

Perpendicular Bisector Equations

To write the equation of the perpendicular bisector you will need to find the slope of the perpendicular bisector and the midpoint of the segment.

Example: Given the points $A(2, 5)$ and $B(-4, 3)$, find the equation of the perpendicular bisector of \overline{AB} .

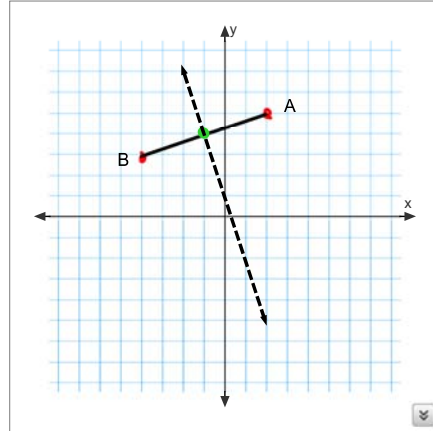
First find slope of segment.

$$m = \frac{3-5}{-4-2} = \frac{-2}{-6} = \frac{1}{3}$$

So the slope of the $\perp = -3$

Next find the midpoint of segment.

$$\begin{aligned} \text{Midpt.} &= \left(\frac{2+(-4)}{2}, \frac{5+3}{2} \right) \\ &= \left(\frac{-2}{2}, \frac{8}{2} \right) = (-1, 4) \end{aligned}$$



Now we are ready to write the equation of the Perpendicular Bisector with a slope of -3 and a midpoint of $(-1, 4)$

$$m = -3 \text{ and midpt. } (-1, 4)$$

Point-slope form

$$y - y_1 = m(x - x_1)$$

$$y - 4 = (-3)(x - (-1))$$

$$y - 4 = -3(x + 1)$$

So the equation of the perpendicular bisector in slope-intercept form is

$$y - 4 = -3x - 3 \text{ distribute and simplify}$$

$$y = -3x + 1$$

In standard form the equation will be

$$3x + y = 1$$

Example #2: Given the points P (2, -5) and H (-6, 7), find the equation of the perpendicular bisector of \overline{PH} .

First find slope of segment.

Therefore slope of perpendicular is:

$$m = \frac{7 - (-5)}{-6 - 2} = \frac{7 + 5}{-8} = \frac{12}{-8} = -\frac{3}{2} \quad \longrightarrow \quad m_{\perp} = \frac{2}{3}$$

Second, find the Midpoint of the segment.

$$\text{Midpt.} = \left(\frac{2 + (-6)}{2}, \frac{-5 + 7}{2} \right) = \left(\frac{-4}{2}, \frac{2}{2} \right) = (-2, 1)$$

Finally, the equation of the perp. bisector

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \left(\frac{2}{3} \right) (x - (-2))$$

$$y - 1 = \left(\frac{2}{3} \right) (x + 2)$$

point-slope form

$$y - 1 = \frac{2}{3}x + \frac{4}{3}$$

$$y = \frac{2}{3}x + \frac{7}{3}$$

slope-intercept form

$$3 \left(y = \frac{2}{3}x + \frac{7}{3} \right)$$

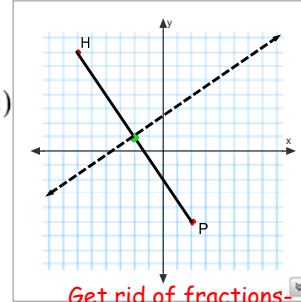
$$3y = 2x + 7$$

$$-2x + 3y = 7$$

$$-1(-2x + 3y = 7)$$

$$2x - 3y = -7$$

Standard form



Get rid of fractions

multiply by common denominator

Example #3 Given the point L (5, 8) and W (-3, -7), find the equation of the perpendicular bisector of \overline{LW} .

First find slope of segment.

Therefore slope of perpendicular is:

$$m = \frac{-7 - 8}{-3 - 5} = \frac{-15}{-8} = \frac{15}{8} \quad \longrightarrow \quad m_{\perp} = -\frac{8}{15}$$

Second, find the Midpoint of the segment.

$$\text{Midpt.} = \left(\frac{5 + (-3)}{2}, \frac{8 + (-7)}{2} \right) = \left(\frac{2}{2}, \frac{1}{2} \right) = \left(1, \frac{1}{2} \right)$$

Finally, the equation of the perp. bisector

$$y - y_1 = m(x - x_1)$$

$$y - \frac{1}{2} = \left(-\frac{8}{15} \right) (x - (1))$$

$$y - \frac{1}{2} = -\frac{8}{15}(x - 1)$$

point-slope form

$$y - \frac{1}{2} = -\frac{8}{15}x + \frac{8}{15}$$

$$y = -\frac{8}{15}x + \frac{31}{30}$$

slope-intercept form

$$30 \left(y = -\frac{8}{15}x + \frac{31}{30} \right)$$

$$30y = -16x + 31$$

$$16x + 30y = 31$$

Standard form

