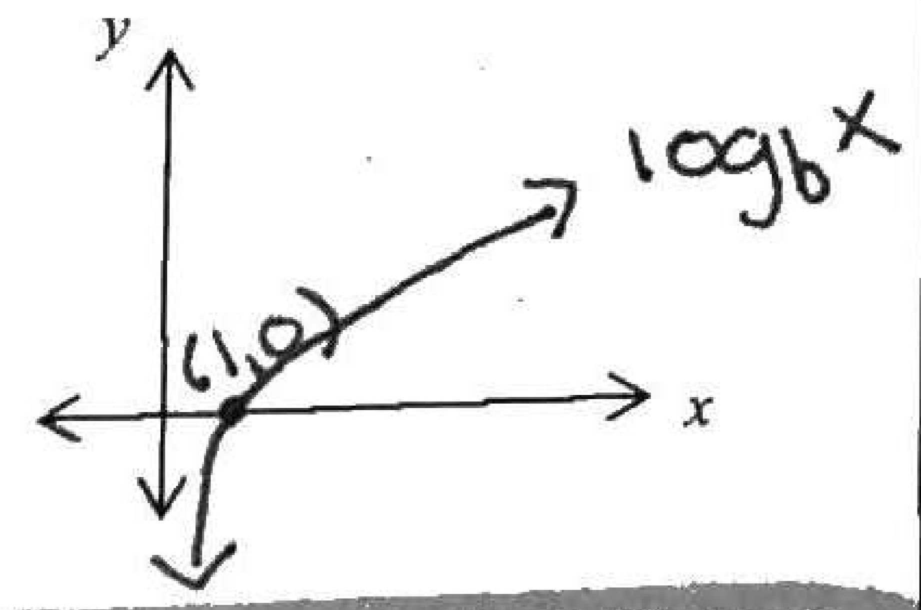
Date ______________  Notes 5.5 Properties of Logs

*Review from Algebra II*

**Properties of Logarithms:** Given $b > 0,\ b \neq 1$

1. The domain of $f(x) = \log_b x$ is $(0, \infty)$.
   [This means you can take the log of positive #'s only.]
2. $\log_b 1 = 0$ and $\log_b b = 1$
3. Property of Inverses
   a) $\log_b b^k = k$
   b) $b^{\log_b v} = v$
4. Product Law: $\log_b (vw) = \log_b v + \log_b w$
5. Quotient Law: $\log_b \left(\frac{v}{w}\right) = \log_b v - \log_b w$
6. Power Law: $\log_b (v^k) = k \log_b v$




Remember: logs are exponents. If you mult. two #'s and the bases are the same, you add the exponet.

If you divide 2 #'s, you subtract the exponents.

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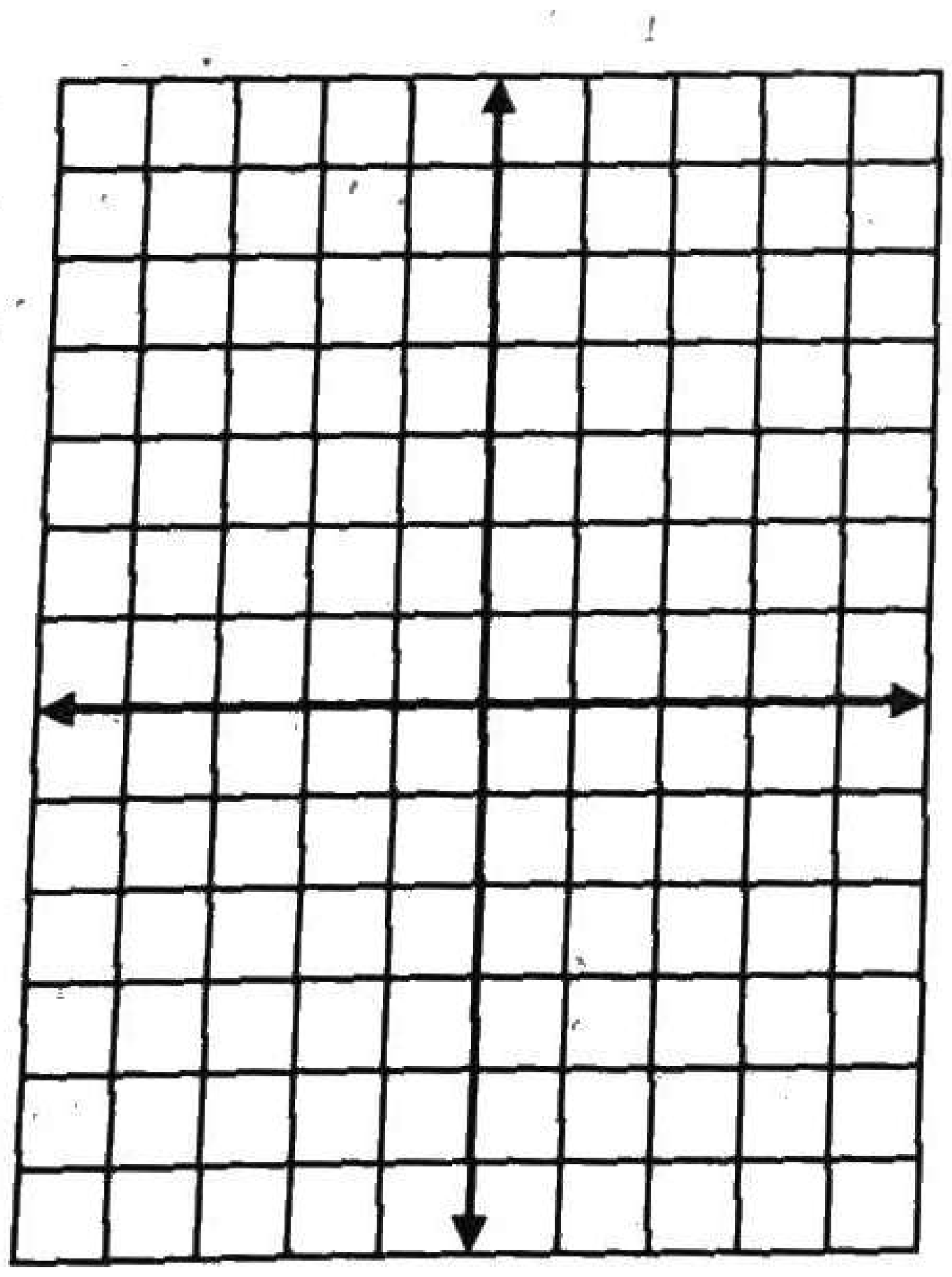
**Graphical analysis:**
Use your TI to graph the two sets of equations below in the same viewing window.

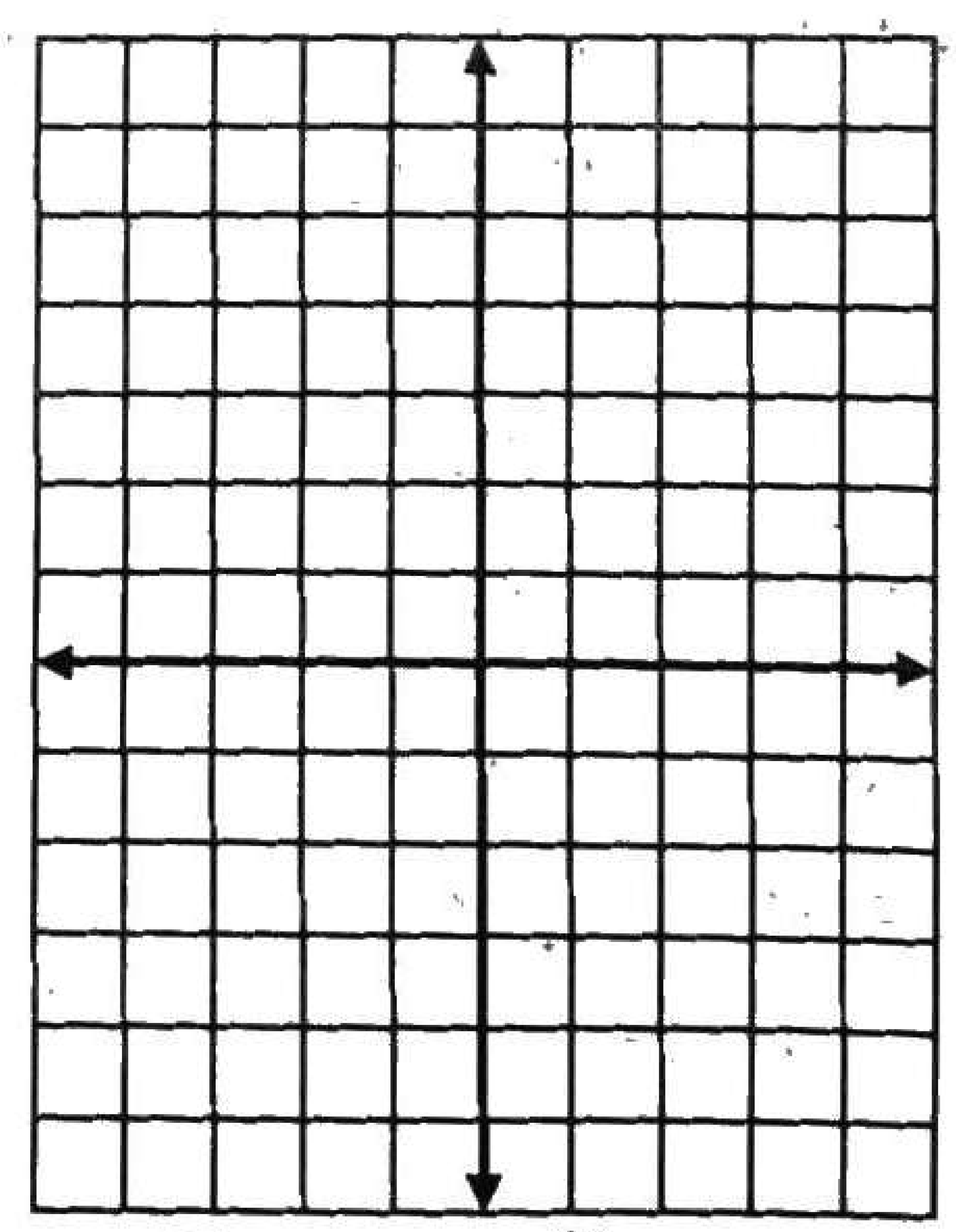
Make $y_2$ a "thick" graph.

Verify your conclusions algebraically. Assume $x > 0$. Sketch each graph.

a) $y_1 = \ln \frac{x^2}{5}$ and $y_2 = 2\ln x - \ln 5$

b) $y_1 = \ln\left[x^3(x+4)\right]$ and $y_2 = 3\ln x + \ln(x+4)$





$y_1 = \ln\frac{x^2}{5} = \ln x^2 - \ln 5$
$= 2\ln x - \ln 5$

$y_2 = 2\ln x - \ln 5$

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$= \ln x^3 + \ln(x+4)$
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Date ____________

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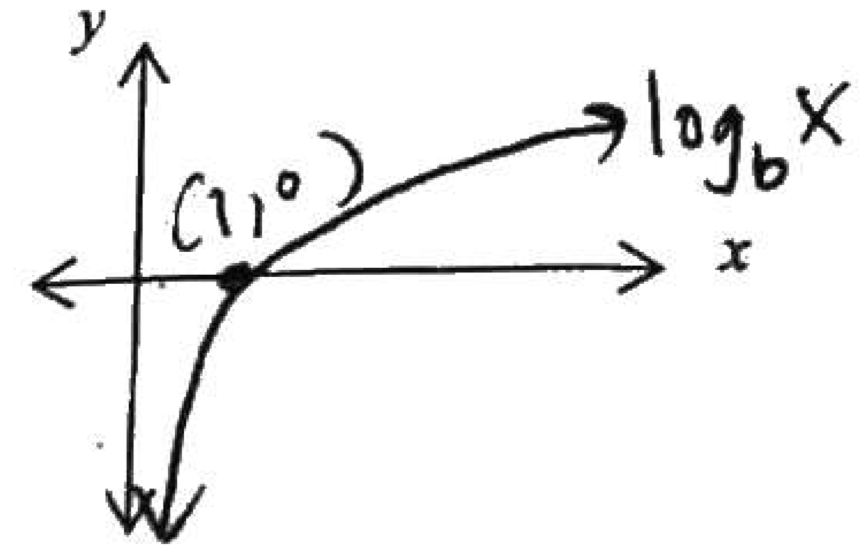


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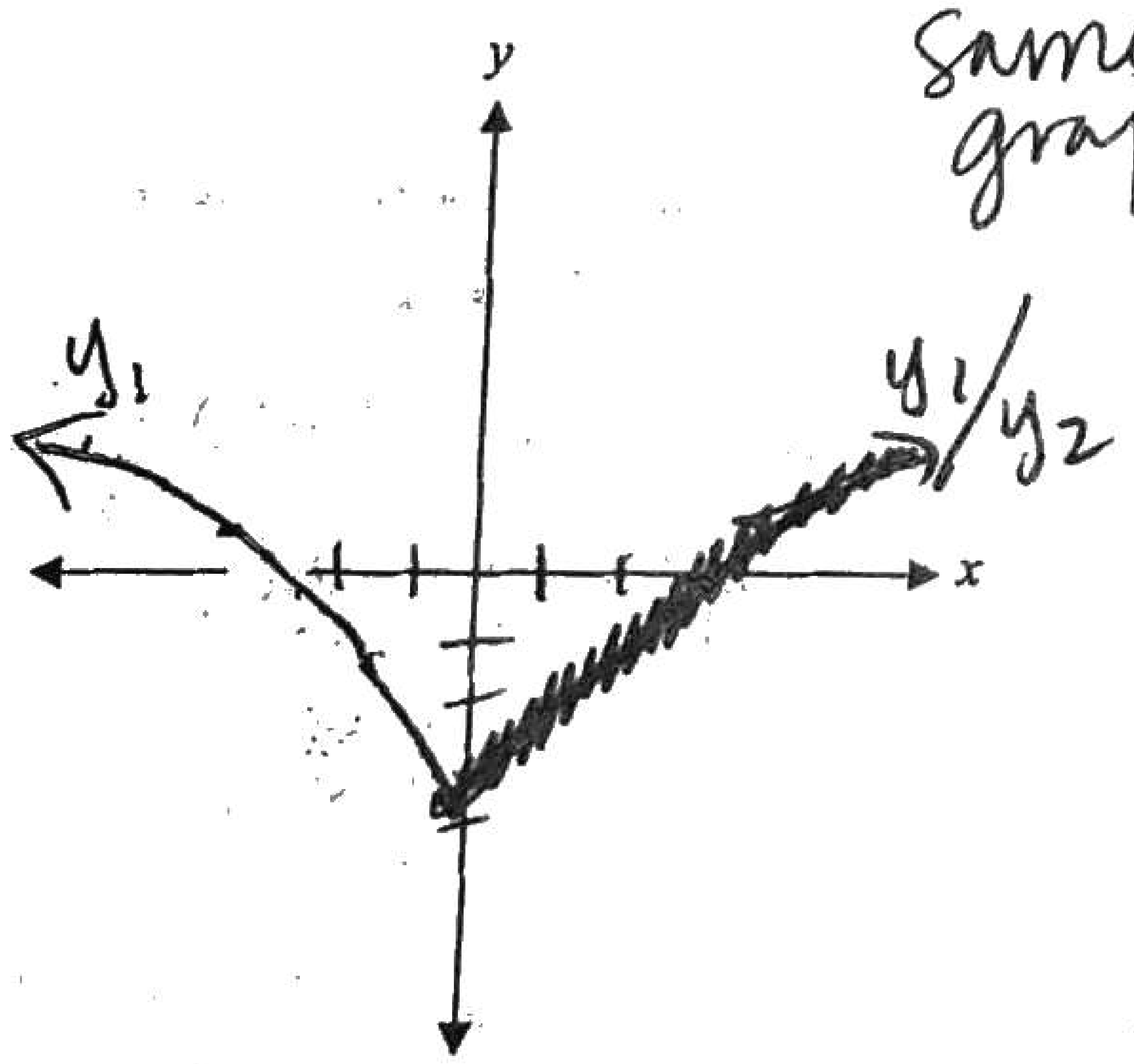
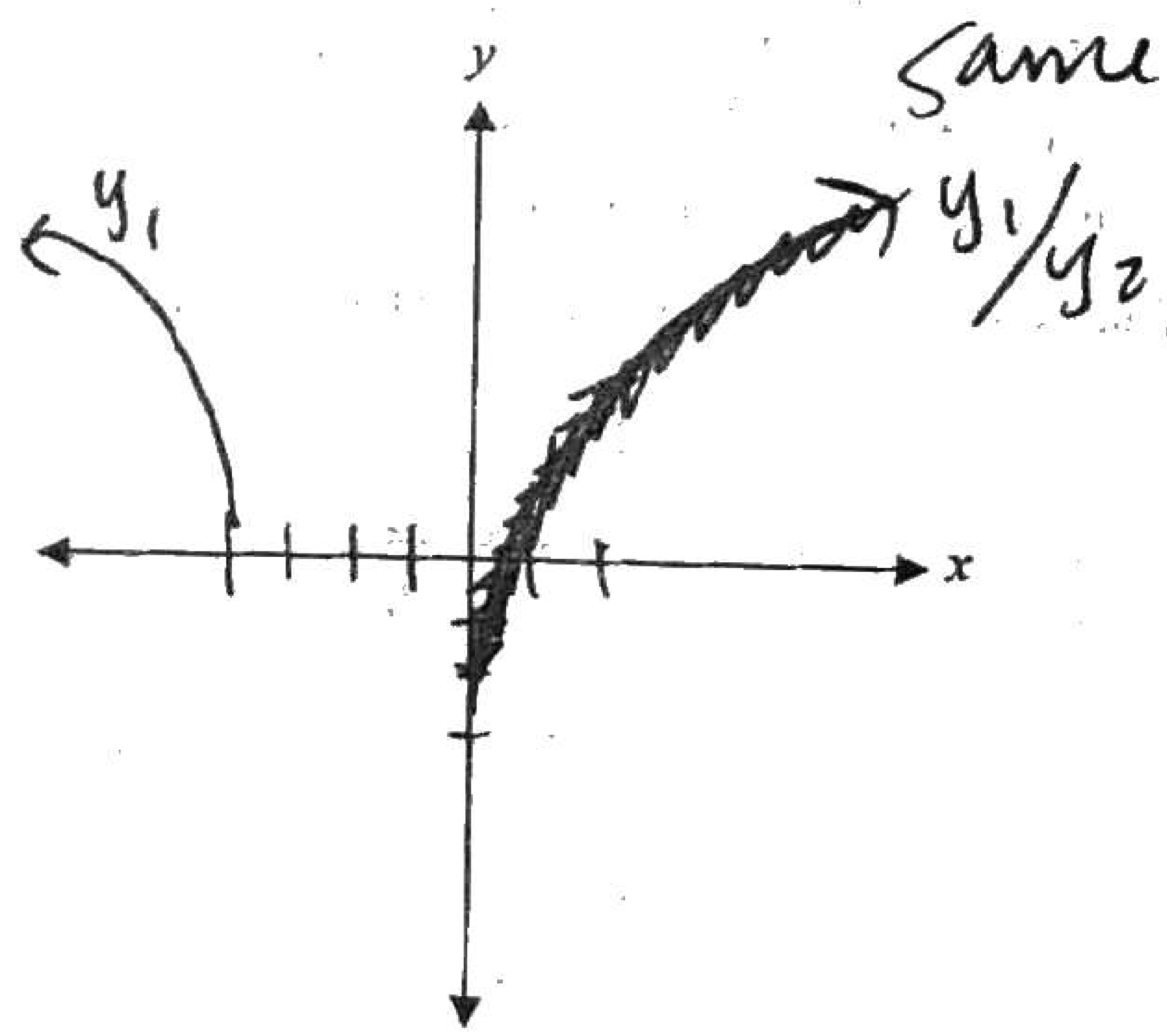
same graph!




[doesn't graph $x<0$ when props are used]

same!

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$$= \ln x^3 + \ln(x+4)$$
$$= \ln\left[x^3(x+4)\right]$$

Use the properties of logarithms to write the expressions below as a single logarithm.

c) $\log 3x + 2\log y$

$\log 3x + \log y^2$

$\boxed{\log 3xy^2}$

d) $\log_3(x+5) - \log_3(x^2-25)$

$\log_3 \frac{x+5}{x^2-25}$

$\log_3 \frac{\cancel{x+5}}{(\cancel{x+5})(x-5)} = \boxed{\log_3 \frac{1}{x-5}}$

or $\log_3 [x-5]^{-1} = \boxed{-\log_3(x-5)}$

e) $2(\log_3 x + \log_3 5)$

$2(\log_3 5x)$

$\log_3 (5x)^2$

$\boxed{\log_3 25x^2}$

f) $\ln 2x - 2\ln x - \ln 3y$

$\ln 2x - \ln x^2 - \ln 3y$

$\ln \frac{2}{x} - \ln 3y$

$\boxed{\ln \frac{2}{3xy}}$

Bonus? g) $\ln(e^2 x) + \ln(ey) - 3$

$\ln e^3 xy - 3$

$\ln e^3 + \ln xy - 3$

$3\ln e^{1} + \ln xy - 3$

$\cancel{3} + \ln xy - \cancel{3} = \boxed{\ln xy}$

h) $2 - \log_5(25z)$

$2 - (\log_5 5^2 + \log_5 z)$

$2 - \cancel{\log_5 5^2} - \log_5 z$

$\cancel{2} - \cancel{2} - \log_5 z$

$\boxed{-\log_5 z}$

Use the properties of logarithms to write the expression as a sum, difference, and/or constant multiple of logarithms. Assume all variables are positive.

i) $\ln \sqrt[3]{t} = \ln t^{1/3}$

$\boxed{\frac{1}{3}\ln t}$

j) $\ln z(z-1)^2,\ z > 1$

$\ln z + \ln(z-1)^2$

$\boxed{\ln z + 2\ln(z-1)}$

k) $\ln\sqrt{\frac{x^2}{y^3}} = \ln\left(\frac{x^2}{y^3}\right)^{1/2}$

$\frac{1}{2}[\ln x^2 - \ln y^3]$

$\frac{1}{2}[2\ln x - 3\ln y] = \boxed{\ln x - \frac{3}{2}\ln y}$

Given: $\log_7 5 \approx 0.8271$, $\log_7 9 \approx 1.1292$. Approximate the following:

p) $\log_7 45 = \log_7(5 \cdot 9)$

$\log_7 5 + \log_7 9 = .8271 + 1.1292$

$= \boxed{1.9563}$

q) $\log_7 63 = \log_7(7 \cdot 9)$

$\cancel{\log_7 7}^{1} + \log_7 9$

$1 + 1.1292 = \boxed{2.1292}$

r) $\log_7 \frac{5}{81} = \log_7 5 - \log_7 9^2$

$= \log_7 5 - 2(\log_7 9)$

$= .8271 - 2(1.1292) =$

$\boxed{-1.4313}$

Evaluate:

$\log_5 27 = x$

$\log 5^x = \log 27$

$x \log 5 = \log 27$

$x = \frac{\log 27}{\log 5} \approx \boxed{2.048}$

Check:

$5^{2.048} \approx 27$

**Change of Base Formula:** This formula allows you to find the logs for *any* base by using either common or natural logarithms on the TI.

$\log_b v = \frac{\log v}{\log b}$ $\qquad$ $\log_b v = \frac{\ln v}{\ln b}$

Use the change of base formula to find $\log_7 5$

a) common logs

$\frac{\log 5}{\log 7}$

$\approx \boxed{.827}$

b) natural logs

$\frac{\ln 5}{\ln 7}$

$\approx \boxed{.827}$