Chapter 11.1 Notes:PERMUTATIONS

Definition of PERMUTATION: an arrangement of objects ***IN A SPECIFIC ORDER***

For example, listening to jazz,classical, then rock music would be DIFFERENT than listening to classical, rock and then jazz music.

Permutations of n objects ..... n! $n! = n \cdot (n-1) \cdot (n-2) \cdots 1$

Ex. 1: In a family of 5 children, each child must have a bite of food before a child gets a second bite. How many different orders can the parent feed the child?

child $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5! = 120$

Ex. 2: On a baseball team, 9 players are designated as the starting lineup. Before the game, the manager announces the order in which the 9 players will bat. How many different orders are possible?

$9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 9! = 362,880$

Permutations of n objects taken r at a time:

$$P(n,r) = {}_nP_r = \frac{n!}{(n-r)!} \quad \textit{where } n > r$$

EX. 3: Find the number of ways to listen to 4 CDs from a selection of 7 CDs

$P(7,4) = {}_7P_4 = \frac{7!}{(7-4)!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} = 7 \cdot 6 \cdot 5 \cdot 4 = 840 \text{ ways}$

Ex. 4: Find the number of ways (permutations) of 9 objects taken 6 at a time.

$P(9,6) = {}_9P_6 = \frac{9!}{(9-6)!} = \frac{9!}{3!} = 60,480$

Ex. 5a : Calculate ${}_6P_2$

$= \frac{6!}{(6-2)!} = \frac{6!}{4!} = \frac{6 \cdot 5 \cdot 4!}{4!} = 30$

Ex. 5b : Calculate ${}_6P_4$

$\frac{6!}{2!} = 6 \cdot 5 \cdot 4 \cdot 3 = 30 \cdot 12 = 360$

Ex. 6 Make up a word problem for #5a

Ex. 7 Find the number of ways to watch 4 videos from a selection of 12 videos.

${}_{12}P_4 = \frac{12!}{(12-4)!} = \frac{12!}{8!} = 11,880 \text{ ways}$

Permutations with identical objects:

The number of DISTINCT permutations of n objects with r identical objects is given by: $\frac{n!}{r!}$

The number of distinct permutations of n objects with r_1 identical objects, r_2 identical objects of another kind, r_3 identical objects of another kind.....and r_k identical objects of another kind is given by:

$$\frac{n!}{r_1!r_2!r_3!\ldots r_k!}$$

Ex. 6 Ms Wilson wants to arrange the letters N and two A s in as many different ways as she can. Please write down her options.

NAA
AAN
ANA

3 letters

$\frac{3!}{2!} = \frac{3 \cdot 2 \cdot 1}{2 \cdot 1} = 3$

Ex. 7 Find the number of permutations of the letters in each word:

a) Attic $= \frac{5!}{2!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = 60$

b) banana $= \frac{6!}{(3! \cdot 2!)} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 2 \cdot 1} = 60$

CIRCULAR PERMUTATION -----> (n-1)!

Ex. 8 In how many ways can you arrange 4 different candies on a circular tray?

$(4-1)! = 3! = 3 \cdot 2 \cdot 1 = 6$

Ex. 9: Eight principals are sitting around a circular table. In how many ways can they be arranged?

$7! = 5040$ ways.