Natural Logarithms

$\boxed{y = \log_e x \text{ is the inverse of natural exponential function } y = e^x}$

Some information about *e.*

1. The natural base, e, is used to estimate the ages of artifacts.
2. The value of $1\left(1+\dfrac{1}{n}\right)^n$ when n is large approaches 2.71828...= *e*
3. The 18th century Swiss mathematician, Leonhard Euler. "discovered" the natural exponential, so named after him. (the "e")
4. *e* is an irrational number; it's decimal value is approximately 2.71828182845904.

$\boxed{y = \log_e x \text{ is abbreviated } y = \ln x \text{ ("y equals the natural log of x")}}$

Warning: If you eventually progress to much-more advanced mathematics, you may find that sometimes "*log*(*x*)" means the base-*e* log or even base-2 log, rather than the common log.

Write each expression as a single natural logarithm.

1. $\ln 32 - \ln 8$

$= \ln \frac{32}{8}$

$= \ln 4$

2. $3\ln 6 + \ln 5$

$= \ln 6^3 + \ln 5$

$= \ln(6^3 \cdot 5)$

$= \ln 1080$

Solve each equation. Check your answers.

3. $\ln(x-4)=5$

$\log_e(x-4)=5$

$e^5=x-4$

$x=e^5+4$ exact answer

decimal approx.

$x \approx 152.41$

Check in calc

$\ln(e^5+4-4)\stackrel{?}{=}5$ ✓

4. $\ln 2x + \ln 3 = 2$

$\ln(3 \cdot 2x)=2$

$\ln 6x = 2$

go to exp. form

$\frac{e^2}{6}=\frac{6x}{6}$

exact $x=\frac{e^2}{6}$

decimal approx. $x \approx 1.23$

Use natural logarithms to solve each equation.

5. $e^{x-2}=12$ write in log form

$\log_e 12 = x-2$

$\ln 12 = x-2$

exact $x=\ln 12 + 2$

decimal approx $x \approx 4.48$

6. $e^{3x}+5=15$

$\quad\;\; -5 \quad -5$ isolate var. first

$e^{3x}=10$

$\log_e 10 = 3x$ go to log form

$\frac{\ln 10}{3}=\frac{3x}{3}$

exact $x=\frac{\ln 10}{3}$

decimal approx $x \approx 0.77$

Simplify each expression.

7. $\ln e = \log_e e^1$

$= 1$

8. $\ln e^3 = \log_e e^3$

$= 3$

9. $\frac{\ln e^2}{6}$

$= \frac{2}{6} = \boxed{\frac{1}{3}}$