Solving Rational Equations 8.6

When solving rational equations, you need to find the *lowest common denominator*.

Example 1

$\frac{1}{x}+\frac{1}{2x}=\frac{1}{3}$ What is the LCD for this problem?

LCD: $6x$ Any domain restrictions??? $x\neq 0$

The long, drawn-out way of solving by getting a common denominator first:

$\left(\frac{6}{6}\right)\frac{1}{x}+\left(\frac{3}{3}\right)\frac{1}{2x}=\left(\frac{2x}{2x}\right)\frac{1}{3}$

$\frac{6}{6x}+\frac{3}{6x}=\frac{2x}{6x}$

$\frac{9}{6x}=\frac{2x}{6x}$ ← If bottoms are = tops are =

$9=2x$

$\frac{9}{2}=x$

The shorter way to clear denominators: **Mult each term of the equation by LCD**

$6x\left(\frac{1}{x}\right)+6x\left(\frac{1}{2x}\right)=6x\left(\frac{1}{3}\right)$

$6+3=2x$

$9=2x$

$\frac{9}{2}=x$

Isn't this easier??!!??

DID YOU CHECK YOUR ANSWER(S)?

Example 2 $\frac{x}{x-3}+\frac{2x}{x+3}=\frac{18}{x^2-9}$ What's the LCD? $(x+3)(x-3)$

Domain restrictions?? $x\neq\pm 3$

$(x+3)(x-3)\frac{x}{(x-3)}+(x+3)(x-3)\frac{2x}{(x+3)}=\frac{18}{(x+3)(x-3)}(x+3)(x-3)$

$(x+3)x+(x-3)2x=18$

$x^2+3x+2x^2-6x=18$

$3x^2-3x=18$

$3x^2-3x-18=0$

$3(x^2-x-6)=0$

$3(x-3)(x+2)=0$

$x-3=0$ or $x+2=0$

$x=3$ or $x=-2$

DR $x\neq 3$ $x=3$ YIKES! Pay attention! not possible

DID YOU CHECK YOUR ANSWER(S)

SO $x=-2$

Example 3 $\frac{x}{x+2}+\frac{8}{x^2-4}=\frac{2}{x-2}$ LCD? Domain restrictions?

LCD $(x+2)(x-2)$

DR $x\neq\pm 2$

$(x+2)(x-2)\frac{x}{x+2}+(x+2)(x-2)\frac{8}{(x+2)(x-2)}=\frac{2}{x-2}(x+2)(x-2)$

$(x-2)x+(8)=2(x+2)$

$x^2-2x-8=2x+4$

$-2x\quad -4\quad -2x-4$

$x^2-4x-12=0$

$(x-6)(x+2)=0$

$x-6=0$ or $x+2=0$

$x=6$ $x=-2$ not possible

DID YOU CHECK YOUR ANSWER(S)?

SO $x=6$

Example 4

$$\frac{x}{x+5}+2=\frac{-42}{x^2-25}$$

LCD?Domain restrictions?

$(x+5)(x-5)$ $x\neq\pm5$

$$((x+5)(x-5))\left(\frac{x}{x+5}\right)+((x+5)(x-5))2=\frac{-42}{(x+5)(x-5)}((x+5)(x-5))$$

$(x-5)x+(x^2-25)2=-42$

$x^2-5x+2x^2-50=-42$

$3x^2-5x-50=-42$

$+42 \quad +42$

$3x^2-5x-8=0$

$(3x-8)(x+1)=0$

$3x-8=0$ or $x+1=0$

$3x=8$ $x=-1$

$x=\frac{8}{3}$ or

-24, -8, $+3$, -5

	x	$+1$
$3x$	$3x^2$	$+3x$
-8	$-8x$	-8

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Ex.5 $\dfrac{2}{x+3}-\dfrac{3}{4-x}=\dfrac{2x-2}{x^2-x-12}$

careful!

LCD

$(x-4)(x+3)$

DR $x\neq4,-3$

$(x-4)(x+3)\dfrac{2}{(x+3)}+\dfrac{-3(x-4)(x+3)}{-(x-4)}=\dfrac{2x-2}{(x-4)(x+3)}(x-4)(x+3)$

$(x-4)2+3(x+3)=2x-2$

$2x-8+3x+9=2x-2$

$5x+1=2x-2$

$-2x \quad -1 \quad -2x-1$

$3x=-3$

$x=-1$

DID YOU CHECK YOUR ANSWER(S)?