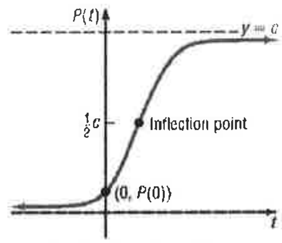
Logistic Model

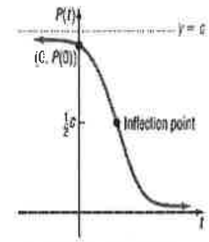
In a logistic model, the population P after time t is given by the function

$$P(t) = \frac{c}{1 + ae^{-bt}} \qquad (6)$$

where a, b, and c are constants with $a > 0$ and $c > 0$. The model is a growth model if $b > 0$; the model is a decay model if $b < 0$.



Logistics Growth



Logistics Decay

$t=0$ for initial value

$C =$ limit to growth or carrying capacity

$$\boxed{P(t) = \frac{c}{1 + ae^{-bt}}}$$

if not 1, then divide everything by that # — so limit to growth becomes $\frac{C}{\#}$

HA: always $y=0$, $y=C$

Properties of the Logistic Growth Function

1. The domain is the set of all real numbers. The range is the interval $(0, c)$, where c is the carrying capacity.
2. There are no x-intercepts; the y-intercept is $P(0)$.
3. There are two horizontal asymptotes: $y = 0$ and $y = c$.
4. $P(t)$ is an increasing function if $b > 0$ and a decreasing function if $b < 0$.
5. There is an **inflection point** where $P(t)$ equals $\frac{1}{2}$ of the carrying capacity. The inflection point is the point on the graph where the graph changes from being curved upward to curved downward for growth functions and the point where the graph changes from being curved downward to curved upward for decay functions.
6. The graph is smooth and continuous, with no corners or gaps.

EXAMPLE **Fruit Fly Population**

5. Fruit flies are placed in a half-pint milk bottle with a banana (for food) and yeast plants (for food and to provide a stimulus to lay eggs). Suppose that the fruit fly population after t days is given by

$$P(t) = \frac{230}{1 + 56.5e^{-0.37t}}$$

a) at $t \to \infty$, $P(t) \to \frac{230}{1} = 230$

$C = 230$ growth rate $= 37\%$

(a) State the carrying capacity and the growth rate.
(b) Determine the initial population.
(c) What is the population after 5 days?
(d) How long does it take for the population to reach 180?
(e) Use a graphing utility to determine how long it takes for the population to reach one-half of the carrying capacity by graphing $Y_1 = P(t)$ and $Y_2 = 115$ and using INTERSECT.

b) $P(0) = \dfrac{230}{1+56.5e^{-0.37(0)}} = \dfrac{230}{1+56.5} = \dfrac{230}{57.5} = 4$ fruit flies

c) $P(5) = \dfrac{230}{1+56.5e^{-0.37(5)}} \approx 23$ fruit flies

d) $180 = \dfrac{230}{1+56.5e^{-0.37(t)}}$ (can solve graphically or alg.)

$180 + 10170e^{-0.37t} = 230$

$10170e^{-.37t} = 50$

$e^{-.37t} = 50/10170$

$\to -.37t = \ln \frac{50}{10170}$

$t \approx 14.4$ days

e) $\underset{y_1}{115} = \underset{y_2}{\dfrac{230}{1+56.5e^{-0.37t}}}$

$t \approx 10.9$ days

EXAMPLE **Wood Products**

6. The EFISCEN wood product model classifies wood products according to their life-span. There are four classifications: short (1 year), medium short (4 years), medium long (16 years), and long (50 years). Based on data obtained from the European Forest Institute, the percentage of remaining wood products after t years for wood products with long life-spans (such as those used in the building industry) is given by

$$P(t) = \frac{100.3952}{1 + 0.0316e^{0.0581t}}$$

(a) What is the decay rate?

(b) What is the percentage of remaining wood products after 10 years?

(c) How long does it take for the percentage of remaining wood products to reach 50%?

(d) Explain why the numerator given in the model is reasonable.

a) 5.81 %

b) $P(10) = \dfrac{100.3952}{1 + 0.0316e^{0.0581(10)}} \approx 95.03\%$

c) $50 = \dfrac{100.3952}{1 + .0316e^{.0581t}}$

$50 + 1.58e^{0.0581t} = 100.3952$

$1.58e^{0.0581t} = 50.3952$

$e^{.0581t} = \dfrac{50.3952}{1.58}$

$.0581t = \ln\left(\dfrac{50.3952}{1.58}\right)$

$t \approx 59.60$ years

d) because the maximum % of wood products that remains is $\approx 100\%$