Suppose that you have \$1000 and a bank offers to pay you 3% annual interest on a savings account with interest compounded monthly. In one year:

$$A = \$1000\left(1 + \frac{0.03}{12}\right)^{12} \quad \text{Use } A = P\left(1+\frac{r}{n}\right)^n \text{ with } P = \$1000, r = 0.03, n = 12.$$
$$= \$1030.42$$

So the interest earned is \$30.42. Using $I = Prt$ with $t = 1$, $I = \$30.42$, and $P = \$1000$, we find the annual simple interest rate is $0.03042 = 3.042\%$. This interest rate is known as the *effective rate of interest.*

$30.42 = 1000 \cdot r \cdot 1$

The **effective rate** of interest is the equivalent annual simple rate of interest that would yield the same amount as compounding after 1 year.

comp. yrly - 1 yr   compound n times 1 year

$1 + r = \left(1 + \frac{r}{n}\right)^n$

APY

APR

Effective Rate of Interest
The effective rate of interest r_e of an investment earning an annual interest rate r is given by
Compounding n times per year: $r_e = \left(1 + \frac{r}{n}\right)^n - 1$
Continuous compounding: $r_e = e^r - 1$

EXAMPLE **Computing the Effective Rate of Interest—Which Is the Best Deal?**

1. Suppose you want to open a money market account. You visit three banks to determine their money market rates. Bank A offers you 4% annual interest compounded daily, Bank B offers you 4.1% compounded monthly, and Bank C offers 3.95% compounded continuously. Determine which bank is offering the best deal.

Bank A 4% n=4	Bank B 4.1% n=12	Bank C 3.95%
$r_e = \left(1+\frac{.04}{365}\right)^{365} - 1$	$r_e = \left(1+\frac{.041}{12}\right)^{12} - 1$	$r_e = e^r - 1$
4.081%	4.178%	$r_e = e^{.0395} - 1$ 4.029%

Bank B

Present Value Formulas

The present value P of A dollars to be received after t years, assuming a per annum interest rate r compounded n times per year, is

$$P = A \cdot \left(1 + \frac{r}{n}\right)^{-nt} \qquad (5)$$

If the interest is compounded continuously,

$$P = Ae^{-rt} \qquad (6)$$

Could just use $A = P(1+\frac{r}{n})^{nt}$ + $A = Pe^{rt}$

EXAMPLE

2. A zero-coupon (noninterest-bearing) bond can be redeemed in 10 years for \$1000. How much should you be willing to pay for it now if you want a return of

(a) 7% compounded monthly? (b) 6% compounded continuously?

$A = P(1+\frac{r}{n})^{nt}$

$1000 = P\left(1+\frac{.07}{12}\right)^{12 \cdot 10}$

$P = \$497.60$

$P = 1000\left(1+\frac{.07}{12}\right)^{-120}$

$1000 = Pe^{+.06(10)}$

$P = \$548.81$

$P = 1000e^{-.06(10)}$