Direct Variation:

1. If y varies directly as x and $y = 25$ when $x = 15$, find x when $y = 40$.

$y = kx$

$\frac{25}{15} = \frac{k(15)}{15}$

$\frac{5}{3} = k$

$y = \frac{5}{3}x$

$40 = \frac{5}{3}x$

$x = 40 \cdot \frac{3}{5}$ $\boxed{x = 24}$

2. If y varies directly as x and $y = 15$ when $x = 10$, find y when $x = 14$.

$y = kx$

$\frac{15}{10} = \frac{k(10)}{10}$

$1.5 = k$

$y = 1.5x$

$y = (1.5)(14)$

$\boxed{y = 21}$

3. a is directly proportional to the cube of b. If $a = 10$ when $b = 2$, find a when $b = 4$.

$a = kb^3$

$10 = k(2)^3$

$\frac{10}{8} = \frac{k(8)}{8}$

$k = 5/4$

$a = \frac{5}{4}b^3$

$= \frac{5}{4}(4)^3$

$\boxed{a = 80}$

4. y is directly proportional to the square of x. If $y = 12$ when $x = 4$, find y when $x = 6$.

$y = kx^2$

$12 = k(4)^2$

$\frac{12}{16} = \frac{k(16)}{16}$

$\frac{3}{4} = k$

$y = \frac{3}{4}x^2$

$= \frac{3}{4}(6)^2$

$= \frac{3}{4}(36)$

$\boxed{y = 27}$

Inverse Variation:

5. If y varies inversely as q and $y = 22$ when $q = 6$, find q when $y = 15$.

$y = \frac{k}{q}$

$22 = \frac{k}{6}$

$k = 6 \cdot 22 = 132$

$y = \frac{132}{q}$

$15 = \frac{132}{q}$

$\frac{15q}{15} = \frac{132}{15}$

$q = 8.8$

6. If c is inversely proportional to d and $c = 2$ ~~when $d =$~~ 3.6, ...

$c = \frac{k}{d}$

$2 = \frac{k}{3.6}$

$k = (3.6)(2)$

$k = 7.2$

$c = \frac{7.2}{d}$

a. find c when $d = 4.5$

$c = \frac{7.2}{4.5}$

$c = 1.6$

b. find d when ~~$c =$~~ $c = 6.4$

$6.4 = \frac{7.2}{d}$

$\frac{6.4d}{6.4} = \frac{7.2}{6.4}$

$d = 1.125$

Joint Variation:

7. If r varies jointly as s and t and inversely as u, and $r = 18$ when $s = 2$, $t = 3$, and $u = 4$, find s when $r = 6$, $t = 2$, and $u = 4$.

$r = k\frac{s \cdot t}{u}$

$18 = k\frac{(2)(3)}{4}$

$18 = k\left(\frac{3}{2}\right)$

$k = 18 \cdot \frac{2}{3}$

$k = 12$ constant of variation

$r = 12\frac{st}{u}$ eq.

$6 = 12\frac{s(2)}{4}$

$6 = \frac{24}{4}s$

$\frac{6}{6} = \frac{6}{6}s$

$s = 1$

#8 is below

8. Suppose r varies jointly as t and s, and inversely as the square of v. When $t = 3$, $s = 18$, and $v = 5$, $r = 3.78$. Find r when $t = 4$, $s = 12$, and $v = 4$.

$r = k\frac{t \cdot s}{v^2}$

$3.78 = k\frac{(3)(18)}{(5)^2}$

$3.78 = k\left(\frac{54}{25}\right)$

$k = 3.78\left(\frac{25}{54}\right)$

$k = 1.75$

$r = 1.75\frac{t \cdot s}{v^2}$

$r = 1.75\frac{(4)(12)}{4^2}$

$= 1.75\frac{(48)^3}{16}$

$r = 5.25$