Population and Money doubles/triples

Example 1:
Mary invested $800 at 3.5% compounded quarterly. $n=4$

at the end

a) How long will it take for her investment to double in value? Round to nearest tenth.

$$A(t)=P\left(1+\frac{r}{n}\right)^{nt}$$

$$\frac{1600}{800}=\frac{800}{800}\left(1+\frac{.035}{4}\right)^{(4)t}$$

$$2=\left(1+\frac{.035}{4}\right)^{4t}$$

$$\log 2=\log\left(1+\frac{.035}{4}\right)^{4t}$$

$$\log 2=4t\log\left(1+\frac{.035}{4}\right)$$

$$\frac{4t}{4}=\frac{\frac{\log 2}{\log\left(1+\frac{.035}{4}\right)}}{4}$$

$$\boxed{t\approx 19.9\text{ years}}$$

b) How long will it take for her investment to triple in value? Round to nearest tenth.

$$\frac{2400}{800}=\frac{800}{800}\left(1+\frac{.035}{4}\right)^{4t}$$

$$3=\left(1+\frac{.035}{4}\right)^{4t}$$

$$3=(1.00875)^{4t}$$

$$\ln 3=\ln(1.00875)^{4t}$$

$$\frac{\ln 3}{\ln(1.00875)}=\frac{4t\,\cancel{\ln(1.00875)}}{\cancel{\ln(1.00875)}}$$

$$\frac{4t}{4}=\frac{\frac{\ln(3)}{\ln(1.00875)}}{4}$$

Remember to end parentheses in calc.

$$\boxed{t\approx 31.5\text{ years}}$$

Example 2:
The United States public debt, in billions of dollars, has been estimated with the model $y=0.051517(1.1306727)^x$.
The exponent represents the number of years since 1990.

a) How long will it take to double the public debt?(Sometimes called the doubling time.)

$$2=(1.1306727)^x$$

$$\log 2=\log(1.1306727)^x$$

$$\frac{\log 2}{\log(1.1306727)}=\frac{x\,\cancel{\log(1.1306727)}}{\cancel{\log(1.1306727)}}$$

$$\boxed{x\approx 5.6\text{ years}}$$

a) How long will it take to triple the public debt?

$$3=(1.1306727)^x$$

exp form. you could go to log form

$$\log_{1.1306727}(3)=x$$

$$\frac{\log 3}{\log 1.136727}=x$$

change of base formula.

$$\boxed{x\approx 8.9\text{ years}}$$

Example 3

A certain bacteria grows at a rate of 31.5% per hour. $B = I(1 + .315)^h$

a) How long will it take to double the population?

$2 = (1.315)^h$

go to log form $\log_{1.315} 2 = h$

change of base formula $h = \frac{\log 2}{\log 1.315}$

$\boxed{h \approx 2.5 \text{ hours}}$

b) How long will it take to triple the population?

$3 = 1.315^h$

take log of each side $\log 3 = \log 1.315^h$

power property $\frac{\log 3}{\log 1.315} = \frac{h \cancel{\log 1.315}}{\cancel{\log 1.315}}$

$\boxed{h \approx 4.0 \text{ hours}}$