Adv Alg 2 Week 9 Block Day Notes

Deriving a formula for Geometric Series

LOOK-don't copy

Sum of terms of Geometric sequence → $S_n = a_1 + a_1r + a_1r^2 + a_1r^3 + ... + a_1r^{n-2} + a_1r^{n-1}$

multiply by r → $rS_n = a_1r + a_1r^2 + a_1r^3 + ... + a_1r^{n-1} + a_1r^n$

subtract → $S_n - rS_n = a_1 + 0 + 0 + 0 + ... + 0 - a_1r^n$

simplify → $S_n - rS_n = a_1 - a_1r^n$

factor → $S_n(1-r) = a_1(1-r^n)$

divide → $S_n = \frac{a_1(1-r^n)}{(1-r)}$

$$\boxed{S_n = \frac{a_1(1-r^n)}{(1-r)}}$$

COPY this!

r = common ratio

a_1 = first term

n= number of terms in geometric series

Examples using Geometric Series formula

1. Find S_{10} of series 2 + 5 + 12.5 + 31.25 + ...

$r = \frac{5}{2} = \frac{12.5}{5} = \frac{31.25}{12.5}$

$r = 2.5$

$S_n = \frac{a_1(1-r^n)}{(1-r)}$

$S_{10} = \frac{2(1-(2.5)^{10})}{(1-2.5)} =$

2. Evaluate $\sum_{k=1}^{6} 3(2^{k-1})$ ← **This is *NOT* one of our 3 formulas**

First, find term 1,2 and 3 to figure out a pattern

$a_1 = 3(2^{1-1})$ $a_2 = 3(2^{2-1})$ $a_3 = 3(2^{3-1})$

$3(2^0)$ $=3(2)$ $= 12$

$3(1) = 3$ 6

3, 6, 12 ← **What kind of pattern is it?**

Geo $r = 2$

$S_n = \frac{a_1(1-r^n)}{(1-r)}$

$S_6 = \frac{3(1-(2)^6)}{(1-2)}$

$S_6 = 189$

3. Given $a_1 = 5$, $r = -3$, find S_{10}.

$S_n = \frac{a_1(1-r^n)}{(1-r)}$

$S_{10} = \frac{5(1-(-3)^{10})}{(1-(-3))}$

$= \frac{5(1-59049)}{4} = -73810$

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4. Identify a_1, r, a_n, and evaluate $\sum_{k=1}^{12} 2^k$

First, find term 1,2 and 3 to figure out a pattern

$a_1 = 2^1 = 2$ $a_2 = 2^2 = 4$ $a_3 = 2^3 = 8$

$a_1 = 2$ $a_n = 2^n$

$r = 2$

$S_n = \frac{a_1(1-r^n)}{1-r}$

$S_{12} = \frac{2(1-2^{12})}{(1-2)}$

$= 8190$

5. Evaluate the sum of the finite geometric series.

-8 - 24 - 72 - 216 + ... $\boxed{-17496}$

Geo

$r=3$

Find $\underline{n}$ (how many)

$a_n = a_1 r^{n-1}$

$\frac{-17496}{-8} = \frac{-8(3)^{n-1}}{-8}$

$\rightarrow 2187 = 3^{n-1}$

same base $\rightarrow 3^7 = 3^{n-1}$

so exponents equal

$7 = n-1$

$+1 \quad +1$

$n = 8$

$S_n = \frac{a_1(1-r^n)}{(1-r)}$

$S_8 = \frac{-8(1-3^8)}{(1-3)}$

$= -26240$

6. How many terms must be added together for this geometric series to equal 728?

2 + 6 + 18 + ...

$S_n = \frac{a_1(1-r^n)}{(1-r)}$

Solve for n

$728 = \frac{2(1-3^n)}{(1-3)}$

$728 = \frac{2(1-3^n)}{-2}$

$728 = -1(1-3^n)$

$728 = -1 + 3^n$

$+1 \quad +1$

$729 = 3^n$

$3^6 = 3^n$

$\boxed{n=6}$

7. Given 2 + (-6) + 18 + (-54) +..., find S_9.

$S_9 = \frac{2(1-(-3)^9)}{(1-(-3))}$

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