Law of Sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

(LOS)

Law of Cosines (side): $a^2 = b^2 + c^2 - 2bc\cos A$

(LOC)

$b^2 = a^2 + c^2 - 2ac\cos B$

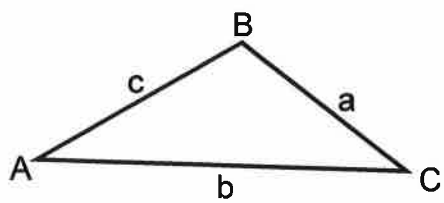
$c^2 = a^2 + b^2 - 2ab\cos C$

Law of Cosines (angle):

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$



Generic non right triangle

Largest angle should always be across from largest side

smallest angle always across from smallest side

Solve the following triangles: (find all missing sides and angles)

Round angles to the nearest minute and sides to 4 significant digits.

Make sure ∠s add up to 180°

When using law of sines and you have a choice of ∠s to find, find the smallest.

1. A=30°, B=80°, a=12

A= 30°	a= 12
B= 80°	b= 23.64
C= 70°	c= 22.55

$\angle C = 180° - 30° - 80° = 70°$

$\frac{\sin 80°}{b} = \frac{\sin 30°}{12}$

$b\sin 30° = 12\sin 80°$

$b = \frac{12\sin 80°}{\sin 30°}$

$\frac{\sin 70°}{c} = \frac{\sin 30°}{12}$

$c \approx 22.55$

2. A=38°, b=6, c=10

A= 38°	a= 6.437
B= 35°1'	b= 6
C= 106°59'	c= 10

$a^2 = 6^2 + 10^2 - 2\cdot 6\cdot 10\cdot\cos 38°$

$a \approx 6.437$

$\frac{\sin B}{6} = \frac{\sin 38°}{6.437...}$

$\sin B = 0.5738$

$B = 35°1'$

$\angle C = 180 - 38 - 35°1'$

$C = 106°59'$

When using law of Cosines and you have a choice of angles to find, find the largest angle first.

3. a=11, b=13, c=15

Whenever you are given 3 sides you must find the largest angle first. How do we know which one is the largest angle?

A= 45°34′	a= 11
B= 57°34′	b= 13
C= 76°52′	c= 15

$$\cos C = \frac{11^2+13^2-15^2}{2\cdot 11\cdot 13}$$

$$\cos C \approx 0.22727\ldots$$

$$C = 76°52'$$

You can stick with LOC or switch to LOS.

$$\frac{\sin A}{11} = \frac{\sin 76°52'}{15}$$

$$\sin A = 0.7141\ldots$$

$$A \approx 45°34'$$

$$B = 180 - A - C$$

$$\approx 57°34'$$

4. A=83°10', a=80, b=70

A= 83°10′	a= 80
B= 60°19′	b= 70
C= 36°31′	c=47.95

$$\frac{\sin B}{70} = \frac{\sin 83°10'}{80}$$

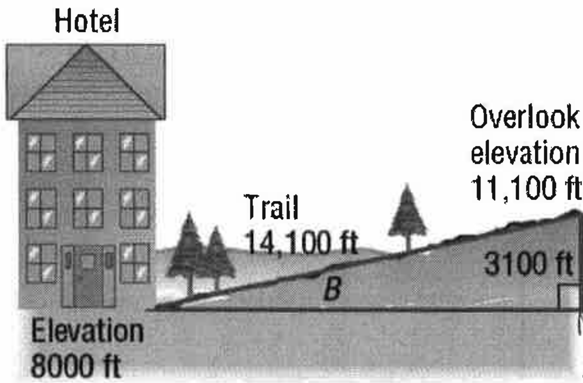
$$B \approx 60°19'$$

$$C = 180 - A - B = 36°31'$$

$$\frac{\sin 36°31'}{c} = \frac{\sin 83°10'}{80}$$

$$c \approx 47.95$$

A straight trail leads from the Alpine Hotel, elevation 8000 feet, to a scenic overlook, elevation 11,100 feet. The length of the trail is 14,100 feet. What is the inclination (grade) of the trail? That is, what is the angle B in Figure 4?



right △

$$\sin B = \frac{3100}{14100}$$

$$B \approx 12°42'$$

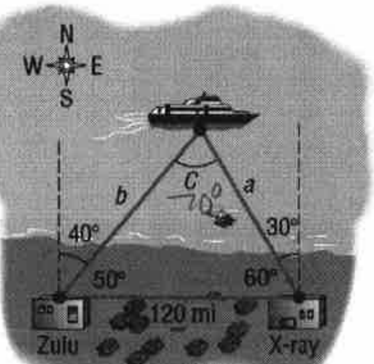
The inclination of the trail is 12°42′

Coast Guard Station Zulu is located 120 miles due west of Station X-ray. A ship at sea sends an SOS call that is received by each station. The call to Station Zulu indicates that the bearing of the ship from Zulu is N40°E (40° east of north). The call to Station X-ray indicates that the bearing of the ship from X-ray is N30°W (30° west of north).

(a) How far is each station from the ship?

(b) If a helicopter capable of flying 200 miles per hour is dispatched from the nearest station to the ship, how long will it take to reach the ship?

$\angle C = 70°$



a) $\dfrac{\sin 70°}{120} = \dfrac{\sin 60°}{b}$ $b \approx 110.59$ miles to Zulu

$\dfrac{\sin 50°}{a} = \dfrac{\sin 70°}{120}$ $a = 97.82$ miles to X-ray

b) $\dfrac{200\text{mi}}{1\text{hr.}} = \dfrac{97.82\text{mi}}{x}$ $x = .489\ldots$ hrs.

X-ray is closest station

≈ 29 minutes

A motorized sailboat leaves Naples, Florida, bound for Key West, 150 miles away. Maintaining a constant speed of 15 miles per hour, but encountering heavy crosswinds and strong currents, the crew finds, after 4 hours, that the sailboat is off course by 20°.

(a) How far is the sailboat from Key West at this time?

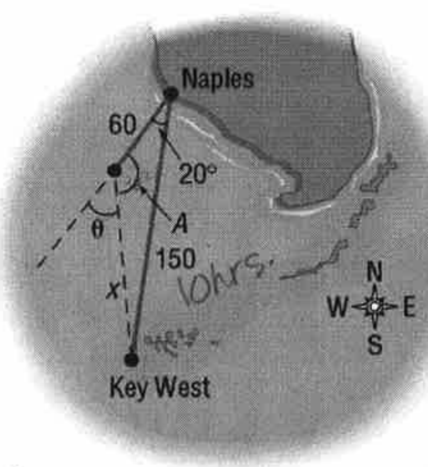
(b) Through what angle should the sailboat turn to correct its course?

(c) How much time has been added to the trip because of this? (Assume that the speed remains at 15 miles per hour.)

a) $x^2 = 60^2 + 150^2 - 2 \cdot 60 \cdot 150 \cos 20°$

$x^2 \approx 9185.53\ldots$

$\boxed{x \approx 95.84 \text{ miles}}$



b) $\dfrac{\sin A}{150} = \dfrac{\sin 20°}{95.84\ldots}$

$A = 32°22'$

Turn through: $180 - 32°22' = \boxed{147°38'}$

$\dfrac{150}{x} = \dfrac{15}{1}$ $x = 10$ hrs

$\dfrac{15\text{ mi}}{1\text{hr}} = \dfrac{60\text{mi}}{x}$ $x = 4$ hrs

c) $\dfrac{15\text{ mi}}{1\text{ hr.}} = \dfrac{95.84\text{mi}}{x}$

$6.39\text{hrs} + 4\text{ hrs} = 10.39\text{ hrs.}$ so 0.39 hrs. or ≈ 23 min. longer