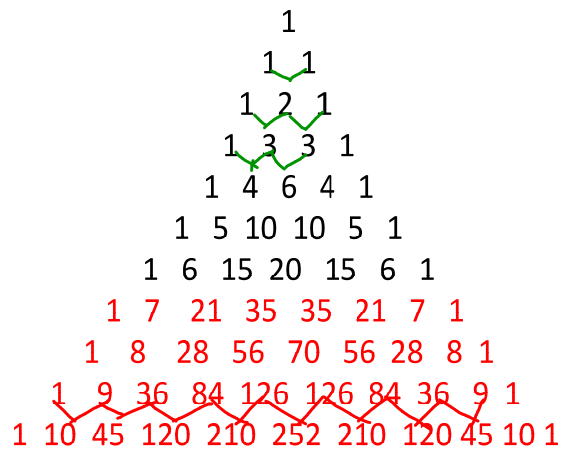


5.7 PASCAL'S TRIANGLE. Notes

Look for a pattern to add 4 more rows.



Row

0	1
1	1 1
2	1 2 1
3	1 3 3 1
4	1 4 6 4 1
5	1 5 10 10 5 1
6	1 6 15 20 15 6 1
7	1 7 21 35 35 21 7 1
8	1 8 28 56 70 56 28 8 1
9	1 9 36 84 126 126 84 36 9 1
10	1 10 45 120 210 252 210 120 45 10 1

1. Remember that  $\binom{n}{r} = {}_n C_r$ . Find each of the following.

$\binom{6}{0} = \frac{6!}{6! \cdot 0!} = 1$    
 $\binom{6}{1} = \frac{6!}{5! \cdot 1!} = 6$    
 $\binom{6}{2} = \frac{6!}{4! \cdot 2!} = 15$    
 $\binom{6}{3} = \frac{6!}{3! \cdot 3!} = 20$    
 $\binom{6}{4} = \frac{6!}{2! \cdot 4!} = 15$    
 $\binom{6}{5} = \frac{6!}{1! \cdot 5!} = 6$    
 $\binom{6}{6} = \frac{6!}{0! \cdot 6!} = 1$

2. Look at Pascal's Triangle. Which row has these numbers?

Row 6

3. Find each of the following:

$\binom{3}{0} = 1$    
 $\binom{3}{1} = 3$    
 $\binom{3}{2} = 3$    
 $\binom{3}{3} = 1$

Which row has these numbers?

3RD Row.

## Binomial Theorem

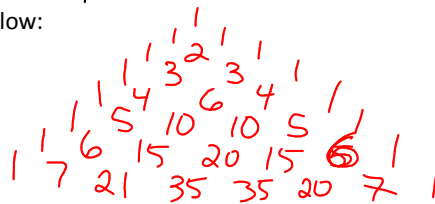
Multiply and simplify the following.

$$\begin{aligned}
 & \left. \begin{aligned}
 1. (x+y)^0 &= 1 \\
 2. (x+y)^1 &= x+y \\
 3. (x+y)^2 &= \underline{x^2} + \underline{2xy} + \underline{y^2} \\
 4. (x+y)^3 &= x^3 + 3x^2y + 3xy^2 + y^3 \\
 5. (x+y)^4 &= x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4 \\
 6. (x+y)^5 &= x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5 \\
 7. (x+y)^6 &= x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6
 \end{aligned} \right\} \text{copy}
 \end{aligned}$$

Analyze carefully. When expanding  $(x+y)^n$ :

- 1) *It's Pascal's Δ*  
The powers of  $x$  decrease from  $n$  to  $0$
- 2) The powers of  $y$  increase from  $0$  to  $n$
- 3) A given term the powers of  $x$  and  $y$
- 4) ~~the powers of  $y$  add up to  $n$~~
- 5)  $n+1$  terms  
*looks like Pascal's Δ*

There is also a pattern in the coefficients. So, there must be an easier way to raise a binomial to a power. Write the coefficients of the polynomials in 1 through 7 below:



What does this look like? Pascal's Triangle!

So if you want the coefficient of the 12<sup>th</sup> term of  $(x+y)^{15}$  you would

$${}^{15}C_{11} = \binom{15}{11} = \frac{15!}{11!4!}$$

### Binomial Theorem

$$(x+y)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} y^k$$

Examples. Expand each of the following.

10th Row: 1 10 45 120 210 252 210 120 45 10 1

1)  $(x + y)^{10}$

$$\begin{aligned} & 1x^{10}y^0 + 10x^9y^1 + 45x^8y^2 + 120x^7y^3 + 210x^6y^4 \\ & + 252x^5y^5 + 210x^4y^6 + 120x^3y^7 + 45x^2y^8 \\ & + 10x^1y^9 + 1x^0y^{10} \end{aligned}$$

1/1

Examples. Expand each of the following.

5th Row 1 5 10 10 5 1

2)  $(x - y)^5$

$$\begin{aligned} & x^5 + 5x^4(-y)^1 + 10x^3(-y)^2 + 10x^2(-y)^3 + 5x(-y)^4 \\ & + 1x^0(-y)^5 \\ = & \boxed{x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5} \end{aligned}$$

Examples. Expand each of the following.

4th Row 1 4 6 4 1

3)  $(2x - 3y)^4$

$$\begin{aligned} & \underline{1}(2x)^4 + \underline{4}(2x)^3(-3y)^1 + \underline{6}(2x)^2(-3y)^2 \\ & \quad + 4(2x)^1(-3y)^3 + \underline{1}(2x)^0(-3y)^4 \\ = & 16x^4 - 96x^3y + 216x^2y^2 \\ & \quad - 216xy^3 + 81y^4 \end{aligned}$$

4) Find the 7<sup>th</sup> term of  $(x - y)^6$

$$\begin{aligned} & \binom{6}{6} x^0 (-y)^6 \\ & \quad 1(1) y^6 \\ & \quad = y^6 \end{aligned}$$

5) Find the 6<sup>th</sup> term of  $(x+4)^9$

$$\binom{9}{5} (x^{9-5}) (4)^5$$

$$126 x^4 (1024)$$

$$= \boxed{129024 x^4}$$