Pre Calculus
Chapter 5 Worksheet #2

Name Key

In Problems 1 - 10, solve each equation algebraically. Express irrational solutions in exact form and as a decimal rounded to 3 decimal places. Verify your results using a graphing utility.

1. $4^{1-2x} = 2$

$2^{2-4x} = 2$

$2-4x = 1$

$-4x = -1$

$x = 1/4$

2. $4^{x-x^2} = \frac{1}{2}$

$2^{2x-2x^2} = 2^{-1}$

$-2x^2+2x = -1$

$2x^2-2x-1 = 0$

$\frac{2 \pm \sqrt{4-4(2)(-1)}}{2(2)} = \frac{2 \pm \sqrt{12}}{4} = \frac{2 \pm 2\sqrt{3}}{4}$

$x = \left\{\frac{1 \pm \sqrt{3}}{2}\right\} \approx \{1.366, -0.366\}$

3. $\log_{\sqrt{2}} x = -6$

$\sqrt{2}^{-6} = x$

$x = \frac{1}{2^{6/2}} = \frac{1}{8}$

4. $5^{x+2} = 7^{x-2}$

$(x+2)\ln 5 = (x-2)\ln 7$

$x\ln 5 + 2\ln 5 = x\ln 7 - 2\ln 7$

$2\ln 5 + 2\ln 7 = x\ln 7 - x\ln 5$

$\ln(25 \cdot 49) = x\left(\ln \frac{7}{5}\right)$

$x = \frac{\ln 1225}{\ln 7/5} = 21.133$

5. $9^{2x} = 27^{3x-4}$

$3^{4x} = 3^{9x-12}$

$4x = 9x-12$

$12 = 5x$

$x = 12/5$

6. $2^{x+1} \cdot 8^{-x} = 4$

$2^{x+1} \cdot 2^{-3x} = 2^2$

$2^{-2x+1} = 2^2$

$-2x+1 = 2$

$-2x = 1$

$x = -1/2$

7. $\log(7x-12) = 2\log x$

$7x-12 = x^2$

$x^2-7x+12 = 0$

$(x-4)(x-3) = 0$

$x = \{4, 3\}$

8. $\log_2 x + \log_2(x+2) = 3$

$\log_2(x^2+2x) = 3$

$x^2+2x = 8$

$x^2+2x-8 = 0$

$(x+4)(x-2) = 0$

$x = \{2\}$

9. $\ln e^{1-2x} = \ln 4$

$1-2x = \ln 4$

$-2x = -1+\ln 4$

$2x = 1-\ln 4$

$x = \frac{1-\ln 4}{2} \approx -0.193$

10. $(4^x - 14 \cdot 4^{-x} = 5)4^x$

$4^{2x} - 14 = 5 \cdot 4^x$

$4^{2x} - 5 \cdot 4^x - 14 = 0$

$(4^x - 7)(4^x + 2) = 0$

$\ln 4^x = \ln 7$ ~~$4^x = -2$~~

$x\ln 4 = \ln 7$

$x = \frac{\ln 7}{\ln 4} \approx 1.404$

In Problems 11 and 12, use the following result: If x is the atmospheric pressure (measured in millimeters of mercury), then the formula for the altitude h(x) (measured in meters above sea level) is $h(x) = (30T + 8000)\log\left(\frac{P_o}{x}\right)$

where T is the temperature (in degrees Celsius) and P_o is the atmospheric pressure at sea level, which is approximately 760 millimeters of mercury.

11. At what height is a Piper Cub whose instruments record an outside temperature of 0° C and a barometric pressure of 300 millimeters of mercury?

$h(300) = (30(0) + 8000)\log\left(\frac{760}{300}\right)$

$= 8000 \log\left(\frac{760}{300}\right) \approx 3229.54$ meters

12. How high is a mountain if instruments place on its peak record a temperature of 5° C and a barometric pressure of 500 millimeters of mercury?

$h(500) = (30 \cdot 5 + 8000)\log\left(\frac{760}{500}\right)$

$= 8150 \log\left(\frac{760}{500}\right) \approx 1482.03$ m

13. A child's grandparents wish to purchase a bond that matures in 18 years to be used for her college education. The bond pays 4% interest compounded semiannually. How much should they pay so that the bond will be worth $85,000 at maturity? $n = 2$

$85000 = P\left(1 + \frac{.04}{2}\right)^{2 \cdot 18}$

$P = \$41,668.97$

14. The half-life of radioactive cobalt is 5.27 years. If 100 grams of radioactive cobalt is present now, how much will be present in 20 years? In 40 years?

$\ln \frac{1}{2} = \ln e^{k(5.27)}$

$\ln \frac{1}{2} = 5.27k$

$k \approx -.1315$

$N(t) = N_0 e^{-.1315t}$

$N(20) = 100 e^{-.1315(20)}$

≈ 7.204 grams

$N(40) = 100 e^{-.1315(40)}$

≈ 0.519 g

15. Suppose the population of a newly discovered insect grows according to the logistic growth model $P(t) = \frac{50000}{1+25e^{-0.04t}}$ where P represents the population and t represents the time in years.

(a) How many insects were originally discovered?

$P(0) = \frac{50000}{1+25} \approx 1923$ insects

(b) Determine the maximum population of the insect population.

50 000

(c) Use a graphing utility, graph $P = P(t)$.

(d) When will the population reach 20,000 insects?

$20000 = \frac{50000}{1+25e^{-0.04t}}$

$1+25e^{-0.04t} = \frac{5}{2}$

$25e^{-0.04t} = \frac{3}{2}$

$e^{-0.04t} = \frac{3}{50}$

$t = \frac{\ln 3/50}{-0.04} \approx 70.34$ yrs.

16. The following data represent the value of an IRA invested in a variety of mutual funds.

Year	Account Value
0	\$3000
1	\$3165
2	\$3299
3	\$3563
4	\$3926
5	\$4170

(a) Using a graphing utility, draw a scatter diagram for the data.

(b) Using a graphing utility, build an exponential model from the data.

$y = 2951.976957\,(1.07005667)^x$

(c) Based on the model, predict the value of the account after 10 years.

\$5810.06