

• **Rational Root Theorem**

Let P be a polynomial function with integer coefficients in standard form.
 If $\frac{p}{q}$ is a root of $P(x)=0$, then p is a factor of the constant term P
 and q is a factor of the leading coefficient of P.

Example 1 $x^3 - 9x^2 + 3x^2 - x + 8 = 0$ Standard form?

GCF? Factor? ANYTHING???

COPY this! $\left\{ \begin{array}{l} \text{IF there is a rational root, then it will be in the form of } \frac{p}{q}, \\ \text{where p is a factor of the constant term and q is a factor} \\ \text{of the leading coefficient.} \end{array} \right.$

factors of leading coefficient is on bottom
 factors of last term are on top.

What are all of the POSSIBLE rational roots?

$$\frac{\pm 8, \pm 4, \pm 2, \pm 1}{\pm 1} = \pm 8, \pm 4, \pm 2, \pm 1$$

Ex. 2 State all possible rational roots.

a) $x^3 + 4x^2 + 7x - 12 = 0$

b) $x^3 + 9x^2 - 2x + 10 = 0$

Possible rational roots: $\pm 12, \pm 6, \pm 4, \pm 3, \pm 2, \pm 1$

Possible rational roots: $\pm 10, \pm 5, \pm 2, \pm 1$

Let's use all of our knowledge to find the roots of the following problems.

Ex. 3 $x^3 + 2x^2 - 7x - 12 = 0$

GCF?
 Grouping/X-Box?
 Quad Formula?

NOW WHAT??

Handwritten synthetic division work for $x^3 + 2x^2 - 7x - 12 = 0$:

- 1) $\begin{array}{r|rrrr} 1 & 1 & 2 & -7 & -12 \\ & & 1 & 3 & -4 \\ \hline & 1 & 3 & -4 & -16 \end{array}$
- 2) $\begin{array}{r|rrrr} -1 & 1 & 2 & -7 & -12 \\ & & -1 & -1 & 8 \\ \hline & 1 & 1 & -8 & -4 \end{array}$
- 3) $\begin{array}{r|rrrr} -2 & 1 & 2 & -7 & -12 \\ & & -2 & 0 & 14 \\ \hline & 1 & 0 & -7 & 2 \end{array}$
- 4) $\begin{array}{r|rrrr} 3 & 1 & 2 & -7 & -12 \\ & & 3 & 15 & 24 \\ \hline & 1 & 5 & 8 & 12 \end{array}$
- 5) $\begin{array}{r|rrrr} -3 & 1 & 2 & -7 & -12 \\ & & -3 & 3 & 12 \\ \hline & 1 & -1 & -4 & 0 \end{array}$

Final result: $x^2 - x - 4$

Imm... I wish there was a way to get help with this.

Possible Rational Roots

$$\pm 12, \pm 6, \pm 4, \pm 3, \pm 2, \pm 1$$

Remainder needs to be = 0 for it to be a root

$$(x^2 - x - 4)(x + 3) = 0$$

$$x^2 - x - 4 = 0 \text{ or } x + 3 = 0$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-4)}}{2(1)}$$

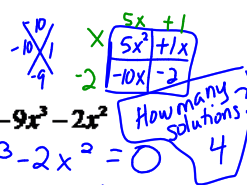
$$x = \frac{1 \pm \sqrt{1+16}}{2}$$

$$x = \frac{1 \pm \sqrt{17}}{2}$$

$$\left\{ -3, \frac{1 \pm \sqrt{17}}{2} \right\}$$

Yes, -3 is a root!

Find all zeros of the polynomial function.



Ex. 4 $W(x) = x^3 - 2x^2 - 3x + 10$

$0 = x^3 - 2x^2 - 3x + 10$
Possible Rational Roots $\pm 10, \pm 5, \pm 2, \pm 1$
graph $y = x^3 - 2x^2 - 3x + 10$
in calculator
crosses x-axis at $x = -2$

$$\begin{array}{r|rrrr} -2 & 1 & -2 & -3 & 10 \\ & & -2 & 8 & -10 \\ \hline & 1 & -4 & 5 & 0 \end{array}$$

$0 = (x+2)(x^2 - 4x + 5)$
 $x+2=0$ or $x^2 - 4x + 5 = 0$
 $x = -2$ $x^2 - 4x + 4 = -5 + 4$
 $(x-2)^2 = -1$
 $x-2 = \pm \sqrt{-1}$
 $x-2 = \pm i$
 $x = 2 \pm i$

$\{-2, 2 \pm i\}$

Ex. 5 $J(x) = 5x^4 - 9x^3 - 2x^2$

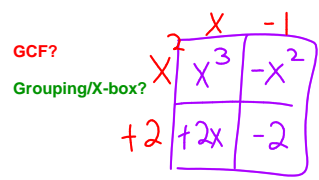
$5x^4 - 9x^3 - 2x^2 = 0$
 $x^2(5x^2 - 9x - 2) = 0$
 $x^2(5x+1)(x-2) = 0$

$x^2 = 0$ or $5x+1=0$ or $x-2=0$
 $x=0$ (multiplicity 2) $\frac{5x}{5} = \frac{-1}{5}$ $x=2$
 $x = \left\{ 0 \text{ (mult 2)}, -\frac{1}{5}, 2 \right\}$

Ex. 6 $P(x) = x^3 - x^2 + 2x - 2$

$0 = x^2(x-1) + 2(x-1)$
 $0 = (x-1)(x^2+2)$
 $x-1=0$ or $x^2+2=0$
 $x=1$ $x^2 = -2$
 $x = \pm \sqrt{-2}$
 $x = \pm i\sqrt{2}$

$x = \{1, \pm i\sqrt{2}\}$



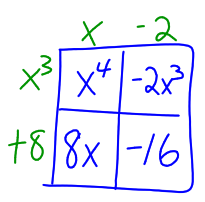
Ex. 7 $P(x) = x^4 - 2x^3 + 8x - 16$

$0 = x^3(x-2) + 8(x-2)$
 $0 = (x-2)(x^3+8)$

$0 = (x-2)(x+2)(x^2-2x+4)$

$x-2=0$ or $x+2=0$
 $x=2$ or $x=-2$

$\{\pm 2, 1 \pm i\sqrt{3}\}$



solve by completing square

$x^2 - 2x + 4 = 0$
 $x^2 - 2x + 1 = -4 + 1$
 $(x-1)^2 = -3$
 $x-1 = \pm \sqrt{-3}$
 $x-1 = \pm i\sqrt{3}$
 $x = 1 \pm i\sqrt{3}$