Date ________ 5.4 Notes Logarithmic Functions

Review: Exponential functions are strictly increasing or strictly decreasing ⇒ exponential functions are one-to-one and therefore, they must have an inverse. The inverse of an exponential function is a logarithmic function

Definition of a Logarithmic Function: For $x > 0$, and $a > 0$, $a \neq 1$

$$\underbrace{f(x) = \log_a x \quad \text{or} \quad y = \log_a x}_{\textit{Logarithmic form}} \quad \Leftrightarrow \quad \underbrace{x = a^y}_{\textit{Exponential form}}$$

Remember a logarithm is the exponent needed on a specific base to create a given number.

Examples:

1. $\log_2 32 =$ 5 This is the same as "What is the exponent needed on the base 2 to create 32? ..or $2^? = 32$

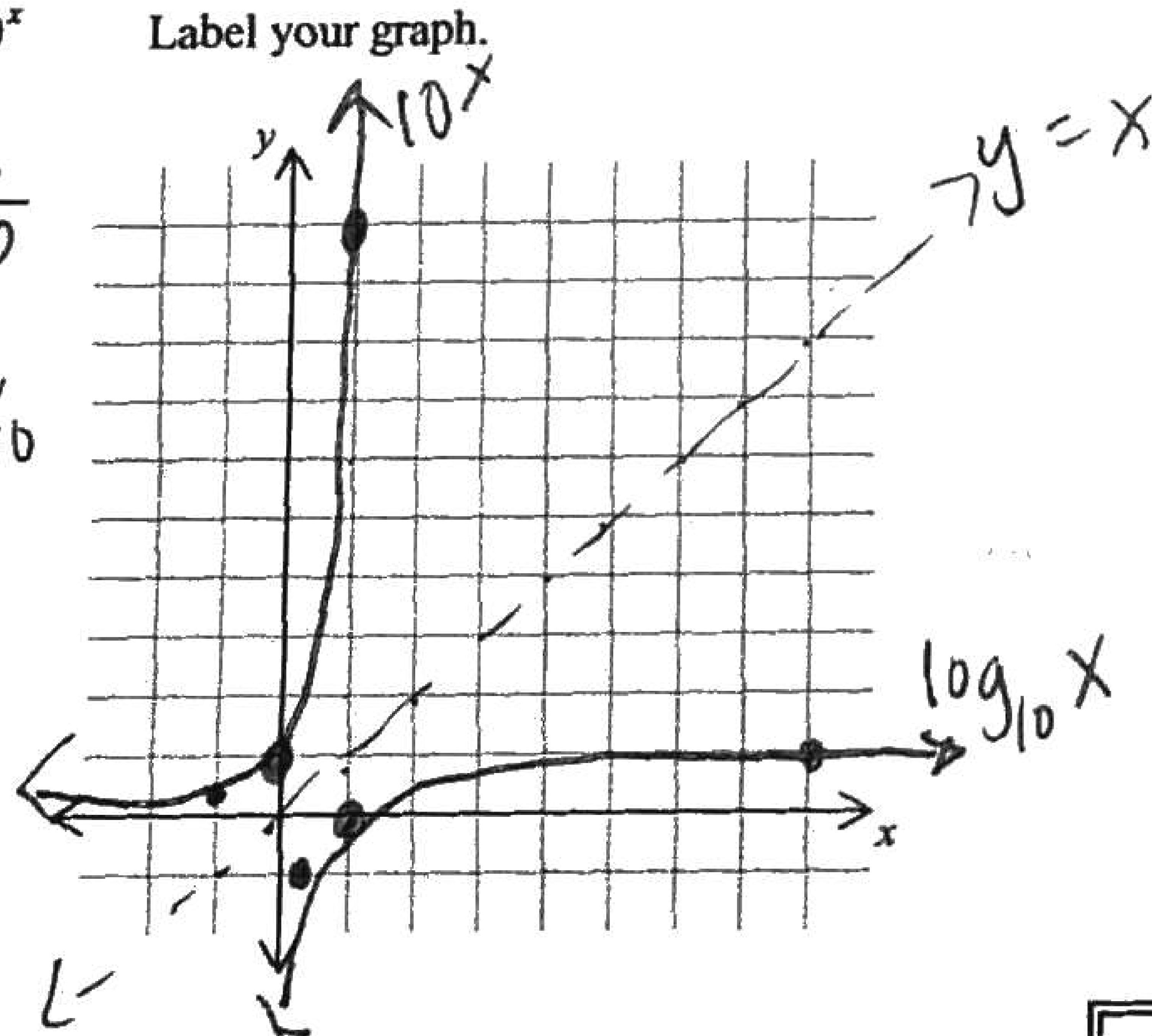
2. $\log_{10} 100 =$ 2 or $10^? = 100$ 3. $\log_{49} 7 =$ $\frac{1}{2}$ or $49^? = 7$ 4. $\log_{10} \frac{1}{100} =$ -2 or $10^? = \frac{1}{100}$

5. Write in logarithmic form: $10^{7k} = r \Leftrightarrow$ $\log_{10} r = 7k$ 6. Write in exponential form: $\log_{27}(a+c) = d \Leftrightarrow$ $27^d = a + c$

Graphical connection between *exponential* functions and *logarithmic* functions.

Complete the table and sketch the graph of $y = 10^x$ Label your graph.

x	y
1	10
0	1
-1	1/10



Find the inverse of $y = 10^x$. $x = 10^y$

change to log form → $\boxed{\log_{10} x = y}$

Use color to draw the line $y = x$.

What transformation is used to generate the inverse?

reflection over $y = x$

Write the corresponding ordered pairs for the inverse, and sketch the graph. Label the function.

x	y
10	1
1	0
1/10	-1

Def. of a **common logarithm**: Always has a base 10.

$\log v = u \Leftrightarrow$ $10^u = v$

Answer the following *without* a calculator:

7. $\log \frac{1}{10} =$ -1 8. $\log 1 =$ 0 9. $\log 10 =$ 1

Use the equivalent exponential form to check your answers.

$10^x = \frac{1}{10}$ $10^x = 1$ $10^x = 10$

Answers these using your TI. (3 dec. places)

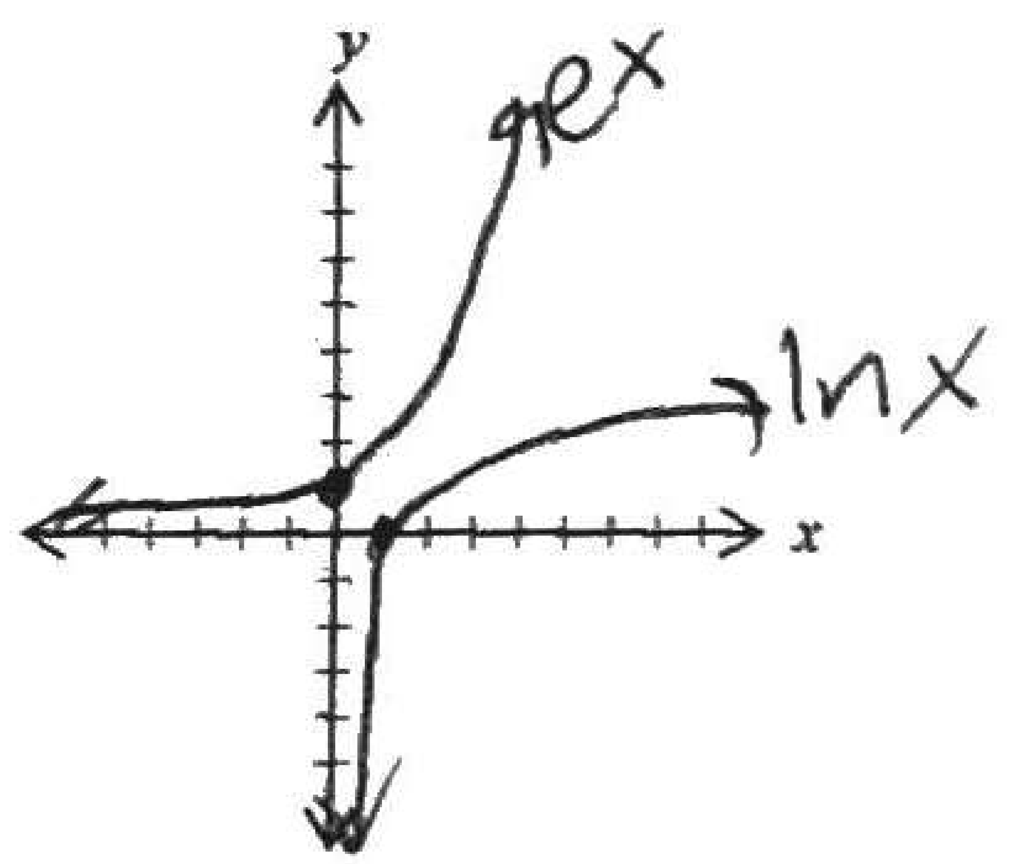
10. $\log 2 =$ .301 11. $\log 5 =$ .699 12. $\log 8 =$ .903

Exponential Function	**Logarithmic Function**
Ex: $f(x) = 10^x$	Ex: $f(x) = \log_{10} x$ or $\log x$
* Domain = $(-\infty, \infty)$	* Domain = $(0, \infty)$
* Continuous	[Always to the right of y-axis].
* Range = $(0, \infty)$	* continuous
[Always above x-axis]	* Range = $(-\infty, \infty)$
* Contains the *key* point: $(0, 1)$	* Contains the *key* point: $(1, 0)$
* Horiz. asymptote: $y = 0$	* Vert. asymptote: $x = 0$
* Description: inc @ inc rate	* Description: inc @ dec rate

Connection between *natural base e exponential* functions and *natural logarithmic* functions...

Sketch and label the graph of $y = e^x$

$\log_a b = c \quad a^c = b$



Find the inverse of $y = e^x$. Sketch on the same axes (diff. color) and label.

$x = e^y \longrightarrow \log_e x = y$

$[\log_e = \ln] \longrightarrow \boxed{\ln x = y}$

Use your TI to find the following. Round to 3 decimal places.
Then write the equivalent natural log expression or exponential expression.

$e^2 = 7.389$ $\quad$ $e^{-1.5} = 0.223$

$\ln 7.389 \approx 2$ $\quad$ $\ln 0.223 \approx -1.5$

$\ln 0.789 = -.237$ $\quad$ $\ln 14.2 = 2.653$

$e^{-.237} \approx .789$ $\quad$ $e^{2.653} \approx 14.2$

Def. of a natural logarithm: Always has a base e.

$\ln v = u \Leftrightarrow v = e^u$

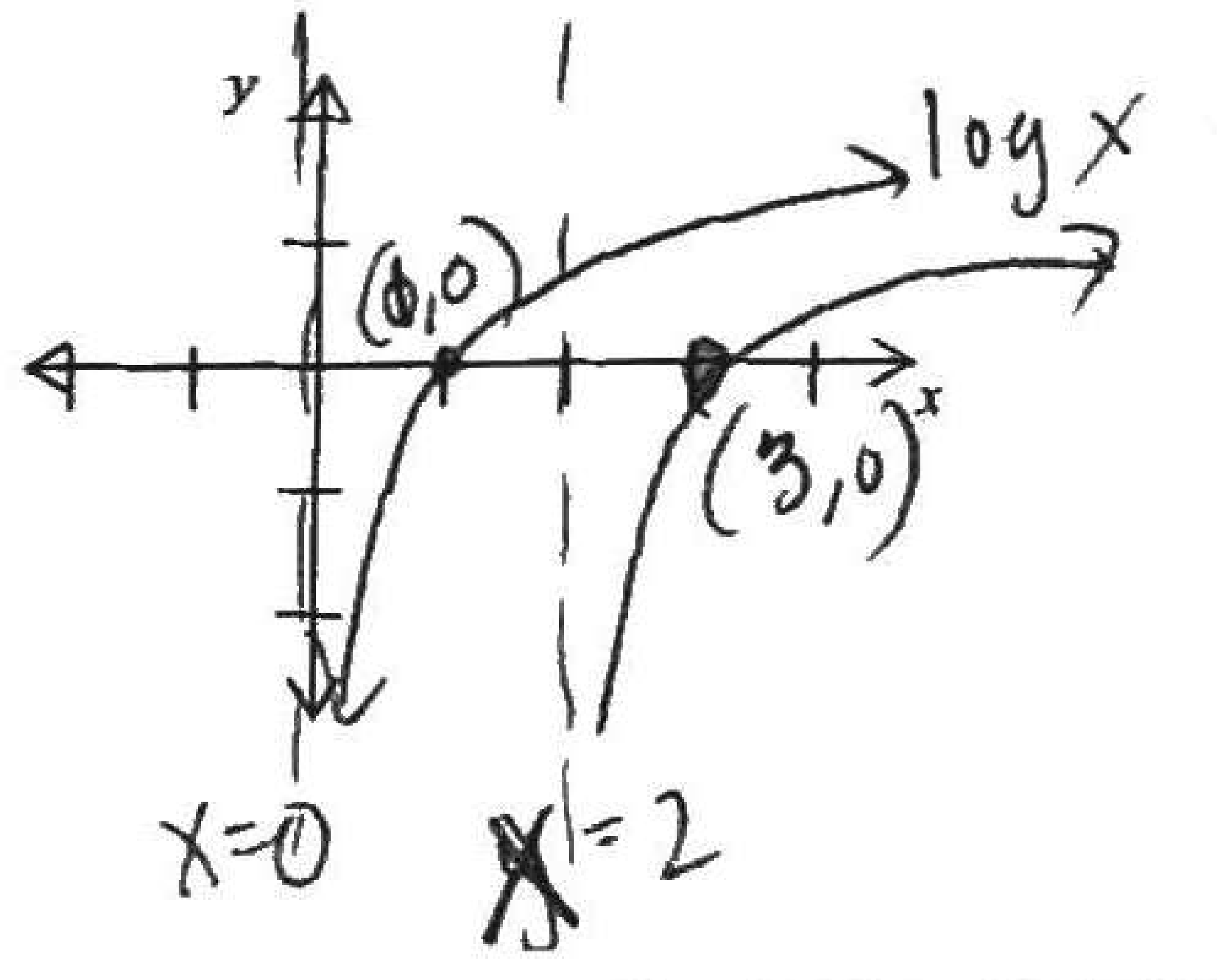
Transformations:

Describe the transformation from the parent function. Give the domain and range of the new function. Sketch.
Pay special attention to the key points and the equations of asymptotes.

$f(x) = \log x$ to $g(x) = \log(x-2)$

Transformation: horiz rt 2

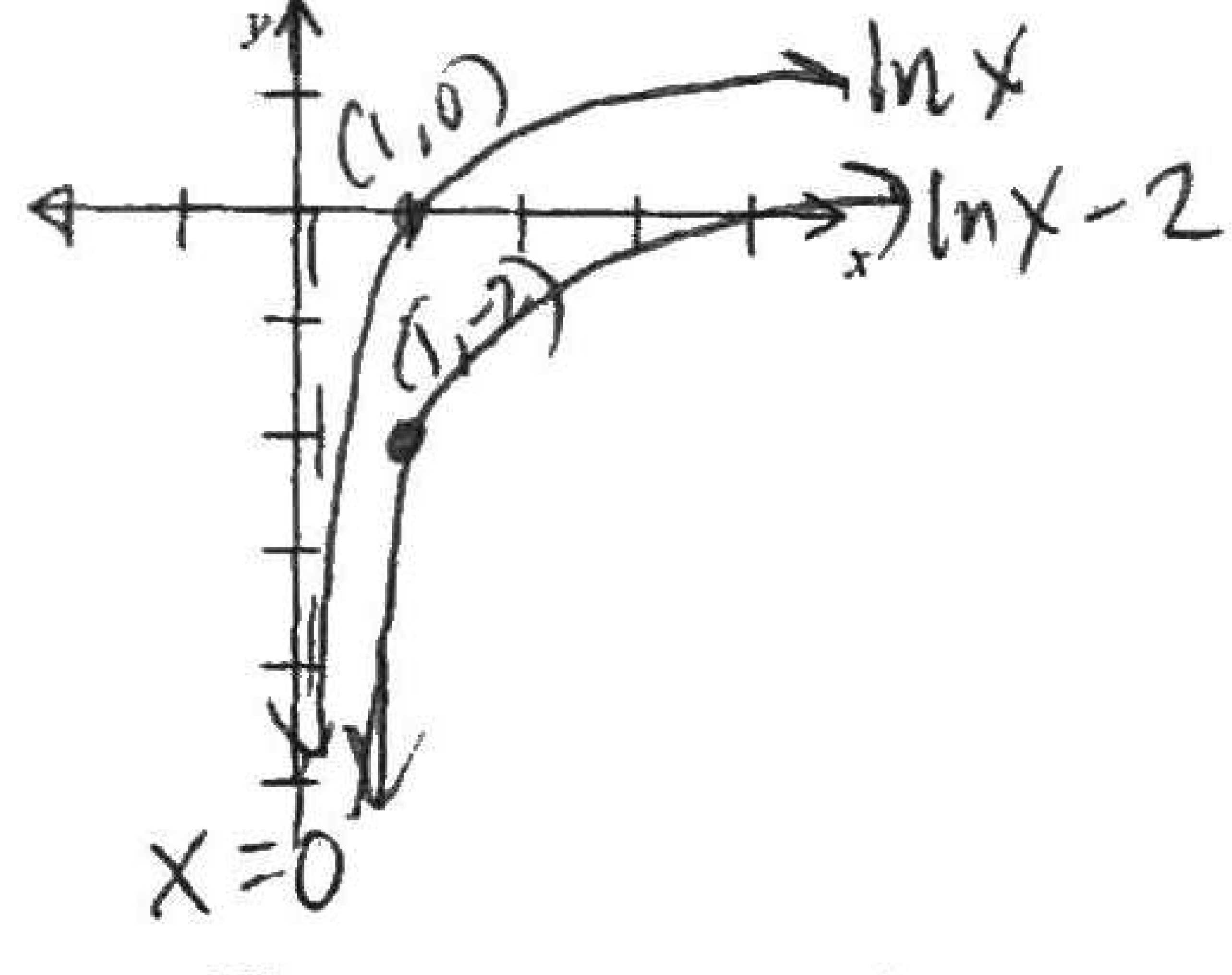
Domain: $(2, \infty)$ Range: $(-\infty, \infty)$



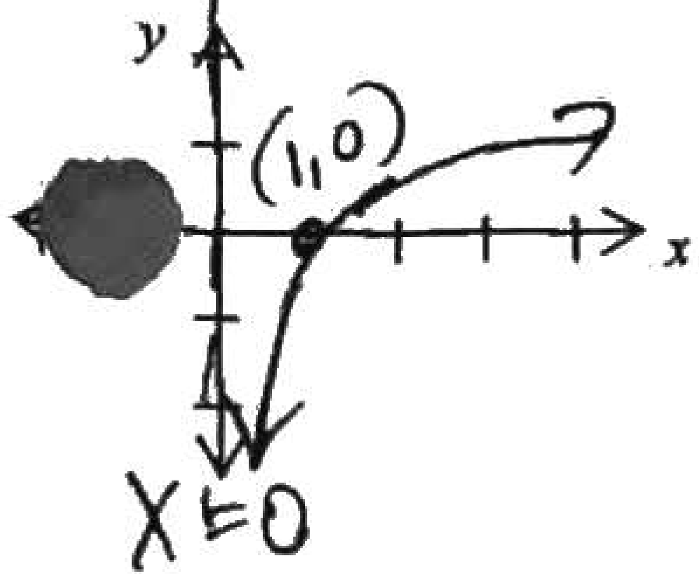
$f(x) = \ln x$ to $g(x) = \ln x - 2$

Transformation: vert down 2

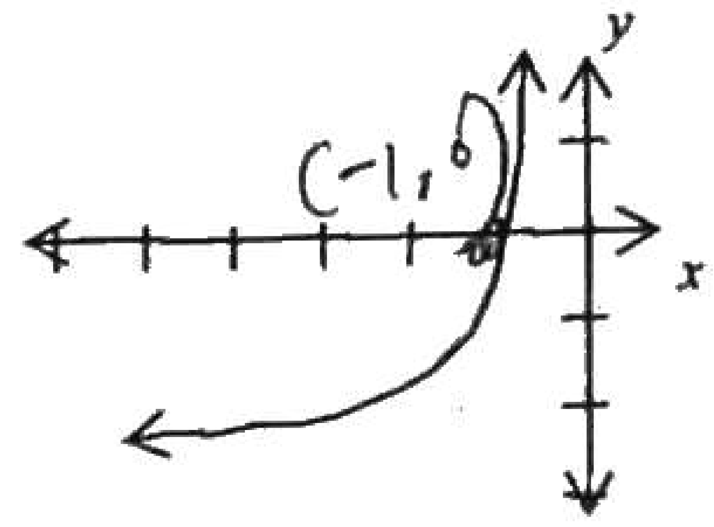
Domain: $(0, \infty)$ Range: $(-\infty, \infty)$



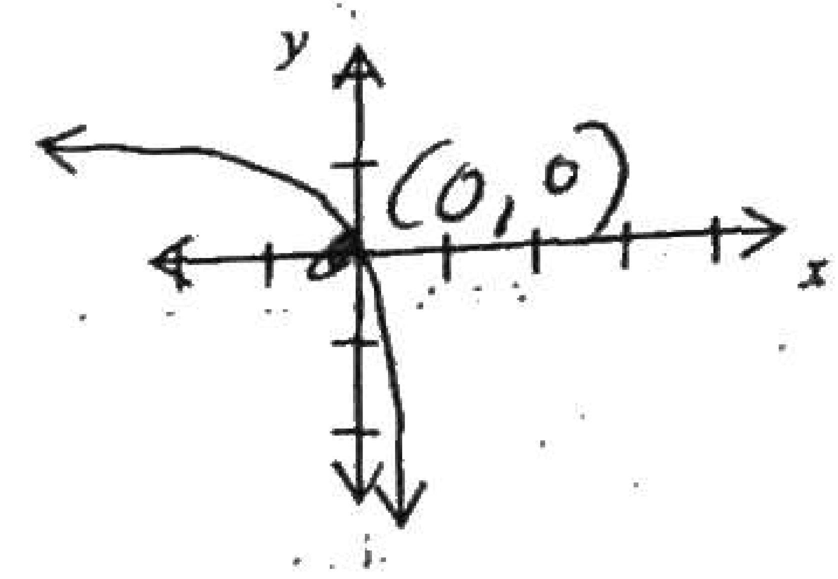
Sketch $y = \log_2 x$ Describe the transformation(s) from the parent graph. Then match it to one of the graphs given below.



a)

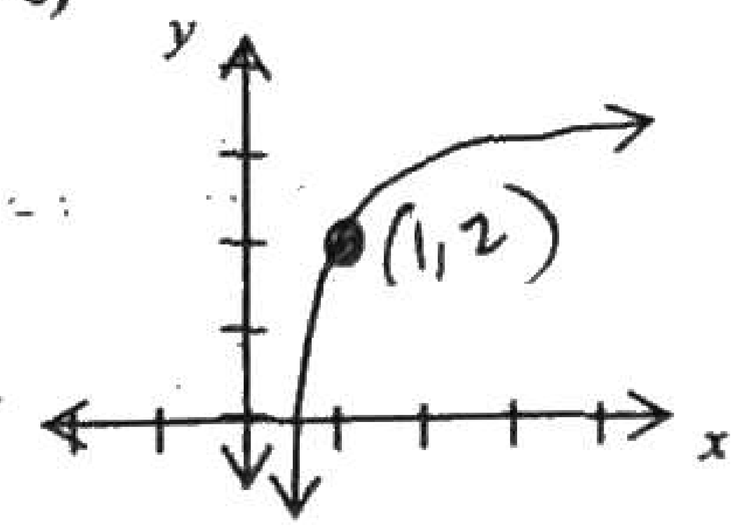


b)

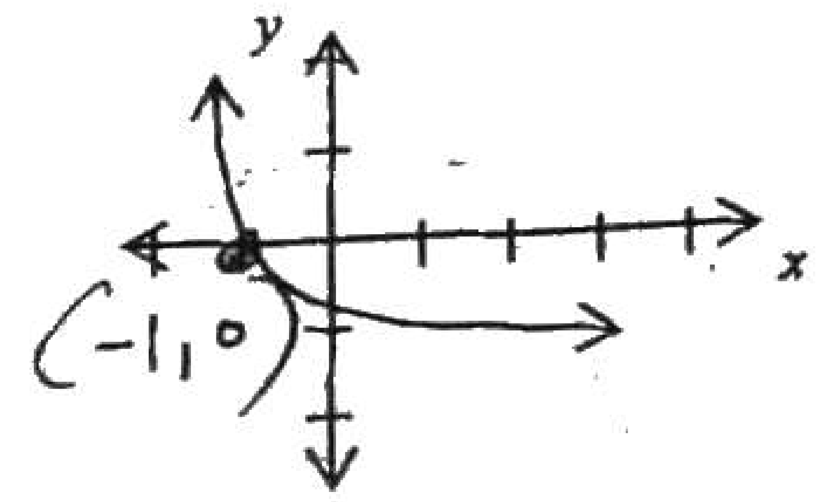


1. $f(x) = \log_2 x + 2$
   up 2 (c)

c)



d)

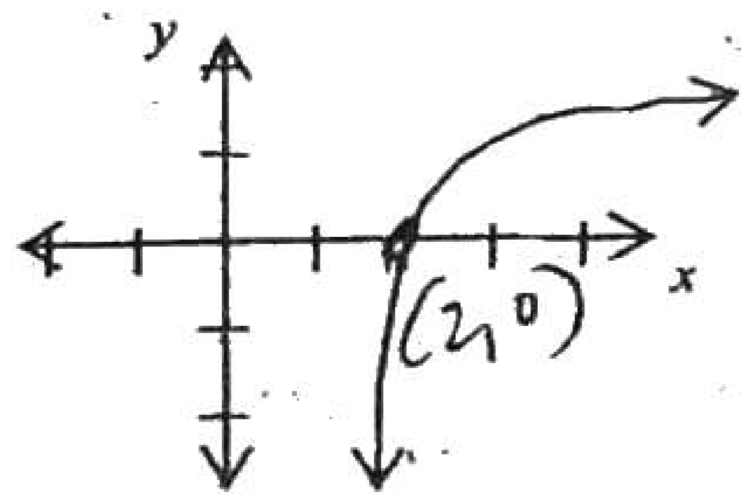


2. $f(x) = -\log_2 x$
   reflect over x-axis (f)

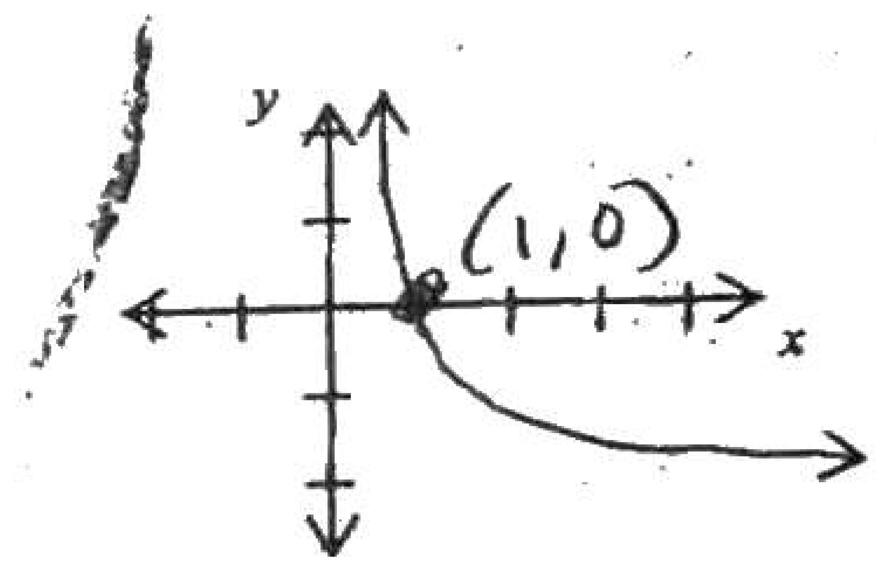
3. $f(x) = -\log_2(x+2)$ (d)
   left 2
   reflect x-axis

4. $f(x) = \log_2(x-1)$
   rt. 1 (e)

e)



f)



5. $f(x) = \log_2(1-x) = \log_2[-(x-1)]$ $(-x+1)$ $-(x-1)$
   reflect y-axis
   rt. 1 (b)

6. $f(x) = -\log_2(-x)$
   reflect y-axis
   reflect x-axis (a)

Use the *Property of Inverses* to discover an important property for logarithms. Remember, a logarithmic function is the inverse of an exponential function, provided the bases are the same. If f and g are inverses then $f \circ g = g \circ f = x$

$f(x) = 3^x$ and $g(x) = \log_3 x$

$f \circ g = f(g(x))$
$f(\log_3 x)$
$= 3^{\log_3 x}$
$= \boxed{x}$

$g \circ f = g(f(x))$
$g(3^x) = \log_3 3^x$
$= \boxed{x}$

*we know these have to equal x because of P of Inv!

In general, Property of Inverses, $a > 0$, $a \neq 1$	
$a^{\log_a x} = x$	$\log_a a^x = x$
$e^{\ln x} = x$	$\ln e^x = x$

Rewrite to match one of the properties of inverses and evaluate w/o your TI:

7. $\log_2 2^4 = \boxed{4}$
8. $\log 1000 =$ $\log_{10} 10^3 = \boxed{3}$
9. $3^{\log_3 x^2} = \boxed{x^2}$
10. $\ln e^{3.78} = \boxed{3.78}$
11. $e^{\ln\sqrt{x+3}} = \boxed{\sqrt{x+3}}$

Solve for x.

12. $\log_2\left(\frac{1}{8}\right) = x$
    $\log_2 2^{-3} = x$
    $\boxed{x = -3}$

13. $\log 0.001 = x$
    $\log_{10} \frac{1}{1000} = \log_{10} 10^{-3}$
    $\boxed{x = -3}$

14. $\ln e^{\sqrt{5}} = x$
    $\boxed{x = \sqrt{5}}$