

Reference Angles

<http://www.mathopenref.com/reference-angle.html>

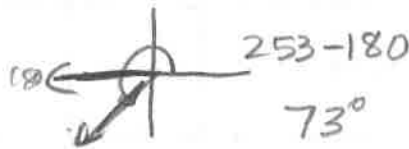
A reference angle is a positive acute angle. To find a reference angle, graph the given angle in standard position and then find the measure of the angle between the terminal side and the x-axis. Always the x-axis and never the y-axis.

Find the reference angle for each of the following.

1.  $127^\circ$



2.  $253^\circ$



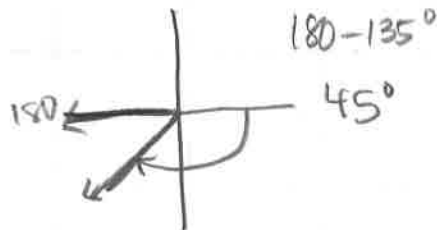
3.  $342^\circ$



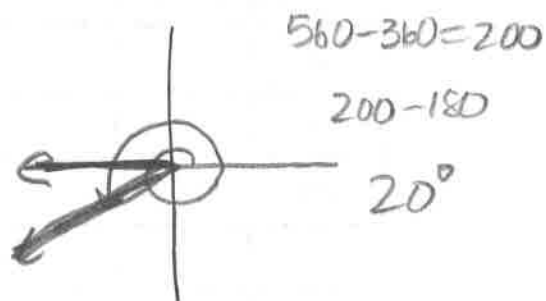
4.  $72^\circ$



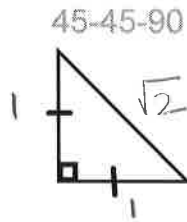
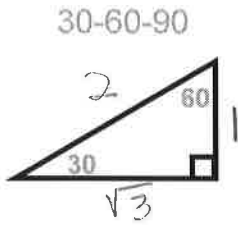
5.  $-135^\circ$



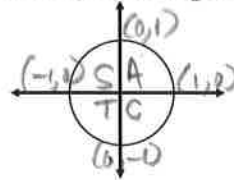
6.  $560^\circ$



### Exact Values for Special Angles



Quadrantal Angles



unit circle  
r = 1

$$(x, y) = (\cos \theta, \sin \theta)$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \text{ or } \frac{y}{x}$$

	Angle	sin	cos	tan	cot	sec	csc
	0°	0	1	0	und.	1	und.
Quad I	30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$	2
	45°	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
	60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	2	$\frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$
	90°	1	0	und.	0	und.	1
Quad II	120° <small>ref. = 60°</small>	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	$-\frac{\sqrt{3}}{3}$	-2	$\frac{2\sqrt{3}}{3}$
	135° <small>45°</small>	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1	-1	$-\sqrt{2}$	$\sqrt{2}$
	150° <small>30°</small>	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	$-\sqrt{3}$	$-\frac{2\sqrt{3}}{3}$	2
	180°	0	-1	0	und.	-1	und.
Quad III	210° <small>30°</small>	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$-\frac{2\sqrt{3}}{3}$	-2
	225° <small>45°</small>	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1	-1	$-\sqrt{2}$	$-\sqrt{2}$
	240° <small>60°</small>	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	$\frac{\sqrt{3}}{3}$	2	$-\frac{2\sqrt{3}}{3}$
	270°	-1	0	und.	0	und.	-1
Quad IV	300° <small>60°</small>	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$	$-\frac{\sqrt{3}}{3}$	2	$-\frac{2\sqrt{3}}{3}$
	315° <small>45°</small>	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1	-1	$\sqrt{2}$	$-\sqrt{2}$
	330° <small>30°</small>	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	$-\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	-2
	360°	0	1	0	und.	1	und.

← use 60° + ASTC

← use 45° + ASTC

← use 30° + ASTC