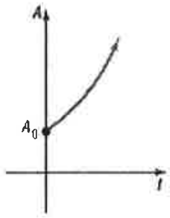
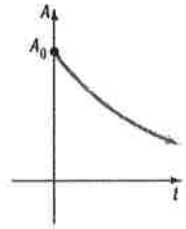


(a) $A(t) = A_0e^{kt}, k > 0$

Exponential Growth



(b) $A(t) = A_0e^{kt}, k < 0$

Exponential Decay

Uninhibited Growth of Cells

A model that gives the number N of cells in a culture after a time t has passed (in the early stages of growth) is

$$N(t) = N_0e^{kt} \qquad k > 0 \tag{2}$$

where N_0 is the initial number of cells and k is a positive constant that represents the growth rate of the cells.

Uninhibited Radioactive Decay

The amount A of a radioactive material present at time t is given by

$$A(t) = A_0e^{kt} \qquad k < 0 \tag{3}$$

where A_0 is the original amount of radioactive material and k is a negative number that represents the rate of decay.

really the same formulas

EXAMPLE Bacterial Growth

1. A colony of bacteria grows according to the law of uninhibited growth according to the function $N(t) = 150e^{0.035t}$, where N is measured in grams and t is measured in days.

a) Determine the initial amount of bacteria. $t = 0$ $N(0) = 150e^{0.035(0)} = 150$

b) What is the growth rate of the bacteria? 3.5%

c) Graph the function using a graphing utility.

d) What is the population after 7 days?

$N(7) = 150e^{.035(7)} \approx 191.64$ grams

e) How long will it take for the population to reach 200 grams?

$200 = 150e^{.035t}$

$\frac{4}{3} = e^{.035t}$ $\ln\frac{4}{3} = .035t$ $t = 8.22$ days

f) What is the doubling time for the population?

started with 150

$300 = 150e^{.035t}$

$2 = e^{.035t}$

$\ln 2 = .035t$

$t \approx 19.80$ days

2. A colony of bacteria increases according to the law of uninhibited growth.
   (a) If N is the number of cells and t is the time in hours, express N as a function of t.
   (b) If the number of bacteria doubles in 2 hours, find the function that gives the number of cells in the culture.
   (c) How long will it take for the size of the colony to triple?
   (d) How long will it take for the population to double a second time (that is increase four times)?

a) $N(t)=Ne^{kt}$

b) $2=1e^{k(2)}$ find k
$2=e^{2k}$
$\ln 2=2k$ $k\approx 0.3466$

c) $3=1e^{.3466t}$
$\ln 3=.3466t$
$t\approx 3.17$ hours

d) $4=e^{.3466t}$
$\ln 4=.3466t$
$t\approx 4$ hours

EXAMPLE Estimating the Age of Ancient Tools

3. Traces of burned wood along with ancient stone tools in an archeological dig in Chile were found to contain approximately 1.67% of the original amount of carbon 14. If the half-life of carbon 14 is 5600 years, approximately when was the tree cut and burned?

$\frac{1}{2}=1e^{k(5600)}$
$\ln\frac{1}{2}=5600k$
$k\approx -.0001238$

$.0167=1e^{-0.0001238t}$
$\ln .0167=-0.0001238t$
$t\approx 33056.11$ yrs ago

Newton's Law of Cooling

The temperature u of a heated object at a given time t can be modeled by the following function:

$$u(t)=T+(u_0-T)e^{kt} \qquad k<0 \qquad (4)$$

where T is the constant temperature of the surrounding medium, u_0 is the initial temperature of the heated object, and k is a negative constant.

EXAMPLE Using Newton's Law of Cooling

4. A cake is heated to 350° F and is then allowed to cool in a room whose air temperature is 70° F.
   (a) If the temperature of the cake is 300° F after 5 minutes, when will its temperature be 200° F?

$300=70+(350-70)e^{k(5)}$
$-70 \quad -70$
$230=280e^{5k}$

$\frac{230}{280}=e^{5k}$
$\ln\frac{23}{28}=5k$
$k\approx -0.0393$

$200=70+280e^{-.0393t}$
$130=280e^{-.0393t}$
$\ln\frac{130}{280}=-.0393t$
$t\approx 19.50$ minutes

(b) Determine the elapsed time before the temperature of the cake is 100 degrees F.

$100=70+280e^{-0.0393t}$
$30=280e^{-.0393t}$

$\frac{30}{280}=e^{-.0393t}$
$\ln\frac{30}{280}=-.0393t$
$t\approx 56.77$ min.

(c) What do you notice about the temperature as time passes?

as t increases, $e^{-.0393t}$ approaches 0, so the temperature will approach 70° (the room temp.)