Notes for fractional exponents

If m and n are integers and $n \neq 0$, then

$$a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$$

Simplify

1. $64^{\frac{2}{3}}$

$(\sqrt[3]{64})^2$

$(4)^2$

$= \boxed{16}$

2. $64^{\frac{3}{2}}$

$(\sqrt{64})^3$

$(8)^3$

$= \boxed{512}$

3. $216^{\frac{1}{3}}$

$(\sqrt[3]{216})^1 = \sqrt[3]{216}$

$= \boxed{6}$

4. a) $-64^{\frac{2}{3}}$

$-(\sqrt[3]{64})^2$

$-(4)^2 = \boxed{-16}$

b) $(-64)^{\frac{2}{3}}$

$= (\sqrt[3]{-64})^2$

$= (-4)^2 = \boxed{16}$

5. $4^{2.5}$

$= 4^{\frac{5}{2}}$

$= (\sqrt{4})^5$

$= 2^5$

$= \boxed{32}$

6. $36^{-\frac{1}{2}}$

$= \frac{1}{36^{1/2}}$

$= \frac{1}{\sqrt{36}}$

$= \boxed{\frac{1}{6}}$

7. $81^{-\frac{3}{4}}$

$= \frac{1}{81^{3/4}}$

$= \frac{1}{(\sqrt[4]{81})^3} = \frac{1}{3^3} = \boxed{\frac{1}{27}}$

Simplify

8. $7^{\frac{1}{2}} \bullet 7^{\frac{1}{2}}$

$= 7^{\frac{1}{2}+\frac{1}{2}}$

$= 7^1$

$= \boxed{7}$

9. $2^{\frac{1}{2}} \bullet 32^{\frac{1}{2}}$

need same base

$2^{\frac{1}{2}} \cdot (2^5)^{1/2} = 2^{\frac{1}{2}}\, 2^{\frac{5}{2}}$

$= 2^{\frac{1}{2}+\frac{5}{2}} = 2^{\frac{6}{2}} = 2^3$

$= \boxed{8}$

10. $\left(\frac{x^{-\frac{2}{3}}}{y^{-\frac{1}{3}}}\right)^{15}$

$= \frac{x^{-\frac{2}{\cancel{3}}\cdot\frac{\cancel{15}\,5}{1}}}{y^{-\frac{1}{\cancel{3}}\cdot\frac{\cancel{15}\,5}{1}}}$

$= \frac{x^{-10}}{y^{-5}} = \boxed{\frac{y^5}{x^{10}}}$

11. $y^{\frac{1}{2}} \bullet y^{\frac{3}{10}}$

$= y^{\frac{1}{2}+\frac{3}{10}}$

$= y^{\frac{5}{10}+\frac{3}{10}} = y^{\frac{8}{10}}$

$= y^{4/5}$

or

$\sqrt[5]{y^4}$

12. $\dfrac{x^{\frac{1}{2}} \bullet y^{-\frac{1}{3}}}{x^{\frac{3}{4}} \bullet y^{\frac{1}{2}}}$

$= x^{\frac{1}{2}-\frac{3}{4}}\, y^{-\frac{1\cdot 2}{3\cdot 2}-\frac{1\cdot 3}{2\cdot 3}}$

$= x^{\frac{2}{4}-\frac{3}{4}}\, y^{-\frac{2}{6}-\frac{3}{6}}$

$= x^{-\frac{1}{4}}\, y^{-5/6}$

$= \frac{1}{x^{\frac{1}{4}}\, y^{5/6}}$

or

$\frac{1}{\sqrt[4]{x}\,\sqrt[6]{y^5}}$

Write in exponential form

13. $\sqrt{5xy^3}$

$= (5xy^3)^{1/2}$

$= 5^{1/2} x^{1/2} (y^3)^{1/2}$

$= \boxed{5^{1/2} x^{1/2} y^{3/2}}$

14. $\sqrt[6]{10x^2y^3}$

$= (10x^2y^3)^{\frac{1}{6}}$

$= 10^{1/6}(x^2)^{1/6}(y^3)^{1/6}$

$10^{\frac{1}{6}} x^{\frac{2}{6}} y^{\frac{3}{6}}$ simplify!

$\boxed{10^{\frac{1}{6}} x^{\frac{1}{3}} y^{\frac{1}{2}}}$

NEW LESSON PART 1---SOLVING RADICAL EQUATIONS

Solve. Check for extraneous solutions.

***Ex.*1** $3+\sqrt{2x-3}=8$

$-3 \qquad -3$

Isolate the radical (get the radical all alone)

$(\sqrt{2x-3})^2 = (5)^2$

$2x-3=25$

$+3 \quad +3$

$\frac{2x}{2} = \frac{28}{2}$

$\boxed{x=14}$

Always check in the **original** problem

Check:

$3+\sqrt{2x-3}=8$

$3+\sqrt{2(14)-3} \stackrel{?}{=} 8$

$3+\sqrt{28-3} = 8$

$3+\sqrt{25} = 8$

$3+5 = 8$

$8=8$ ✓

*Ex.*2 $17 - 4\sqrt[3]{x-1} = 5$

Isolate radical

$$\begin{aligned} 17 - 4\sqrt[3]{x-1} &= 5 \\ -17 \qquad\quad &\;\; -17 \end{aligned}$$

$$\frac{-4\sqrt[3]{x-1}}{-4} = \frac{-12}{-4}$$

$$\left(\sqrt[3]{x-1}\right)^3 = (3)^3$$

$$\begin{aligned} x - 1 &= 27 \\ +1 &\quad +1 \end{aligned}$$

$$\boxed{x = 28}$$

Check:

$17 - 4\sqrt[3]{x-1} = 5$

$17 - 4\sqrt[3]{28-1} = 5$

$17 - 4\sqrt[3]{27} = 5$

$17 - 4(3) = 5$

$17 - 12 = 5$

$5 = 5$ ✓