## Conditional Probability

The conditional probability of event $B$, given event $A$, denoted by $P(B \mid A)$, is given by:

$$
\text { Probabality of } B \text { given } A \text {. }
$$



Example 1: Find the probability that a randomly selected student:
a) runs track given that the student is in shop. $P(T \mid S)=\frac{P(T \text { and } S)}{\frac{9}{23}}=\frac{\frac{9}{58}}{\frac{23}{58}}$
$=\frac{9}{58} \div \frac{23}{58}$
$=\frac{9}{58} \cdot \frac{58}{23}$
$=9 / 23$
$P(B \mid A)=\frac{P(A \text { and } B)}{P(A)} \quad$ where $\mathrm{P}(\mathrm{A}) \neq 0$

b) is in auto given that the student golfs. $P(A \mid G)=\frac{P(A \text { and } G)}{P(G)}=\frac{\frac{12}{46}\left(\cdot \frac{26}{46}\right)=\frac{12}{46}\left(\frac{10}{25}\right.}{46}$
c) $P(S T T)=\frac{9}{44}$
d) $P(G \mid A)=\frac{12}{33}$

$$
P(B \mid A)=\frac{P(A \text { and } B)}{P(A)} \quad \text { where } \mathrm{P}(\mathrm{~A}) \neq 0
$$

Example 2:
a) Given $P(B \mid A)=\frac{\mathbf{2}}{7}$ and $p(A$ and $B)=\frac{\mathbf{1}}{\mathbf{4}}$, find $P(A)$

$$
\begin{aligned}
P(B \mid A) & =\frac{P(A \text { and } B)}{P(A)} \\
\frac{2}{7} & =\frac{\frac{1}{4}}{x} \\
\frac{7}{2} \cdot \frac{2}{7} x & =\frac{1}{4} \cdot \frac{7}{2}
\end{aligned}
$$

$$
\theta(A)=\frac{7}{a^{8}} \quad \frac{2}{7}=\frac{\frac{1}{4}}{x}
$$

b) Given $P(A$ and $B)=\frac{2}{5}$ and $P(A)=\frac{1}{2}$ find $P(B \mid A)$

$$
A(B / 4)=\frac{\frac{2}{5}}{\frac{1}{2}} \cdot \frac{10}{10}=\frac{4}{5}
$$

Example 3:

$$
4+7+3+2=16
$$

A bag contains 4 red, 7 blue, 3 purple and 2 white marbles. On two consecutive draws, find the probability of:
a) drawing a red, then a blue with replacement $\frac{4}{16} \cdot \frac{7}{16}=\frac{1}{4} \cdot \frac{7}{16}=\frac{7}{64}$
b) drawing a red, then a blue without replacement $\frac{4}{16} \cdot \frac{7}{15}=\frac{1}{4} \cdot \frac{7}{15}=\frac{7}{60}$
c) drawing 2 purples without replacement

$$
\frac{3}{16} \cdot \frac{22}{15}=\frac{1}{40}
$$

d) drawing a blue, then a white with replacement 8

$$
\begin{aligned}
& \frac{7}{76} \cdot \frac{2}{16}=\frac{7}{128} \\
& \frac{7}{16} \cdot \frac{3}{15}=\frac{7}{80}
\end{aligned}
$$

