

## Notes Sigma Formulas

$$\text{Ex. 1 } \sum_{j=1}^4 3 = 3 + 3 + 3 + 3 = 12$$

$$\text{Ex. 2 } \sum_{i=1}^5 6 = 6 + 6 + 6 + 6 + 6 = 30$$

Do you see a shortcut???

A shortcut for ex. 1 and ex. 2

$$\sum_{k=1}^n c = nc$$

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$$\text{Ex. 3 } \sum_{j=1}^4 j = 1+2+3+4 = 10$$

$$\begin{aligned} \text{Ex. 4 } \sum_{k=1}^3 3k &= 3(1)+3(2)+3(3) \\ &= 3+6+9 = 18 \\ &= 3(1+2+3) \\ &= 3(6) = 18 \end{aligned}$$

$$\begin{aligned} \text{Ex. 5 } \sum_{n=1}^3 4n^2 &= 4(1)^2 + 4(2)^2 + 4(3)^2 \\ &= 4(1^2 + 2^2 + 3^2) \\ &= 4(1+4+9) = 56 \\ &= 4(14) \end{aligned}$$

$$\begin{aligned} \text{Ex. 6 } \sum_{n=1}^3 (n+n^2) &= (1+1^2) + (2+2^2) \\ &\quad + (3+3^2) \\ &= (1+2+3) + (1^2+2^2+3^2) = \frac{6+14}{20} \end{aligned}$$

### Summation Properties

$$1. \sum_{k=1}^n ca_k = c \sum_{k=1}^n a_k$$

$$2. \sum_{k=1}^n (a_k + b_k) = \sum_{k=1}^n a_k + \sum_{k=1}^n b_k$$

# Notes Sigma Formulas

## Summation Formulas

Constant Series

$$\sum_{k=1}^n c = nc$$

Linear Series

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}$$

Quadratic Series

$$\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$$

Ex. 1  $\sum_{m=1}^5 (2m^2 + 3m + 2)$  *constant in front & split into parts*

$$= 2 \sum_{m=1}^5 m^2 + 3 \sum_{m=1}^5 m + \sum_{m=1}^5 2$$

$$= 2 \left[ \frac{n(n+1)(2n+1)}{6} \right] + 3 \left[ \frac{n(n+1)}{2} \right] + n(2)$$

$$= 2 \left[ \frac{5(5+1)(2(5)+1)}{6} \right] + 3 \left[ \frac{5(5+1)}{2} \right] + 5(2)$$

$$= 2 \left[ \frac{5(6)(11)}{6} \right] + 3 \left[ \frac{5(6)}{2} \right] + 10$$

$$= 110 + 45 + 10 = \boxed{165}$$

Ex. 2  $\sum_{j=1}^{50} (-j^2 + 2j + 5)$   $n=50$

$$= - \sum_{j=1}^{50} j^2 + 2 \sum_{j=1}^{50} j + \sum_{j=1}^{50} 5$$

$$= - \left[ \frac{n(n+1)(2n+1)}{6} \right] + 2 \left[ \frac{n(n+1)}{2} \right] + n(5)$$

$$= - \left[ \frac{50(50+1)(2(50)+1)}{6} \right] + 2 \frac{(50)(50+1)}{2} + 50(5)$$

$$= - \left[ \frac{50(51)(101)}{6} \right] + 50(51) + 250$$

$$= - (42925) + 2550 + 250$$

$$= \boxed{-40,125}$$