

Solving Radical Equations Continued (6.5)

Solve. Check for extraneous roots.

CHECK:

$$\text{Ex. 1 } \sqrt{x+7} - 5 = x$$

Isolate the radical first

$$\sqrt{x+7} = x+5$$

$$(\sqrt{x+7})^2 = (x+5)^2$$

$$\begin{array}{r} x+7 = x^2 + 10x + 25 \\ -x \quad -7 \quad \quad -x \quad -7 \end{array}$$

$$0 = x^2 + 9x + 18$$

$$0 = (x+3)(x+6)$$

$$x+3=0 \quad x+6=0$$

$$x=-3 \quad \text{or} \quad x=-6$$

CHECK!

$$\sqrt{x+7} - 5 = x$$

$$\sqrt{(-3)+7} - 5 = (-3) \quad \sqrt{(-6)+7} - 5 = (-6)$$

$$\sqrt{4} - 5 = -3 \quad \sqrt{1} - 5 = -6$$

$$2 - 5 = -3 \quad 1 - 5 = -6$$

$$-3 = -3 \checkmark \quad -4 \neq -6$$

Answer:

$$\{-3\}$$

UGH! Two radicals

$$\text{Ex. 2 } \sqrt{x+5} = \sqrt{2x}$$

But...each is isolated so it's easy!!

$$(\sqrt{x+5})^2 = (\sqrt{2x})^2$$

$$\begin{array}{r} x+5 = 2x \\ -x \quad \quad -x \end{array}$$

$$5 = x$$

Should we bother to check?

YES!!!

Answer:
{5}

CHECK:

$$\sqrt{x+5} = \sqrt{2x}$$

$$\sqrt{5+5} = \sqrt{2(5)}$$

$$\sqrt{10} = \sqrt{10} \checkmark$$

★ **LOVE this one...hint**

Ex.3 $\sqrt{2x+1} - \sqrt{x} = 1$ You cannot isolate both of these at the same time, so choose one to isolate

$(\sqrt{2x+1} - \sqrt{x})^2 = 1^2$
 LOOK! This is better. We only have one radical now...isolate it!
 $2x+1 = x + 2\sqrt{x} + 1$
 $x = 2\sqrt{x}$
 $x^2 = (2\sqrt{x})^2 \rightarrow 2\sqrt{x} \cdot 2\sqrt{x} = 4\sqrt{x^2} = 4x$

$x^2 = 4x$ Now, solve...how many possible answers?

$x^2 - 4x = 0$
 $x(x-4) = 0$
 $x = 0$ or $x - 4 = 0$

Should we check?

ABSOLUTELY!!

$x=0$
 $\sqrt{2(0)+1} - \sqrt{0} = 1$
 $\sqrt{1} - 0 = 1$
 $1 - 0 = 1$
 $1 = 1$ ✓

$x=4$
 $\sqrt{2(4)+1} - \sqrt{4} = 1$
 $\sqrt{9} - 2 = 1$
 $3 - 2 = 1$
 $1 = 1$ ✓

$x = \{0, 4\}$

Ex.4 $(x+5)^{\frac{2}{3}} = 4$

Really...this is a radical equation, if you wanted it to be!

$\sqrt[3]{(x+5)^2} = 4$

$\left((x+5)^{\frac{2}{3}}\right)^{\frac{3}{2}} = (4)^{\frac{3}{2}}$

Remember, we are just solving, so we don't have to write this in radical form. Use your exponent rules.

$(\sqrt[3]{(x+5)^2})^3 = (4)^3$

$(x+5)^2 = \sqrt[3]{64}$

$x+5 = \pm 8$

$x = -5 \pm 8$
 $x = -5 + 8 = 3$
 $x = -5 - 8 = -13$

CHECK!!!

$x=3$
 $(3+5)^{\frac{2}{3}} = 4$
 $(8)^{\frac{2}{3}} = 4$
 $(\sqrt[3]{8})^2 = 4$
 $(2)^2 = 4$
 $4 = 4$

$x=-13$
 $(-13+5)^{\frac{2}{3}} = 4$
 $(-8)^{\frac{2}{3}} = 4$
 $(\sqrt[3]{-8})^2 = 4$
 $(-2)^2 = 4$
 $4 = 4$

check:

$\sqrt[3]{(x+5)^2} = 4$

$x=3$
 $\sqrt[3]{(3+5)^2} = 4$
 $\sqrt[3]{8^2} = 4$
 $\sqrt[3]{64} = 4$
 $4 = 4$ ✓

$x=-13$
 $\sqrt[3]{(-13+5)^2} = 4$
 $\sqrt[3]{(-8)^2} = 4$
 $\sqrt[3]{64} = 4$
 $4 = 4$ ✓

$$\text{Ex.5} \quad \frac{3(x-2)^{\frac{3}{4}}}{3} = \frac{24}{3}$$

Divide by 3 first, then...

$$(x-2)^{\frac{3}{4}} = 8$$

$$\left((x-2)^{\frac{3}{4}}\right)^{\frac{4}{3}} = (8)^{\frac{4}{3}}$$

$$x-2 = (\sqrt[3]{8})^4$$

$$x-2 = 16$$

$$x = 18 \quad \text{CHECK!}$$

$$3(x-2)^{\frac{3}{4}} = 24$$

$$3(18-2)^{\frac{3}{4}} = 24$$

$$3(16)^{\frac{3}{4}} = 24$$

$$3(\sqrt[4]{16})^3 = 24$$

$$3(2)^3 = 24$$

$$3(8) = 24$$

$$24 = 24 \quad \checkmark$$