- ## Rational Root Theorem

Let P be a polynomial function with integer coefficients in standard form.

If $\dfrac{p}{q}$ is a root of P(x)=0, then p is a factor of the constant term P and q is a factor of the leading coefficient of P.

Example 1 $\quad x^5-9x^4+3x^2-x+8=0$ $\quad$ Standard form?

GCF?Factor?ANYTHING???

COPY this!

IF there is a rational root, then it will be in the form of $\dfrac{p}{q}$,

where p is a factor of the constant term and q is a factor of the leading coefficient.

factors of leading coefficient is on bottom
factors of last term are on top.

What are all of the *POSSIBLE* rational roots?

$$\frac{\pm 8, \pm 4, \pm 2, \pm 1}{\pm 1} = \pm 8, \pm 4, \pm 2, \pm 1$$

Ex. 2 State all *possible* rational roots.

a) $1x^3+4x^2+7x-12=0$ $\quad$ ↑factors

Possible rational roots:
$\pm 12, \pm 6, \pm 4, \pm 3, \pm 2, \pm 1$

b) $x^5+9x^4-2x^3+10=0$

Possible rational roots:
$\pm 10, \pm 5, \pm 2, \pm 1$

Let's use all of our knowledge to find the roots of the following problems.

Ex. 3 $\quad 1x^3+2x^2-7x-12=0$

GCF?
Grouping/X-Box?
Quad Formula?

NOW WHAT??

Hmm... I wish there was a way to get help with this.

Possible Rational Roots
$\pm 12, \pm 6, \pm 4, \pm 3, \pm 2, \pm 1$

$$\begin{array}{r|rrrr} 1 & 1 & 2 & -7 & -12 \\ & \downarrow & 1 & 3 & -4 \\ \hline & 1 & 3 & -4 & -16 \end{array}$$

$$\begin{array}{r|rrrr} -1 & 1 & 2 & -7 & -12 \\ & \downarrow & -1 & -1 & 8 \\ \hline & 1 & 1 & -8 & -4 \end{array}$$

Remainder needs to be = 0 for it to be a root

$$\begin{array}{r|rrrr} -2 & 1 & 2 & -7 & -12 \\ & \downarrow & -2 & 0 & 14 \\ \hline & 1 & 0 & -7 & 2 \end{array}$$

$$\begin{array}{r|rrrr} 3 & 1 & 2 & -7 & -12 \\ & \downarrow & 3 & 15 & 24 \\ \hline & 1 & 5 & 8 & 12 \end{array}$$

$$\begin{array}{r|rrrr} -3 & 1 & 2 & -7 & -12 \\ & \downarrow & -3 & 3 & 12 \\ \hline & 1 & -1 & -4 & 0 \end{array}$$

x^2-x-4

Yes, -3 is a root!

$$(x^2-x-4)(x+3)=0$$

$$x^2-x-4=0 \quad \text{or} \quad x+3=0$$
$$x=-3$$

$$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(-4)}}{2(1)}$$

$$x=\frac{1\pm\sqrt{1+16}}{2}$$

$$x=\frac{1\pm\sqrt{17}}{2}$$

$$\boxed{\left\{-3, \frac{1\pm\sqrt{17}}{2}\right\}}$$

Find **all** zeros of the polynomial function.

Ex. 4 $W(x) = x^3 - 2x^2 - 3x + 10$

$0 = x^3 - 2x^2 - 3x + 10$

Possible Rational Roots: $\pm 10, \pm 5, \pm 2, \pm 1$

graph $y = x^3 - 2x^2 - 3x + 10$ in calculator

crosses x-axis at x=-2

$$\begin{array}{r|rrrr} -2 & 1 & -2 & -3 & 10 \\ & \downarrow & -2 & 8 & -10 \\ \hline & 1 & -4 & 5 & 0 \end{array}$$

$0 = (x+2)(x^2 - 4x + 5)$

$x + 2 = 0$ or $x^2 - 4x + 5 = 0$

$x = -2$ $\quad x^2 - 4x + 4 = -5 + 4$

$(x-2)^2 = -1$

$x - 2 = \pm\sqrt{-1}$

$x - 2 = \pm i$

$x = 2 \pm i$

$\boxed{\{-2,\ 2 \pm i\}}$

Ex.5 $J(x) = 5x^4 - 9x^3 - 2x^2$

How many solutions? 4

$$\begin{array}{c|cc} & 5x & +1 \\ \hline x & 5x^2 & +1x \\ -2 & -10x & -2 \end{array}$$

$5x^4 - 9x^3 - 2x^2 = 0$

$x^2(5x^2 - 9x - 2) = 0$

$x^2(5x+1)(x-2) = 0$

$x^2 = 0$ or $5x + 1 = 0$ or $x - 2 = 0$

$x = 0$ (multiplicity 2) $\quad \frac{5x}{5} = \frac{-1}{5}$ $\quad x = 2$

$x = -\frac{1}{5}$

$x = \{0\ (\text{mult } 2),\ -\frac{1}{5},\ 2\}$

Ex. 6 $P(x) = x^3 - x^2 + 2x - 2$

GCF?

Grouping/X-box?

	x	-1
x^2	x^3	$-x^2$
$+2$	$+2x$	-2

$0 = x^2(x-1) + 2(x-1)$

$0 = (x-1)(x^2+2)$

$x - 1 = 0$ or $x^2 + 2 = 0$

$x = 1$ $\quad x^2 = -2$

$x = \pm\sqrt{-2}$

$x = \pm i\sqrt{2}$

$\boxed{x = \{1,\ \pm i\sqrt{2}\}}$

Ex. 7 $P(x) = x^4 - 2x^3 + 8x - 16$

How many solutions?

or put in box

	x	-2
x^3	x^4	$-2x^3$
$+8$	$8x$	-16

$0 = x^3(x-2) + 8(x-2)$

$0 = (x-2)(x^3+8)$

factor

$0 = (x-2)(x+2)(x^2 - 2x + 4)$

$x - 2 = 0$ or $x + 2 = 0$

$x = 2$ or $x = -2$

solve by completing square

$x^2 - 2x + 4 = 0$

$x^2 - 2x + 1 = -4 + 1$

$(x-1)^2 = -3$

$x - 1 = \pm\sqrt{-3}$

$x - 1 = \pm i\sqrt{3}$

$x = 1 \pm i\sqrt{3}$

$\boxed{\{\pm 2,\ 1 \pm i\sqrt{3}\}}$