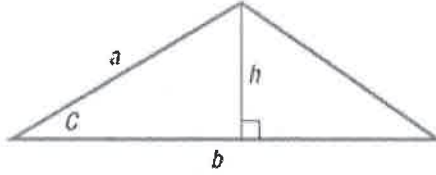Area of a Triangle given two sides and the angle between the two sides.

$$K = \frac{1}{2}ab \sin C$$

$$K = \frac{1}{2}bc \sin A$$

$$K = \frac{1}{2}ac \sin B$$

The area A of a triangle equals one-half the product of two of its sides times the sine of their included angle.

Handwritten:

If you start with
$A = \frac{1}{2}bh$ and use trig
to find h, then you can
get to this formula.

$\sin C = \frac{h}{a}$ so $h = a \sin C$

$A = \frac{1}{2}bh$
$A = \frac{1}{2}b(a \sin C)$
so $A = \frac{1}{2}ab \sin C$

Area of a Triangle given three sides of the triangle.
THEOREM

Heron's Formula

The area K of a triangle with sides a, b, and c is

$$K = \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = \frac{1}{2}(a+b+c)$.

Find the area of each triangle. Round answers to 2 decimal places.

1. $a = 3$, $b = 5$, $c = 6$

$S = \frac{3+5+6}{2} = \frac{14}{2} = 7$

$A = \sqrt{7(7-3)(7-5)(7-6)}$

$A = \sqrt{7 \cdot 4 \cdot 2 \cdot 1}$

$A = \sqrt{56}$

$\approx 7.48\, u^2$

2. $a = 8$, $b = 6$, $C = 30°$

$A = \frac{1}{2} a \cdot b \cdot \sin C$

$A = \frac{1}{2} \cdot 8 \cdot 6 \cdot \sin 30°$

$A = 12\, u^2$

3. $b = 12.3$, $c = 15.2$, $A = 52°15'$

$A = \frac{1}{2} \cdot b \cdot c \cdot \sin A$

$A = \frac{1}{2}(12.3)(15.2) \sin 52°15'$

$A \approx 73.91\, u^2$

4. $a = 8$, $b = 15$, $c = 10$

$S = \frac{8+15+10}{2} = 16.5$

$A = \sqrt{16.5(16.5-8)(16.5-15)(16.5-10)}$

$A = \sqrt{16.5(8.5)(1.5)(6.5)}$

$A \approx 36.98\, u^2$