5.7 PASCAL'S TRIANGLE. Notes

Look for a pattern to add 4 more rows.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1

Row	
0	1
1	1 1
2	1 2 1
3	1 3 3 1
4	1 4 6 4 1
5	1 5 10 10 5 1
6	1 6 15 20 15 6 1
7	1 7 21 35 35 21 7 1
8	1 8 28 56 70 56 28 8 1
9	1 9 36 84 126 126 84 36 9 1
10	1 10 45 120 210 252 210 120 45 10 1

1. Remember that $\binom{n}{r} = {}_nC_r$. Find each of the following. $\frac{n!}{(n-r)!\,r!}$

$\binom{6}{0}$ $\binom{6}{1}$ $\binom{6}{2}$ $\binom{6}{3}$ $\binom{6}{4}$ $\binom{6}{5}$ $\binom{6}{6}$

${}_6C_0 = \frac{6!}{6!0!} = 1$ $\quad \frac{6!}{5!1!}$ $\frac{6!}{4!2!}$ $\frac{6!}{3!3!}$ $\frac{6!}{4!2!}$ $\frac{6!}{5!1!}$ $\frac{6!}{6!}$

1 6 15 20 15 6 1

2. Look at Pascal's Triangle. Which row has these numbers?

Row 6

3. Find each of the following:

$\binom{3}{0}$ $\binom{3}{1}$ $\binom{3}{2}$ $\binom{3}{3}$ Which row has these numbers?

1 3 3 1 3RD Row.

Binomial Theorem

Multiply and simplify the following.

copy

1. $(x+y)^0 = 1$
2. $(x+y)^1 = x+y$
3. $(x+y)^2 = x^2 + 2xy + y^2$
4. $(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$
5. $(x+y)^4 = x^4 + 4x^3y^1 + 6x^2y^2 + 4xy^3 + y^4$
6. $(x+y)^5 = x^5 + 5x^4y^1 + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$
7. $(x+y)^6 = x^6 + 6x^5y^1 + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6x^1y^5 + y^6$

Analyze carefully. When expanding $(x+y)^n$:

It's pascal's Δ

1) The powers of x decrease from n to 0
2) The powers of y increase from 0 to n
3) A given term the powers of x and
4) ~~the powers of y add up to n~~ n+1 terms
5) looks like pascal's Δ.

There is also a pattern in the coefficients. So, there must be an easier way to raise a binomial to a power. Write the coefficients of the polynomials in 1 through 7 below:

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 20 7 1

What does this look like? Pascal's Triangle!

So if you want the coefficient of the 12th term of $(x+y)^{15}$ you would

$_{15}C_{11}$ $\binom{15}{11} = \frac{15!}{11!4!}$

Binomial Theorem

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

Examples. Expand each of the following.

10th Row: 1 10 45 120 210 252 210 120 45 10 1

1) $(x+y)^{10}$

$1x^{10}\cancel{y^0} + 10x^9y^1 + 45x^8y^2 + 120x^7y^3 + 210x^6y^4$
$+ 252x^5y^5 + 210x^4y^6 + 120x^3y^7 + 45x^2y^8$
$+ 10x^1y^9 + 1\cancel{x^0}y^{10}$

1 1 1

Examples. Expand each of the following.

5th Row 1 5 10 10 5 1

2) $(x-y)^5$

$x^5 + 5x^4(-y)^1 + 10x^3(-y)^2 + 10x^2(-y)^3 + 5x^1(-y)^4$
$+ 1x^0(-y)^5$

$= \boxed{x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5}$

Examples. Expand each of the following.

4th Row 1 4 6 4 1

3) $(2x - 3y)^4$

$$1(2x)^4 + 4(2x)^3(-3y)^1 + 6(2x)^2(-3y)^2$$
$$+ 4(2x)^1(-3y)^3 + 1(2x)^0(-3y)^4$$
$$= 16x^4 - 96x^3y + 216x^2y^2$$
$$- 216xy^3 + 81y^4$$

4) Find the 7th term of $(x - y)^6$

$$\binom{6}{6}x^0(-y)^6$$
$$1(1)\ y^6$$
$$= y^6$$

5) Find the 6^{th} term of $(x+4)^9$

$$\binom{9}{5}(x^{9-5})(4)^5$$

$$126\,x^4(1024)$$

$$= \cancel{8}\ \boxed{129024x^4}$$