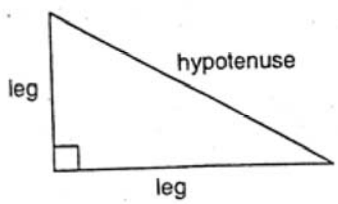
8.10

Link to Introduction to Trig

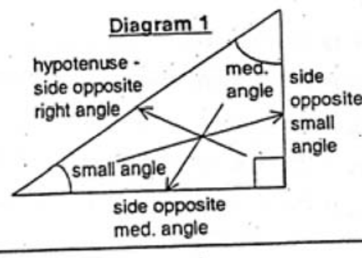
"HYPADJOPP"

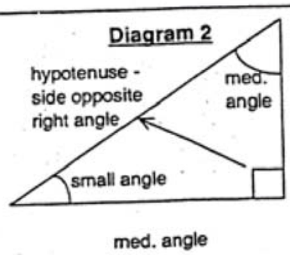
We already know that right triangles are important because the Pythagorean relationship applies to them in a special way. (The sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.) Right triangles are important for other reasons, too, which we will begin to explore in the activity after this one. But first, let's learn the vocabulary we will need, and practice using it.



The most common way to label the sides of right triangles is to call the two sides that form the right angle the legs, and the remaining side (the longest one) the hypotenuse. This labeling system does not allow us to distinguish one leg from the other. Mathematicians have developed a way to tell the legs apart based on their position relative to one of the acute angles in the triangle. The words they use to describe the legs are **opposite** and **adjacent**.

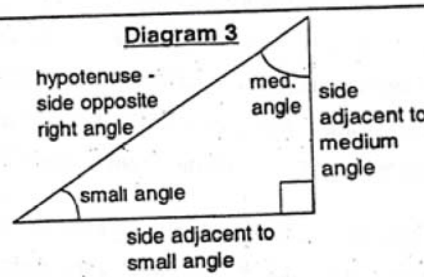
First, let's look at the term *opposite*. Every side is opposite, or across from, one of the angles of the triangle. In fact, the smallest side is always opposite the smallest angle, the medium side is always opposite the medium angle, and the longest side is always opposite the largest angle. (See Diagram 1.)

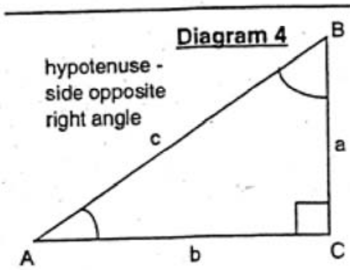




Since the longest side of the triangle will always be opposite the largest angle (the right angle) that side has a special name, the hypotenuse. (See Diagram 2.)

Now let's examine the term *adjacent*. In normal English, the word adjacent means "next to." That is the same sense that we use the word here. Each angle actually has two sides that are adjacent, or next to it. One of those sides is already called the hypotenuse, so the other one is the one that gets the name *adjacent*. (See Diagram 3.)



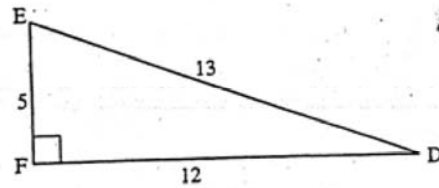


Let's review what we've learned. The longest side of the triangle, which is opposite the right angle, is always called the hypotenuse. The two legs of the triangle (the sides that form the right angle) are each *opposite* one of the angles, and *adjacent* to the other. For example, in Diagram 4 leg *a* is *opposite* angle A and *adjacent* to angle B. Leg *b* is *opposite* angle B and *adjacent* to angle A.

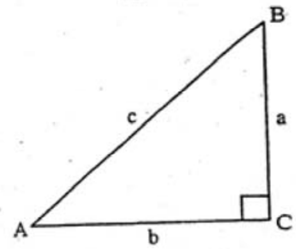
8.11

Let's practice using this vocabulary with some sample right triangles.

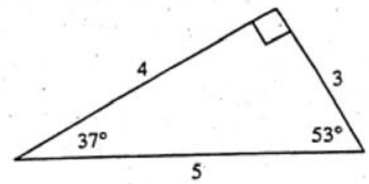
1. How long is the *hypotenuse*? ________
   How long is the side *opposite* angle D? ________
   How long is the side *adjacent* to angle D? ________
   How long is the side *opposite* angle E? ________
   How long is the side *adjacent* to angle E? ________



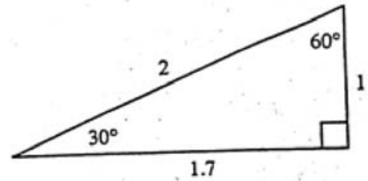
2. How long is the *hypotenuse*? ________
   How long is the side *opposite* angle A? ________
   How long is the side *adjacent* to angle A? ________
   How long is the side *opposite* angle B? ________
   How long is the side *adjacent* to angle B? ________



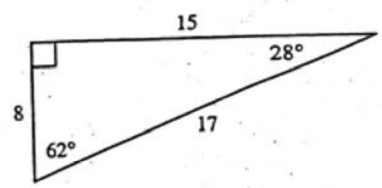
3. How long is the *hypotenuse*? ________
   How long is the side *opposite* the 53° angle? ________
   How long is the side *adjacent* to the 53° angle? ________
   How long is the side *opposite* the 37° angle? ________
   How long is the side *adjacent* to the 37° angle? ________



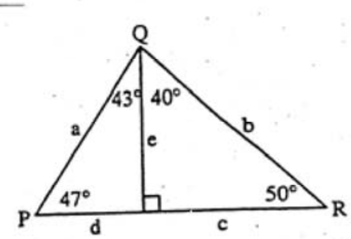
4. How long is the *hypotenuse*? ________
   How long is the side *opposite* the 30° angle? ________
   How long is the side *adjacent* to the 60° angle? ________
   How long is the side *opposite* the 60° angle? ________
   How long is the side *adjacent* to the 30° angle? ________



5. How long is the *hypotenuse*? ________
   How long is the side *opposite* the 28° angle? ________
   How long is the side *adjacent* to the 28° angle? ________
   How long is the side *opposite* the 62° angle? ________
   How long is the side *adjacent* to the 62° angle? ________



6. How long is the *hypotenuse* of the right triangle on the left? ________
   How long is the *hypotenuse* of the right triangle on the right? ________
   How long is the side *opposite* the 43° angle? ________
   How long is the side *adjacent* to the 43° angle? ________
   How long is the side *opposite* the 50° angle? ________
   How long is the side *adjacent* to the 50° angle? ________
   How long is the side *opposite* the 47° angle? ________
   How long is the side *adjacent* to the 40° angle? ________



SOH-CAH-TOA

We found some interesting patterns in our data for right triangles. (See page 9.10.4). More specifically, we found that: (1) even when the sides have different lengths, if the angles are the same, then dividing certain sides together always produces the same ratio; and (2) changing the angles in the triangle changed the ratios, but when we compared the same sides in two different triangles that have the same angles, the ratios were equal.

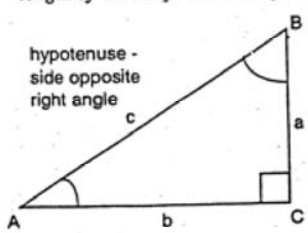
The ratios we have been looking at are called **trigonometric ratios.** The word "*trigonometric*" comes from two Greek words, "trigon", for triangle, and "meter", for measure. The ancient Greeks discovered these ratios thousands of years ago by doing precisely what we did - measuring the sides of triangles! The name of this branch of mathematics is, "**trigonometry**", or "**trig**".

The names given to the three ratios we studied in the data table are the **sine** ratio (*sin* for short), the **cosine** ratio (**cos** for short), and the **tangent** ratio (**tan** for short.) Here are their definitions, along with a diagram to help the definitions make sense:

The **sine** of angle A = $\dfrac{\textit{length of side opposite angle A}}{\textit{length of hypotenuse}}$, or $\sin A = \dfrac{opp}{hyp} = \dfrac{a}{c}$

The **cosine** of angle A = $\dfrac{\textit{length of side adjacent to angle A}}{\textit{length of hypotenuse}}$, or $\cos A = \dfrac{adj}{hyp} = \dfrac{b}{c}$

The **tangent** of angle A = $\dfrac{\textit{length of side opposite angle A}}{\textit{length of side adjacent to angle A}}$, or $\tan A = \dfrac{opp}{adj} = \dfrac{a}{b}$



The *sine* of angle B = $\dfrac{\textit{length of side opposite angle B}}{\textit{length of hypotenuse}}$, or $\sin B = \dfrac{opp}{hyp} = \dfrac{b}{c}$

The *cosine* of angle B = $\dfrac{\textit{length of side adjacent to angle B}}{\textit{length of hypotenuse}}$, or $\cos B = \dfrac{adj}{hyp} = \dfrac{a}{c}$

The *tangent* of angle B = $\dfrac{\textit{length of side opposite angle B}}{\textit{length of side adjacent to angle B}}$, or $\tan B = \dfrac{opp}{adj} = \dfrac{b}{a}$

Since angle C is the right angle, it does not have the same relationship to the sides of the triangle that the other two angles do. For example, the side that is opposite the right angle is already called the *hypotenuse*. Not only that, but there are two sides that could both be the *adjacent* side to the right angle, since neither one is the hypotenuse. For this reason, these ratios are only defined for the two acute angles in the triangle.

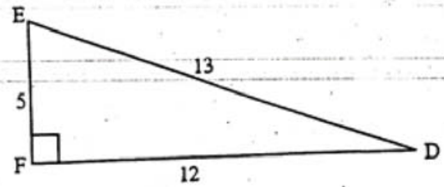
8.13

Practice finding the indicated ratios for each triangle below. Leave your answers in fraction form.

1. sin D = —   cos D = —

   tan D = —   sin E = —

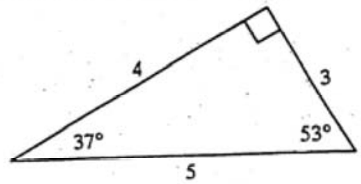
   cos E = —   tan E = —



2. sin 37° = —   cos 37° = —

   tan 37° = —   sin 53° = —

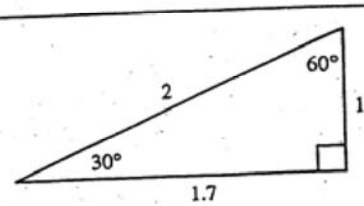
   cos 53° = —   tan 53° = —



3. sin 30° = —   cos 30° = —

   tan 30° = —   sin 60° = —

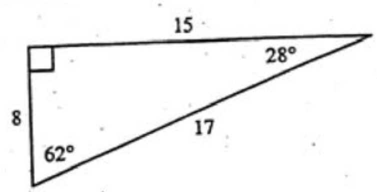
   cos 60° = —   tan 60° = —



4. sin 28° = —   cos 28° = —

   tan 28° = —   sin 62° = —

   cos 62° = —   tan 62° = —



5. sin P = —   cos P = —

   tan P = —   sin R = —

   cos R = —   tan R = —

   sin 43° = —   cos 40° = —

   tan 47° = —   sin 50° = —

   cos 43° = —   tan 40° = —