

5.8/5.9 Practice

Advanced Pre-Calculus

Key

1. 500 grams of a certain radioactive material decays to 450 grams in 1000 years. Find how long it would take to become 250 grams (in other words, find the half-life).

$$\frac{450}{500} = \frac{500e^{k(1000)}}{500} \rightarrow \ln \frac{9}{10} = 1000k$$

$$\frac{9}{10} = e^{1000k}$$

$$k = -.0001$$

$$250 = 500e^{-.0001t}$$

$$\frac{1}{2} = e^{-.0001t}$$

$$\frac{\ln \frac{1}{2}}{k} = \frac{-.0001t}{k} \rightarrow t \approx 6578.81 \text{ years}$$

2. In exercise 1, find the amount left after 2000 years.

$$A(2000) = 500 e^{-.0001(2000)}$$

$$= 405.0 \text{ grams}$$

3. A typical nuclear power plant produces about 10 pounds of Krypton-85 per year. The half-life of Krypton-85 is 11 years. How long must the Krypton-85 be contained so that only 0.1 pounds remain?

$$\frac{1}{2} = e^{k(11)}$$

$$\ln \frac{1}{2} = 11k$$

$$k \approx -.0630$$

$$\frac{.1}{10} = \frac{10e^{-.0630t}}{10}$$

$$.01 = e^{-.0630t}$$

$$\frac{\ln .01}{k} = \frac{-.0630t}{k} \rightarrow t \approx 73.08 \text{ years}$$

4. A culture of 300 bacteria increases to 2000 bacteria in 36 hours. How long would it take to become 600 bacteria (in other words, what is its doubling time)?

$$\frac{2000}{300} = \frac{300e^{k(36)}}{300} \rightarrow \ln \frac{20}{3} = 36k$$

$$\frac{20}{3} = e^{36k}$$

$$k \approx .0527$$

$$600 = 300e^{.0527t}$$

$$2 = e^{.0527t}$$

$$\frac{\ln 2}{k} = \frac{.0527t}{k} \rightarrow t \approx 13.15 \text{ hours}$$

5. A culture of 450 bacteria increases to 3000 bacteria in 4 days. How much bacteria would be present after 7 days?

$$\frac{3000}{450} = \frac{450e^{k(4)}}{450} \rightarrow k = .4743$$

$$\frac{20}{3} = e^{4k}$$

$$\ln \frac{20}{3} = 4k$$

$$N(7) = 450e^{.4743(7)} \approx 12447 \text{ bacteria}$$

6. A cake is removed from an oven when its temperature is 375°F in a room with a constant temperature of 72°F. It takes 5 minutes for it to reach 345°F. How long will it take for the cake to cool down to 150°F?

$$345 = 72 + (375 - 72)e^{k(5)}$$

$$\frac{273}{303} = \frac{303e^{5k}}{303}$$

$$\frac{273}{303} = e^{5k}$$

$$\ln \frac{273}{303} = 5k$$

$$k \approx -.0209$$

$$150 = 72 + (375 - 72)e^{-.0209t}$$

$$\frac{78}{303} = \frac{303e^{-.0209t}}{303}$$

$$\frac{78}{303} = e^{-.0209t}$$

$$\frac{\ln \frac{78}{303}}{k} = \frac{-.0209t}{k} \rightarrow t \approx 65.08 \text{ minutes}$$