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A Mini-Deductive System Involving Parallel Lines

UNDEFINED TERMS — Point , line and plane

DEFINITIONS:

Vertical Angles:
Two non-adjacent angles formed by two intersecting lines.

Corresponding Angles:
Two angles on the same side of the transversal where one is interior, one is exterior, and they are not adjacent.

Alternate Interior Angles:
Two interior angles on different sides of the transversal that are not supplementary.

Alternate Exterior Angles:
Two exterior angles on different sides of the transversal that are not supplementary.

Same Side Interior Angles:
Two interior angles on the same side of the transversal

POSTULATES:

Linear Pair Postulate:
If two adjacent angles have exterior sides forming a line, then the angles sum to 180°.

Corresponding Angles Postulate
If two parallel lines are cut by a transversal, then corresponding angles are equal.

Converse of Corresponding Angles Postulate
If two lines are cut by a transversal and corresponding angles are equal, then the lines are parallel.

Substitution Property:
If two quantities are equal, you can replace one with the other.

Ex.1: If $\angle 1 = \angle 2$ and $\angle 2 + \angle 4 = 90$, then $\angle 1 + \angle 4 = 90$

Ex.2: If $\angle 1 + \angle 1 = 180$, then $2(\angle 1) = 180$

THEOREMS:

Vertical Angles Theorem
If two lines intersect, the vertical angles formed are equal.

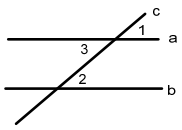
Supplementary Angles Theorem
If two adjacent angles have exterior sides forming a line, the angles are supplementary.

Supplements of Equal Angles Theorem
Two angles supplementary to the same angle are equal.

Proofs Involving Parallel Lines 3.20

Prove the following theorems:

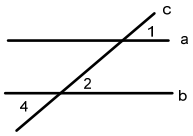
1. Given: a ll b
   Prove: $m\angle 2 = m\angle 3$



Statements	Reasons
1. a ll b	1.
2. <1=<2	2.
3. <1=<3	3.
4. <2=<3	4.

Conclusion If lines are parallel, then alternate interior angles are equal.

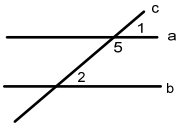
2. Given: a ll b
   Prove: $m\angle 1 = m\angle 4$



Statements	Reasons
1.	1. Given
2.	2. Corresponding Angles Postulate
3.	3. Vertical angles theorem
4.	4. Substitution

Conclusion

3. Given: a ll b
   Prove: $m < 2$ and $m < 5$ are supplementary



Statements	Reasons
1. a ll b	1.
2. <1 = <2	2.
3. <1 + <5 = 180	3.
4. <2 + <5 = 180	4.
5. <2 and <5 are supplementary	5.

Conclusion

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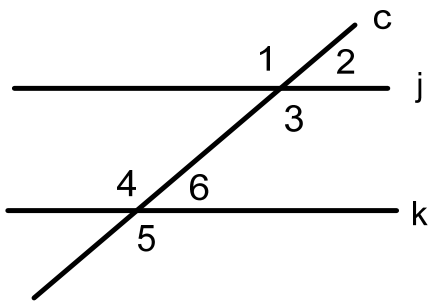

Given: $j \parallel k$

Prove: As many statements as you can using the numbered angles.

Statements	Reasons

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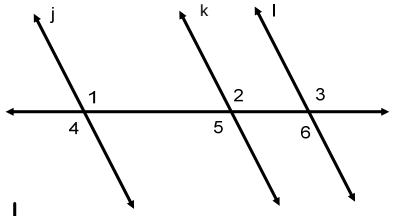
Given: The plane figure
j ll k, k ll j

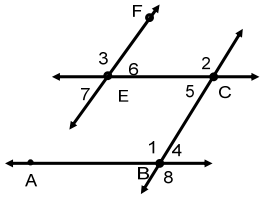
Prove: <1 = < 3



Statements	Reasons
1. *j ll k, k ll j*	1.
2. <1=<2, <2=<3	2.
3. <1 = < 3	3.

Given: The plane figure
$\overleftrightarrow{AB}$ II $\overleftrightarrow{EC}$; $\overleftrightarrow{BC}$ II $\overleftrightarrow{EF}$

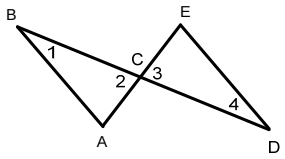
Prove: <1=<3



Statements	Reasons
1. $\overleftrightarrow{AB}$ II $\overleftrightarrow{EC}$; $\overleftrightarrow{BC}$ II $\overleftrightarrow{EF}$	1. Given
2.	2. Corresponding Angles Postulate
3.	3. Substitution Property

Given: $\overline{AB}$ II $\overline{DE}$; <1=<2

Prove: <4=<3



Statements	Reasons
1. $\overline{AB}$ II $\overline{DE}$; <1=<2	1.
2. <1=<4	2.
3. <2=<4	3.
4. <2=<3	4.
5. <4=<3	5.