable function, and let  $f$  be the function defined by  $f(x) = h(x^2 - 3)$ . Which of the following is equal to  $f'(2)$ ?

- A)  $h'(1)$       B)  $4h'(1)$       C)  $4h'(2)$       D)  $h'(4)$       E)  $4h'(4)$

4. In the  $xy$ -plane, the line  $x+y=k$ , where  $k$  is a constant, is tangent to the graph of  $y = x^2 + 3x + 1$ . What is the value of  $k$ ?

- A)  $-3$       B)  $-2$       C)  $-1$       D)  $0$       E)  $1$

5.  $\int \frac{7x}{(2x-3)(x+2)} dx =$

- A)  $\frac{3}{2} \ln|2x-3| + 2 \ln|x+2| + C$       B)  $3 \ln|2x-3| + 2 \ln|x+2| + C$   
 C)  $3 \ln|2x-3| - 2 \ln|x+2| + C$       D)  $-\frac{6}{(2x-3)^2} - \frac{2}{(x+2)^2} + C$       E)  $-\frac{3}{(2x-3)^2} - \frac{2}{(x+2)^2} + C$

6. What is the sum of the series  $1 + \ln 2 + \frac{(\ln 2)^2}{2!} + \dots + \frac{(\ln 2)^n}{n!} + \dots$ ?

- A)  $\ln 2$       B)  $\ln(1 + \ln 2)$       C)  $2$       D)  $e^2$       E) The series diverges

$x$	0	1
$f(x)$	2	4
$f'(x)$	6	-3
$g(x)$	-4	3
$G'(x)$	2	-1

7. The table above gives values of  $f, f', g,$  and  $g'$  for selected values of  $x$ . If  $\int_0^1 f'(x)g(x)dx = 5$ , then

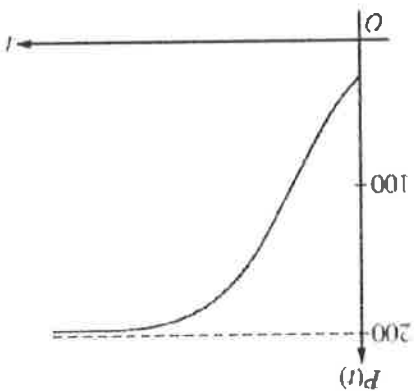
$\int_0^1 f(x)g'(x)dx =$

- A)  $-14$       B)  $-13$       C)  $-2$       D)  $7$       E)  $15$

8. If  $f(x) = x \sin(2x)$ , which of the following is the Taylor series for  $f$  about  $x=0$ ?

- A)  $x - \frac{x^3}{5} + \frac{4x^5}{61} - \frac{2x^7}{71} + \dots$   
 B)  $x - \frac{4x^3}{16x^5} + \frac{2x^5}{41} - \frac{4x^7}{64x^7} + \dots$   
 C)  $2x - \frac{8x^3}{32x^5} + \frac{3x^5}{51} - \frac{7x^7}{128x^7} + \dots$   
 D)  $2x^2 - \frac{2x^4}{2x^6} + \frac{3x^6}{51} - \frac{7x^8}{128x^8} + \dots$   
 E)  $2x^2 - \frac{8x^4}{8x^4} + \frac{3x^6}{32x^6} - \frac{7x^8}{128x^8} + \dots$

9. Which of the differential equations for a population  $P$  could model the logistic growth shown in the figure below



- A)  $\frac{dP}{dt} = 0.2P - 0.001P^2$  B)  $\frac{dP}{dt} = 0.1P - 0.001P^2$  C)  $\frac{dP}{dt} = 0.2P^2 - 0.001P$   
 D)  $\frac{dP}{dt} = 0.1P^2 - 0.001P$  E)  $\frac{dP}{dt} = 0.1P^2 + 0.001P$

10. In the  $xy$ -plane, a particle moves along the parabola  $y = x^2 - x$  with a constant speed of  $2\sqrt{10}$  units per second. If  $\frac{dx}{dt} > 0$ , what is the value of  $\frac{dy}{dt}$  when the particle is at the point  $(2,2)$ ?

- A)  $\frac{3}{2}$  B)  $\frac{2\sqrt{10}}{3}$  C) 3 D) 6 E)  $6\sqrt{10}$

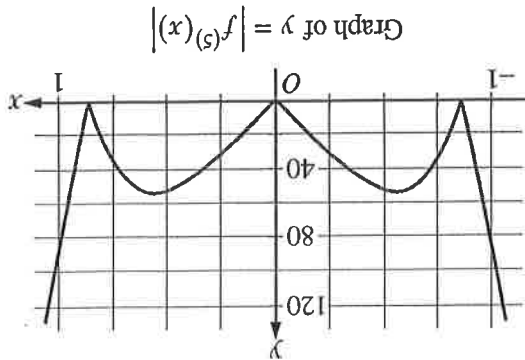
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6. The function  $g$  has derivatives of all orders, and the Maclaurin series for  $g$  is

$$\sum_{n=0}^{\infty} (-1)^n x^{2n+1} = \frac{3}{x} - \frac{3}{x^3} + \frac{7}{x^5} - \dots$$

- (a) Using the ratio test, determine the interval of convergence of the Maclaurin series for  $g$ .
- (b) The Maclaurin series for  $g$  evaluated at  $x = \frac{2}{17}$  is an alternating series whose terms decrease in absolute value to 0. The approximation for  $g\left(\frac{2}{17}\right)$  using the first two nonzero terms of this series is  $\frac{120}{17}$ . Show that this approximation differs from  $g\left(\frac{2}{17}\right)$  by less than  $\frac{1}{200}$ .
- (c) Write the first three nonzero terms and the general term of the Maclaurin series for  $g'(x)$ .



6. Let  $f(x) = \sin(x^2) + \cos x$ . The graph of  $y = |f^{(5)}(x)|$  is shown above.

- (a) Write the first four nonzero terms of the Taylor series for  $\sin x$  about  $x = 0$ , and write the first four nonzero terms of the Taylor series for  $\sin(x^2)$  about  $x = 0$ .
- (b) Write the first four nonzero terms of the Taylor series for  $\cos x$  about  $x = 0$ . Use this series and the series for  $\sin(x^2)$ , found in part (a), to write the first four nonzero terms of the Taylor series for  $f$  about  $x = 0$ .
- (c) Find the value of  $f^{(6)}(0)$ .

(d) Let  $P_4(x)$  be the fourth-degree Taylor polynomial for  $f$  about  $x = 0$ . Using information from the graph of  $y = |f^{(5)}(x)|$  shown above, show that  $\left|P_4\left(\frac{1}{4}\right) - f\left(\frac{1}{4}\right)\right| > \frac{1}{3000}$ .

