SECTION 5.2

Write a polynomial function in standard form with the given zeros.

1. $x = 2, 1$

$x = 2$ or $x = 1$

$x - 2 = 0$ or $x - 1 = 0$

$(x-2)(x-1) = 0$

$x^2 - 3x + 2 = 0$

$y = x^2 - 3x + 2$

2. $x = -1, \frac{3}{2}$

$x = -1$ or $x = \frac{3}{2}$

$x + 1 = 0$ $\quad 2x = 3$

$2x - 3 = 0$

$(x+1)(2x-3) = 0$

$2x^2 - x - 3 = 0$

$y = 2x^2 - x - 3$

3. $x = -1, 2, 3$

$x = -1$ or $x = 2$ or $x = 3$

$x + 1 = 0$ or $x - 2 = 0$ or $x - 3 = 0$

$(x+1)(x-2)(x-3) = 0$

$(x^2 - x - 2)(x-3) = 0$

	x^2	$-x$	-2
x	x^3	$-x^2$	$-2x$
-3	$-3x^2$	$+3x$	$+6$

$y = x^3 - 4x^2 + x + 6$

4. $x = 0, 0, 3, 4$ (multiplicity of 2)

$y = x^2(x-3)(x-4)$

$y = x^2(x^2 - 7x + 12)$

$y = x^4 - 7x^3 + 12x^2$

How do you factor $x^4 - 3x^2 + 2$?

well.....

How do you factor $x^2 - 3x + 2$? factor it $(x-2)(x-1)$

Do you see a similarity? Do you see that the middle term's exponent is half of the leading terms's exponent?

What if you made a substitution?

$x^2 = m$ $\quad x^4 - 3x^2 + 2$ $\quad (x^2)^2 - 3(x^2) + 2$

then you get: $m^2 - 3m + 2$

and can you factor that? it is the same as the one above.....except that you made the substitution.....now substitute back and solve.

$(m-2)(m-1)$

$(x^2-2)(x^2-1)$

but...are we done? do you see the difference of squares?

What is the final answer? $(x^2-2)(x^2-1) =$

$(x^2-2)(x+1)(x-1)$

Try this:

Let $m = x^2$

1. $x^4 + 7x^2 + 10$ $\quad m^2 + 7m + 10$ $\quad (m+2)(m+5)$

substitute back in → $(x^2+2)(x^2+5)$

2. $2x^6 - 3x^3 - 20$ $\quad$ Let $m = x^3$ so $2m^2 - 3m - 20$

$(m-4)(2m+5)$

$(x^3-4)(2x^3+5)$

-40, -8, 5, -3

	$2m$	$+5$
m	$2m^2$	$5m$
-4	$-8m$	-20

3. $x^4 - 5x^2 - 24$ $\quad$ Let $m = x^2$

$m^2 - 5m - 24$

$(m-8)(m+3)$

$(x^2-8)(x^2+3)$

Find the real or imaginary solutions of each equation by factoring.

Find all solutions.

Ex. 1 $2x^3 - 7x^2 + 3x = 0$

$x(2x^2 - 7x + 3) = 0$

$x(2x - 1)(x - 3) = 0$

$x = 0$ or $2x - 1 = 0$ or $x - 3 = 0$

$2x = 1$ $\quad x = 3$

$x = \frac{1}{2}$

$x = \left\{0, \frac{1}{2}, 3\right\}$

1. Factor, factor, factor!
2. Zero product property
3. Check answer

$2(0)^3 - 7(0)^2 + 3(0) =$

$0 - 0 + 0 = 0$ ✓

$2\left(\frac{1}{2}\right)^3 - 7\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) \stackrel{?}{=} 0$

$2\left(\frac{1}{8}\right) - 7\left(\frac{1}{4}\right) + \frac{3}{2} \stackrel{?}{=} 0$

$\frac{1}{4} - \frac{7}{4} + \frac{6}{4}$

$0 = 0$ ✓

$2(3)^3 - 7(3)^2 + 3(3) \stackrel{?}{=} 0$

$2(27) - 7(9) + 9 = 0$

$54 - 63 + 9 = 0$

$-9 + 9 = 0$

$0 = 0$ ✓

Ex. 2 $5x^3 - 12x^2 + 4x = 0$

$x(5x^2 - 12x + 4) = 0$

$x(5x - 2)(x - 2) = 0$

20, -10, -2, -12

	$5x$	-2
x	$5x^2$	$-2x$
-2	$-10x$	4

$x = 0$ or $5x - 2 = 0$ or $x - 2 = 0$

$5x = 2$ $\quad x = 2$

$x = \frac{2}{5}$

$x = \left\{0, \frac{2}{5}, 2\right\}$

Find the real or imaginary solutions of each equation by factoring.

How many solutions? 4

Ex. 3 $x^4 - 8x^2 + 16 = 0$

Let $m = x^2$ $\quad m^2 - 8m + 16 = 0$

$(m - 4)(m - 4) = 0$

$(x^2 - 4)(x^2 - 4) = 0$

$(x + 2)(x - 2)(x + 2)(x - 2) = 0$

$(x + 2)^2 (x - 2)^2 = 0$

$x + 2 = 0$ or $x - 2 = 0$

$x = -2$ or $x = 2$

multiplicity of 2 $\quad$ multiplicity of 2

$x = \{-2, 2\}$

Ex. 4 $x^4 - 2x^2 - 15 = 0$

Let $m = x^2$ $\quad m^2 - 2m - 15 = 0$

$(m - 5)(m + 3) = 0$

$(x^2 - 5)(x^2 + 3) = 0$

$x^2 - 5 = 0$ or $x^2 + 3 = 0$

$x^2 = 5$ $\quad x^2 = -3$

$x = \pm\sqrt{5}$ $\quad x = \pm\sqrt{-3}$

$x = \pm i\sqrt{3}$

$\left\{\pm\sqrt{5}, \pm i\sqrt{3}\right\}$

Find the real or imaginary solutions.

Ex. 5 $x^3 - 8 = 0$

$(x)^3 - (2)^3$

$(x-2)(x^2+2x+4) = 0$

$x - 2 = 0$ or $x^2 + 2x + 4 = 0$

$x = 2$

$x^2 + 2x + 1 = -4 + 1$

$(x+1)^2 = -3$

$x + 1 = \pm\sqrt{-3}$

$x + 1 = \pm i\sqrt{3}$

$x = -1 \pm i\sqrt{3}$

$x = \{2, -1 \pm i\sqrt{3}\}$

Ex. 6 $8x^3 = 1$

$a^3 - b^3 = (a-b)(a^2+ab+b^2)$

$8x^3 - 1 = 0$

$(2x-1)((2x)^2 + (2x)(1) + (1)^2)$

$(2x)^3 - (1)^3$

$(2x-1)(4x^2 + 2x + 1) = 0$

$2x - 1 = 0$ or $4x^2 + 2x + 1 = 0$

$2x = 1$

$x = \frac{1}{2}$

$x = \frac{-2 \pm \sqrt{2^2 - 4(4)(1)}}{2(4)}$ Use Quadratic formula

$x = \frac{-2 \pm \sqrt{4-16}}{8} = \frac{-2 \pm \sqrt{-12}}{8}$ simplify

$= \frac{-2 \pm \sqrt{-1}\sqrt{4}\sqrt{3}}{8}$

$= \frac{-2 \pm 2i\sqrt{3}}{8} = \frac{2(-1 \pm i\sqrt{3})}{8\ 4}$

$x = \frac{-1 \pm i\sqrt{3}}{4}$

$x = \left\{\frac{1}{2}, \frac{-1 \pm i\sqrt{3}}{4}\right\}$

Find the real or imaginary solutions of each equation.

Ex. 7 $x^4 - 16 = 0$ Use subst or factor directly.

Let $m = x^2$

$m^2 - 16 = 0$

$(m+4)(m-4) = 0$

subst back in $(x^2+4)(x^2-4) = 0$

$(x^2+4)(x+2)(x-2) = 0$

$x^2 + 4 = 0$ or $x + 2 = 0$ or $x - 2 = 0$

$-4 \quad -4$

$x^2 = -4$ $\quad x = -2$ $\quad x = 2$

$x = \pm\sqrt{-4}$

$x = \pm 2i$

$x = \{\pm 2, \pm 2i\}$